PROFESSOR JEFFREY R. L. WEBB

Jeff studied for his Ph.D. under the supervision of David Edmunds at Sussex University between 1967 and 1970. This was followed by a one year research fellowship in Rome and a two year postdoctoral fellowship with David.

During these years at Sussex, David Edmunds was the leader of an outstanding Nonlinear Analysis group, which included Bert Peletier, Peter Bushell, Robin Dyer and Charles Stuart among the staff, Sir John Ball FRS, Frances Goldman, A.J.B. Potter and John Toland FRS among the doctoral students. David also organized visits from many eminent mathematicians during those years including F.V. Atkinson, Haim Brezis, Felix E. Browder, Gaetano Fichera, Robert Finn, Jaques-Louis Lions, Robert H. Martin, M.K.V. Murthy, Louis Nirenberg, Paul Rabinowitz, James Serrin, Sergio Spagnolo and Guido Stampacchia. During his Ph.D. years Jeff began his study of various types of nonlinear maps, which later became known as accretive, condensing and A-proper maps. His first paper was published in 1969 and proved that semi-contractive maps were condensing, provided the ball measure of non-compactness was used.

In 1970, after completing his Ph.D., Jeff spent a year in Rome at the Istituto per le applicazioni del Calcolo, at that time led by Guido Stampacchia. In Rome, Jeff met two other visitors: Bruce Calvert from New Zealand and Akira Ono from Japan (who passed away a few yeas ago). Jeff and Bruce had enough common interests to write a joint paper. Apart from that, he was working on extensions of his thesis work, including proving a connection between k-ball contractions and A-proper maps.

Jeff spent the next two years as a postdoc back at Sussex and worked with David Edmunds and Vincenzo Moscatelli, who was a recently appointed member of the staff. Jeff began his interest in strongly nonlinear elliptic problems at that time, thanks to David.

In 1973 Jeff moved to Glasgow University, where he was appointed as a Lecturer, and has remained there ever since. At Glasgow he was the sole person doing research in Nonlinear Analysis so most of his early work was done alone. Jeff said that typing papers was harder than now, corrections being a particular nuisance, but this had the positive effect of encouraging publication of a finished result, rather than a series of intermediate steps. He always presented new results in conference proceedings, unfortunately this meant some of his papers did not receive the attention they merited.

Also collaboration was not so easy before the days of e-mail and $\mathbb{L}^{T}_{E}X$! Jeff said that he became too used to working alone, but regrets not having made a greater effort to have better contacts with the Italian group involving Alfonso Vignoli and Massimo Furi, who were active in similar fields at that time. His love of Italy could have grown stronger sooner.

Jeff had a number of research students, from different nationalities, in Glasgow between 1981 and 2005; an M.Sc. student, Atta Abu Hany (Palestine) and several Ph.D. students, namely Stewart C. Welsh (Scotland), now in Texas State University (San Marcos, USA), Weiyu Zhao (China), Zongming Guo (China), now in Henan Normal University (China), Wenying Feng (China), now in Trent University (Canada), Casey Cremins (USA), now in University of Maryland (USA), Kunquan Lan (China), now in Ryerson University (Canada), Wenming Bian (China), now in University of Southampton (UK), Gennaro Infante (Italy), now in University of Calabria (Italy) and Rahmat Ali Khan (Pakistan), now in NUST (Pakistan).

Jeff is recognised by his research students in simple words as a *great supervisor*, well known for his limitless patience, invaluable assistance and constant encouragement, that led them into the field of Nonlinear Analysis. Frequently his precious advices went beyond mathematics and his duties of a thesis advisor, helping his students in their new Glaswegian life, perhaps due to his 'Roman holiday'. Jeff keeps a strong connection with most of his students, still collaborating with some of them today.

Jeff had took some study leave during his appointment in Glasgow. In 1978 he spent a sabbatical year in Indiana University, where George Minty was among the staff and Lars Hedberg a visitor. In 1982 he had a sabbatical semester in Tulane University, where he was invited by Jerry Goldstein, who was giving a graduate course on nonlinear semigroups. In 1994 he spent a sabbatical term in Rutgers University, where he worked with Walter Petryshyn and went to a course given by Roger Nussbaum. In 2003 Jeff had a sabbatical year in Glasgow and Calabria, where Jeff was invited by Espedito De Pascale and worked with Gennaro Infante.

Jeff had many visitors over the years in Glasgow, namely Genaro Lopez Acedo, Espedito De Pascale, Daniel Franco, Messoud Efendiev, Mike Fitzpatrick, Gennaro Infante, Larry Payne, Igor Skrypnik, Liu Zhenhai and Mirosława Zima.

Jeff was the secretary of the North British Differential Equations Seminar for a number of years and, during his time as secretary, among the distinguished speakers in Glasgow were Haim Brezis, Jack Hale, Jim Keener and Jacques-Louis Lions.

Jeff was also co-organiser with Michael Grinfeld of Strathclyde University of an Applied Analysis Seminar, with the venue varying between the two departments.

Jeff, John Graef and Kunquan Lan, organized a special session 'Topological Methods for Boundary Value Problems' in various 'Dynamical Systems and Differential Equations' and 'Dynamic Systems and Applications' conferences.

Jeff's work was recognised in 1984 by his becoming a Fellow of the Royal Society of Edinburgh. He was promoted to Professor in Glasgow University in 1987. In June 2006, at the University of Poitiers, Messoud Efendiev, Gennaro Infante and Kunquan Lan organized a special session at the Sixth International Conference on 'Dynamical Systems and Differential Equations', to celebrate the 60th birthday of Jeff.

For many years Jeff wrote reviews for Zentralbatt für Mathematik. He has written eight book reviews for several journals. He does refereeing work for many journals and editorial work for the book series 'Differential Equations and Dynamical Systems', AIMS, and for the journals Nonlinear Analysis TMA and Electronic Journal of Qualitative Theory of Differential Equations. He previously did editorial work for the London Mathematical Society, Fixed Point Theory and Applications, Glasgow Mathematical Journal and Proc. A of the Royal Society of Edinburgh.

During his years at Glasgow, apart from teaching, he also did several administrative jobs over the years, including Head of the Teaching Committee for the Department and Vice Dean of Undergraduate Studies for the Faculty. His dislike of Committees meant that these jobs were not done out of love, but he carried them out diligently, giving them up as quickly as possible!

In 2009 Jeff fully retired from Glasgow and, although retired, Jeff taught one course in 2010.

1. MAIN STRANDS OF JEFF'S RESEARCH

Jeff has been the author or co-author of nearly 100 research papers and one textbook. The following description is not complete, but gives a hint of the main strands of his research.

1.1. Classes of maps, degree and fixed point index theory. In the early 1970s Jeff wrote a number of papers on condensing type maps, including one which showed that semi-contractive maps (with a weaker hypothesis than used by F.E. Browder at that time) fall into this class provided one uses the ball-measure of non-compactness [2]. He developed a degree theory for k-ball contractions [7] and for set-valued 'limit compact' operators [15], via an approximation by single-valued ones, and he showed a uniqueness result [16]. He proved that, in Hilbert spaces, the classes of linear k-set contractions and k-ball contractions coincide [12]. Years later, a paper on some connections between *nonlinear* condensing maps defined via different measures of non-compactness was done in collaboration with Weiyu Zhao [35]. In the 1990s, in collaboration with Kunquan Lan, he developed theories of fixed point index for weakly inward A-proper maps [47] and for 'generalized inward' condensing type maps [46].

1.2. Strongly nonlinear elliptic problems. Jeff studied strongly nonlinear problems during the 1970s, mainly in unbounded domains, for semilinear problems having non-polynomial growth in their lowest order terms. This was done in a Sobolev space setting, using theory related to pseudo-monotone operators [11, 13, 17, 20]. In 1979, while visiting Indiana University, he met co-visitor Lars Hedberg (Sweden). In discussions with him he found that Hedberg had a result on approximation of functions in higher order Sobolev spaces which allowed removal of some technical conditions, resulting in a definitive paper in J. London Math Soc. [21]. More recently, Jeff has collaborated with Zongming Guo on quasilinear problems (*p*-Laplacian) in bounded domains discussing uniqueness [40], problems with spikes [60, 64] and solutions that blow-up at the boundary, [70, 71].

1.3. A-proper maps. Jeff's first contribution to A-proper map theory was in a joint paper with David Edmunds [1] where they proved that semi-accretive maps were Aproper and proved a Leray-Schauder continuation theorem for A-proper maps. Jeff later on investigated the A-properness of accretive and condensing type maps. Jeff showed that a map I - f is A-proper for a condensing map f provided the *ball* measure of non-compactness is used [9]. In several papers, many authors had often imposed the condition that the duality map J be 'weakly continuous' which held in ℓ^p spaces but not in L^p spaces, so was rather restrictive. In the early 80s, Jeff proved a property of duality maps which can replace this in suitable circumstances, and enabled Jeff to prove results in L^p spaces. This was used to prove A-properness of accretive type maps [24]. A more recent paper by Jeff and Kunquan Lan [55] refined this. Jeff also proved [28], with an elementary but new method, a connection between weakly inward contractive maps and A-proper maps, partially answering an open question of Petryshyn. He was by now a leading expert on A-proper maps and gave a series of lectures on this work at a conference in Pisek in 1982 [25]. Stewart Welsh studied bifurcation theory for A-proper maps for his Ph.D. and he and Jeff collaborated in a few papers on this topic. Later Casey Cremins also did a Ph.D. with Jeff on fixed point index theory for semilinear A-proper maps.

1.4. Nonlinear spectral theory. Jeff did some joint work on nonlinear spectral theory, firstly with David Edmunds [27] making some remarks on the Furi-Martelli-Vignoli spectrum, secondly with Wenying Feng [57] on a new theory for semilinear maps, thirdly with Gennaro Infante [61] for A-proper maps. These contributions appear in the recent book *Nonlinear Spectral Theory* by Appell, De Pascale and Vignoli, (Birkhauser 2004).

1.5. Nonlocal boundary value problems for ODEs. Jeff started studying these problems around 1997 with some collaborations with Wenying Feng [44], where coincidence degree theory of Mawhin was used. This was followed by a number of collaborations with Kunquan Lan [51] and Gennaro Infante. These papers used classical fixed point index to study existence of multiple positive solutions (under suitable conditions on the nonlinearity). The work on nonlocal problems is continuing at present. Jeff and Kunquan [81] gave some sharp existence results by using the principal eigenvalue of a related linear problem. Jeff and Gennaro gave a new unified method [80, 93] which allowed the treatment of many of the nonlocal BCs that have been studied in recent years by one single method, which also gives real improvements on previous work! Jeff investigated some of the constants that occur in this theory and obtained some optimal values [72, 73]. Jeff and Gennaro have studied problems with sign changing kernels obtaining solutions that are positive on some subinterval [66]. In particular they studied some models of a thermostat and established results where loss of positivity occurs as a parameter changes [75]. Using similar techniques Jeff collaborated with John Graef [87] on third order problems and with Mirosława Zima on second order resonant problems [94]. Jeff also wrote a paper on systems in collaboration with Daniel Franco [74]. In some joint work with Rahmat Ali Khan [77, 82] he used upper and lower solution techniques. Jeff has studied a class of positive linear operators and used its properties to derive new results for the existence of positive solutions for local and nonlocal BVPs [97].

1.6. Some other work. Jeff collaborated with Larry Payne [36] on some applications of the maximum principle. He also did some work on extensions of Gronwall's inequality, giving a new version with a simple proof, which contains a large number of versions that appear in the literature, [56, 67]. Jeff and Wenming Bian also did some work on operator inclusions [54] and open mapping problems with applications to the controllability of certain nonlinear systems [53].

REFERENCES

- D. E. Edmunds and <u>J. R. L. Webb</u>, A Leray-Schauder theorem for a class of nonlinear operators, *Math. Ann.*, 182 (1969), 207–212.
- [2] J. R. L. Webb, Fixed point theorems for non-linear semicontractive operators in Banach spaces, J. London Math. Soc. (2), 1 (1969), 683–688.
- [3] J. R. L. Webb, Mapping and fixed-point theorems for non-linear operators in Banach-spaces, Proc. London Math. Soc. (3), 20 (1970), 451–468.
- [4] J. R. L. Webb, Topics in Nonlinear Functional Analysis, Ph.D. thesis, University of Sussex, Brighton, UK (1970).
- [5] B. Calvert and J. R. L. Webb, An existence theorem for quasimonotone operators, Atti Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Natur. (8), 50 (1971), 690–696.
- [6] D. E. Edmunds and J. R. L. Webb, Nonlinear operator equations in Hilbert spaces, J. Math. Anal. Appl., 34 (1971), 471–478.
- [7] J. R. L. Webb, A fixed point theorem and applications to functional equations in Banach spaces, Boll. Un. Mat. Ital. (4), 4 (1971), 775–788.
- [8] J. R. L. Webb, On a characterisation of k-set contractions, Atti Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Natur. (8), 50 (1971), 686–689.
- [9] J. R. L. Webb, Remarks on k-set contractions, Boll. Un. Mat. Ital. (4), 4 (1971), 614–629.
- [10] J. R. L. Webb, Existence theorems for operators of monotone type, in Conference on the Theory of Ordinary and Partial Differential Equations (Univ. Dundee, Dundee, 1972), Springer, Berlin, 1972, pp. 358–362. Lecture Notes in Math., Vol. 280.
- [11] D. E. Edmunds and J. R. L. Webb, Quasilinear elliptic problems in unbounded domains, Proc. Roy. Soc. (London) Ser. A, 334 (1973), 397–410.
- [12] J. R. L. Webb, On seminorms of operators, J. London Math. Soc. (2), 7 (1973), 337–342.
- [13] D. E. Edmunds, V. B. Moscatelli and <u>J. R. L. Webb</u>, Strongly nonlinear elliptic operators in unbounded domains, *Publ. Math. Univ. Bordeaux Année* (1973/74), 6–32.
- [14] D. E. Edmunds, V. B. Moscatelli and J. R. L. Webb, Opérateurs elliptiques fortement non linéaires dans des domaines non bornés, C. R. Acad. Sci. Paris Sér. A, 278 (1974), 1505– 1508.
- [15] J. R. L. Webb, On degree theory for multivalued mappings and applications, Boll. Un. Mat. Ital. (4), 9 (1974), 137–158.
- [16] J. R. L. Webb, On uniqueness of topological degree for set-valued mappings, Proc. Roy. Soc. Edinburgh Sect. A, 74 (1974/75), 225–229 (1976).
- [17] J. R. L. Webb, On the Dirichlet problem for strongly non-linear elliptic operators in unbounded domains, J. London Math. Soc. (2), 10 (1975), 163–170.
- [18] J. F. Toland, Global bifurcation theory via Galerkin's method, Nonlinear Anal., 1 (1976/77), 305–317, with an appendix by J. R. L. Webb.
- [19] D. E. Edmunds and J. R. L. Webb, Some generalizations of the Borsuk-Ulam theorem, Math. Proc. Cambridge Philos. Soc., 82 (1977), 119–125.
- [20] J. R. L. Webb, Strongly nonlinear elliptic equations, in Journées d'Analyse Non Linéaire (Proc. Conf., Besançon, 1977), volume 665 of Lecture Notes in Math., Springer, Berlin, 1978, pp. 242–256.
- [21] J. R. L. Webb, Boundary value problems for strongly nonlinear elliptic equations, J. London Math. Soc. (2), 21 (1980), 123–132.

- [22] J. R. L. Webb, On the homotopy property of degree for multivalued noncompact maps, Glasgow Math. J., 21 (1980), 125–130.
- [23] J. R. L. Webb, Existence theorems for sums of K-ball contractions and accretive operators via A-proper mappings, Nonlinear Anal., 5 (1981), 891–896.
- [24] J. R. L. Webb, On a property of duality mappings and the A-properness of accretive operators, Bull. London Math. Soc., 13 (1981), 235–238.
- [25] J. R. L. Webb, Approximation solvability of nonlinear equations, in Nonlinear analysis, function spaces and applications, Vol. 2 (Písek, 1982), volume 49 of Teubner-Texte zur Math., Teubner, Leipzig, 1982, pp. 234–257.
- [26] J. R. L. Webb, Mappings of accretive and pseudo-A-proper type, J. Math. Anal. Appl., 85 (1982), 146–152.
- [27] D. E. Edmunds and J. R. L. Webb, Remarks on nonlinear spectral theory, Boll. Un. Mat. Ital. B (6), 2 (1983), 377–390.
- [28] J. R. L. Webb, A-properness and fixed points of weakly inward mappings, J. London Math. Soc. (2), 27 (1983), 141–149.
- [29] J. R. L. Webb and S. C. Welsh, A-proper maps and bifurcation theory, in Ordinary and partial differential equations (Dundee, 1984), volume 1151 of Lecture Notes in Math., Springer, Berlin, 1985, pp. 342–349.
- [30] J. R. L. Webb, Topological degree and A-proper operators, in Proceedings of the symposium on operator theory (Athens, 1985), volume 84, 1986, pp. 227–242.
- [31] J. R. L. Webb and S. C. Welsh, Topological degree and global bifurcation, in Nonlinear functional analysis and its applications, Part 2 (Berkeley, Calif., 1983), volume 45 of Proc. Sympos. Pure Math., Amer. Math. Soc., Providence, RI, 1986, pp. 527–531.
- [32] J. R. L. Webb, Maximum principles for functionals associated with the solution of semilinear elliptic boundary value problems, Z. Angew. Math. Phys., 40 (1989), 330–338.
- [33] J. R. L. Webb and S. C. Welsh, Existence and uniqueness of initial value problems for a class of second-order differential equations, J. Differential Equations, 82 (1989), 314–321.
- [34] J. R. L. Webb and W. Zhao, An L^p inequality, Proc. Roy. Soc. Edinburgh Sect. A, 116 (1990), 129–131.
- [35] J. R. L. Webb and W. Zhao, On connections between set and ball measures of noncompactness, Bull. London Math. Soc., 22 (1990), 471–477.
- [36] L. E. Payne and J. R. L. Webb, Comparison results in second order quasilinear Dirichlet problems, Proc. Roy. Soc. Edinburgh Sect. A, 118 (1991), 91–103.
- [37] J. R. L. Webb, Functions of several real variables., Ellis Horwood Series in Mathematics and its Applications. New York etc.: Ellis Horwood, xiii, 289 p., 1991.
- [38] J. R. L. Webb and W. Zhao, On the maximal monotonicity and the range of the sum of nonlinear maximal monotone operators, Proc. Edinburgh Math. Soc. (2), 34 (1991), 143–153.
- [39] L. E. Payne and J. R. L. Webb, Spatial decay estimates for second order partial differential equations, Nonlinear Anal., 18 (1992), 143–156.
- [40] Z. M. Guo and J. R. L. Webb, Uniqueness of positive solutions for quasilinear elliptic equations when a parameter is large, Proc. Roy. Soc. Edinburgh Sect. A, 124 (1994), 189–198.
- [41] J. R. L. Webb, Solutions of semilinear equations in cones and wedges, in World Congress of Nonlinear Analysts '92, Vol. I-IV (Tampa, FL, 1992), de Gruyter, Berlin, 1996, pp. 137–147.
- [42] J. R. L. Webb, Zeros of weakly inward accretive mappings via A-proper maps, in Theory and applications of nonlinear operators of accretive and monotone type, volume 178 of Lecture Notes in Pure and Appl. Math., Dekker, New York, 1996, pp. 289–297.

- [43] W. Feng and J. R. L. Webb, Solvability of *m*-point boundary value problems with nonlinear growth, J. Math. Anal. Appl., **212** (1997), 467–480.
- [44] W. Feng and J. R. L. Webb, Solvability of three point boundary value problems at resonance, in Proceedings of the Second World Congress of Nonlinear Analysts, Part 6 (Athens, 1996), volume 30, 1997, pp. 3227–3238.
- [45] W. Feng and J. R. L. Webb, Surjectivity results for nonlinear mappings without oddness conditions, Comment. Math. Univ. Carolin., 38 (1997), 15–28.
- [46] K. Q. Lan and <u>J. R. L. Webb</u>, A fixed point index for generalized inward mappings of condensing type, *Trans. Amer. Math. Soc.*, **349** (1997), 2175–2186.
- [47] K. Q. Lan and J. R. L. Webb, A fixed point index for weakly inward A-proper maps, Nonlinear Anal., 28 (1997), 315–325.
- [48] W. V. Petryshyn and <u>J. R. L. Webb</u>, Existence and multiplicity results for nonstandard semilinear biharmonic boundary value problems, *Nonlinear Anal.*, 28 (1997), 965–981.
- [49] J. R. L. Webb and K. Q. Lan, Positive solutions and the theory of fixed point index, Nonlinear Bound. Value Probl., 7 (1997), 189–197.
- [50] K. Q. Lan and J. R. L. Webb, New fixed point theorems for a family of mappings and applications to problems on sets with convex sections, *Proc. Amer. Math. Soc.*, **126** (1998), 1127–1132.
- [51] K. Q. Lan and J. R. L. Webb, Positive solutions of semilinear differential equations with singularities, J. Differential Equations, 148 (1998), 407–421.
- [52] K. Q. Lan and <u>J. R. L. Webb</u>, Variational inequalities and fixed point theorems for PM-maps, J. Math. Anal. Appl., **224** (1998), 102–116.
- [53] W. Bian and J. R. L. Webb, Constrained open mapping theorems and applications, J. London Math. Soc. (2), 60 (1999), 897–911.
- [54] W. Bian and J. R. L. Webb, Solutions of nonlinear evolution inclusions, Nonlinear Anal., 37 (1999), 915–932.
- [55] K. Q. Lan and J. R. L. Webb, A-properness and fixed point theorems for dissipative type maps, Abstr. Appl. Anal., 4 (1999), 83–100.
- [56] J. R. L. Webb, An extension of Gronwall's inequality., Nonlinear Bound. Value Probl., 9 (1999), 196–204.
- [57] W. Feng and J. R. L. Webb, A spectral theory for semilinear operators and its applications, in *Recent trends in nonlinear analysis*, volume 40 of *Progr. Nonlinear Differential Equations Appl.*, Birkhäuser, Basel, 2000, pp. 149–163.
- [58] J. R. L. Webb, Positive solutions of a three point boundary value problem., Nonlinear Bound. Value Probl., 10 (2000), 208–212.
- [59] J. R. L. Webb, Positive solutions of some three point boundary value problems via fixed point index theory, in *Proceedings of the Third World Congress of Nonlinear Analysts*, Part 7 (Catania, 2000), volume 47, 2001, pp. 4319–4332.
- [60] Z. Guo and J. R. L. Webb, Large and small solutions of a class of quasilinear elliptic eigenvalue problems, J. Differential Equations, 180 (2002), 1–50.
- [61] G. Infante and J. R. L. Webb, A finite-dimensional approach to nonlinear spectral theory, Nonlinear Anal., 51 (2002), 171–188.
- [62] G. Infante and <u>J. R. L. Webb</u>, Nonzero solutions of Hammerstein integral equations with discontinuous kernels, J. Math. Anal. Appl., **272** (2002), 30–42.
- [63] K. Q. Lan and J. R. L. Webb, A-properness of contractive and condensing maps, Nonlinear Anal., 49 (2002), 885–895.

- [64] Z. Guo and J. R. L. Webb, Profile of solutions with sharp layers to some singularly perturbed quasilinear Dirichlet problems, *Commun. Contemp. Math.*, 5 (2003), 883–920.
- [65] G. Infante and J. R. L. Webb, Positive solutions of some nonlocal boundary value problems, Abstr. Appl. Anal. (2003), 1047–1060.
- [66] G. Infante and J. R. L. Webb, Three-point boundary value problems with solutions that change sign, J. Integral Equations Appl., 15 (2003), 37–57.
- [67] J. R. L. Webb, Extensions of Gronwall's inequality with logarithmic terms., Barletta, Elisabetta (ed.), *Lecture notes of Seminario Interdisciplinare di Matematica*. Vol. II. Potenza: Università degli Studi della Basilicata, Dipartimento di Matematica. 149-159 (2003).
- [68] J. R. L. Webb, Remarks on positive solutions of some three point boundary value problems, *Discrete Contin. Dyn. Syst.* (2003), 905–915, dynamical systems and differential equations (Wilmington, NC, 2002).
- [69] R. A. Khan and J. R. L. Webb, Generalized quasilinearization technique for a three-point nonlinear boundary value problem, *Dynam. Systems Appl.*, 13 (2004), 187–202.
- [70] Z. Guo and J. R. L. Webb, Structure of boundary blow-up solutions for quasi-linear elliptic problems. II. Small and intermediate solutions, J. Differential Equations, 211 (2005), 187–217.
- [71] Z. Guo and J. R. L. Webb, Structure of boundary blow-up solutions for quasilinear elliptic problems. I. Large and small solutions, Proc. Roy. Soc. Edinburgh Sect. A, 135 (2005), 615– 642.
- [72] J. R. L. Webb, Multiple positive solutions of some nonlinear heat flow problems, Discrete Contin. Dyn. Syst. (2005), 895–903.
- [73] J. R. L. Webb, Optimal constants in a nonlocal boundary value problem, Nonlinear Anal., 63 (2005), 672–685.
- [74] D. Franco and <u>J. R. L. Webb</u>, Collisionless orbits of singular and non singular dynamical systems, *Discrete Contin. Dyn. Syst.*, **15** (2006), 747–757.
- [75] G. Infante and J. R. L. Webb, Loss of positivity in a nonlinear scalar heat equation, NoDEA Nonlinear Differential Equations Appl., 13 (2006), 249–261.
- [76] G. Infante and J. R. L. Webb, Nonlinear non-local boundary-value problems and perturbed Hammerstein integral equations, Proc. Edinb. Math. Soc. (2), 49 (2006), 637–656.
- [77] R. A. Khan and <u>J. R. L. Webb</u>, Existence of at least three solutions of a second-order threepoint boundary value problem, *Nonlinear Anal.*, 64 (2006), 1356–1366.
- [78] R. A. Khan and J. R. L. Webb, Existence of at least two solutions of second order nonlinear three point boundary value problems, *Dynam. Systems Appl.*, 15 (2006), 119–132.
- [79] J. R. L. Webb, Fixed point index and its application to positive solutions of nonlocal boundary value problems, in *Seminar of Mathematical Analysis*, Univ. Sevilla Secr. Publ., Seville, 2006, pp. 181–205.
- [80] J. R. L. Webb and G. Infante, Positive solutions of nonlocal boundary value problems: a unified approach, J. London Math. Soc. (2), 74 (2006), 673–693.
- [81] J. R. L. Webb and K. Q. Lan, Eigenvalue criteria for existence of multiple positive solutions of nonlinear boundary value problems of local and nonlocal type, *Topol. Methods Nonlinear Anal.*, 27 (2006), 91–115.
- [82] R. A. Khan and J. R. L. Webb, Existence of at least three solutions of nonlinear three point boundary value problems with super-quadratic growth, J. Math. Anal. Appl., 328 (2007), 690–698.
- [83] J. R. L. Webb, A unified approach to nonlocal boundary value problems, in *Dynamic Systems and Applications. Vol. 5*, Dynamic Publishers, Atlanta, GA, 2008, pp. 510–515.

- [84] J. R. L. Webb, Uniqueness of the principal eigenvalue in nonlocal boundary value problems, Discrete Contin. Dyn. Syst. Ser. S, 1 (2008), 177–186.
- [85] J. R. L. Webb and G. Infante, Positive solutions of nonlocal boundary value problems involving integral conditions, NoDEA Nonlinear Differential Equations Appl., 15 (2008), 45–67.
- [86] J. R. L. Webb, G. Infante and D. Franco, Positive solutions of nonlinear fourth-order boundary-value problems with local and non-local boundary conditions, *Proc. Roy. Soc. Edinburgh Sect. A*, **138** (2008), 427–446.
- [87] J. R. Graef and <u>J. R. L. Webb</u>, Third order boundary value problems with nonlocal boundary conditions, *Nonlinear Anal.*, **71** (2009), 1542–1551.
- [88] J. R. L. Webb, Boundary value problems with vanishing Green's function, Commun. Appl. Anal., 13 (2009), 587–595.
- [89] J. R. L. Webb, Higher order non-local (n 1, 1) conjugate type boundary value problems, in Mathematical models in engineering, biology and medicine, volume 1124 of AIP Conf. Proc., Amer. Inst. Phys., Melville, NY, 2009, pp. 332–341.
- [90] J. R. L. Webb, Nonlocal conjugate type boundary value problems of higher order, Nonlinear Anal., 71 (2009), 1933–1940.
- [91] J. R. L. Webb, Positive solutions of some higher order nonlocal boundary value problems, Electron. J. Qual. Theory Differ. Equ. (2009), No. 29, 15.
- [92] J. R. L. Webb, Remarks on u_0 -positive operators, J. Fixed Point Theory Appl., 5 (2009), 37–45.
- [93] J. R. L. Webb and G. Infante, Non-local boundary value problems of arbitrary order, J. Lond. Math. Soc. (2), 79 (2009), 238–258.
- [94] J. R. L. Webb and M. Zima, Multiple positive solutions of resonant and non-resonant nonlocal boundary value problems, *Nonlinear Anal.*, **71** (2009), 1369–1378.
- [95] J. R. L. Webb, Remarks on a non-local boundary value problem, Nonlinear Anal., 72 (2010), 1075–1077.
- [96] J. R. L. Webb, Remarks on nonlocal boundary value problems at resonance, Appl. Math. Comput., 216 (2010), 497–500.
- [97] J. R. L. Webb, Solutions of nonlinear equations in cones and positive linear operators, J. Lond. Math. Soc. (2), 82 (2010), 420–436.
- [98] J. R. L. Webb and G. Infante, Semi-positone nonlocal boundary value problems of arbitrary order, *Commun. Pure Appl. Anal.*, 9 (2010), 563–581.
- [99] J. R. L. Webb, Positive solutions of a boundary value problem with integral boundary conditions, *Electron. J. Differential Equations* (2011), No. 55, 10 pp. (electronic).

Messoud A. Efendiev Institute of Biomathematics and Biometry, Helmholtz Zentrum München Ingolstädter Landstr 1, 85764 Neuherberg, Germany E-mail: messoud.efendiyev@helmholtz-muenchen.de

> Gennaro Infante Dipartimento di Matematica, Università della Calabria 87036 Arcavacata di Rende, Cosenza, Italy E-mail: g.infante@unical.it