Communications in Applied Analysis **13** (2009), no. 3, 317–326

PARAMETERIZED NONLINEAR EQUATIONS ON DIRICHLET FORMS

ALEXANDRU KRISTÁLY[1], DONAL O'REGAN[2], AND CSABA VARGA[3]

[1]Babeş-Bolyai University, Department of Economics
400591 Cluj-Napoca, Romania
E-mail: alexandrukristaly@yahoo.com

[2]Department of Mathematics, National University of Ireland
Galway, Ireland
E-mail: donal.oregan@nuigalway.ie

[3]Babeş-Bolyai University, Department of Mathematics and Computer Sciences
400084 Cluj-Napoca, Romania
E-mail: csvarga@cs.ubbcluj.ro

ABSTRACT. By using variational techniques we prove the existence of two nontrivial solutions for a parameterized nonlinear equation relative to Dirichlet forms.

AMS (MOS) Subject Classification. 31C25, 35B38

1. INTRODUCTION AND PRELIMINARIES

The theory of Dirichlet forms is an active area of research, having its genesis in the stochastic process theory that uses energy functionals to study a Markov process from a quantitative point of view. In particular, Dirichlet forms can be used to study Markov processes taking values in spaces of fractional dimension, i.e. fractals, see Fukushima [6], Kusuoka and Yin [9]. Other applications can be found in Brownian motions on Euclidean spaces, Ornstein-Uhlenbeck processes on Hilbert spaces, Fleming-Viot processes on spaces of probability measures, particle processes on configuration spaces, etc.

In the recent years many results were obtained concerning the existence of solutions for semilinear problems involving Dirichlet forms by using *variational techniques*. For completeness we will just mention the papers of Biroli, Mataloni and Matzeu [1], Biroli and Tersian [2], Lisei and Varga [10], Matzeu [11] (for the general framework), as well as the papers of Falconer [3], Falconer and Hu [4], [5], Hu [8] as well as Grigor'yan, Hu and Lau [7] for particular fractals.

Received January 15, 2009 1083-2564 $15.00 ©Dynamic Publishers, Inc.

The purpose of the present paper is to give a multiplicity result for a parameterized nonlinear equation on Dirichlet forms. Except, for the papers [8] and [10] where Ricceri's three critical points theorem or genus-type arguments are exploited, the aforementioned papers give only the existence of at least one nontrivial solution for certain semilinear equations relative to a Dirichlet form. Combining a global minimization technique with a Mountain Pass argument, we are able to guarantee the existence of at least two nontrivial solutions for the studied problem for certain parameters, see Theorem 2.2.

In the rest of this section we recall the general framework of Dirichlet forms where we are going to work in. In the next Section we state our main result Theorem 2.2, while in the last Section we prove it.

We consider X to be a locally compact separable Hausdorff space, whose topology is endowed with a pseudodistance d, and let m be a positive Radon measure supported on the whole of X. Suppose that X is complete with respect to d.

Let $\mathcal{E}(\cdot,\cdot)$ be a strongly local, regular, symmetric Dirichlet form on $L^2(X,m)$, where $D[\mathcal{E}]$ denotes its domain. It has an integral representation

$$\mathcal{E}(u,v) = \int_X \mu(u,v)(dx) \quad \text{for all } u,v \in D[\mathcal{E}]$$

where $\mu(u,v)$ is the signed Radon measure on X (called the energy density of the form $\mathcal{E}$) uniquely associated with u and v.

We denote by $B(x,r) = \{y \in X : d(x,y) < R\}$ the ball of radius r centered at $x \in X$ with respect to the pseudodistance d. For any open set $A \subset X$ we consider

$$D(A) = \{u : A \to \mathbb{R} : u \in D[\mathcal{E}]\}, \qquad D_0(A) = \overline{D(A) \cap C_0(A)},$$

where the closure is taken in $D[\mathcal{E}]$ with respect to the norm $\|\cdot\|$, where

$$\|u\| = (\mathcal{E}(u,u) + \|u\|^2_{L^2(X,m)})^{1/2}.$$

$D_{loc}(A)$ denotes the set of all measurable functions on X coinciding m-almost everywhere with some function from $D[\mathcal{E}]$ on every compact subset of A.

Throughout the paper, we suppose that the following two properties hold:

• *The Poincaré property*: For every $R_0 > 0$ there exist $c > 0$, $k \geq 1$ such that for every $u \in D_{loc}(B(x,kr))$ and every $r \leq R_0$ we have

$$\int_{B(x,r)} |u - u_{x,r}|^2 m(dx) \leq cr^2 \int_{B(x,kr)} \mu(u,u)(dx), \qquad (PP)$$

where

$$u_{x,r} = \frac{1}{m(B(x,r))} \int_{B(x,r)} u\, m(dx),$$

while c and k are constants independent of x, r (but possibly depending on the number R_0).

• *The duplication property*: there exists $\nu > 0$ such that for every $R_0 > 0$ there exists $c_0 > 0$ such that for $r \leq 2r \leq R \leq R_0$ and $x \in X$

$$c_0 \left(\frac{r}{R}\right)^{\nu} m(B(x,R)) \leq m(B(x,r)). \tag{DP}$$

Moreover suppose that there exist $\xi > 0$ such that $m(B(x,1)) \geq \xi$ for all $x \in X$. Such a triple (X, d, m) is called a *homogeneous space* of dimension ν.

In the sequel, we assume that

(V1) $V \in C(X)$ with $V(x) > 0$ for all $x \in X$ and $V(x) \to \infty$ as $d(0,x) \to \infty$.

We denote by $L^2(X, Vm)$ the L^2-space on X with Radon measure Vm with density V with respect to m. We introduce the following Hilbert space

$$W = \left\{ u \in D[\mathcal{E}] \cap L^2(X, Vm) : \int_X \mu(u,u)(dx) + \int_X V(x)|u(x)|^2 m(dx) < \infty \right\}$$

equipped with the scalar product

$$(u,v)_W = \int_X \mu(u,v)(dx) + \int_X V(x) u(x) v(x) m(dx).$$

Proposition 1.1 ([2] Lemma 3 and Lemma 7)**.** *The following continuous embedding properties hold:*

$$\begin{aligned} &D[\mathcal{E}] \hookrightarrow L^{\frac{2\nu}{\nu-2}}(X,m), \ \ \textit{if } \nu > 2, \\ &D[\mathcal{E}] \hookrightarrow L^p(X,m), \quad \textit{if } p \geq 2 \textit{ and } \nu = 2, \\ &D[\mathcal{E}] \hookrightarrow L^\infty(X,m), \quad \textit{if } \nu < 2. \end{aligned}$$

Moreover, the embedding $W \hookrightarrow L^2(X,m)$ is compact.

We denote by $\|\cdot\|_p$ the norm in the space $L^p(X,m)$, $p \in [1,\infty]$. Observe, that by Proposition 1.1 it follows that there exists a constant $C_p > 0$ such that

$$\|u\|_p \leq C_p \|u\|_W \text{ for all } u \in W,$$

where

$$\begin{cases} p = \dfrac{2\nu}{\nu-2}, & \text{if } \nu > 2, \\ p \geq 2, & \text{if } \nu = 2, \\ p = \infty, & \text{if } \nu < 2. \end{cases}$$

2. MAIN RESULT

We assume that $f, g : \mathbb{R} \to \mathbb{R}$ are continuous function such that:

(FG1) $\lim_{|s|\to\infty} \frac{f(s)}{|s|^\beta} = \lim_{|s|\to\infty} \frac{g(s)}{|s|^\beta} = 0$ where $\beta = \frac{\nu+2}{\nu-2}$, if $\nu > 2$, and $\beta > 1$, if $\nu = 2$.

(FG2) $\lim_{s\to 0} \frac{f(s)}{s} = \lim_{s\to 0} \frac{g(s)}{s} = 0$.

(FG3) There exist $q \in]0, 2[$, $\alpha \in L^{\frac{\beta+1}{\beta+1-q}}(X, m), \gamma \in L^1(X, m)$ such that

$$\max\{|F(u(x))|, |G(u(x))|\} \leq \alpha(x)|u(x)|^q + \gamma(x) \text{ for a.e. } x \in X \text{ and all } u \in W,$$

with $F(u) := \int_0^u f(t)dt$, $G(u) := \int_0^u g(t)dt$ where $\beta = \frac{\nu+2}{\nu-2}$, if $\nu > 2$, and $\beta > 1$, if $\nu = 2$. If $\nu < 2$ we assume $\alpha \in L^1(X, m)$.

For $\lambda > 0$ and $\mu \in \mathbb{R}$, we are concerned with the existence of multiple (nontrivial) solutions $u \in W$ for the following problem $(P_{\lambda,\mu})$:

$$\int_X \mu(u, v)(dx) + \int_X V(x)u(x)v(x)m(dx)$$
$$= \lambda \int_X f(u(x))v(x)m(dx) + \mu \int_X g(u(x))v(x)m(dx), \quad \forall v \in W.$$

In order to achieve our goal, we also assume that

(F4) There exists $u_0 \in W$ such that $\int_X F(u_0(x))m(dx) > 0$.

Remark 2.1.

1) Without **(F4)**, problem $(P_{\lambda,\mu})$ may have only the trivial solution; indeed, choosing $f = g = 0$, the only solution is $u = 0$ for every $\lambda, \mu \in \mathbb{R}$.

2) When $\nu \geq 2$, then by **(FG1)** and **(FG2)**, it follows that for each $\varepsilon > 0$ there exist $c(\varepsilon) > 0$ such that

$$\max\{|f(s)|, |g(s)|\} \leq \varepsilon\Big(|s|^\beta + |s|\Big) + c_\varepsilon |s|^r; \tag{2.1}$$

here, $r \leq \beta$.

3) Let $C > 0$ be arbitrary. From **(FG2)** it follows that there exists $0 < \delta_3 < C$ such that

$$|f(s)| \leq |s| \text{ for all } |s| < \delta_3.$$

Then by the continuity of f we have that there exists $M_C = \frac{1}{\delta_3} \max_{|s|\leq C} |f(s)|$ and

$$|f(s)| \leq (1 + M_C)|s| \text{ for all } |s| \leq C. \tag{2.2}$$

Our main result reads as follows:

Theorem 2.2. *Let $f, g \in C(\mathbb{R})$ satisfying* **(FG1), (FG2), (FG3)** *and* **(F4)**. *Then there exists an $\lambda_0 > 0$ such that for every $\lambda > \lambda_0$ there exists $\mu_\lambda^* > 0$ with the property that for every $\mu \in [-\mu_\lambda^*, \mu_\lambda^*]$, the problem $(P_{\lambda,\mu})$ has at least two distinct nontrivial solutions.*

3. PROOF OF THE MAIN RESULT

We denote by $\Phi : W \to \mathbb{R}$, $J_\mu : W \to \mathbb{R}$ the functions defined by

$$\Phi(u) = \frac{1}{2}\|u\|_W^2 \quad \text{and} \quad J_\mu(u) = \mathcal{F}(u) + \mu\mathcal{G}(u),$$

where

$$\mathcal{F}(u) = \int_X F(u(x))m(dx) \quad \text{and} \quad \mathcal{G}(u) = \int_X G(u(x))m(dx).$$

For every $\lambda, \mu \in \mathbb{R}$ we consider the energy functional $\mathcal{E}_{\lambda,\mu} : W \to \mathbb{R}$ given by

$$\mathcal{E}_{\lambda,\mu}(u) = \Phi(u) - \lambda J_\mu(u).$$

Observe that $\mathcal{E}_{\lambda,\mu} \in C^1$ and for all $u, v \in W$ we have

$$\langle \mathcal{E}'_{\lambda,\mu}(u), v\rangle = (u, v)_W - \lambda \int_X f(u(x))v(x)m(dx) - \lambda\mu \int_X g(u(x))v(x)m(dx).$$

Moreover, if $\lambda > 0$, every critical point of $\mathcal{E}_{\lambda,\frac{\mu}{\lambda}}$ is a solution of problem $(P_{\lambda,\mu})$.

Lemma 3.1. *For every $\lambda, \mu \in \mathbb{R}$ we have that*

(1) *$\mathcal{E}_{\lambda,\mu}$ is coercive, i.e., $\lim\limits_{\|u\|_W \to \infty} \mathcal{E}_{\lambda,\mu}(u) = \infty$;*
(2) *$\mathcal{E}_{\lambda,\mu}$ fulfills the Palais-Smale condition.*

Proof. (1) We first prove that

$$\lim_{\|u\|_W \to \infty} \mathcal{E}_{\lambda,\mu}(u) = \infty.$$

First case: $\nu \geq 2$.

Due to **(FG3)**, we have

$$\begin{aligned} \mathcal{E}_{\lambda,\mu}(u) &\geq \frac{1}{2}\|u\|_W^2 - |\lambda|(1 + |\mu|) \int\limits_X [\alpha(x)|u(x)|^q + \gamma(x)]m(dx) \\ &\geq \frac{1}{2}\|u\|_W^2 - |\lambda|(1 + |\mu|)(\|\alpha\|_{\frac{\beta+1}{\beta+1-q}} C_{\beta+1}^q \|u\|_W^q + \|\gamma\|_1). \end{aligned}$$

Since $q \in]0, 2[$, if $\|u\|_W \to \infty$, we have $\mathcal{E}_{\lambda,\mu}(u) \to \infty$.

Second case: $\nu < 2$.

$$\begin{aligned} \mathcal{E}_{\lambda,\mu}(u) &\geq \frac{1}{2}\|u\|_W^2 - |\lambda|(1 + |\mu|) \int\limits_X [\alpha(x)|u(x)|^q m(dx) + \gamma(x)]m(dx) \\ &\geq \frac{1}{2}\|u\|_W^2 - |\lambda|(1 + |\mu|)[\|\alpha\|_1 C_\infty^q \|u\|_W^q + \|\gamma\|_1]. \end{aligned}$$

Since $q \in]0, 2[$, if $\|u\|_W \to \infty$, we have $\mathcal{E}_{\lambda,\mu}(u) \to \infty$.

(2) In order to prove that the function $\mathcal{E}_{\lambda,\mu} = \Phi - \lambda J_\mu$ fulfils the Palais-Smale condition, let (u_n) be a sequence from W such that $\{\mathcal{E}_{\lambda,\mu}(u_n)\}$ is bounded and $\mathcal{E}'_{\lambda,\mu}(u_n) \to 0$ in W' as $n \to \infty$. Because $\mathcal{E}_{\lambda,\mu}$ is coercive, it follows that the sequence (u_n) is bounded. Therefore, there exists an element $u \in W$ and a subsequence of (u_n) (which we denote also by (u_n)) such that $u_n \rightharpoonup u$ weakly in W and by Proposition 1.1, $u_n \to u$ strongly in $L^2(X,m)$. We clearly have

$$\|u_n - u\|_W^2 = -(u, u_n - u)_W + \langle \mathcal{E}'_{\lambda,\mu}(u_n), u_n - u\rangle + \lambda \int_X f(u_n)(u_n - u)m(dx) + \lambda\mu \int_X g(u_n)(u_n - u)m(dx). \tag{3.1}$$

Because $\mathcal{E}'_{\lambda,\mu}(u_n) \to 0$ and $u_n \rightharpoonup u$ weakly, the first two terms in (3.1) tend to 0 as $n \to \infty$. For the remaining two terms, we distinguish two cases:

First case: $\nu \geq 2$.

By (2.1) (with $r = 1$) and by the Hölder inequality, it follows that

$$\int_X f(u_n)(u_n - u)m(dx) \leq \int_X \Big(\varepsilon|u_n|^\beta + \varepsilon|u_n| + c_\varepsilon|u_n|\Big)|u_n - u|m(dx)$$
$$\leq (\varepsilon + c_\varepsilon)\|u_n - u\|_2\|u_n\|_2 + \varepsilon\|u_n - u\|_{\beta+1}\|u_n\|_{\beta+1}^\beta.$$

Since the embedding $W \hookrightarrow L^{\beta+1}(X,m)$ is continuous, we have that the sequence (u_n) is also bounded in $L^{\beta+1}(X,m)$. Because ε is arbitrary and the embedding $W \hookrightarrow L^2(X,m)$ is compact it follows that

$$\int_X f(u_n)(u_n - u)m(dx) \to 0$$

as $n \to \infty$. In the same way we can prove that

$$\int_X g(u_n)(u_n - u)m(dx) \to 0.$$

Second case: $\nu < 2$.

By Proposition 1.1 it follows that (u_n) is bounded in $L^\infty(X,m)$. Then there exists $\tilde{C} > 0$ such that $\|u_n\|_\infty < \tilde{C}$ for all $n \in \mathbb{N}$. We apply (2.2) (for $C = \tilde{C}$) and the Hölder inequality in order to get

$$\int_X f(u_n)(u_n - u)m(dx) \leq (1 + M_{\tilde{C}}) \int_X |u_n||u_n - u|m(dx)$$
$$\leq (1 + M_{\tilde{C}})\|u_n\|_2\|u_n - u\|_2.$$

Since the embedding $W \hookrightarrow L^2(X,m)$ is compact, it follows that

$$\int_X f(u_n)(u_n - u)m(dx) \to 0$$

as $n \to \infty$. In the same way we have $\int_X g(u_n)(u_n - u)m(dx) \to 0$.

Consequently, in both cases we have that $\|u_n - u\|_W^2 \to 0$ as $n \to \infty$, i.e., $\mathcal{E}_{\lambda,\mu}$ fulfills the Palais-Smale condition. □

Let $u_0 \in W$ be the element from condition **(F4)** and define

$$c_{\mathcal{G}} = \int_X |G(u_0(x))| m(dx), \quad \text{and} \quad \lambda_0 = \frac{\|u_0\|_W^2}{2\mathcal{F}(u_0)}.$$

For every $\lambda > \lambda_0$ set

$$\mu_\lambda^\star = \frac{1}{1 + c_{\mathcal{G}}} (\lambda - \lambda_0) \mathcal{F}(u_0).$$

Lemma 3.2. *Let $\lambda > \lambda_0$ and $\mu \in [-\mu_\lambda^\star, \mu_\lambda^\star]$, then we have* $\inf_{u \in W} \mathcal{E}_{\lambda,\frac{\mu}{\lambda}}(u) < 0$.

Proof. We prove that $\mathcal{E}_{\lambda,\frac{\mu}{\lambda}}(u_0) < 0$ for $\lambda > \lambda_0$ and $\mu \in [-\mu_\lambda^\star, \mu_\lambda^\star]$. We have

$$\begin{aligned}
\mathcal{E}_{\lambda,\frac{\mu}{\lambda}}(u_0) &= \frac{1}{2}\|u_0\|_W^2 - \lambda\mathcal{F}(u_0) - \mu\mathcal{G}(u_0) \\
&\le (\lambda_0 - \lambda)\mathcal{F}(u_0) + |\mu| c_{\mathcal{G}} \\
&\le (\lambda_0 - \lambda)\mathcal{F}(u_0) + \mu_\lambda^* c_{\mathcal{G}} \\
&= \frac{(\lambda - \lambda_0)\mathcal{F}(u_0)}{1 + c_{\mathcal{G}}} \\
&< 0.
\end{aligned}$$

□

Lemma 3.3. *For $\lambda > \lambda_0$ and $\mu \in [-\mu_\lambda^\star, \mu_\lambda^\star]$, the functional $\mathcal{E}_{\lambda,\frac{\mu}{\lambda}}$ satisfies the Mountain Pass geometry.*

Proof. We distinguish two cases:

First case: $\nu \ge 2$.

From Remark 2.1, follows that, for every $\varepsilon > 0$, there exists $c(\varepsilon) > 0$ such that

$$\max\{|\mathcal{F}(u)|, |\mathcal{G}(u)|\} \le \frac{\varepsilon C_2^2}{2}\|u\|_W^2 + \frac{(\varepsilon + c(\varepsilon))C_{\beta+1}^{\beta+1}}{\beta + 1}\|u\|_W^{\beta+1}.$$

Thus, for every $u \in W$, we have

$$\begin{aligned}
\mathcal{E}_{\lambda,\frac{\mu}{\lambda}}(u) &\ge \frac{1}{2}\|u\|_W^2 - \lambda|\mathcal{F}(u)| - |\mu||\mathcal{G}(u)| \\
&\ge \frac{1}{2}\|u\|_W^2 \left[1 - (\lambda + |\mu|)\varepsilon C_2^2\right] - (\lambda + |\mu|)\frac{(\varepsilon + c(\varepsilon))C_{\beta+1}^{\beta+1}}{\beta + 1}\|u\|_W^{\beta+1}.
\end{aligned}$$

We denote by $A = \frac{1}{2}[1 - (\lambda + |\mu|)\varepsilon C_2^2]$, $B = (\lambda + |\mu|)\dfrac{(\varepsilon + c(\varepsilon))C_{\beta+1}^{\beta+1}}{\beta + 1}$ and $\rho = \|u\|_W$. Therefore,

$$\mathcal{E}_{\lambda,\frac{\mu}{\lambda}} \ge A\rho^2 - B\rho^{\beta+1}.$$

If we choose $0 < \varepsilon < \dfrac{1}{(\lambda + |\mu|)} C_2^{-2}$ and $\rho_1 := \rho_1^{\lambda,\mu} \in \,]0, (\frac{A}{B})^{1/(\beta-1)}[$, then for $u \in W$, with $\|u\| = \rho_1$, we have

$$\mathcal{E}_{\lambda,\frac{\mu}{\lambda}}(u) > \rho_1^2(A - B\rho_1^{\beta-1}) > 0.$$

Second case: $\nu < 2$.

Hypothesis **(FG2)** implies that for every $\varepsilon > 0$, there exists $\delta(\varepsilon) > 0$ such that

$$\max\{|F(s)|, |G(s)|\} \leq \varepsilon |s|^2,$$

for every $|s| \leq \delta(\varepsilon)$. Choosing $\|u\|_W = \rho \leq \frac{\delta(\varepsilon)}{C_\infty}$, we have $\|u\|_{L^\infty} \leq \delta(\varepsilon)$, then

$$\mathcal{F}(u) \leq \varepsilon \int_X u^2(x) m(dx) \leq \varepsilon C_2^2 \|u\|_W^2.$$

In the same way we have

$$\mathcal{G}(u) \leq \varepsilon C_2^2 \|u\|_W^2.$$

Therefore $\mathcal{E}_{\lambda,\frac{\mu}{\lambda}}(u) \geq \left[\frac{1}{2} - \varepsilon C_2^2(\lambda + |\mu|)\right] \|u\|_W^2$. If $\varepsilon \in]0, \frac{C_2^{-2}}{2(\lambda+|\mu|)}[$ and correspondingly, we choose $\rho_2 := \rho_2^{\lambda,\mu} \in]0, \min\{\|u_0\|_W, \frac{\delta(\varepsilon)}{C_2}\}[$, then for every $\|u\|_W = \rho_2$ we have

$$\mathcal{E}_{\lambda,\frac{\mu}{\lambda}}(u) \geq \left[\frac{1}{2} - \varepsilon C_2^2(\lambda + |\mu|)\right] \rho_2^2 = M(\rho_2) > 0.$$

Finally, if we choose $\rho := \rho^{\lambda,\mu} \in]0, \min\{\rho_1, \rho_2\}[$ and taking into account that $\mathcal{E}_{\lambda,\frac{\mu}{\lambda}}(u_0) < 0$, see Lemma 3.2, it follows that the functional $\mathcal{E}_{\lambda,\frac{\mu}{\lambda}}$ satisfies the Mountain Pass geometry. ☐

Now, the proof of Theorem 2.2 easily follows. Indeed, for $\lambda > \lambda_0$ and $\mu \in [-\mu_\lambda^\star, \mu_\lambda^\star]$, the functional $\mathcal{E}_{\lambda,\frac{\mu}{\lambda}}$ has

- a global minimum point $u_1^{\lambda,\mu} \in W$, see Lemma 3.2, with $\mathcal{E}_{\lambda,\frac{\mu}{\lambda}}(u_1^{\lambda,\mu}) < 0$,
- a Mountain pass type critical point $u_2^{\lambda,\mu} \in W$, see Lemma 3.3, with

$$\mathcal{E}_{\lambda,\frac{\mu}{\lambda}}(u_2^{\lambda,\mu}) = \inf_{\gamma \in \Gamma} \sup_{t \in [0,1]} \mathcal{E}_{\lambda,\frac{\mu}{\lambda}}(\gamma(t)),$$

where

$$\Gamma = \{\gamma \in C([0,1], W) : \gamma(0) = 0, \gamma(1) = u_0\}.$$

We know from the Mountain Pass Theorem that $\mathcal{E}_{\lambda,\frac{\mu}{\lambda}}(u_2^{\lambda,\mu}) \geq M(\rho_2) > 0$.

Consequently, for $\lambda > \lambda_0$ and $\mu \in [-\mu_\lambda^\star, \mu_\lambda^\star]$, the functional $\mathcal{E}_{\lambda,\frac{\mu}{\lambda}}$ has two distinct, nontrivial critical points (since $\mathcal{E}_{\lambda,\frac{\mu}{\lambda}}(0) = 0$), thus, solutions to our problem $(P_{\lambda,\mu})$.

ACKNOWLEDGMENTS

Alexandru Kristály and Csaba Varga are supported by the Grant PN II, IDEI_527 from CNCSIS.

REFERENCES

[1] M. Biroli, S. Mataloni, M. Matzeu, Stability results for Mountain Pass and linking type solutions of semilinear problems involving Dirichlet forms. *NoDEA: Nonlinear Differential Equations and Applications* **12** (2005), 295–321.

[2] M. Biroli, S. Tersian, On the existence of nontrivial solutions to a semilinear equation relative to a Dirichlet form. *Rendiconti Istituto Lombardo (Rend.Sc.) A* **131** (1998), 151–168.

[3] K. J. Falconer, Semilinear partial differential equations on self-similar fractals. *Comm. Math. Phys.* **206** (1999), 235–245.

[4] K. J. Falconer, J. Hu, Nonlinear elliptical equations on the Sierpinski gasket. *J. Math. Anal. Appl.* **240** (1999), 552–573.

[5] K. J. Falconer, J. Hu, Nonlinear diffusion equations on unbounded fractal domains. *J. Math. Anal. Appl.* **256** (2001), 606–624.

[6] M. Fukushima, Dirichlet forms, diffusion processes and spectral dimensions for nested fractals. *In : Ideas and methods in mathematical analysis, stochastics, and applications* (Oslo, 1988), 151–161. Cambridge University Press: Cambridge, 1992.

[7] A. Grigor'yan, J. Hu, K.-S. Lau, Heat kernels on metric measure spaces and an application to semilinear elliptic equations. *Trans. Am. Math. Soc.* **355** (2003), 2065–2095.

[8] J. Hu, Multiple solutions for a class of nonlinear elliptic equations on the Sierpin'ski gasket. *Sci. China Ser. A* **47** (2004), no. 5, 772–786.

[9] S. Kusuoka, Z. X. Yin, Dirichlet forms on fractals: Poincaré constant and resistance. *Probability Theory and Related Fields,* **93** 169–196, 1992.

[10] H. Lisei, Cs. Varga, Multiple solutions for nonlinear equations involving Dirichlet forms. *Topics in mathematics, computer science and philosophy,* 135–145, Presa Univ. Clujeana (Cluj-Napoca), 2008.

[11] M. Matzeu, Mountain Pass and Linking Type Solutions for Semilinear Dirichlet Forms. *Progr. in Non. Diff. Eq. and Appl.* **40** (2000), 217–231.