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STABILITY RESULTS FOR DELAY POWER SYSTEM UNDER IMPULSIVE PERTURBATIONS

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ABSTRACT. An impulsive delay model for electric power system with control has been proposed. For the proposed model sufficient conditions of asymptotic stability has been established via direct Lyapunov method using discontinuous unbounded Lyapunov function.

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1. INTRODUCTION

Stability analysis of electric power systems has been performed since the middle of the past century [1, 2, 3]. Development of further approaches to power systems stability analysis includes the methods proposed in [4, 5, 6]. There is a gain of popularity of mathematical models of power systems, that include delay in a control circuit [7]. When considering a problem of control of power system it is natural to suppose that this control is provided by using a delayed information about the system [8, 9].

Consideration of pulse effects and the delay while constructing models for electric power systems seems natural because of the essence of this systems. Pulse effects can be understood as momentary failure of generators. Delay is desirable for two reasons: first, it describes more precisely the process of control of the system, and second, it can be used to describe the transients in the circuits, that are inert to a certain extent.

In this paper we discuss the stabilization problem for electric power system with pulse effects and with proportional-derivative controller.

In Section 2 we formulate a result that is needed for the power systems stability analysis. In Section 3 we formulate the mathematical problem and reduce it into the form, that is applicable for the technique. In Section 4 we make the final formulation of the problem, construct a Lyapunov function for it and consider the problem by using auxiliary result. The stability conditions are given in the end of the paragraph. In Section 5 we make a note on the obtained result.

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2. AUXILIARY RESULT

Consider a system with delay and pulse effects

$$\begin{aligned} \frac{dx}{dt} &= f(t, x_t), \quad t \neq \tau_k \\ x(t^+) &= I_k(x), \quad t = \tau_k \end{aligned} \tag{2.1}$$

and the initial conditions

$$x(t) = \varphi_0(t), \quad t \in [t_0 - r, t_0], \tag{2.2}$$

where $x : [-r, +\infty) \to \mathbb{R}^n$ is left-continuous and has no more than a countable number of discontinuities of the first kind, $f : [-r, +\infty) \times \mathbb{E} \to \mathbb{R}^n$ is continuous in the first argument and is Lipschitz in the second, $\mathbb{E}$ is the space of functions $\varphi : [-r, 0] \to \mathbb{R}^n$, that are left-continuous and has no more than a countable number of discontinuities of the first kind, $I_k : [-r, +\infty] \to \mathbb{R}^n$ is continuous, $\tau_k \to \infty$, as $k \to \infty$. Suppose, that the initial problem (2.1), (2.2) has a unique solution on $[t_0, +\infty)$.

Definition 2.1 ([11])**.** A function $v(t, x)$ belongs to class V_0', if the following conditions hold:

(1) $v(t, x)$ is continuously differentiable on a set $\mathcal{T} \times \mathbb{R}^n$, where $\mathcal{T} = [t_0 - r, \infty) \setminus \{\tau_k\}_{k \in \mathbb{N}}$;
(2) there exists a function a of a Hahn's class, for which we have the estimate $a(\|x\|) \leq v(t, x)$ for all $(t, x) \in \mathbb{R}_+ \times \mathbb{R}^n$;
(3) there exist limits

$$\lim_{t \to \tau_k - 0} v(t, x) = v(\tau_k, x), \qquad \lim_{t \to \tau_k + 0} v(t, x) = v(\tau_k + 0, x)$$

for all $k = 1, 2, \ldots$.

Now we formulate a theorem.

Theorem 2.2. *Suppose that for the system* (2.1) *there exists a function* $v(t, x)$ *from* V_0 *and a monotonic function* $g : \mathbb{R}_+ \to \mathbb{R}_+$, $g(0) = 0$, $g(s) > 0$, $s > 0$ *such that:*

(1) $\frac{d}{dt} v(t, x(t))\big|_{(2.1)} \leq -g(v(t, x(t)))$, *if* $v(t, x(t + \zeta)) \leq p(v(t, x(t)))$ $\zeta \in [-r', 0]$ *(Razumikhin condition), where* $p(s) > s$ *while* $s > 0$, $p(0) = 0$, $p(s)$ *is continuous;*
(2) $v(\tau_k, x(\tau_k^+)) \leq v(\tau_k, x(\tau_k))$.

Then the system (2.1) *is asymptotically stable.*

The proof of this theorem is similar to those in [10].

3. FORMULATION OF THE PROBLEM

Consider the equations of dynamics of the system with impulsive effect

$$M_i \frac{d^2\theta_i}{dt^2} = P_{mi} - P_{ei} + P_{\tau i}, \quad t \neq \tau_k, \quad k \in \mathbb{N}, \tag{3.1}$$

$$\begin{aligned} \dot{\theta}_i(\tau_k^+) &= I_i(\theta_i(\tau_k), \dot{\theta}_i(\tau_k)), \\ \theta_i(\tau_k^+) &= \theta_i(\tau_k), \quad i = 1, \dots, n, \end{aligned} \tag{3.2}$$

with the initial conditions

$$\theta_i(t) = \varphi_i(t), \quad t_0 - r \leq t \leq t_0, \tag{3.3}$$

where M_i is the inertia constant, θ_i is the rotation angle of the rotor of the i-th generator, P_{mi} are the constants, defining a mechanical shaft horsepower of the machines, $P_{\tau i} \in C(\mathbb{R}^n, \mathbb{R})$ is a control with delay, that equals $r > 0$, τ_k^+ is a shorthand for $\tau_k + 0$, $t_0 < \tau_1 < \cdots < \tau_k < \cdots, k \in \mathbb{N}$, $\lim_{k\to\infty} \tau_k = \infty$, $I_{ki} \in C^1(\mathbb{R}^2, \mathbb{R})$, $\varphi_i \in C^1([-r, 0], \mathbb{R})$, P_{ei} are active powers, defined by

$$P_{ei} = \sum_{j=1}^{n} E_i E_j Y_{ij} \sin(\theta_i - \theta_j) + E_i U Y_{i,n+1} \sin\theta_i,$$

where E_i is the emf of the i-th machine, Y_{ii} are the intrinsic conductivities of machines, Y_{ij} are the mutual conductivities, and besides $Y_{ij} = Y_{ji}$, $i, j = 1, \dots, n$, $Y_{i,n+1}$ is the mutual conductivity of the i-th generator and DC bus voltage, U is the value of coming voltage.

Note that the considered model of dynamics of electric power system doesn't contain damping. However, introduction of any damping won't cause the loss of stability, therefore, the stability conditions obtained below can be applied also for a system with arbitrary damping [12].

Suppose that the equilibrium coordinates θ_i^0 are known and that

$$P_{mi} = \sum_{j=1}^{n} P_{ij},$$

where $P_{ii} = E_i B_i \sin(\theta_i^0)$ and $P_{ij} = E_i E_j B_{ij} \sin(\theta_i^0 - \theta_j^0)$ when $i \neq j$. Then equations (3.1) can be written as [13]

$$\begin{aligned} M_i \frac{d^2\theta_i}{dt^2} &= E_i B_i (\sin(\theta_i^0) - \sin(\theta_i)) \\ &\quad + \sum_{\substack{j=1 \\ j\neq i}}^{n} E_i E_j B_{ij} (\sin(\theta_i^0 - \theta_j^0) - \sin(\theta_i - \theta_j)) + P_{\tau i}, \quad i = 1, \dots, n, \end{aligned} \tag{3.4}$$

where B_i, B_{ij} are some constants, satisfying the requirement $B_{ij} = B_{ji}$.

Let $x_i = \theta_i - \theta_i^0$ $\theta_{ij}^0 = \theta_i^0 - \theta_j^0$ for all $i, j = 1, \dots, n$. Further denote a state vector of the subsystem $\mathbf{x_i} = (x_i, \dot{x}_i)^T$ and the system $\mathbf{x} = (\mathbf{x}_1^T, \mathbf{x}_2^T, \mathbf{x}_3^T)^T$. Therefore, the equations of perturbed motion of power system (3.4) take the form

$$M_i \frac{d^2 x_i}{dt^2} = Q_i(\sin\theta_i^0 - \sin(x_i + \theta_i^0)) + \sum_{\substack{j=1 \\ j\neq i}}^{n} Q_{ij}(\sin\theta_{ij}^0 - \sin(x_i - x_j + \theta_{ij}^0)), \quad i = 1, \dots, n,$$

where $Q_i = E_i B_i$, $Q_{ij} = E_i E_j B_{ij}$ for $i, j = 1, \dots, n$.

By performing the conversion using the trigonometric identities, the system can be reduced to the form

$$M_i \frac{d^2 x_i}{dt^2} = -2P_i \chi_i(x_i) \sin\frac{x_i}{2} - 2\sum_{\substack{j=1 \\ j\neq i}}^{n} P_{ij}\chi_{ij}(x_i, x_j)\sin\frac{x_i - x_j}{2} + P_{\tau i}, \tag{3.5}$$

$i = 1, \dots, n$, where $P_i = Q_i \cos\theta_i^0$, $\chi_i(x_i) = \frac{\cos(\theta_i^0 + \frac{x_i}{2})}{\cos\theta_i^0}$, $P_{ij} = Q_{ij}\cos\theta_{ij}^0$, $\chi_{ij}(x_i, x_j) = \frac{\cos(\theta_{ij}^0 + \frac{x_i - x_j}{2})}{\cos\theta_{ij}^0}$, $i, j = 1, \dots, n$. Note, that for $\chi_i(x_i) = \chi_{ij}(x_i, x_j) = 1$ the form of these equations would be the same as those of the dynamics equations of unloaded power system. Furthermore, the functions $\chi_i(x_i)$, $\chi_{ij}(x_i, x_j)$ satisfy

$$\lim_{x_i \to 0} \chi_i(x_i) = 1, \quad \lim_{x_i \to x_j} \chi_{ij}(x_i, x_j) = 1, \quad i, j = 1, \dots, n.$$

In addition to the equations (3.5) we consider pulse effects and initial conditions, obtained after conversion of (3.2) and (3.3) respectively:

$$\dot{x}_i(\tau_k^+) = J_i(x_i(\tau_k), \dot{x}_i(\tau_k)), \quad i = 1, \dots, n, \tag{3.6}$$

$$x_i(t) = \psi_i(t), \quad t_0 - r \le t \le t_0, \tag{3.7}$$

where J_i and ψ_i are defined by

$$J_i(x, \dot{x}_i) = I_i(x + \theta_i^0, \dot{x}_i), \quad \psi_i(t) = \varphi_i(t) - \theta_i^0,$$

and suggest that $J_i(0, 0) = 0$.

4. STABILITY ANALYSIS

After linearization (3.5)–(3.7) we get

$$\begin{aligned} M_i \frac{d^2 x_i}{dt^2} &= -P_i x_i - \sum_{\substack{j=1 \\ j\neq i}}^{n} P_{ij}(x_i - x_j) + a_i x(t - r) + b_i \dot{x}_i(t - r), \quad t \ne \tau_k; \\ \dot{x}_i(\tau_k^+) &= c_{ki1} x_i(\tau_k) + c_{ki2}\dot{x}_i(\tau_k), \quad i = 1, \dots, n. \end{aligned} \tag{4.1}$$

$$x_i(t) = \psi_i(t), \quad t_0 - r \le t \le t_0, \tag{4.2}$$

Let

$$\theta = \varliminf_{k\to\infty}(\tau_{k+1}-\tau_k), \quad \theta_\varepsilon = \theta - \varepsilon,$$

where $\varepsilon > 0$ is a parameter.

Suggest that amount of delay r satisfies an estimation

$$2r < \tau_{k+1} - \tau_k, \quad k \in \mathbb{N}.$$

Consider an auxiliary function $V_0(\mathbf{x})$, given by

$$\begin{aligned} V_0(\mathbf{x}) &= \sum_{i,j=1}^{n} v_{0,ij}(\mathbf{x}_i, \mathbf{x}_j) \\ &= \sum_{i=1}^{n}(M_i\dot{x}_i^2 + 2M_i\widetilde{R}x_i\dot{x}_i + P_ix_i^2) + \sum_{\substack{i,j=1\\ i\neq j}}^{n} P_{ij}(x_i - x_j)^2. \end{aligned}$$

By using the function $V_0(\mathbf{x})$ construct discontinuous piecewise exponential function in the form

$$V(\mathbf{x}, t) = V_0(\mathbf{x})e^{\nu(t-\tau_k)}, \quad t \in [\tau_k, \tau_{k+1}), \quad k \in \mathbb{N}_0, \tag{4.3}$$

where $\tau_0 = t_0$, $\nu > 0$ is a constant parameter. This function belongs to the class V_0'. We will require satisfying of conditions of Theorem 1 for this function. These conditions can be formulated as

$$\left.\frac{dV(t, \mathbf{x}(t))}{dt}\right|_{(2.6)} \le -\alpha V(t, \mathbf{x}(t)), \quad t \neq \tau_k, \quad k \in \mathbb{N} \tag{4.4}$$

if

$$V(t, \mathbf{x}(t)) > pV(t+\zeta, \mathbf{x}(t+\zeta)), \quad \zeta \in \Omega_{2r}, \tag{4.5}$$

where $\alpha > 0$, $p \in (0,1)$ are some parameters, $\Omega_{2r} = [\max\{-2r, t_0 - t - r\}, 0)$, and

$$V(\tau_k + 0, \mathbf{x}(\tau_k + 0)) \le V(\tau_k, \mathbf{x}(\tau_k)), \quad k \in \mathbb{N}. \tag{4.6}$$

Substituting into (4.5) and (4.6) expression (4.3) for the function V, we get the conditions for the function V_0 for $t \in (\tau_k, \tau_{k+1})$ in the form:

$$\left.\frac{dV(t, \mathbf{x}(t))}{dt}\right|_{(4.1)} \le -(\alpha+\nu)V(t, \mathbf{x}(t)), \quad t \neq \tau_k, \quad k \in \mathbb{N} \tag{4.7}$$

if

$$\begin{cases} V_0(\mathbf{x}(t)) > pe^{\nu\zeta}V_0(\mathbf{x}(t+\zeta)),\ \zeta \in \Omega_{2r}, & \tau_k - t \notin \Omega_{2r}, \\ V_0(\mathbf{x}(t)) > pe^{\nu(\zeta+\chi(\tau_k-t-\zeta)\Delta\tau_{k-1})}V_0(\mathbf{x}(t+\zeta)),\ \zeta \in \Omega_{2r}, & \tau_k - t \in \Omega_{2r}, \end{cases} \tag{4.8}$$

where $\Delta\tau_k = \tau_{k+1} - \tau_k$. For any $\beta > 0$ a set

$$\{\Delta\tau_k | \Delta\tau_k < \theta_\beta,\ k \in \mathbb{N}\}$$

is finite, therefore, for the study of asymptotic stability the condition (4.8) can be replaced by the condition

$$\begin{cases} V_0(\mathbf{x}(t)) > pe^{\nu\zeta}V_0(\mathbf{x}(t+\zeta)),\ \zeta \in \Omega_{2r}, & \tau_k - t \notin \Omega_{2r}, \\ V_0(\mathbf{x}(t)) > pe^{\nu(\zeta+\chi(\tau_k-t-\zeta)\theta_\varepsilon)}V_0(\mathbf{x}(t+\zeta)),\ \zeta \in \Omega_{2r}, & \tau_k - t \in \Omega_{2r} \end{cases} \tag{4.9}$$

for some sufficiently small $\varepsilon > 0$, given that on a bounded set

$$\{(\tau_{k-1}, \tau_k) | \Delta\tau_k < \theta_\varepsilon,\ k \in \mathbb{N}\}$$

the function $V_0(\mathbf{x}(t))$ can be estimated exponentially with respect to t.

Consider first the case $\tau_k - t \notin \Omega_{2r}$. Then denoting

$$\begin{aligned} K_i(t) &= \int\limits_{t-r}^{t} (L_i(s) + M_i(s))ds, \\ L_i(t) &= -a_i\dot{x}_i(t) + b_i\big(P_i x_i(t) - a_i x_i(t-r) - b_i\dot{x}_i(t-r)\big), \\ M_i(t) &= b_i \sum_{\substack{j=1 \\ j\neq i}}^{n} P_{ij}(x_i(t) - x_j(t)), \end{aligned} \tag{4.10}$$

and considering the Razumikhin condition in (4.9) corresponding to this case, in the notations (3.5) we can estimate

$$\begin{aligned} |K(t)| &< r\lambda_3 W(\mathbf{x}) + \lambda_2 \int\limits_{t-r}^{t} p^{-1}e^{\nu(t-s)}V_0(\mathbf{x}(t))ds \\ &+ \lambda_1 \int\limits_{t-r}^{t} p^{-1}e^{\nu(t+r-s)}V_0(\mathbf{x}(t))ds = r\lambda_\nu^{(\exp)}(p)V_0(\mathbf{x}), \end{aligned} \tag{4.11}$$

where for arbitrary $\eta_{i1}, \eta_{i2} > 0$ the constants λ_l, $l = 1, 2, 3$ are defined by the equalities

$$\begin{aligned} \lambda_1 &= \max_{i=\overline{1,n}} \left\{ \frac{\lambda_{1i} r}{\eta_{i1}}, \frac{r}{4\eta_{i2}} \right\}, \\ \lambda_2 &= \max_{i=\overline{1,n}} \left\{ \lambda_{2i} \frac{r}{\eta_{i1}} \right\}, \\ \lambda_3 &= \frac{1}{4r} \max_{i=\overline{1,n}} \{(2\eta_{i1} + b_i^2 \overline{P}_i \eta_{i2})\lambda_{3i}\}, \end{aligned} \tag{4.12}$$

$$\lambda_{1i} = \frac{a_i^2}{M_i}\Big(1 - \frac{M_i(b_i P_i + \widetilde{R}a_i)^2}{P_i(a_i^2 + 2\widetilde{R}a_i b_i M_i P_i + b_i^2 M_i P_i)}\Big)^{-1}, \tag{4.13}$$

$$\lambda_{2i} = \frac{a_i^2 b_i^2}{M_i}\Big(1 - \frac{M_i(\widetilde{R}b_i - a_i)^2}{P_i b_i^2 - 2\widetilde{R}M_i a_i b_i + M_i a_i^2}\Big)^{-1}, \tag{4.14}$$

$$\lambda_{3i} = \frac{4\widetilde{R}_i(P_i - M_i\widetilde{R}^2)}{-4\widetilde{R}_i(P_i - a_i)(b_i + \widetilde{R}M_i) - (b_i\widetilde{R} + a_i)^2}, \tag{4.15}$$

and the function $\lambda_\nu^{(\exp)}(p)$ is defined by

$$\lambda_\nu^{(\exp)}(p) = \frac{e^{r\nu}-1}{p\nu r}(\lambda_1 e^{r\nu} + \lambda_2) + \lambda_3\mu.$$

In this notation derivative of the function $V_0(\mathbf{x})$ can be estimated by

$$\frac{dV_0(\mathbf{x}(t))}{dt}\Big|_{(4.1)} < (-\mu + r\lambda_\nu^{(\exp)}(p))V_0(\mathbf{x}),$$

where the constant μ is defined by the relations

$$\mu = \min\left\{\widetilde{R}, \{2\mu_i\}_{i=\overline{1,n}}\right\}, \tag{4.16}$$

$$\mu_i = \frac{b_i(P_i - M_i\widetilde{R}^2) - \Delta_i^{1/2}}{2M_i\left(P_i - M_i\widetilde{R}^2\right)}, \tag{4.17}$$

$$\Delta_i = \left(b_i(P_i - M_i\widetilde{R}^2)\right)^2 + M_i\left(P_i - M_i\widetilde{R}^2\right)\left((b_i\widetilde{R} + a_i)^2 + 4\widetilde{R}(P_i - a_i)(b_i + \widetilde{R}M_i)\right).$$

The condition (4.7) gives an inequality

$$r < \frac{\mu - \alpha - \nu}{\lambda_\nu^{(\exp)}(p)},$$

that is true for some $p \in (0,1)$ and $\alpha > 0$.

This implies an inequality

$$r < \frac{\mu - \nu}{\overline{\lambda}_\nu^{(\exp)}}, \tag{4.18}$$

where

$$\overline{\lambda}_\nu^{(\exp)} = \frac{e^{r\nu}-1}{r\nu}(\lambda_1 e^{r\nu} + \lambda_2) + \lambda_3\mu.$$

According to the results of parametric optimization it follows that $\overline{\lambda}_\nu^{(\exp)}$ can be selected in the form

$$\overline{\lambda}_\nu^{(\exp)} = \begin{cases} 2\widetilde{\lambda}_1 + 2\sqrt{\widetilde{\lambda}_3(\widetilde{\lambda}_2 + \widetilde{\lambda}_3)}, & \widetilde{\lambda}_2^2(\widetilde{\lambda}_2 + \widetilde{\lambda}_3) < \widetilde{\lambda}_3\widetilde{\lambda}_1^2, \\ 2\sqrt{(\widetilde{\lambda}_2 + \widetilde{\lambda}_3)(\widetilde{\lambda}_2 + \widetilde{\lambda}_3 + \frac{\widetilde{\lambda}_1^2}{\widetilde{\lambda}_2})}, & \widetilde{\lambda}_2^2(\widetilde{\lambda}_2 + \widetilde{\lambda}_3) \geq \widetilde{\lambda}_3\widetilde{\lambda}_1^2 \end{cases} \tag{4.19}$$

where parameters $\widetilde{\lambda}_i$, $i = 1,2,3$ are defined by

$$\widetilde{\lambda}_1 = \frac{\sqrt{\mu}}{4}\max_{i=\overline{1,n}}\left\{|b_i|\sqrt{\overline{P}_i\lambda_{3i}}\right\},$$

$$\widetilde{\lambda}_2 = e^{r\nu}\sqrt{\frac{(e^{r\nu}-1)\mu}{2r\nu}}\max_{i=\overline{1,n}}\left\{\lambda_{1i}\sqrt{\frac{\lambda_{3i}}{\lambda_{1i}e^{r\nu} + \lambda_{2i}}}\right\},$$

$$\widetilde{\lambda}_3 = \sqrt{\frac{(e^{r\nu}-1)\mu}{2r\nu}}\max_{i=\overline{1,n}}\left\{\lambda_{2i}\sqrt{\frac{\lambda_{3i}}{\lambda_{1i}e^{r\nu} + \lambda_{2i}}}\right\},$$

and constants λ_{1i}, λ_{2i}, λ_{3i}, $i = \overline{1,n}$ by (4.13), (4.14), (4.15).

By similar considerations it can be shown that for $\tau_k - t \in \Omega_{2r}$ the (4.11) is a true estimation.

Now, let $\tau_k - t \in \Omega_r$. It can be shown that

$$K_i(t) = K_{ic}(t) + K_{i\delta}(t), \tag{4.20}$$

where

$$\begin{aligned} K_{ic}(t) = \int\limits_{t-r}^{t} \big(a_i \dot{x}(s) + b_i(P_i x_i(s) + \sum_{\substack{j=1 \\ j\neq i}}^{n} P_{ij} x_{ij}(s))\big) ds \\ - b_i \int\limits_{t-r}^{t} (a_i x_i(s-r) + b_i \dot{x}_i(s-r)) ds, \\ K_{i\delta}(t) = \int\limits_{t-r}^{t} \delta(s-\tau_k)\big(c_{ki1} x_i(s) + (c_{ki2}-1)\dot{x}_i(s)\big) ds. \end{aligned}$$

Then the term in the expression for $K_i(t)$ in (4.20), that describes impulsive effects, can be represented in the form

$$K_{i\delta}(t) = c_{ki1} x_i(\tau_k) + (c_{ki2}-1)\dot{x}_i(\tau_k) \tag{4.21}$$

However,

$$\int\limits_{t-r}^{t} V_0(\mathbf{x}(s-r)) ds < \frac{e^{\nu(r-\theta_\varepsilon)}(e^{\nu r}-1)}{p\nu} V_0(\mathbf{x}(t))$$

and

$$\int\limits_{t-r}^{t} V_0(\mathbf{x}(s)) ds < \frac{e^{\nu r}-1}{p\nu} V_0(\mathbf{x}(t))$$

therefore, we obtain

$$\Big|\sum_{i=1}^{n} (\dot{x}_i + \widetilde{R} x_i) K_{ic}(t)\Big| \le \Big(\frac{e^{r\nu}-1}{p\nu}(\lambda_1 e^{r(\nu-\theta_\varepsilon)} + \lambda_2) + r\lambda_3 \mu\Big) V_0(\mathbf{x}).$$

Now, estimate a similar sum containing the terms $K_{i\delta}(t)$:

$$\begin{aligned} &\Big|\sum_{i=1}^{n} (\dot{x}_i + \widetilde{R} x_i) K_{i\delta}(t)\Big| \\ &= \Big|\sum_{i=1}^{n} (\dot{x}_i + \widetilde{R} x_i)\big((c_{ki2}-1)\dot{x}_i(\tau_k) + c_{ki1} x_i(\tau_k)\big)\Big| \\ &\le \frac{1}{2}\Big(\sum_{i=1}^{n} \eta_{3i}(\dot{x}_i + \widetilde{R} x_i)^2 + \sum_{i=1}^{n} \frac{1}{\eta_{3i}}\big((c_{ki2}-1)\dot{x}_i(\tau_k) + c_{ki1} x_i(\tau_k)\big)^2\Big), \end{aligned} \tag{4.22}$$

where $\eta_{3i} > 0$ are some constants.

We can prove the estimation

$$\big((c_{ki2}-1)\dot{x}_i + c_{ki1}x_i\big)^2 \le \lambda_{4i} v_{0,ii}(\mathbf{x}_i),$$

where

$$\lambda_{4i} = \frac{P_i(c_{ki2}-1)^2 - 2\widetilde{R}M_i(c_{ki2}-1)c_{ki1} + M_i c_{ki1}^2}{M_i(P_i - M_i\widetilde{R}^2)}.$$

Denote further

$$\lambda_4 = \frac{1}{2}\max_i \left\{\frac{\lambda_{4i}}{\eta_{3i}}\right\}, \tag{4.23}$$

and then

$$\frac{1}{2}\sum_{i=1}^{n}\frac{1}{\eta_{3i}}\big((c_{ki2}-1)\dot{x}_i(\tau_k) + c_{ki1}x_i(\tau_k)\big)^2 \le \lambda_4 V_0(\mathbf{x}(\tau_k)).$$

Denote

$$\lambda_5 = \frac{1}{4}\max_{i=\overline{1,n}}\{((2\eta_{1i} + b_i^2\overline{P}_i\eta_{2i})r + 2\eta_{3i})\lambda_{3i}\}, \tag{4.24}$$

then the absolute value of $K(t)$ can be estimated as follows:

$$\begin{aligned}
|K(t)| &< \Big(\frac{e^{r\nu}-1}{p\nu}(\lambda_1 e^{\nu(r-\theta_\varepsilon)} + \lambda_2) + \lambda_5\mu\Big)V_0(\mathbf{x}) + \lambda_4 V_0(\mathbf{x}(\tau_k)) \\
&< \Big(\frac{e^{r\nu}-1}{p\nu}(\lambda_1 e^{\nu(r-\theta_\varepsilon)} + \lambda_2) + \lambda_5\mu\Big)V_0(\mathbf{x}) + \lambda_4 e^{\nu(t-\tau_k)} V_0(\mathbf{x}) \\
&\le \Big(\frac{e^{r\nu}-1}{p\nu}(\lambda_1 e^{\nu(r-\theta_\varepsilon)} + \lambda_2) + \frac{\lambda_4}{p}e^{\nu r} + \lambda_5\mu\Big)V_0(\mathbf{x}).
\end{aligned}$$

Therefore,

$$\frac{dV_0(\mathbf{x}(t))}{dt}\Big|_{(4.1)} \le \Big(-\mu + \frac{e^{r\nu}-1}{p\nu}(\lambda_1 e^{\nu(r-\theta_\varepsilon)} + \lambda_2) + \frac{\lambda_4}{p}e^{\nu r} + \lambda_5\mu\Big)V_0(\mathbf{x}), \tag{4.25}$$

By using (4.7) and (4.25) we get an inequality

$$-\mu + \frac{e^{\nu(r-\theta_\varepsilon)}-1}{p\nu}(\lambda_1 e^{r\nu} + \lambda_2) + \frac{\lambda_4}{p}e^{\nu r} + \lambda_5\mu < -\alpha - \nu.$$

By selecting items $p \in (0,1)$, $\alpha > 0$ this condition can be reformulated as

$$-\mu + \frac{e^{r\nu}-1}{\nu}(\lambda_1 e^{\nu(r-\theta_\varepsilon)} + \lambda_2) + \lambda_4 e^{\nu r} + \mu\lambda_5 < -\nu.$$

Denote

$$\overline{\alpha}_{\nu,\varepsilon}^{(\exp)} = \frac{e^{r\nu}-1}{\nu}(\lambda_1 e^{\nu(r-\theta_\varepsilon)} + \lambda_2) + \lambda_4 e^{\nu r} + \mu\lambda_5.$$

Then, using of parametric optimization, we can write

$$\overline{\alpha}_{\nu,\varepsilon}^{(\exp)} = \begin{cases} 2\widetilde{\lambda}_1 + 2\sqrt{\widetilde{\lambda}_3(\widetilde{\lambda}_2 + \widetilde{\lambda}_3)} + \widetilde{\lambda}_4, & \widetilde{\lambda}_2^2(\widetilde{\lambda}_2 + \widetilde{\lambda}_3) < \widetilde{\lambda}_3\widetilde{\lambda}_1^2, \\ 2\sqrt{(\widetilde{\lambda}_2 + \widetilde{\lambda}_3)(\widetilde{\lambda}_2 + \widetilde{\lambda}_3 + \frac{\widetilde{\lambda}_1^2}{\widetilde{\lambda}_2})} + \widetilde{\lambda}_4, & \widetilde{\lambda}_2^2(\widetilde{\lambda}_2 + \widetilde{\lambda}_3) \ge \widetilde{\lambda}_3\widetilde{\lambda}_1^2 \end{cases} \tag{4.26}$$

where parameters $\widetilde{\lambda}_i$, $i=\overline{1,4}$ are defined by the relations

$$\widetilde{\lambda}_1 = \frac{\sqrt{\mu}}{4}\max_{i=\overline{1,n}}\left\{|b_i|\sqrt{P_i\lambda_{3i}}\right\},$$

$$\widetilde{\lambda}_2 = e^{\nu(r-\theta_\varepsilon)}\sqrt{\frac{(e^{\nu r}-1)\mu}{2r\nu}}\max_{i=\overline{1,n}}\left\{\lambda_{1i}\sqrt{\frac{\lambda_{3i}}{\lambda_{1i}e^{\nu(r-\theta_\varepsilon)}+\lambda_{2i}}}\right\},$$

$$\widetilde{\lambda}_3 = \sqrt{\frac{(e^{\nu r}-1)\mu}{2r\nu}}\max_{i=\overline{1,n}}\left\{\lambda_{2i}\sqrt{\frac{\lambda_{3i}}{\lambda_{1i}e^{\nu(r-\theta_\varepsilon)}+\lambda_{2i}}}\right\},$$

$$\widetilde{\lambda}_4 = e^{\frac{r\nu}{2}}\frac{\sqrt{\mu}}{r}\max_{i=\overline{1,n}}\left\{\sqrt{\lambda_{3i}\lambda_{4i}}\right\}.$$

Stability condition has the form

$$r < \frac{\mu-\nu}{\overline{\alpha}^{(\exp)}_{\nu,\varepsilon}}.$$

We write the condition (4.6), using the substitution (4.3). We obtain:

$$V_0(\mathbf{x}(\tau_k+0)) \le e^{\nu(\tau_k-\tau_{k-1})}V_0(\mathbf{x}(\tau_k)), \quad k\in\mathbb{N}. \tag{4.27}$$

From the systems of inequalities (4.27) for all k, except for those belonging to a finite set, the following estimation holds

$$V_0(\mathbf{x}(\tau_k+0)) \le e^{\nu\theta_\varepsilon}V_0(\mathbf{x}(\tau_k)), \quad k\in\mathbb{N}.$$

We obtain the system of inequalities

$$M_i(c_{ki1}x_i+c_{ki2}\dot{x}_i)^2 + 2M_i\widetilde{R}(c_{ki1}x_i+c_{ki2}\dot{x}_i)x_i + P_ix_i^2 \le e^{\nu\theta_\varepsilon}\big(M_i\dot{x}_i^2+2M_i\widetilde{R}\dot{x}_ix_i+P_ix_i^2\big), \quad i=\overline{1,n}.$$

that can be formulated as

$$(e^{\nu\theta_\varepsilon}-c_{ki2}^2)\dot{x}_i^2 + 2\big(\widetilde{R}(e^{\nu\theta_\varepsilon}-c_{ki2})-c_{ki1}c_{ki2}\big)x_i\dot{x}_i + \Big(\frac{P_i}{M_i}(e^{\nu\theta_\varepsilon}-1)-2\widetilde{R}c_{ki1}-c_{ki1}^2\Big)x_i^2 \ge 0, \quad i=1,n.$$

The requirement of positive definiteness of this form implies the condition

$$(e^{\nu\theta_\varepsilon}-c_{ki2}^2)\Big(\frac{P_i}{M_i}(e^{\nu\theta_\varepsilon}-1)-2\widetilde{R}c_{ki1}-c_{ki1}^2\Big) \ge \big(\widetilde{R}(e^{\nu\theta_\varepsilon}-c_{ki2})-c_{ki1}c_{ki2}\big)^2,$$

it is convenient to formulate it as a polynomial of $e^{\nu\theta_\varepsilon}$:

$$\Big(\frac{P_i}{M_i}-\widetilde{R}^2\Big)e^{2\nu\theta_\varepsilon} + \Big(-\frac{P_i}{M_i}(c_{ki2}^2+1)+2\widetilde{R}c_{ki1}(c_{ki2}-1) - c_{ki1}^2+2\widetilde{R}^2c_{ki2}\Big)e^{\nu\theta_\varepsilon} + c_{ki2}^2\Big(\frac{P_i}{M_i}-\widetilde{R}^2\Big) \ge 0 \tag{4.28}$$

Now we write down all stability conditions for the system (4.1) that have been obtained:

$$
\begin{aligned}
r &< \frac{\mu-\nu}{\overline{\lambda}_{\nu}^{(\exp)}},\\
r &< \frac{\mu-\nu}{\overline{\alpha}_{\nu,\varepsilon}^{(\exp)}},
\end{aligned}
\tag{4.29}
$$

where the terms $\overline{\lambda}_{\nu}^{(\exp)}$, $\overline{\alpha}_{\nu}^{(\exp)}$ are defined by (4.19), (4.26).

Inequality (4.28) and conditions (4.29) guarantee the asymptotic stability of the system (4.1) for any $\varepsilon > 0$. Finally, we obtain stability conditions in the form

$$
\begin{gathered}
b_i < -\widetilde{R}M_i,\\
4\widetilde{R}(b_i+\widetilde{R}M_i)(P_i-a_i)+(b_i\widetilde{R}+a_i)^2<0,\\
r<\frac{\mu-\nu}{\overline{\lambda}_{\nu}^{(\exp)}},\quad r<\frac{\mu-\nu}{\overline{\alpha}_{\nu}^{(\exp)}},\\
\Big(\frac{P_i}{M_i}-\widetilde{R}^2\Big)e^{2\nu\theta}+\Big(-\frac{P_i}{M_i}(c_{ki2}^2+1)+2\widetilde{R}c_{ki1}(c_{ki2}-1)\\
-c_{ki1}^2+2\widetilde{R}^2c_{ki2}\Big)e^{\nu\theta}+c_{ki2}^2\Big(\frac{P_i}{M_i}-\widetilde{R}^2\Big)>0\\
P_i>M_i\widetilde{R}^2,
\end{gathered}
\tag{4.30}
$$

where $\overline{\alpha}_{\nu}^{(\exp)} = \overline{\alpha}_{\nu,0}^{(\exp)}$.

In the inequalities (4.30) there are arbitrary parameters ν and $\widetilde{R}$. Thus, the power system will be asymptotically stable if for given parameters the system of inequalities (4.30) can be solved in the variables ν and $\widetilde{R}$.

5. CONCLUDING REMARKS

The obtained results are based on the assumption that the continuous model of the power system is stable. This seems naturally, because it refuses the possibility of stability achieving by impulses that are supposed destructive.

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