Communications in Applied Analysis **19** (2015), 343–352

EXPONENTIAL STABILITY OF STOCHASTIC COHEN-GROSSBERG-TYPE BAM NEURAL NETWORKS WITH S-TYPE DISTRIBUTED DELAYS

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ABSTRACT. This paper is concerned with exponential stability in mean square for stochastic Cohen-Grossberg-type BAM neural networks with S-type distributed delays. By using Lyapunov functional method and with the help of stochastic analysis technique, the sufficient conditions to guarantee the exponential stability in mean square for the neural networks are obtained. An example is given to demonstrate the advantage and applicability of the proposed results.

AMS (MOS) Subject Classification. 34K30, 34K50, 60H15.

1. Introduction

The Cohen-Grossberg-type BAM neural networks was first proposed in 1983 by Cohen and Grossberg [1]. Because of its wide applications in pattern recognition, signal process, optimization problems and many other fields, the stability of the neural networks with discrete delays or distributed delays, which these applications are largely dependent upon, has been extensively studied [4]–[9]. Recently, the various results have been obtained for the stability of stochastic Cohen-Grossberg-type BAM neural networks with discrete and distributed delays due to signal interference by random factors [10]–[15]. But it has rare reports for the stability of the stochastic neural networks with S-type distributed delays. For the systems with discrete and distributed delays are complementary events, and the system with S-type distributed delays contains systems with discrete and distributed delays [2], [16]. So, it is very meaningful to study the stability of stochastic Cohen-Grossberg-type BAM neural networks with S-type distributed delays. In this paper, we focus on the stability for the stochastic Cohen-Grossberg-type BAM neural networks with S-type distributed delays, which the motivation come from the mathematics and applications of the neural networks. Some sufficient conditions of the exponential stability in mean square are obtained in terms of Lyapunov functional and stochastic analysis technique. An

Received October 20, 2014

example is given to demonstrate the advantage and applicability of the proposed results.

2. Preliminaries

Consider the following stochastic Cohen-Grossberg-type BAM neural networks with S-type distributed

$$\begin{cases} dx_i(t) = -\alpha_i(x_i(t))\Big[a_i(x_i(t)) - \sum_{j=1}^{m} a_{ji} f_j(y_j(t)) \\ \qquad - \sum_{j=1}^{m} b_{ji} \int_{-\infty}^{0} f_j(y_j(t+\theta))d\eta_j^{(1)}(\theta) - I_i\Big]dt + \sum_{j=1}^{m} \sigma_{ji}(y_j(t))dw_j(t), \\ dy_j(t) = -\beta_j(y_j(t))\Big[b_j(y_j(t)) - \sum_{i=1}^{n} c_{ij} g_i(x_i(t)) \\ \qquad - \sum_{i=1}^{n} d_{ij} \int_{-\infty}^{0} g_i(x_i(t+\theta))d\eta_i^{(2)}(\theta) - J_j\Big]dt + \sum_{i=1}^{n} \tau_{ij}(x_i(t))dw_{m+i}(t), \\ x_i(t) = \phi_i(t), \quad t \in (-\infty, 0], \\ y_j(t) = \varphi_j(t), \quad t \in (-\infty, 0], \end{cases} \tag{1}$$

where $\phi_i(t)$ and $\varphi_j(t)$ are bounded in $(-\infty, 0]$, $x_i(t)$ and $y_j(t)$ are the neuron state variable. $\alpha_i(s)$ and $\beta_j(s)$ represent the amplification functions of the ith and jth cell neurons. $a_i(s)$ and $b_j(s)$ are appropriately behaved functions, a_{ji}, b_{ji}, c_{ij}, d_{ij}, $f_j(s)$ and $g_i(s)$ represent interconnection weight coefficients and the neuron activation functions, respectively. $W(t) = (w_1(t), w_2(t), \dots, w_{m+n}(t))^T$ is $n+m$ dimensional Brownian motion defined on a complete probability space (Ω, F_t, P) with a natural filtration F_t. $\sigma_{ji}(s)$ and $\tau_{ij}(s)$ are diffusion coefficients, I_i and J_j denote external inputs to the neurons introduced from outside the network. $\int_{-\infty}^{0} f_j(y_j(t+\theta))d\eta_j^{(1)}(\theta)$, $\int_{-\infty}^{0} g_i(x_i(t+\theta))d\eta_i^{(2)}(\theta)$ are Lebesgue-Stieltjes integrable, and $\eta_j^{(1)}(\theta)$, $\eta_i^{(2)}(\theta)$ are nondecreasing bounded variation functions which satisfy

$$\int_{-\infty}^{0} d\eta_j^{(1)}(\theta) = k_j, \quad \int_{-\infty}^{0} d\eta_i^{(2)}(\theta) = l_i. \tag{2}$$

Throughout this paper, we assume that

(H1) $f_j(s)$, $g_i(s)$, $\sigma_{ji}(s)$, $\tau_{ij}(s)$ are Lipschitz continuous. That is, exist positive constants p_j, q_i, M_{ji}, N_{ij} such that

$$|f_j(t) - f_j(s)| \le p_j|t-s|, |g_i(t) - g_i(s)| \le q_i|t-s|, \tag{3}$$

$$|\sigma_{ji}(t) - \sigma_{ji}(s)| \le M_{ji}|t-s|, |\tau_{ij}(t) - \tau_{ij}(s)| \le N_{ij}|t-s|, \tag{4}$$

for all $t, s \in R$.

(H2) There exists a positive constant λ such that

$$\int_{-\infty}^{0} e^{-2\lambda\theta} d\eta_j^{(1)}(\theta) < +\infty, \quad \int_{-\infty}^{0} e^{-2\lambda\theta} d\eta_i^{(2)}(\theta) < +\infty, \tag{5}$$

(H3) $\alpha_i(s)$ and $\beta_j(s)$ are continuous bounded functions in R, and there exist positive constants $\underline{\alpha}_i$, $\overline{\alpha}_i$, $\underline{\beta}_j$, $\overline{\beta}_j$, such that

$$\underline{\alpha}_i \le \alpha_i(s) \le \overline{\alpha}_i, \underline{\beta}_j \le \beta_j(s) \le \overline{\beta}_j, \quad i = 1, 2, \ldots, n, \quad j = 1, 2, \ldots, m. \tag{6}$$

(H4) There exist positive constants μ_i, ν_j such that

$$\mu_i(2 - 2\underline{\alpha}_i a_i) + \sum_{j=1}^{m} [\overline{\alpha}_i p_j(a_{ji}^+ + b_{ji}^+ k_j)\mu_i + \nu_j \overline{\beta}_j (c_{ij}^+ + d_{ij}^+ l_i) q_i + \nu_j N_{ij}^2] < 0, \tag{7}$$

$$\nu_j(2 - 2\underline{\beta}_j b_j) + \sum_{i=1}^{n} [\overline{\beta}_j q_i(c_{ij}^+ + d_{ij}^+ l_i)\nu_j + \mu_i \overline{\alpha}_i (a_{ji}^+ + b_{ji}^+ k_j) p_j + \mu_i M_{ji}^2] < 0, \tag{8}$$

with $a_{ji}^+ = |a_{ji}|$, $b_{ji}^+ = |b_{ji}|$, $c_{ij}^+ = |c_{ij}|$, $d_{ij}^+ = |d_{ij}|$.
(H5) There exist constants $a_i > 0$, $b_j > 0$, such that

$$a_i(x) - a_i(y) \ge a_i(x - y), b_j(x) - b_j(y) \ge b_j(x - y), \tag{9}$$

for all $x, y \in R$, and $a_i(0) = b_j(0) = 0$.
(H6)

$$a_i - \sum_{j=1}^{m} p_j(a_{ji}^+ + b_{ji}^+ k_j) > 0, \quad b_j - \sum_{i=1}^{n} q_i(c_{ij}^+ + d_{ij}^+ l_i) > 0. \tag{10}$$

Definition 2.1 ([14]). The equilibrium point of system (1) is said to be exponentially stable in mean square, if there exist positive constants K, δ, such that

$$\sum_{i=1}^{n} E|x_i(t) - x_i^*|^2 + \sum_{j=1}^{m} E|y_j(t) - y_j^*|^2 \le K e^{-\delta t}(E \sum_{i=1}^{n} |\phi_i - x_i^*|^2 + E \sum_{j=1}^{m} |\varphi_j - y_j^*|^2), \tag{11}$$

for all $t > 0$. When $|\phi_i - x_i^*| = \sup_{-\infty < \theta \le 0} |\phi_i(\theta) - x_i^*|$, $|\varphi_j - y_j^*| = \sup_{-\infty < \theta \le 0} |\varphi_j(\theta) - y_j^*|$.

Lemma 2.2 ([3]). *For the equation*

$$dx(t) = f(x_t, t)dt + g(x_t, t)dW(t), \quad t_0 \le t, \tag{12}$$

where $x_t = x(t + \theta) : -\tau \le \theta \le 0$ *is regarded as a* $C([-\tau, 0]; R^n)$*-valued stochastic process, and the initial data* $x_{t_0} = \varphi(\theta)$ *is an* F_{t_0}*-measurable* $C([-\tau, 0]; R^n)$ *with* $E|\varphi|^2 < \infty$. *Assume that for any* $b \in (t_0, \infty)$
(1) $f(t, 0) \in L^2([t_0, b]; R^n)$ *and* $g(t, 0) \in L^2([t_0, b]; R^{n\times m})$.
(2) There is a constant $K_n = K_n(b) > 0$ *such that*

$$|f(t, \varphi) - f(t, \psi)| \le K_n\|\varphi - \psi\|, |g(t, \varphi) - g(t, \psi)| \le K_n\|\varphi - \psi\|, \tag{13}$$

for all $t \in [t_0, b]$ *and* $\varphi, \psi : [-\tau, 0] \to R^n$ *with* $\|\varphi\| \vee \|\psi\| \le n$.
(3) There is a function $V(t, x) \in C([t_0-\tau, \infty)\times R^n; R_+)$ *with* $\lim_{|x|\to\infty} \inf_{t_0 \le s < \infty} V(s, x) = \infty$ *such that the following priori estimate is satisfied*

$$EV(t, x(t)) \le L(t), \tag{14}$$

where $L : [t_0, T) \to R_+$ *with* $\sup_{s\in[t_0,t]} L(s) < \infty$ *for any given* $t \in [t_0, \infty)$*, then the solution* $x(t)$ *to* (12) *is unique and exists globally on* $[t_0 - \tau, \infty)$*.*

Let $V(t, x) \in C([t_0 - \tau, \infty) \times R^n; R_+)$ and

$$LV(t, x(t)) = V_t(t, x(t)) + V_x(t, x(t))f(t, x_t) + \frac{1}{2}trg^T(t, x_t)V_{xx}(t, x(t))g(t, x_t). \quad (15)$$

Then, from Itô formula [3], it follows

$$V(t, x(t)) = V(t_0, x(t_0)) + \int_{t_0}^{t} LV(s, x(s))ds + \int_{t_0}^{t} V_x g(s, x_s)dW(s). \quad (16)$$

Remark 2.3. The condition (1) holds obviously if $f(\varphi, t) = f(\varphi)$, $g(\varphi, t) = g(\varphi)$.

3. Main result and its proof

Consider the following model:

$$\begin{cases} dx_i(t) = -\alpha_i(t)\Big[a_i(x_i(t)) - \sum_{j=1}^{m} a_{ji}f_j(y_j(t)) \\ \qquad - \sum_{j=1}^{m} b_{ji}\int_{-\infty}^{0} f_j(y_j(t+\theta))d\eta_j^{(1)}(\theta) - I_i\Big]dt, \\ dy_j(t) = -\beta_j(t)\Big[b_j(y_j(t)) - \sum_{i=1}^{n} c_{ij}g_i(x_i(t)) \\ \qquad - \sum_{i=1}^{n} d_{ij}\int_{-\infty}^{0} g_i(x_i(t+\theta))d\eta_j^{(2)}(\theta) - J_j\Big]dt. \end{cases} \quad (17)$$

In a similar way of proof for the literature [4], under hypotheses (H1)–(H6), we can prove that there exists an equilibrium point $z^* = (x_1^*, x_2^*, \dots, x_n^*, y_1^*, y_2^*, \dots, y_m^*)^T$ of the system (17) by using topological degree and homotopy invariance, i.e.,

$$a_i(x_i^*) - \sum_{j=1}^{m} a_{ji}f_j(y_j^*) - \sum_{j=1}^{m} b_{ji}k_jf_j(y_j^*) - I_i = 0, \quad (18)$$

$$b_j(y_j^*) - \sum_{i=1}^{n} c_{ij}g_i(x_i^*) - \sum_{i=1}^{n} d_{ij}l_ig_i(x_i^*) - J_j = 0. \quad (19)$$

Hypotheses
(H7) $\sigma_{ji}(y_j^*) = \tau_{ij}(x_j^*) = 0$, $i - 1, 2, \dots, n$, $j = 1, 2, \dots, m$.
From (H7),we know that then z^* is an equilibrium point of the system (1).

Theorem 3.1. *Assume that* (H1)–(H7) *hold. Then, the equilibrium point* z^* *of system* (1) *is exponentially stable in mean square.*

Proof. From Lemma 2.2, we can prove that the system (1) has a unique solution $z(t) = (x_1, x_2, \dots, x_n, y_1, y_2, \dots, y_m)^T$, $t \in [0, \infty)$, which the solution belongs to

$M^2([0,\infty);R^{n+m})$. By the condition $\int_{-\infty}^{0} d\eta_j^{(1)}(\theta) = k_j$, $\int_{-\infty}^{0} d\eta_i^{(2)}(\theta) = l_i$ and (H2), we know that, exists a $\overline{\lambda} > 0$, such that [15]

$$\int_{-\infty}^{0} e^{-2\lambda\theta} d\eta_j^{(1)}(\theta) < +\infty, \quad \int_{-\infty}^{0} e^{-2\lambda\theta} d\eta_i^{(2)}(\theta) < +\infty, \tag{20}$$

for all $\lambda \in (0,\overline{\lambda})$, and

$$\lim_{\lambda\longrightarrow\overline{\lambda}} \int_{-\infty}^{0} e^{-2\lambda\theta} d\eta_j^{(1)}(\theta) = \lim_{\lambda\longrightarrow\overline{\lambda}} \int_{-\infty}^{0} e^{-2\lambda\theta} d\eta_i^{(2)}(\theta) = +\infty, \tag{21}$$

Define

$$\begin{aligned} F(\lambda) = 2\mu_i(\underline{\alpha}_i a_i - \lambda) - \sum_{j=1}^{m} \Big[& \overline{\alpha}_i p_j (a_{ji}^+ + b_{ji}^+ k_j)\mu_i \\ & + \nu_j \overline{\beta}_j \left(c_{ij}^+ + d_{ij}^+ \int_{-\infty}^{0} e^{-2\lambda\theta} d\eta_i^{(2)}(\theta) \right) q_i + \nu_j N_{ij}^2 \Big], \end{aligned} \tag{22}$$

$$\begin{aligned} G(\lambda) = 2\nu_j(\underline{\beta}_j b_j - \lambda) - \sum_{i=1}^{n} \Big[& \overline{\beta}_j q_i \left(c_{ij}^+ + d_{ij}^+ l_i \right) \nu_j \\ & + \mu_i \overline{\alpha}_i (a_{ji}^+ + b_{ji}^+ \int_{-\infty}^{0} e^{-2\lambda\theta} d\eta_j^{(1)}(\theta)) p_j + \mu_i M_{ji}^2 \Big], \end{aligned} \tag{23}$$

then, by (H3) and (H4), we get $F(0) > 0$, $G(0) > 0$, and we also have $\lambda \longrightarrow \frac{\overline{\lambda}}{2}$. Hence exists a $\lambda^* \in (0, \frac{\overline{\lambda}}{2})$ such that [16]

$$F(\lambda^*) \geq 0, l_i < \int_{-\infty}^{0} e^{-2\lambda\theta} d\eta_i^{(2)}(\theta)) < +\infty, \tag{24}$$

$$G(\lambda^*) \geq 0, k_j < \int_{-\infty}^{0} e^{-2\lambda\theta} d\eta_j^{(1)}(\theta)) < +\infty. \tag{25}$$

Let

$$\begin{aligned} V_1 = & \sum_{i=1}^{n} \mu_i e^{2\lambda^* t} |x_i(t) - x_i^*|^2 \\ & + \sum_{i=1}^{n}\sum_{j=1}^{m} \overline{\alpha}_i b_{ji}^+ p_j \mu_i \int_{-\infty}^{0}\int_{t+\theta}^{t} e^{-2\lambda^*(s-\theta)} |y_j(s) - y_j^*|^2 ds d\eta_j^{(1)}(\theta), \end{aligned} \tag{26}$$

$$\begin{aligned} V_2 = & \sum_{j=1}^{m} \nu_j e^{2\lambda^* t} |y_j(t) - y_j^*|^2 \\ & + \sum_{i=1}^{n}\sum_{j=1}^{m} \overline{\beta}_j d_{ij}^+ q_i \nu_j \int_{-\infty}^{0}\int_{t+\theta}^{t} e^{-2\lambda^*(s-\theta)} |x_i(s) - x_i^*|^2 ds d\eta_i^{(2)}(\theta). \end{aligned} \tag{27}$$

From (4), (5), (24), (25), (H5), Itô formula and Jensen inequality, we have

$$LV_1 = 2\lambda^* e^{2\lambda^* t} \sum_{i=1}^{n} \mu_i |x_i(t) - x_i^*|^2$$

$$
\begin{aligned}
&+\sum_{i=1}^{n}\sum_{j=1}^{m} e^{2\lambda^* t}\overline{\alpha}_i b_{ji}^{+} p_j \mu_i \int_{-\infty}^{0} (e^{-2\lambda^*\theta}|y_j(t)-y_j^*|^2 - |y_j(t+\theta)-y_j^*|^2) d\eta_j^{(1)}(\theta) \\
&+ 2e^{2\lambda^* t}\sum_{i=1}^{n} \mu_i (x_i(t)-x_i^*)[-\alpha_i(x_i(t))(a_i(x_i(t)) - \sum_{j=1}^{m} a_{ji} f_j(y_j(t)) \\
&- \sum_{j=1}^{m} b_{ji} \int_{-\infty}^{0} f_j(y_j(t+\theta)) d\eta_j^{(1)}(\theta) - I_i)] + e^{2\lambda^* t}\sum_{i=1}^{n}\sum_{j=1}^{m} \mu_i \sigma_{ji}^2(y_j(t)) \\
= {}& 2\lambda^* e^{2\lambda^* t}\sum_{i=1}^{n} \mu_i |x_i(t)-x_i^*|^2 \\
&+\sum_{i=1}^{n}\sum_{j=1}^{m} e^{2\lambda^* t}\overline{\alpha}_i b_{ji}^{+} p_j \mu_i \int_{-\infty}^{0} (e^{-2\lambda^*\theta}|y_j(t)-y_j^*|^2 - |y_j(t+\theta)-y_j^*|^2) d\eta_j^{(1)}(\theta) \\
&- 2e^{2\lambda^* t}\sum_{i=1}^{n} \mu_i (x_i(t)-x_i^*)\alpha_i(x_i(t))[a_i(x_i(t)) - a_i(x_i^*)] \\
&+ 2e^{2\lambda^* t}\sum_{i=1}^{n} \mu_i (x_i(t)-x_i^*)\alpha_i(x_i(t))\sum_{j=1}^{m} a_{ji}(f_j(y_j(t)) - f_j(y_j^*)) \\
&+ 2e^{2\lambda^* t}\sum_{i=1}^{n} \mu_i (x_i(t)-x_i^*)\alpha_i(x_i(t))\sum_{j=1}^{m} b_{ji}\int_{-\infty}^{0} (f_j(y_j(t+\theta)) - f_j(y_j^*)) d\eta_j^{(1)}(\theta) \\
&+ e^{2\lambda^* t}\sum_{i=1}^{n}\sum_{j=1}^{m} \mu_i(\sigma_{ji}^2(y_j(t)) - \sigma_{ji}^2(y_j^*)) \\
\le {}& 2\lambda^* e^{2\lambda^* t}\sum_{i=1}^{n} \mu_i |x_i(t)-x_i^*|^2 \\
&+\sum_{i=1}^{n}\sum_{j=1}^{m} e^{2\lambda^* t}\overline{\alpha}_i b_{ji}^{+} p_j \mu_i \int_{-\infty}^{0} (e^{-2\lambda^*\theta}|y_j(t)-y_j^*|^2 - |y_j(t+\theta)-y_j^*|^2) d\eta_j^{(1)}(\theta) \\
&- 2e^{2\lambda^* t}\sum_{i=1}^{n} \mu_i \underline{\alpha}_i a_i |x_i(t)-x_i^*|^2 + 2e^{2\lambda^* t}\sum_{i=1}^{n} \mu_i\overline{\alpha}_i |x_i(t)-x_i^*| \sum_{j=1}^{m} a_{ji}^{+} p_j |y_j(t)-y_j^*| \\
&+ 2e^{2\lambda^* t}\sum_{i=1}^{n}\sum_{j=1}^{m} \mu_i\overline{\alpha}_i b_{ji}^{+} p_j |x_i(t)-x_i^*| \int_{-\infty}^{0} |y_j(t+\theta)-y_j^*| d\eta_j^{(1)}(\theta) \\
&+ 2e^{2\lambda^* t}\sum_{i=1}^{n}\sum_{j=1}^{m} \mu_i M_{ji}^2 |y_j(t)-y_j^*|^2 \\
\le {}& e^{2\lambda^* t}\sum_{i=1}^{n} \mu_i |x_i(t)-x_i^*|^2 \left[2(\lambda^* - \underline{\alpha}_i a_i) + \sum_{j=1}^{m} \overline{\alpha}_i p_j (a_{ji}^{+} + b_{ji}^{+} k_j) \right] \\
&+ e^{2\lambda^* t}\sum_{j=1}^{m} |y_j(t)-y_j^*|^2 \left[\sum_{i=1}^{n} \mu_i \overline{\alpha}_i p_j \Big(a_{ji}^{+} + b_{ji}^{+}\int_{-\infty}^{0} e^{-2\lambda^*\theta} d\eta_j^{(1)}(\theta)\Big) + \sum_{i=1}^{n} \mu_i M_{ji}^2 \right].
\end{aligned}
\tag{28}
$$

$$LV_2 \leq e^{2\lambda^* t}\sum_{j=1}^{m}\nu_j|y_j(t)-y_j^*|^2\left[2(\lambda^*-\underline{\beta}_j b_j)+\sum_{i=1}^{n}\overline{\beta}_j q_i(c_{ij}^{+}+d_{ij}^{+}l_i)\right]$$
$$+ e^{2\lambda^* t}\sum_{i=1}^{n}|x_i(t)-x_i^*|^2\left[\sum_{j=1}^{m}\overline{\beta}_j\nu_j q_i(c_{ij}^{+}+d_{ij}^{+}\int_{-\infty}^{0}e^{-2\lambda^*\theta}d\eta_i^{(2)}(\theta))+\sum_{j=1}^{m}\nu_j N_{ji}^2\right]. \tag{29}$$

From (24), (25), (28) and (29), we have

$$LV(t,z(t)) = LV_1 + LV_2$$
$$= -e^{2\lambda^* t}\left(\sum_{i=1}^{n}|x_i(t)-x_i^*|^2 F(\lambda^*)+\sum_{j=1}^{m}|y_j(t)-y_j^*|^2 G(\lambda^*)\right)\leq 0. \tag{30}$$

From (16), it follows

$$V(t,z(t)) = V(0,z(0)) + \int_0^t LV(s,z(s))ds$$
$$+\int_0^t 2e^{2\lambda^* s}\sum_{i=1}^{n}\sum_{j=1}^{m}\mu_i|x_i(s)-x_i^*|\sigma_{ji}(y_j(s))dw_j(s)$$
$$+\int_0^t 2e^{2\lambda^* s}\sum_{i=1}^{n}\sum_{j=1}^{m}\mu_i|y_j(s)-y_j^*|\tau_{ij}(x_i(s))dw_{m+i}(s). \tag{31}$$

From (30) and (31), we can get

$$EV(t,z(t)) = EV(0,z(0)) + \int_0^t ELV(s,z(s))ds \leq EV(0,z(0))$$
$$\leq E\sum_{i=1}^{n}\mu_i|\phi_i-x_i^*|^2 + E\sum_{i=1}^{n}\sum_{j=1}^{m}\mu_i\overline{\alpha}_i b_{ji}^{+}p_j\int_{-\infty}^{0}\int_{\theta}^{0}e^{2\lambda^*(s-\theta)}|y_j(s)-y_j^*|^2 ds d\eta_j^{(1)}(\theta)$$
$$+E\sum_{j=1}^{m}\nu_j|\varphi_j-y_j^*|^2 + E\sum_{i=1}^{n}\sum_{j=1}^{m}\nu_j\overline{\beta}_j d_{ij}^{+}q_i\int_{-\infty}^{0}\int_{\theta}^{0}e^{2\lambda^*(s-\theta)}|x_i(s)-x_i^*|^2 ds d\eta_j^{(2)}(\theta)$$
$$\leq E\sum_{i=1}^{n}\mu_i|\phi_i-x_i^*|^2 + E\frac{1}{2\lambda^*}\sum_{i=1}^{n}\sum_{j=1}^{m}\mu_i\overline{\alpha}_i b_{ji}^{+}p_j\int_{-\infty}^{0}(e^{-2\lambda^*\theta}-1)d\eta_j^{(1)}(\theta)|\varphi_j-y_j^*|^2$$
$$+E\sum_{j=1}^{m}\nu_j|\varphi_j-y_j^*|^2 + E\frac{1}{2\lambda^*}\sum_{i=1}^{n}\sum_{j=1}^{m}\nu_j\overline{\beta}_j d_{ij}^{+}q_i\int_{-\infty}^{0}(e^{-2\lambda^*\theta}-1)d\eta_i^{(2)}(\theta)|\phi_i-x_i^*|^2$$
$$= E\sum_{i=1}^{n}|\phi_i-x_i^*|^2\left[\mu_i+\frac{1}{2\lambda^*}\sum_{j=1}^{m}\nu_j\overline{\beta}_j d_{ij}^{+}q_i\int_{-\infty}^{0}(e^{-2\lambda^*\theta}-1)d\eta_i^{(2)}(\theta)\right]$$
$$+E\sum_{j=1}^{m}|\varphi_j-y_j^*|^2\left[\nu_j+\frac{1}{2\lambda^*}\sum_{i=1}^{n}\mu_i\overline{\alpha}_i b_{ji}^{+}p_j\int_{-\infty}^{0}(e^{-2\lambda^*\theta}-1)d\eta_j^{(1)}(\theta)\right]. \tag{32}$$

From (26), (27) and (32), we obtain

$$\rho e^{2\lambda^* t}\left(\sum_{i=1}^{n} E|x_i(t)-x_i^*|^2+\sum_{j=1}^{m} E|y_j(t)-y_j^*|^2\right)\le EV(t,z(t))\le EV(0,z(0))$$
$$\le K\left(E\sum_{i=1}^{n}|\phi_i-x_i^*|^2+E\sum_{j=1}^{m}|\varphi_j-y_j^*|^2\right), \tag{33}$$

where

$$K=\max_{1\le i\le n,1\le j\le m}\Big\{\mu_i+\frac{1}{2\lambda^*}\sum_{j=1}^{m}\nu_j\overline{\beta}_j d_{ij}^{+}q_i\int_{-\infty}^{0}(e^{-2\lambda^*\theta}-1)d\eta_i^{(2)}(\theta),$$
$$\nu_j+\frac{1}{2\lambda^*}\sum_{i=1}^{n}\mu_i\overline{\alpha}_i b_{ji}^{+}p_j\int_{-\infty}^{0}(e^{-2\lambda^*\theta}-1)d\eta_j^{(1)}(\theta)\Big\},$$
$$\rho=\min\{\mu_i,\nu_j\},1\le i\le n,1\le j\le m. \tag{34}$$

That is

$$\sum_{i=1}^{n}E|x_i(t)-x_i^*|^2+\sum_{j=1}^{m}E|y_j(t)-y_j^*|^2\le\frac{K}{\rho}e^{-2\lambda^* t}(E\sum_{i=1}^{n}|\phi_i-x_i^*|^2+E\sum_{j=1}^{m}|\varphi_j-y_j^*|^2). \tag{35}$$

So, the system (1)is exponentially stable in mean square. □

Remark 3.2. When $\sigma_{ji}(s)=\tau_{ij}(s)=0$, then system (1) becomes to neural networks without random disturbance.

Remark 3.3. When $\alpha_i(s)=\beta_j(s)=1$, and $\sigma_{ji}(s)=\tau_{ij}(s)=0$,the system (1) is simplified to the general BAM with S-type distributed delays

$$\begin{cases}\dot{x}_i(t)=-a_i(x_i(t))+\sum\limits_{j=1}^{m}a_{ji}f_j(y_j(t))+\sum\limits_{j=1}^{m}b_{ji}\int_{-\infty}^{0}f_j(y_j(t+\theta))d\eta_j^{(1)}(\theta)+I_i,\\ \dot{y}_j(t)=-b_j(y_j(t))+\sum\limits_{i=1}^{n}c_{ij}g_i(x_i(t))+\sum\limits_{i=1}^{n}d_{ij}\int_{-\infty}^{0}g_i(x_i(t+\theta))d\eta_j^{(2)}(\theta)+J_j,\end{cases} \tag{36}$$

which is the model in literature [4].

Remark 3.4. When $\alpha_i(s)=\beta_j(s)=1$, and

$$\eta_j^{(1)}(\theta)=\begin{cases}1, & -\rho_j\le\theta\le 0\\ 0, & -\infty<\theta<-\rho_j\end{cases}$$

$$\eta_i^{(2)}(\theta)=\begin{cases}1, & -\tau_i\le\theta\le 0\\ 0, & -\infty<\theta<-\tau_i\end{cases}$$

then

$$\int_{-\infty}^{0}f_j(y_j(t+\theta))d\eta_j^{(1)}(\theta)=f_j(y_j(t-\rho_j)),\int_{-\infty}^{0}g_i(x_i(t+\theta))d\eta_i^{(2)}(\theta)=g_i(x_i(t-\tau_i)),$$

the system (1) becomes to the model of literature [13]. So, this paper includes of results of Wang and Xu (2002), and Li and Fu (2011) as a special case.

4. Example

Consider the Cohen-Grossberg-type BAM neural networks (1) with the following parameters

$$\alpha_i(x_i(t)) = \beta_j(y_j(t)) = 2 - \cos(t), i = 1, 2, j = 1, 2,$$
$$a_1(x) = 4x, a_2(x) = 5x, b_1(y) = 6y, b_2(y) = 3.5y,$$
$$a_{11} = 0.25, a_{21} = 0.125, a_{12} = -0.25, a_{22} = -0.125,$$
$$b_{11} = 0.5, b_{21} = 0.25, b_{12} = -0.5, b_{22} = -0.25,$$

$$c_{11} = 0.1, c_{21} = 0.2, c_{12} = -0.1, c_{22} = -0.2,$$
$$d_{11} = 0.15, d_{21} = 0.3, d_{12} = -0.15, d_{22} = -0.3,$$
$$\sigma_{11}(y_1(t)) = 0.5y_1(t), \sigma_{21}(y_2(t)) = 0, \sigma_{12}(y_1(t)) = 0, \sigma_{22}(y_2(t)) = 0.2y_2(t),$$
$$\tau_{11}(x_1(t)) = x_1(t), \tau_{21}(x_2(t)) = 0, \tau_{12}(x_1(t)) = 0, \tau_{22}(x_2(t)) = 0.6x_2(t),$$
$$f_j(x) = g_i(x) = \sin(x), \eta_j^{(1)} = \eta_i^{(2)} = e^{\theta}.$$

That it is obvious that $p_j = q_i = k_j = l_i = 1$, $\underline{\alpha}_i = \underline{\beta}_j = 1$, $\overline{\alpha}_i = \overline{\beta}_j = 3$, $M_{11} = 0.5, M_{22} = 0.2, N_{11} = 1, N_{22} = 0.6$, we also assume $\mu_i = \nu_j = 1$, then, we get

$$\begin{cases} \mu_1(2 - 2\underline{\alpha}_1 a_1) + \sum\limits_{j=1}^{2}[\overline{\alpha}_1 p_j(a_{j1}^+ + b_{j1}^+ k_j)\mu_1 + \nu_j\overline{\beta}_j(c_{1j}^+ + d_{1j}^+ l_1)q_1 + \nu_j N_{1j}^2] < -0.125 < 0, \\ \mu_2(2 - 2\underline{\alpha}_2 a_2) + \sum\limits_{j=1}^{2}[\overline{\alpha}_2 p_j(a_{j2}^+ + b_{j2}^+ k_j)\mu_2 + \nu_j\overline{\beta}_j(c_{2j}^+ + d_{2j}^+ l_2)q_2 + \nu_j N_{2j}^2] < -1.39 < 0, \\ \nu_1(2 - 2\underline{\beta}_1 b_1) + \sum\limits_{i=1}^{2}[\overline{\beta}_1 q_i(c_{i1}^+ + d_{i1}^+ l_i)\nu_1 + \mu_i\overline{\alpha}_i(a_{1i}^+ + b_{1i}^+ k_1)p_1 + \mu_i M_{1i}^2] < -2 < 0, \\ \nu_2(2 - 2\underline{\beta}_2 b_2) + \sum\limits_{i=1}^{2}[\overline{\beta}_2 q_i(c_{i2}^+ + d_{i2}^+ l_i)\nu_2 + \mu_i\overline{\alpha}_i(a_{2i}^+ + b_{2i}^+ k_2)p_2 + \mu_i M_{2i}^2] < -0.46 < 0, \end{cases}$$

Therefore, $[0, 0, 0, 0]^T$ is the equilibrium point of system (1), which is exponentially stable in mean square.

ACKNOWLEDGMENTS

The work was supported by the National Natural Science Foundation of China (11171374) and Natural Science Foundation of Shandong Province (ZR2011AZ001).

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