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# ON THE APPROACHES TO A NUMERICAL MODELING OF LANDSLIDE MECHANISM OF TSUNAMI WAVE GENERATION

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**ABSTRACT:** In this work the results obtained in numerical studies of landslide mechanism of tsunami generation on the basis of a complex of multiparameter calculations with the help of algorithms, based on hierarchy of models of wave hydrodynamics, are presented. The dependences of the process of wave formation on length and width of landslide, on the depth of its bedding, on the law of motion and on the bottom slope angle are determined. The solutions, obtained on the approximate and complete models, are compared, the degree of influence of vertical structure of flow is estimated, and the region of the applicability of the approximate models is determined.

**AMS (MOS) Subject Classification:**

## 1. INTRODUCTION

In recent years, the results of numerical and laboratory experiments, simulating mechanism of tsunami wave generation by rigid solid body moving along the slope, are discussed in a number of papers, including those of authors of present article. It has been shown (Grilli and Watts [5], Watts et al [14]) that such approach, with appropriate parametrization, represents adequate schematization of real landslide process over a wide range of key parameters variation. The availability of approximate mathematical hydrodynamic models for description of landslide mechanism of surface waves generation has been examined (Watts et al [14], Watts et al [13], Eletsky et al [3], Chubarov et al [1]); thereat, the necessity of vertical flow structure accounting has been analyzed. It has been shown (Watts et al [14], Eletsky et al [3], Chubarov et al [1]) that in the initial stage of process, in case of long thin landslide, all models, from classical shallow water equations to complete model of inviscid flow, properly describe quintessential characteristics of wave generation.

Moreover, it has been shown that wave hydrodynamic models, which properly describe dispersion and account for process inhomogeneity in the vertical direction, are required to detailed quantitative and qualitative phenomenon description in big basins

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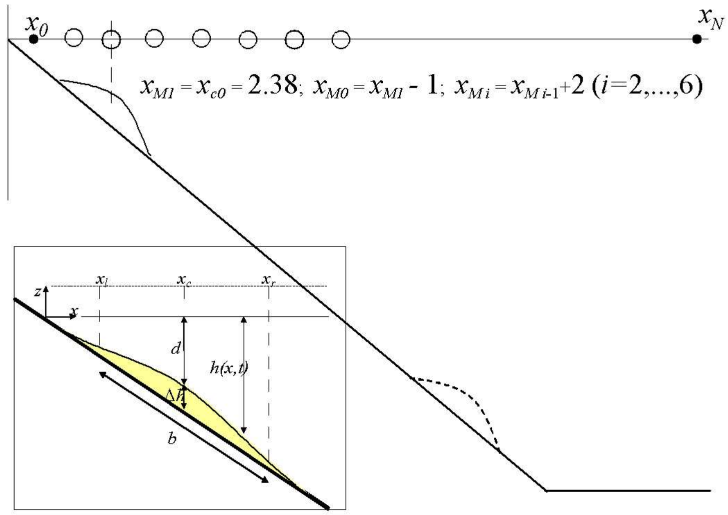



Figure 1. Scheme of model basin

for prolonged period. As for complete model, the capability of using it as a "reference" model, whose solutions are the closest to laboratory results, was demonstrated (Watts et al [14], Watts et al [13], Eletsky et al [3]); however, its implementation for multiple calculations is too expensive yet.

For detailed study of wave generation logic, it is necessary to investigate dependence of process characteristics on the main parameters of the problem: length and width of landslide, depth of its embedding, law of its motion and basin slope angle. This necessity initiated complex of multiparameter calculations, executed with the help of hierarchy of models of wave hydrodynamics, including the shallow water equations in approximations, considering nonlinear and dispersive effects, and complete equations of ideal fluid hydrodynamics.

## 2. SCHEMES OF MODEL BASIN AND LANDSLIDE MECHANISMS

To investigate tsunami wave generation by moving landslide, the model basin, the scheme of which is shown on the Figure 1, is suggested. The water is assumed to be still in the start time, the shoreline coincides with origin of Cartesian coordinates $xOz$. In the near-shore zone, bottom presents the slope with the slope angle $\varphi$; in the point $x_* = h_{const}\cot\varphi$ this zone joins with off-shore part of water area of constant depth. The landslide is modeled as moving rigid body. Such landslide is situated at the slope and its upper surface is away from the level of still water at a distance (Lynett and Liu [7]):

$$h(x,t) = x\tan\varphi - \Delta h \frac{\left[1+\tanh\left(2\frac{x-x_c(t)}{\cos\varphi}+b\right)\right]\left[1-\tanh\left(2\frac{x-x_c(t)}{\cos\varphi}-b\right)\right]}{\left[1+\tanh(b)\right]^2}\,.$$

Here $\Delta h$ is the maximum landslide thickness, $b$ is equal to distance between flex points of landslide surface, $x_c(t)$ is coordinate of maximum landslide thickness point. The value $d$ characterizes initial landslide embedding.

It is shown (Watts et al [14]) that underwater slides and slumps are convenient end members of the general range of possible underwater landslide motions. So, only these two model types of landslides are usually considered in mathematical modeling.

Slides are modeled as rigid thin bodies, moving for long distances along a straight incline, with center of mass motion $s(t) = s_0 \ln[\cosh(t/t_0)]$ parallel to the incline (Watts et al [14], Watts et al [13]). For such law of motion, initial acceleration and terminal velocity can be presented as follows: $a_0 = 0.3g\sin(\varphi)$, $u_{term} = 1.16\sqrt{bg\sin(\varphi)}$. As it has been shown by Watts et al [14], at these assumptions the simple equation $s(t) = a_0t^2/2$ ensures good approximation of initial body acceleration, after that the motion becomes uniform and continues till calculation finishing or any stop of landslide.

Slumps move an angle $\phi$ along a circular arc of radius $R$. Assuming that $\phi$ is small and $R$ is relatively big, arc can be approximated by straight line. So, the law of motion can be presented as follows: $s(t) = s_0[1 - \cosh(t/t_0)]$, where $s_0 = 0.5R\phi$, $t_0 = 1.84\sqrt{R/g}$; the traveling time is $\pi t_0$.

To investigate restructuring of the wave process at deceleration and stop of landslide, authors of present paper reconstituted wave regimes, induced by various versions of two types of landslide mechanisms indicated above (Figure 2). Thus, for slide, authors have studied the case of sudden body stop during uniform motion, and variant, when deceleration occurred gradually up to stop. When the motion law provided for sudden stop or the end of deceleration stage, this stage began at the moment of $t_{stop} = 30.0$ (except for one case).

For landslide motion simulation the following parameter values were employed:

$$\Delta h = 0.05, \quad b = 1.0, \quad h_{const} = 2.3,$$

$$x_{c0} = x_c(t = 0) = 2.38, \quad \varphi = 6^\circ, \quad g = 1.0$$

(free fall acceleration), $R = 2.0$, $\phi = 0.35$, unless otherwise stated. In more details these types of motion are stated in the paper of Shokin et al [12]. To fix simulation results, seven mareographs were placed to the points with following coordinates: $x_{M0} = x_{c0} - 1$, $x_{M1} = x_{c0}$, $x_{Mi} = x_{Mi-1} + 2$, $i = 2, ..., 6$.

## 3. MATHEMATICAL MODELS

In this paper, the hierarchy of models, which consider bottom time history, were utilized: linear and nonlinear shallow water equations, weakly nonlinear dispersive models, which were derived by Dorfman and Yagovdik [2] and which coincide with known Mei-Mehaute and Peregrine models (Mei and Le Mehaute [9], Peregrine [11])

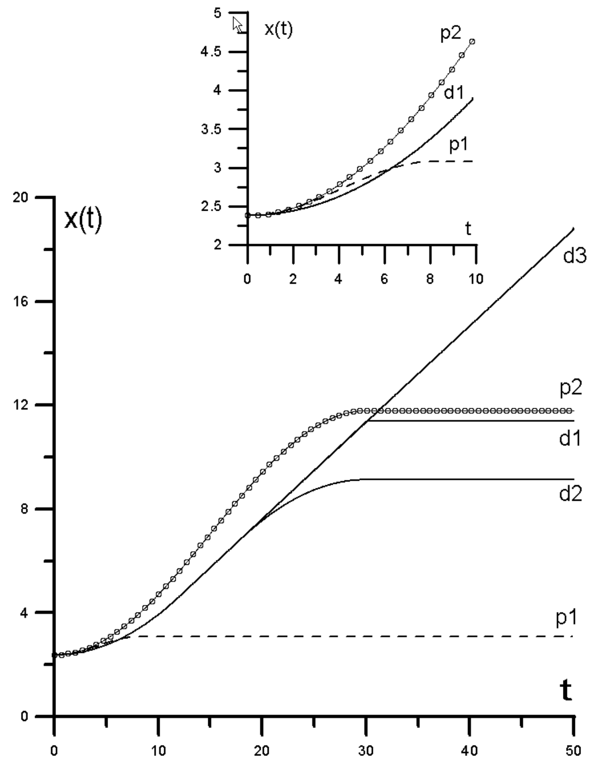



FIGURE 2. Types of landslide motion: “slide 1” (d1) – acceleration, uniform motion, stop, rest; “slide 2” (d2) – acceleration, uniform motion, deceleration, rest; “slide 3” (d3) – acceleration, uniform motion; “slump 1” (p1) – acceleration, deceleration, rest; “slump 2” (p2) – acceleration, deceleration, rest.

in case of even bottom, simplified models of Green-Naghdi (Green and Naghdi [4]) and Nwogu [10], one-and two-layer nonlinear dispersive models of Lynett and Liu [8]) and complete model of ideal inviscid flow (Khakimzyanov et al [6]). The complete description of above models is also given in the paper of Shokin et al Shokin et al [12].

Attempts to construct models with dispersion relations, approximating dispersion relation of the complete model in the best way, led Lynett and Liu [8] to results with more exact reproduction of vertical velocity profile. At the same time linear and nonlinear properties were improved. Derivation of this model required insertion of parameters controlling approximated properties of dispersion curves. These parameters determine depths of virtual layers. Thus, virtual interfaces appear. Lynett and Liu published results for $(2N-1)$-parameter model (with $N$ layers), but in numerical experiments only one- and two-layer models are utilized. On the certain assumptions, one-layer model reduces to nonlinear-dispersive model of Nwogu.

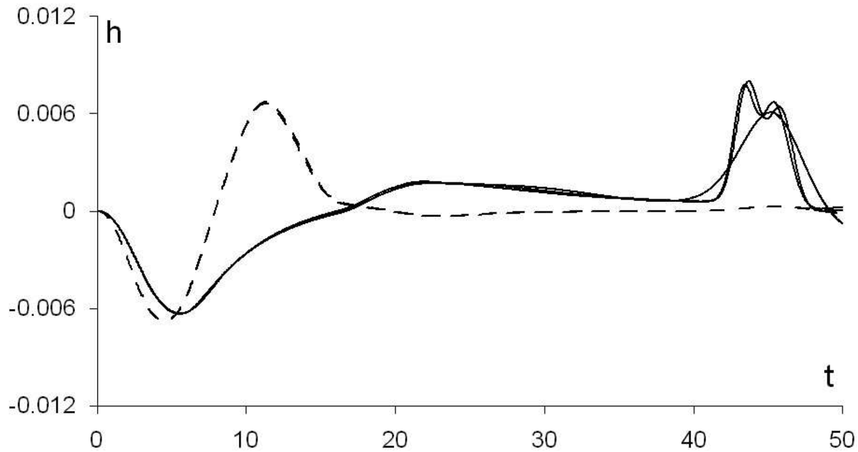



FIGURE 3. Marigrams of all models for “slide 1” (solid line) and “slump 1” (dash line) motion types in the first point.

## 4. NUMERICAL ALGORITHMS

As numerical algorithms, simple but effective finite-difference schemes were employed. For hyperbolic systems the numerical method, based on MacCormack scheme, was utilized. For nonlinear-dispersive equations second-order schemes were developed, which keep a number of operating parameters allowing for selective smoothing.

For numerical solving of Nwogu and Liu-Lynett equations, the finite-difference analog of Adams fourth-order scheme for temporal and spatial approximation (Wei and Liu [15]) was utilized. In case of nonlinear-dispersive equations this scheme has second order of spatial approximation. For approximation of complete hydrodynamic model, schemes on curvilinear net, adapting to the geometry of calculation region (Khakimzyanov et al [6]), were utilized.

## 5. COMPUTATIONAL EXPERIMENTS

The domain boundaries are located in the points $x_0 = 1.0$ and $x_N = 41.0$. Calculations last till $t_{final} = 50.0$. On the left boundary, the presence of vertical impermeable wall is assumed; the right boundary is open.

For comparison of different models, the series of experiments on simulation of wave process, generated by slide and slump, were carried out.

On Figures 3–8, marigrams of the first and the seventh points are represented in different combinations. Just in these points characteristic features of wave field become apparent in the most complete way. The first mareograph records the wave, propagating towards the coast, and the seventh one – propagating off-shore.

As Figure 3 shows, under the “slide 1” motion type, the negative wave associated with the acceleration stage comes initially to the first point, then the long flat positive wave associated with the stage of uniform motion follows, and finally waves, generated

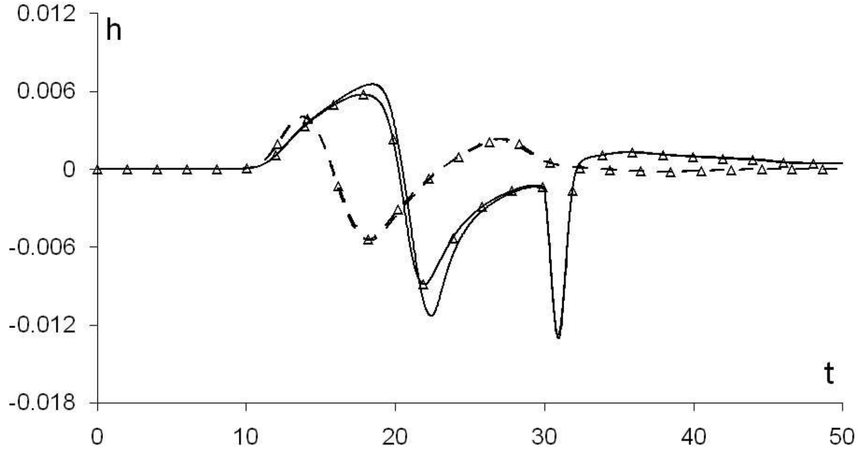



FIGURE 4. Marigrams of linear (marked line) and nonlinear shallow water models for "slide 1" (solid line) and "slump 1" (dash line) motion types in the seventh point.

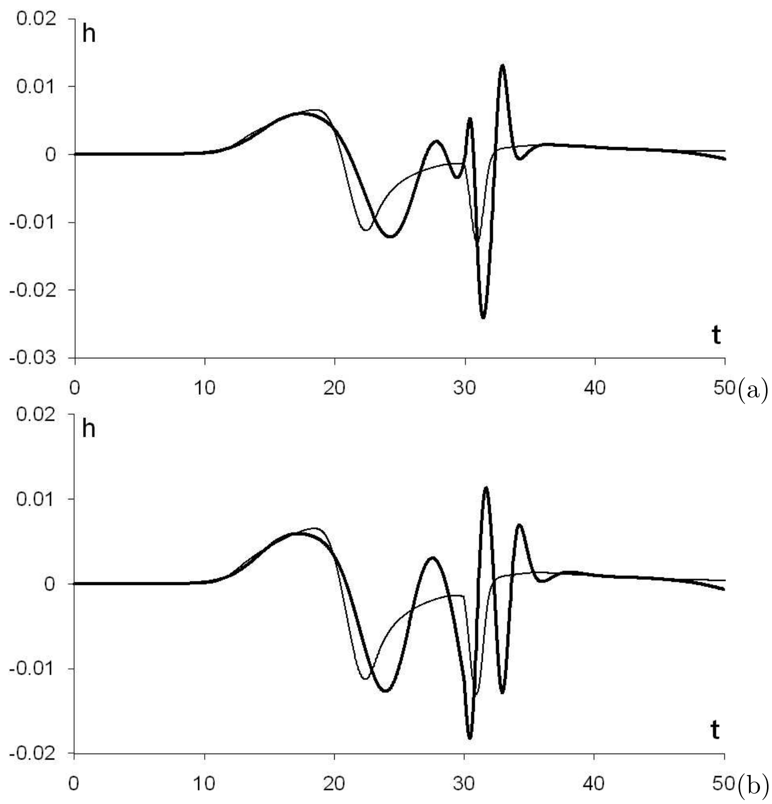



FIGURE 5. Marigrams of nonlinear shallow water model (thin line) and models with linear dispersion (heavy line) for "slide 1" motion type in the seventh point: (a) – Mei-Mehaute model, (b) – reduced Nwogu model, (c) – Peregrine model, (d) – Green-Naghdi model.

by sudden landslide stop as well as by reflection from coast wall, appear almost simultaneously (Figure 3). The dashed lines on this figure ought to be noted. They correspond to the "slump 1" law of motion, which differs by another law of acceleration

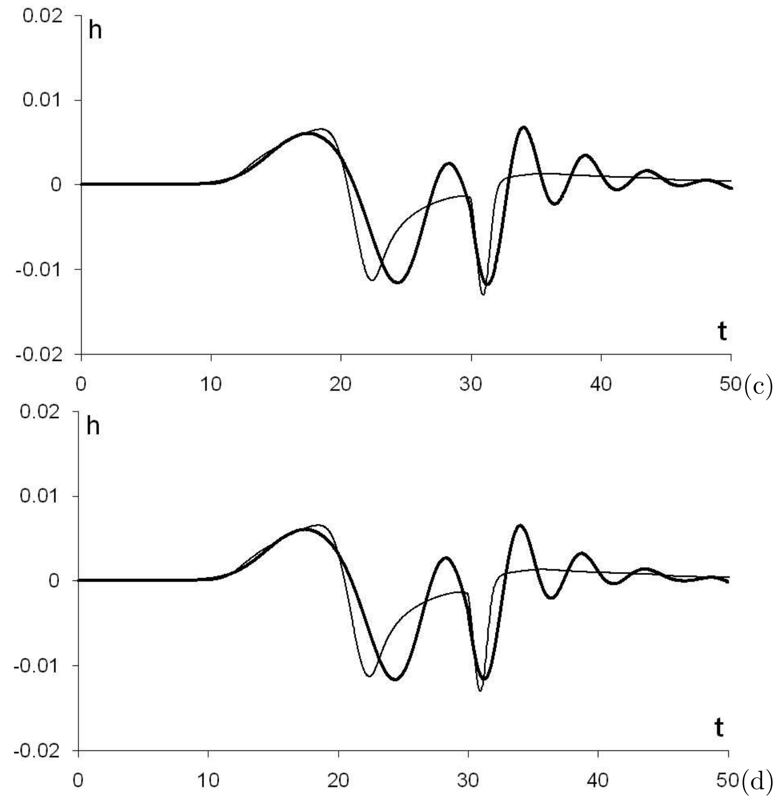



FIGURE 5. Continuation: Marigrams of nonlinear shallow water model (thin line) and models with linear dispersion (heavy line) for "slide 1" motion type in the seventh point: (a) – Mei-Mehaute model, (b) – reduced Nwogu model, (c) – Peregrine model, (d) – Green-Naghdi model.

and by deficiency of uniform motion and sudden stop stages, so there are no uniform motion and stop waves.

As curves show, in the near-shore zone, even the simplest mathematical models give undistinguished results, besides stop and reflection waves, which are reproduced by shallow water model especially precisely. The vital difference between waves, propagating on landslide way, can be seen on Figure 4, which illustrates the seventh point's marigrams of linear and nonlinear shallow water equations. To this point, the positive wave arrives first, then the negative wave follows, concerned with passing of the rear part of landslide (both for "slide 1" and "slump 1" motion types). The marigram of "slide 1" motion type is finished by sharp negative wave associated with uniform motion and stop.

There are no such effects in the case of "slump 1". For this type of motion, all models give similar results in the seventh point, too; i.e. "slump 1" landslide causes wave motions without pronounced vertical effects. Though, beside the wave, nicely reproduced by the simplest models, the small-scale oscillations associated with dispersion effects are apparent.

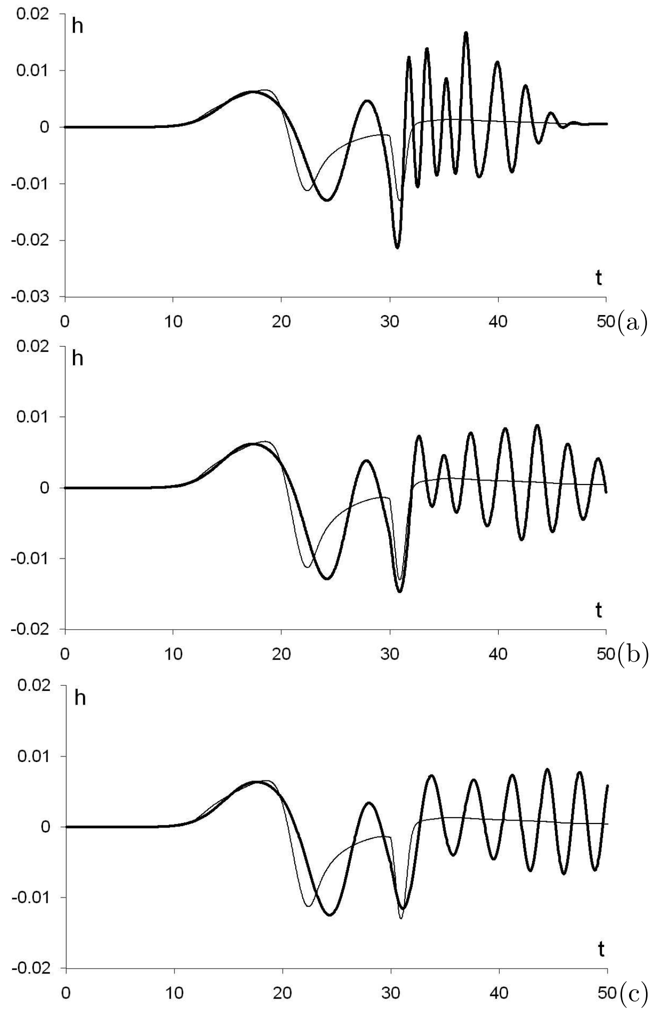



Figure 6. Marigrams of nonlinear dispersion models and complete model for "slide 1" motion type in the seventh point (heavy lines): (a) – one-layer Liu-Lynett model, (b) – two-layer Liu-Lynett model, (c) – complete model; thin lines – nonlinear shallow water model.

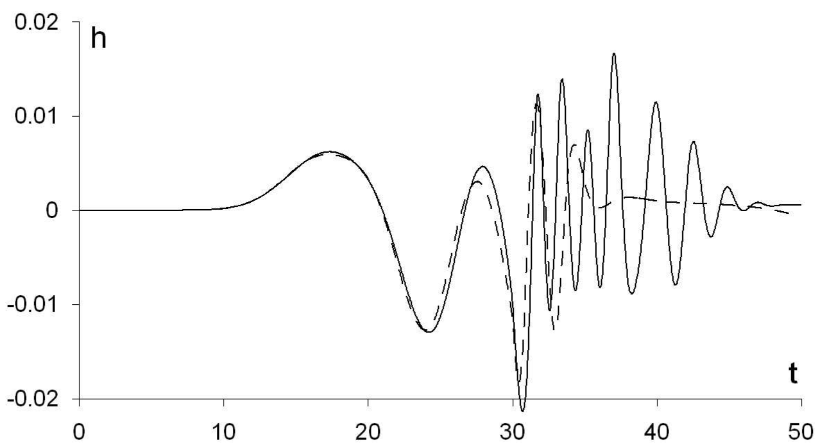



Figure 7. Marigrams of linear (dash line) and nonlinear (solid line) dispersion Nwogu models for "slide 1" motion type in the seventh point.

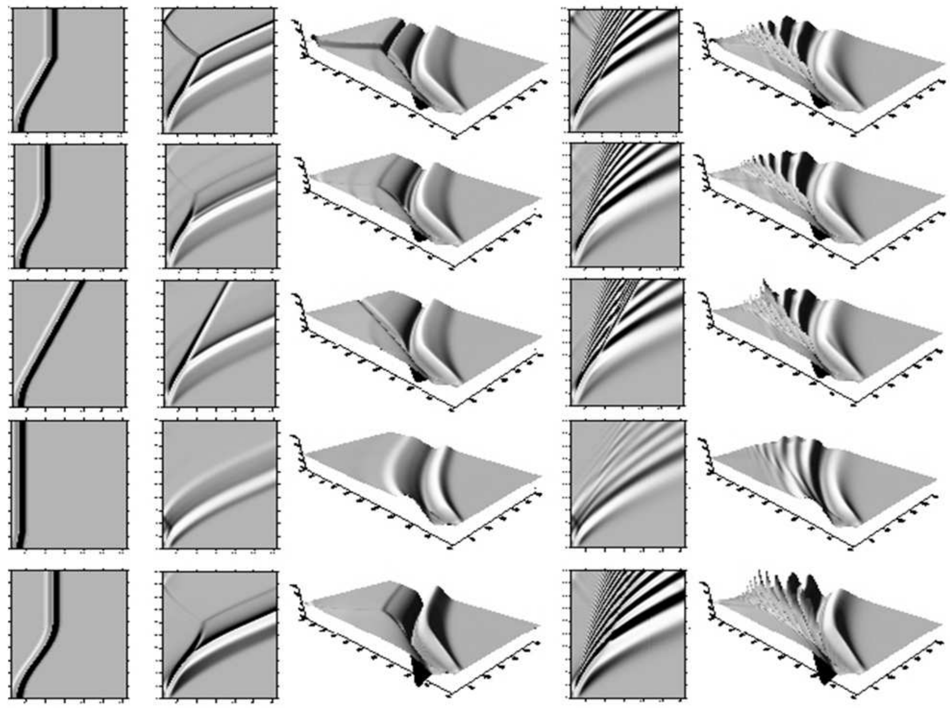


FIGURE 8. Wave patterns for different laws of landslide motion: the first row – "slide 1", the second one – "slide 2", third – "slide 3", fourth – "slump 1", fifth – "slump 2". The first column – landslide movement (top view), second and third – nonlinear shallow water model (top and perspective views), forth and fifth – two-layer Liu-Lynett model. $X$-direction – natural distance, $Y$-direction – time.

To choose the most adequate approximate model, we should analyze marigrams of "slide 1" motion type in the distant point. Toward this end, the results of nonlinear-dispersive models with linear dispersive terms should be examined at first (Figure 5). As we see, taking account of dispersion, even in its linear presentation, leads to essential complication of wave pattern. The proximity of results, obtained by Peregrine and Green-Naghdi models, whose linear analogs coincide, is apparent.

To determine the importance of taking account of nonlinear dispersive terms, the results of models with nonlinear dispersion should be considered first (in comparison with the results of nonlinear shallow water model on Figure 6), and then they should be compared with the results of complete model. Even preliminary analysis of marigrams validates Liu-Lynett's approach of improvement in accuracy of reproducing the process under studying by increasing number of layers.

One can receive pictorial view on importance of nonlinear dispersion accounting by comparison of results, calculated on base (with nonlinear dispersion) and reduced (with linear one) Nwogu models (see Figure 7). Models both describe equally significant part of wave process, but they differ essentially in modeling of dispersive wave train.

The final stage of analysis to choose the model for the subsequent investigations consists in comparison of promising nonlinear dispersion models with the complete one. Calculation results evidently point to the two-layer Liu-Lynett model and to the appropriateness of using of its simplified (one-layer) version for investigation of phenomenon's initial stage.

The wave conditions features caused by generation mechanisms were studied with the help of the two-layer Liu-Lynett model. To extract nonlinear dispersion effects, the similar calculations were done using shallow water theory (Figure 8).

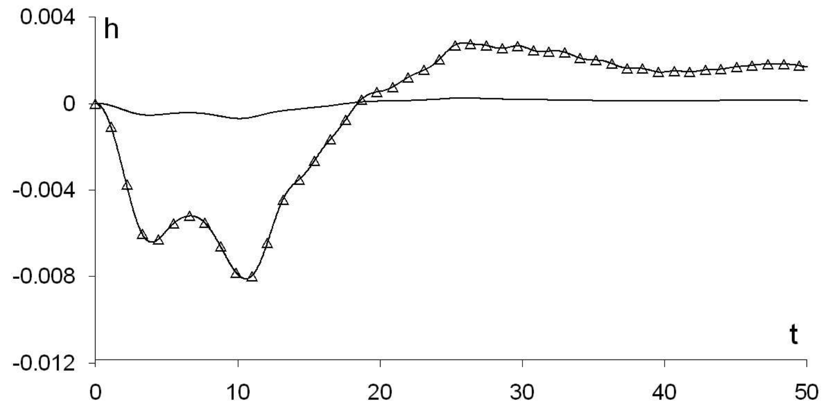
As it can be seen from the figures, at the beginning of motion, a small negative wave propagates towards the shore. The positive wave generates in front of body. During acceleration, this wave, keeping its amplitude, lengthens due to the fact that its leading edge propagates off-shore with the velocity of $\sqrt{gH}$ and the trailing one moves together with landslide. At uniform motion of landslide this wave separates and goes away. The generated negative wave goes with body, and at sudden stop of landslide it also separates from body and propagates off-shore. This stop generates negative wave, propagating towards shallow water. Every restructuring of motion causes waves with different amplitudes and characteristic horizontal dimensions, propagating towards and backwards the shore.

The above effects become apparent obviously for the "slide 1" motion type. The "slide 2" motion type demonstrates effect of absence of stopping, and the "slide 3" one shows effect of durational uniform motion.

The "slump 1" motion type characterizes by high wave formation efficiency for even transient landslide acceleration phase. Herewith, the increase of duration of this stage ("slump 2") results in increase of amplitudes of waves going off-shore, while the amplitude of wave propagating towards the shore occurs higher with shorter acceleration ("slump 1").

The two-layer model generally reproduces the same effects, but thanks to dispersion it demonstrates the more complex flow pattern with the wave train, propagating off-shore. The restructuring of wave pattern over changing of the law of motion becomes complicated, too.

The final part of the present work is devoted to the definition of wave characteristics dependence on the landslide and basin geometrical parameters for "slide 1" and "slide 3" motion types. For investigation of the landslide thickness dependence (Figure 9), the values $h = 0.01$ and $h = 0.1$ were examined. The calculations have been done with the help of the two-layer Liu-Lynett model, complete model, and linear and nonlinear shallow water models. The marigrams show that the wave characteristics change in the same way for both waves propagating towards the shore and those recorded in the distant point. Herewith, nonlinear effects are dominant, that leads to noticeable segregation of the linear model results.



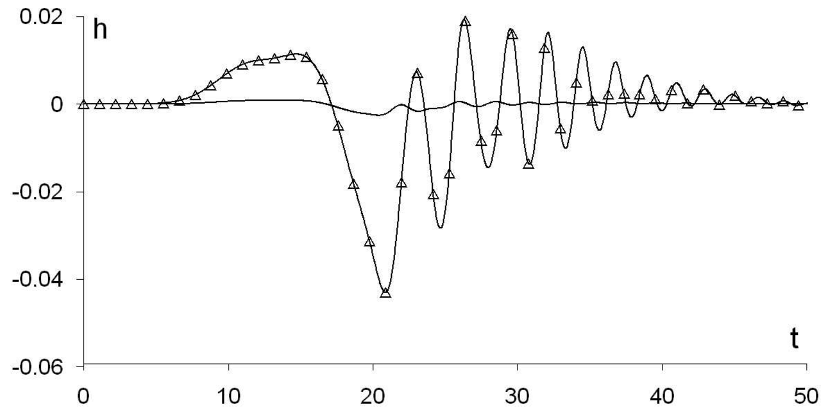




FIGURE 9. Marigrams of two-layer Liu-Lynett model for "slide 3" motion type in the first (a) and the fifth (b) points for different landslide thickness: $\Delta h = 0.01$ – solid line, $\Delta h = 0.1$ – marked line. $x_{c0} = 3.38$, $b = 1.0$.

Increase of landslide length from $b = 1.0$ to $b = 3.0$ (Figure 10) causes amplitude increase with retaining of principal trends in all points.

Changing of mass centre embedding (Figure 11) by increasing of its initial horizontal coordinate from $x_c = 2.38$ to $x_c = 4.38$ results in complication of wave process and decreasing of wave amplitude in the coastal point. In the distant point, amplitudes also decrease, but frequency characteristics keep the same.

Finally, the wave characteristics dependence on the angle of bottom slope for "slide 1" motion type is investigated. For this set of calculations, landslide starts on the depth $d(t = 0) = 0.5$ and stops when $d = 1.5$. Increase of slope angle $\varphi$ from $6°$ to $15°$ (Figure 12) results in amplitude increasing in the coastal points. For bigger angle, the wave, generated by transition from uniformly accelerated motion to steady one, becomes well-marked in the results of nonlinear shallow water model. In the distant points, amplitudes of positive waves keep the same for all models, while amplitude of negative wave becomes notably bigger for nonlinear shallow water model.

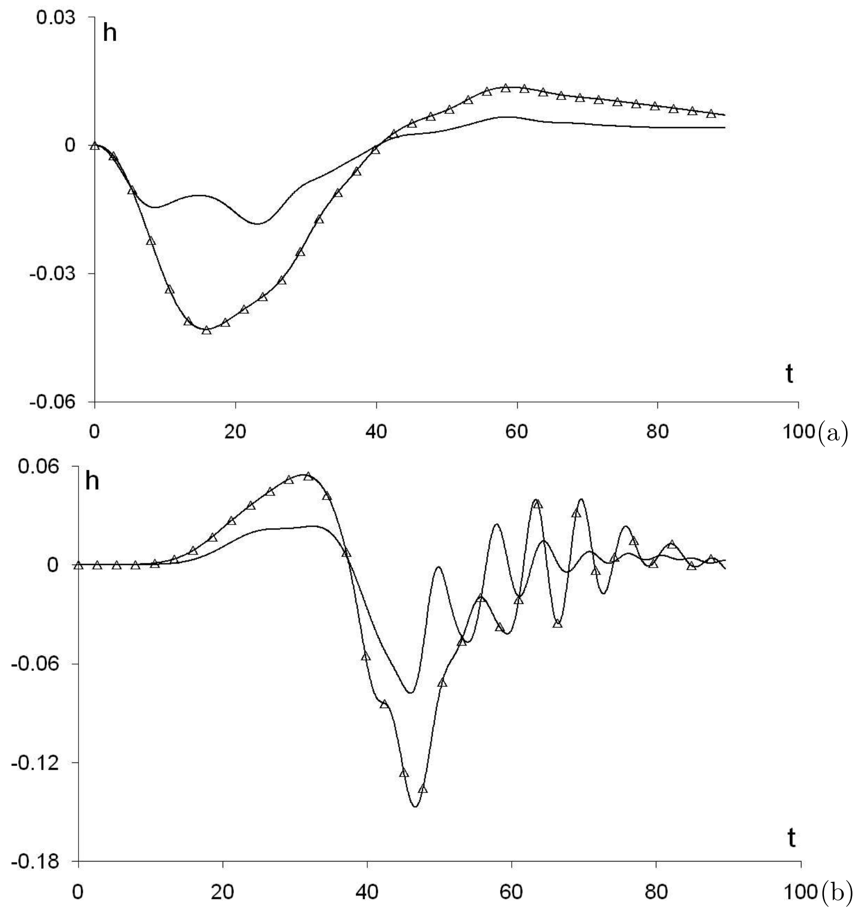



FIGURE 10. Marigrams of two-layer Liu-Lynett model for "slide 3" motion type in the first (a) and the fifth (b) points for different lengths of landslide: $b = 1$ – solid line, $b = 3$ – marked line. $x_c = 3.38$, $\Delta h = 0.05$.

## 6. CONCLUSION

The results of numerical simulation of tsunami wave generation by landslides in near-shore zones are presented. This range of problems is directly connected with the investigation of characteristics of waves generated by the movement of sediments concentrated in the large rivers' outlets. Such motions can be initiated even by small earthquakes, which are not able to cause tsunami waves only by seismic mechanism. The main results of this work are comparative study of different approximate models of wave hydrodynamics and determination of two-layer Liu-Lynett model as the most adequate mathematical model for solving the problems of the specified range. With the help of this model, the dependences of general properties of wave modes from law of landslide motion and from its geometrical parameters were studied.

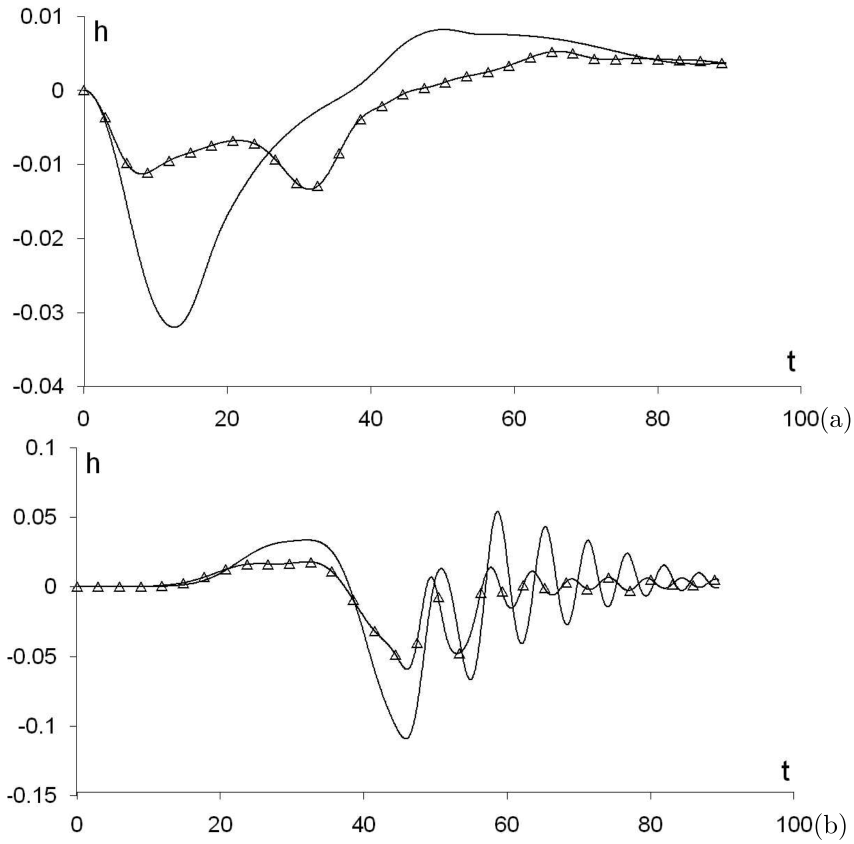



FIGURE 11. Marigrams of two-layer Liu-Lynett model for "slide 3" motion type in the first (a) and the fifth (b) points for different landslide embedding: $x_{c0} = 2.38$ – solid line, $x_{c0} = 4.38$ – marked line. $\Delta h = 0.05$, $b = 1.0$.

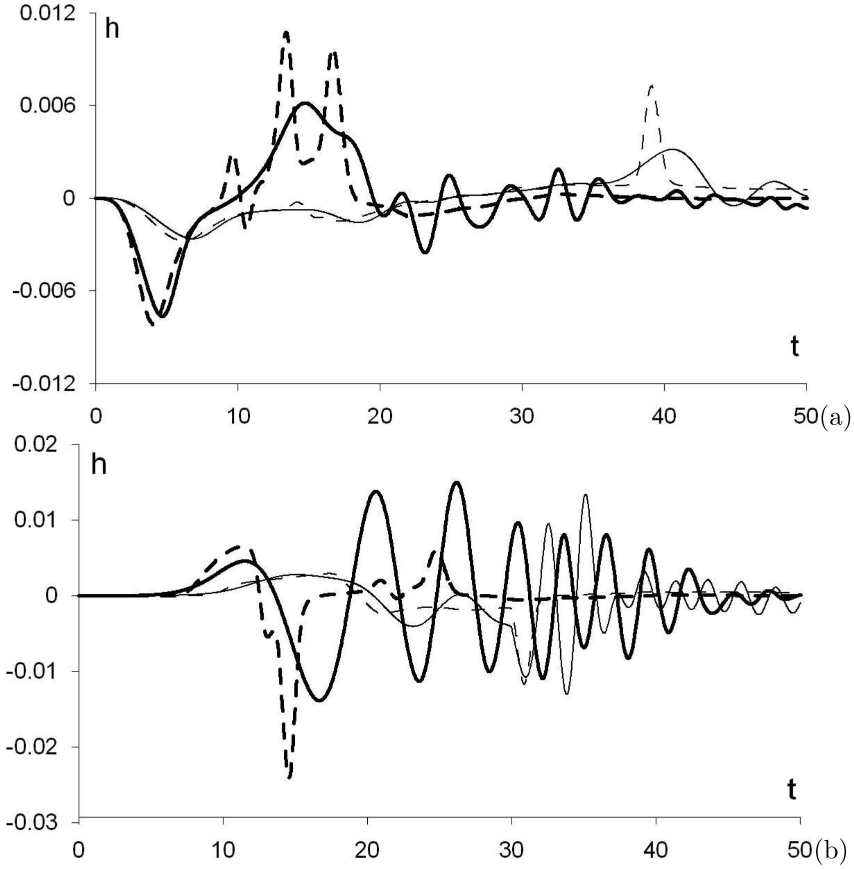



FIGURE 12. Marigrams of nonlinear shallow water (dash line) and two-layer Liu-Lynett (solid line) models for "slide 1" motion type in the second (a) and the seventh (b) points for different slope angles: $\varphi = 6$ – thin line, $\varphi = 15$ – thick line.

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