Communications in Applied Analysis 11 (2007) 419-428

BASIC RESULTS AND STABILITY CRITERIA FOR SET VALUED DIFFERENTIAL EQUATIONS ON TIME SCALES

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ABSTRACT. We develop some basic results for set valued differential equations on time scales. Sufficient conditions for the stability of the trivial solution of set valued differential equations on time scales are also discussed.

Key Words: Set valued differential equations, time scales, existence theory, Lyapunov functions

1. INTRODUCTION

A more realistic approach to model a physical phenomenon is to use a dynamic system on time scales, which replaces the notion of dependence on a continuous time parameter t with a more general notion of a time scale or dynamic chain (an arbitrary closed set of real numbers) which may contain regions with specific types of discontinuities. In other words, we can say that a dynamic system incorporates both continuous and discrete times. In fact the theory of dynamic systems has recently gained impetus as it puts together the theories of continuous and discrete dynamic systems [1]. Another important subject of recent interest is that of setvalued differential equations and has been addressed by many authors, for instance, see [2-7] and the references therein. The interesting feature of the setvalued differential equations is that the results obtained in this new framework become the corresponding results of ordinary differential equations as the Hukuhara derivative and the integral used in formulating the set differential equations reduce to the ordinary vector derivative and integral when the set under consideration is a single valued mapping. In reference [8], the author has discussed the existence and uniqueness of the solution of an initial value problem involving set valued differential equations on time scales. However, it is just the beginning of the study of set differential equations on time scales and many more aspects of this subject need to be addressed. It is imperative to note that time scales take their domain values from specific subsets of $\mathbb{R}$, not necessarily from the whole set $\mathbb{R}$ (they can, however, map to any value in $\mathbb{R}$) while the set valued functions

Received July 10, 2007

have no restrictions on their domain values in $\mathbb{R}$ but they map to sets composed of whole collection of points.

In this paper, we develop the basic theory of set valued differential equations on time scales and employ it to establish the stability criteria in terms of Lyapunov-like functions for set valued differential equations on time scales.

2. PRELIMINARIES

Let $\mathbb{T}$ be a time scale (any nonempty closed subset of the real numbers with order and topological structure defined in a canonical way) with $t_0 \geq 0$ as the minimal element and no maximal element. Since a time scale may or may not be connected, a pair of jump operators needs to be defined.

Definition 2.1 The mappings $\sigma, \ \rho : \mathbb{T} \to \mathbb{T}$ defined by

$$\sigma(t) = \inf\{s \in \mathbb{T} : s > t\} \text{ and } \rho(t) = \sup\{s \in \mathbb{T} : s < t\}$$

are called respectively forward and backward jump operators (or simply jump operators). It is worth remarking that the jump operators are crucial to establish the concept of derivatives. Observe that $\rho(t) \leq t \leq \sigma(t), \ \forall \ t \in \mathbb{T}$.

Definition 2.2 A nonmaximal point $t_0 \in \mathbb{T}$ is called right-scattered (rs) if $\sigma(t_0) > t_0$ and right-dense (rd) if $\sigma(t_0) = t_0$. A nonminimal point $t_0 \in \mathbb{T}$ is called left-scattered (ls) if $\rho(t_0) < t_0$ and left-dense (ld) if $\rho(t_0) = t_0$.

Let $K(\mathbb{R}^n)$ denote the collection of nonempty, compact and convex subsets of $\mathbb{R}^n$. We define the Hausdorff metric as

$$D[X, Y] = \max[\sup_{y \in Y} d(y, X), \sup_{x \in X} d(x, Y)],$$

where $d(y, X) = \inf[d(y, x) : x \in X]$ and X, Y are bounded subsets of R^n. Notice that $K(\mathbb{R}^n)$ with the metric is a complete metric space. Moreover, $K(\mathbb{R}^n)$ equipped with the natural algebraic operations of addition and nonnegative scalar multiplication becomes a semilinear metric space which can be embedded as a complete cone into a corresponding Banach space. This space contains a zero element 0, namely the set consisting of a single specific element, the zero vector of $\mathbb{R}^n$. The Hausdorff metric satisfies the following properties:

$$D[U + W, V + W] = D[U, V] \text{ and } D[U, V] = D[V, U],$$

$$D[\mu U, \mu V] = \mu D[U, V],$$

$$D[U, V] \leq D[U, W] + D[V, W],$$

$\forall \ U, V, W \in K(\mathbb{R}^n)$ and $\mu \in \mathbb{R}_+$. Note that the space $K(\mathbb{R}^n)$ does not have the familiar properties of subtraction that are common in $\mathbb{R}^n$ and instead we have the Hukuhara-based subtraction of the elements of $K(\mathbb{R}^n)$.

Definition 2.3 The set $Z \in K(\mathbb{R}^n)$ satisfying $X = Y + Z$ is known as the Hukuhara difference of the sets X and Y in $K(\mathbb{R}^n)$ and is denoted as $X - Y$.

Definition 2.4 A set valued function $F : \mathbb{T} \to K(\mathbb{R}^n)$ is called $rd-$continuous if and only if for every right-dense point $t_0 \in \mathbb{T}$, we have $\lim_{t \to t_0^+} F(t) = F(t_0) (\in K(\mathbb{R}^n))$.

Remark 2.1 $C_{rd}[\mathbb{T}, K(\mathbb{R}^n)]$ will denote the set of $rd-$continuous functions from $\mathbb{T}$ to $K(\mathbb{R}^n)$.

Definition 2.5 The forward derivative of $F : \mathbb{T} \to K(\mathbb{R}^n)$, denoted by F^Δ, is defined by

$$F^\Delta(t) = \frac{1}{\sigma(t) - t} \cdot [F(\sigma(t)) - F(t)] \text{ if } t \text{ is right-scattered}$$

and

$$F^\Delta(t) = \lim_{s \to t^+} \frac{1}{s - t} \cdot [F(s) - F(t)] \text{ if } t \text{ is right-dense.}$$

Analogously, the backward derivative F^∇ is defined as

$$F^\nabla(t) = \frac{1}{t - \rho(t)} \cdot [F(t) - F(\rho(t))] \text{ if } t \text{ is left-scattered}$$

and

$$F^\nabla(t) = \lim_{s \to t^-} \frac{1}{t - s} \cdot [F(t) - F(s)] \text{ if } t \text{ is left-dense.}$$

Remark 2.2 The usual scalar subtraction occurs in the scalar co-efficient at the beginning of the above expressions and the Hukuhara set difference occurs inside the brackets. Clearly, the existence of Hukuhara difference ensures the existence of the derivative in Definition 2.5 In the forthcoming analysis, we will be considering the forward derivative as the results for backward derivative follow immediately with suitable changes. The backward derivative has the following properties:

(i) Let $F,\ G : \mathbb{T} \to K(\mathbb{R}^n)$ be differentiable at t, then $(F + G)^\Delta(t) = F^\Delta(t) + G^\Delta(t)$;

(ii) $(\alpha F)^\Delta(t) = \alpha F^\Delta(t)$, $\alpha \in \mathbb{R}_+$.

Definition 2.6 Let $F : \mathbb{T} \to K(\mathbb{R}^n)$. Then $F_s : \mathbb{T} \to \mathbb{R}^n$ is a selector of F if $F_s(t) \in F(t)$ for every t. F_s is an integrable selector of F if F_s is a selector of F and F_s is integrable. Let $S(F)$ denote the set of all integrable selectors of F.

Given $a,\ b \in \mathbb{T}$, the definite integral from a to b of a function $F : \mathbb{T} \to K(\mathbb{R}^n)$ is defined as

$$\int_a^b F(t)\Delta t = \overline{\{\int_a^b F_s(t)\Delta t : F_s \in S(F)\}}.$$

Remark 2.3 If the sets in $K(\mathbb{R}^n)$ are singletons only, then there is only one selector possible, namely F itself. In this case, the integral reduces to the generalized integral from time scales into $(\mathbb{R}^n)$. The assumption $n = 1$ generalizes into the integral for time scales. When $\mathbb{T}$ is everywhere right-dense, then we have $\Delta t = dt$

which results into the conventional formulation for integration of set valued functions. Moreover, for $a,\ b,\ c \in \mathbb{T}$, we have

(i) $\int_a^c F(t)\Delta t = \int_a^b F(t)\Delta t + \int_b^c F(t)\Delta t$,

(ii) $\int_a^b F(t)\Delta t$ is closed and convex, but need not be necessarily compact.

Definition 2.7 Let $F,\ G : \mathbb{T} \to K(\mathbb{R}^n)$. Then the Hausdorff distance between F and G, $D[F(.), G(.)] : \mathbb{T} \to \mathbb{R}_+$ is $\Delta-$ integrable and

$$D[\int_{t_0}^t F(s)\Delta s,\ \int_{t_0}^t G(s)\Delta s] \leq \int_{t_0}^t D[F(s), G(s)]\Delta s.$$

3. SOME BASIC RESULTS

Consider the initial value problem

$$X^{\Delta}(t) = F(t, X),\ X(t_0) = X_0, \tag{1}$$

where $F \in C_{rd}[\mathbb{T} \times K(\mathbb{R}^n), K(\mathbb{R}^n)]$.

Now we prove some comparison and existence results for the IVP (1).

Theorem 3.1. Assume that $F \in C_{rd}[\mathbb{T} \times K(\mathbb{R}_+), K(\mathbb{R}_+)]$ and

$$D[F(t, X), F(t, Y)] \leq g(t, D[X, Y]), \quad t \in \mathbb{T}, \quad X, Y \in K(\mathbb{R}^n),$$

where $g \in C_{rd}[\mathbb{T} \times \mathbb{R}_+, \mathbb{R}_+]$. Further, the maximal solution $r(t) = r(t, t_0, w_0)$ of the scalar differential equation

$$w^{\Delta}(t) = g(t, w),\ w(t_0) = w_0 \geq 0,$$

exists for $t \geq t_0$. If $X(t) = X(t, t_0, X_0),\ Y(t) = Y(t, t_0, Y_0)$ are any two solutions of (1) with $X(t_0) = X_0$, $Y(t_0) = Y_0$ (X_0, $Y_0 \in K(\mathbb{R}^n)$) existing for $t \geq t_0$, then

$$D[X(t), Y(t)] \leq r(t, t_0, w_0),\ t \geq t_0,$$

provided $D[X_0, Y_0] \leq w_0$.

Proof. Since $X(t)$ and $Y(t)$ are the solutions of (1), the Hukuhara difference $X(\sigma(t)) - X(t)$, $Y(\sigma(t)) - Y(t)$ exist if t is right-scattered. We set $m(t) = D[X(t), Y(t)]$. Then

$$\begin{aligned}
m(\sigma(t)) - m(t) &= D[X(\sigma(t)), Y(\sigma(t))] - D[X(t), Y(t)] \\
&\leq D[X(\sigma(t)), X(t) + (\sigma(t) - t)F(t, X)] \\
&+ D[X(t) + (\sigma(t) - t)F(t, X), Y(t) + (\sigma(t) - t)F(t, Y)] \\
&+ D[Y(t) + (\sigma(t) - t)F(t, Y), Y(\sigma(t))] - D[X(t), Y(t)] \\
&\leq D[X(\sigma(t)), X(t) + (\sigma(t) - t)F(t, X)] \\
&+ D[Y(t) + (\sigma(t) - t)F(t, Y), Y(\sigma(t))] \\
&+ (\sigma(t) - t)D[F(t, X), F(t, Y)].
\end{aligned}$$

Thus

$$\begin{aligned}\frac{m(\sigma(t))-m(t)}{\sigma(t)-t} &\leq D[\frac{X(\sigma(t))-X(t)}{\sigma(t)-t}, F(t,X)] \\ &+ D[F(t,Y), \frac{Y(\sigma(t))-Y(t)}{\sigma(t)-t}] + D[F(t,X), F(t,Y)],\end{aligned}$$

which, in view of Definition 2.5, yields

$$m^{\Delta}(t) \leq D[X^{\Delta}(t), F(t,X)] + D[F(t,Y), Y^{\Delta}(t)] + D[F(t,X), F(t,Y)].$$

Consequently, we have

$$m^{\Delta}(t) \leq D[F(t,X), F(t,Y)] \leq g(t, D[X,Y]) = g(t, m(t)).$$

Since $m(t_0) = D[X(t_0), Y(t_0)] = D[X_0, Y_0] \leq w_0$, therefore, by Theorem 1.4.1 [9], it follows that $m(t) \leq r(t, t_0, w_0)$, that is, $D[X(t), Y(t)] \leq r(t, t_0, w_0)$, $t \geq t_0$. This completes the proof.

Now, we prove an existence and uniqueness result under the assumptions more general than the Lipschitz condition [8].

Theorem 3.2. Assume that

(**i**) Let $B_0 \subset \mathbb{T} \times K(\mathbb{R}^n)$ and $F \in C_{rd}[B_0, K(\mathbb{R}^n)]$, and there exist values $a,\ b$ such that $|t-t_0| \leq a$, $D[X, X_0] \leq b$, $t \in J = [t_0, t_0+a]$. Further, there exists M_0 such that $\max_{(t,X)\in B_0} \|F(t,X)\| = M_0$.

(**ii**) $g \in C_{rd}[J \times [0,2b], R_+]$ with $g(t,w) \leq M_1$ on $J \times [0,2b]$, $g(t,0) \equiv 0$, $g(t,w)$ is nondecreasing in w for each $t \in J$ and $w(t) \equiv 0$ is the only solution of

$$w^{\Delta}(t) = g(t,w),\ w(t_0) = 0 \text{ on } J. \tag{2}$$

(**iii**) $D[F(t,X), F(t,Y)] \leq g(t, D[X,Y])$ on B_0.

Then the successive approximations given by

$$X_{n+1}(t) = X_0 + \int_{t_0}^{t} F(s, X_n(s))\Delta s,\ X(t_0) = X_0,\ n = 0,1,2,\ldots,$$

exist on B_0 for all values of $t \in J_0 = [t_0, t_0+h]$, where $h = \min\{a, b/M_2\}$, $M_2 = \max(M_0, M_1)$ and converge uniformly to the unique solution of (1) on J_0.

Proof. As a first step, we show that the family of successive approximations exists on the region B_0 for all $t \in J_0$. Since

$$\begin{aligned}&D[X_{n+1}(t), X_0] = D[X_0 + \int_{t_0}^{t} F(s, X_n(s))\Delta s, X_0] = D[\int_{t_0}^{t} F(s, X_n(s))\Delta s, 0] \\ &= \|\int_{t_0}^{t} F(s, X_n(s))\Delta s\| \leq \int_{t_0}^{t} \|F(s, X_n(s))\|\Delta s \leq M_0(t-t_0) \leq M_0 b/M_2 \leq b,\end{aligned}$$

therefore, $(t, X_n(t)) \in B_0$ for all $t \in J_0$. Hence the successive approximations $\{X_n\}$ are well defined on J_0.

Next, we define the successive approximations for (2) as

$$w_0(t) = M_2(t - t_0),\ w_{n+1}(t) = \int_{t_0}^{t} g(s, w_n(s))\Delta s,\ t \in J_0,\ n = 0, 1, 2, \ldots$$

In view of the monotone character of $g(t, w)$ in w, it follows by an easy induction (which holds on time scales [1]) that the successive approximations $\{w_n\}$ are well defined on J_0 and $0 \leq w_{n+1} \leq w_n$, $n = 0, 1, 2, \ldots$ on J_0. As $|w_n^{\Delta}(t)| \leq g(t, w_{n-1}) \leq M_1$, therefore by Arzela-Ascoli theorem together with the monotonicity of $\{w_n\}$, we conclude that $\lim_{n\to\infty} w_n(t) = w(t)$ uniformly on J_0. Clearly $w(t)$ satisfies (2) and by the assumption (ii), we have $w(t) \geq 0, t \in J_0$.

Now we prove the uniform convergence of the successive approximations $\{X_n(t)\}$ on J_0. Observe that

$$D[X_1(t), X_0] \leq \int_{t_0}^{t} \|[F(s, X_0(s))\|\Delta s \leq M_2(t - t_0) = w_0(t).$$

For $k > 1$, we assume that

$$D[X_k(t), X_{k-1}(t)] \leq w_{k-1}(t).$$

In view of (iii) and the monotone character of $(g(t, w)$, we find that

$$\begin{aligned} D[X_{k+1}(t), X_k(t)] &\leq \int_{t_0}^{t} D[F(s, X_k(s)), F(s, X_{k-1}(s))]\Delta s \\ &\leq \int_{t_0}^{t} g(s, w_{k-1}(s))\Delta s = w_k(t). \end{aligned}$$

Thus, by mathematical induction, the estimate

$$D[X_{n+1}(t), X_n(t)] \leq w_n(t),\ t \in J_0, \tag{3}$$

is true for all n.

Letting $x(t) = D[X_{n+1}(t), X_n(t)]$, $t \in J_0$ and repeating the arguments used in the proof of Theorem 3.1, it follows that

$$x^{\Delta}(t) \leq g(t, D[X_n(t), X_{n-1}(t)]) \leq g(t, w_{n-1}(t)),\ t \in J_0.$$

Now, for $n \leq p$, we set $y(t) = D[X_n(t), X_p(t)]$, $t \in J_0$. Employing the method of proof of Theorem 3.1 together with the monotonicity of of $g(t, w)$ in w and the fact that $w_{p-1} \leq w_{n-1}$ ($\{w_n\}$ is a decreasing sequence), we obtain

$$\begin{aligned} y^{\Delta}(t) &\leq D[X_n^{\Delta}(t), X_p^{\Delta}(t)] = D[F(t, X_n(t)), F(t, X_p(t)) \\ &\leq D[F(t, X_n(t)), F(t, X_{n-1}(t)) + D[F(t, X_{n-1}(t)), F(t, X_{p-1}(t)) \\ &+ D[F(t, X_p(t)), F(t, X_{p-1}(t)) \\ &\leq g(t, w_{n-1}(t)) + g(t, w_{p-1}(t)) + g(t, D[X_n(t), X_p(t)]) \\ &\leq g(t, y(t)) + 2g(t, w_{n-1}(t)),\ t \in J_0, \end{aligned}$$

which, by the Comparison Theorem 1.4.1 [9], yields

$$y(t) \leq r_n(t), \ t \in J_0,$$

where $r_n(t)$ is the maximal solution of

$$r_n^{\Delta}(t) = g(t, r_n(t)) + 2g(t, w_{n-1}(t)), \ r_n(t_0) = 0, \ \text{for each } n.$$

As $2g(t, w_{n-1}(t)) \to 0$ uniformly on J_0 as $n \to \infty$, it follows by Lemma 1.3.1 [9] that $r_n(t) \to 0$ uniformly on J_0 as $n \to \infty$. Thus, from the inequality (3) and the definition of $y(t)$, we deduce that $X_n(t)$ converges uniformly to $X(t)$ and hence $X(t)$ is a solution of (1).

In order to establish the uniqueness of the solution, let $Y(t)$ be another solution of (1) on J_0. Setting $m(t) = D[X(t), Y(t)]$ and noting that $m(t_0) = 0$, and applying the arguments of proof and conclusion of Theorem 3.1, we obtain

$$m^{\Delta}(t) \leq g(t, m(t)), \ t \in J_0,$$

and $m(t) \leq r(t, t_0, 0)$ $t \in J_0$. By the assumption $r(t, t_0, 0) = 0$, we obtain $X(t) \equiv Y(t)$ on J_0. This completes the proof.

4. STABILITY CRITERIA

Definition 4.1 For $V \in C_{rd}[\mathbb{T} \times K(\mathbb{R}^n), R_+]$, we define $V^{\Delta}(t, X(t))$ as: given any $\epsilon > 0$, there exists a neighbourhood N_ϵ of $t \in \mathbb{T}$, that is, $N_\epsilon = (t - \delta, t + \delta) \cap \mathbb{T}$ for some $\delta > 0$ such that

$$|[V(\sigma(t), X(\sigma(t))) - V(s, X(\sigma(t)))] - (\sigma(t) - s)[F(t, X(t)) + V^{\Delta}(t, X(t))]| \leq \epsilon |\sigma(t) - s|,$$

for each $s \in N_\epsilon$, $s > t$. If t is right-scattered and $V(t, X(t))$ is continuous at t, then the above definition takes the form

$$V^{\Delta}(t, X(t)) = \frac{V(\sigma(t), X(\sigma(t))) - V(t, X(t))}{\sigma(t) - t}.$$

The following comparison theorem provides a basis to investigate the stability criteria of set differential equation on time scales in term of Lyapunov-like functions.

Theorem 4.1 Assume that

($\mathbf{A_1}$) $V \in C_{rd}[\mathbb{T} \times K(\mathbb{R}^n), R_+]$ and $|V(t, X) - V(t, Y)| \leq KD[X, Y]$ for each right-dense $t \in \mathbb{T}$, $X, Y \in K(R^n)$ and locally Lipschitz constant K;

($\mathbf{A_2}$) $g \in C_{rd}[\mathbb{T} \times R_+, R_+]$ and for $X \in K(R^n)$, $t \in \mathbb{T}$,

$$V^{\Delta}(t, X(t)) \leq g(t, V(t, X(t)));$$

($\mathbf{A_3}$) there exists the maximal solution $r(t, t_0, w_0)$ on $\mathbb{T}$ of

$$w^{\Delta}(t) = g(t, w(t)), \ w(t_0) = w_0 \geq 0.$$

Then, if $V(t_0, X_0) \leq w_0$, we have $V(t, X(t)) \leq r(t, t_0, w_0)$, $t \in \mathbb{T}$, $t \geq t_0$.

Proof. Define $m(t) = V(t, X(t))$ so that $m(t_0) = V(t_0, X_0) \leq w_0$ and consider

$$\begin{aligned} m(\sigma(t)) - m(t) &= V(\sigma(t), X(\sigma(t))) - V(t, X(t)) \\ &= V(\sigma(t), X(\sigma(t))) - V(\sigma(t), X(t) + (\sigma(t) - t)F(t, X(t)) \\ &+ V(\sigma(t), X(t) + (\sigma(t) - t)F(t, X(t)) - V(t, X(t)) \\ &\leq KD[X(\sigma(t)), X(t) + (\sigma(t) - t)F(t, X(t))] \\ &+ V(\sigma(t), X(t) + (\sigma(t) - t)F(t, X(t)) - V(t, X(t)). \end{aligned}$$

Let $Z(t)$ be the Hukuhara difference of $X(\sigma(t))$ and $X(t)$ which is assumed to exist if t is right-scattered, that is, $X(\sigma(t)) = X(t) + Z(t)$. Then we have

$$m(\sigma(t)) - m(t) \leq KD[Z(t), (\sigma(t) - t)F(t, X(t))] + V(\sigma(t), X(\sigma(t))) - V(t, X(t)),$$

which in view of definition 2.5 and assumption (A_2), implies that

$$\begin{aligned} \frac{m(\sigma(t)) - m(t)}{\sigma(t) - t} &\leq KD[\frac{X(\sigma(t)) - X(t)}{\sigma(t) - t}, F(t, X)] \\ &+ \frac{V(\sigma(t), X(\sigma(t))) - V(t, X(t))}{\sigma(t) - t} \\ &= V^{\Delta}(t, X(t)) \leq g(t, V(t, X(t))). \end{aligned}$$

Thus, we have

$$m^{\Delta}(t) \leq g(t, m(t), \ m(t_0) \leq w_0,$$

which, as before (in the proof of Theorem 3.1), provides the desired estimate

$$V(t, X(t)) \leq r(t, t_0, w_0), \ t \in \mathbb{T}, \ t \geq t_0.$$

This proves the assertion of the theorem.

Remark 4.1 In order to match the behavior of the solution of set differential equations with the corresponding solutions of ordinary differential equations, we suppose that the Hukuhara difference W_0 exists for any given initial values $X_0, Y_0 \in K(R^n)$, that is, $X_0 = W_0 + Y_0$ and then consider the stability of the solution $X(t, t_0, X_0 - Y_0) = X(t, t_0, W_0)$ of (1).

We are now in a position to formulate the stability criteria for the trivial solution of (1) as follows:

Theorem 4.2. Assume that the assumptions (A_1) and (A_2) of Theorem 4.1 hold on $\mathbb{T} \times \Omega(\rho)$ instead of $K(R^n)$, where $\Omega(\rho) = [X \in K(R^n) : \|X\| < \rho]$. Further, suppose that $b(\|X\|) \leq V(t, X) \leq a(\|X\|)$ on $\mathbb{T} \times \Omega(\rho)$, where $a, b \in [[0, \rho], R_+]$ are the usual $\mathcal{K}$ class functions. Then the stability properties of the trivial solution of (2) imply the corresponding properties of the trivial solution of (1) subject to the condition $X(t, t_0, X_0 - Y_0) = U(t, t_0, W_0)$.

Proof. We only provide the outline of the proof. By Theorem 4.1 and using the standard method of proof of known results [9], the conclusion of the theorem can be established in a straightforward way.

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