Dynamic Systems and Applications 17 (2008) 303-312

SOLUTIONS AND POSITIVE SOLUTIONS TO SEMIPOSITONE DIRICHLET BVPS ON TIME SCALES

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ABSTRACT. In this paper, we are concerned with the following Dirichlet boundary value problem on a time scale $\mathbb{T}$

$$\begin{cases} -u^{\Delta\Delta}(t) = g(t, u(t)),\ t \in [0, T]_{\mathbb{T}}, \\ u(0) = 0 = u(\sigma^2(T)), \end{cases}$$

where $g : [0, T]_{\mathbb{T}} \times [-\sigma(T)\sigma^2(T)M, +\infty) \to [-M, +\infty)$ is continuous and $M > 0$ is a constant, which implies that this problem is semipositone. For an arbitrary positive integer n, some existence results for n solutions and/or positive solutions are established by using the well-known Guo-Krasnosel'skii fixed point theorem. Our conditions imposed on g are local. An example is also included to illustrate the importance of the results obtained.

AMS (MOS) Subject Classification. 34B15, 39A10.

1. INTRODUCTION

Let $\mathbb{T}$ be a time scale (arbitrary nonempty closed subset of the real numbers $\mathbb{R}$). For each interval $\mathbf{I}$ of $\mathbb{R}$, we denote by $\mathbf{I}_{\mathbb{T}} = \mathbf{I} \cap \mathbb{T}$. For more details on time scales, one can refer to [1, 3, 7, 8]. In this paper, we consider solutions and positive solutions to the nonlinear Dirichlet boundary value problem (BVP for short) on a time scale $\mathbb{T}$

$$\begin{cases} -u^{\Delta\Delta}(t) = g(t, u(t)),\ t \in [0, T]_{\mathbb{T}}, \\ u(0) = 0 = u(\sigma^2(T)), \end{cases} \tag{1.1}$$

where $T > 0$ is fixed and $0,\ T \in \mathbb{T}$. Here, the solution u of the BVP (1.1) is called positive if $u(t) > 0$, $t \in (0, \sigma^2(T))_{\mathbb{T}}$. Throughout this paper, we assume that $g : [0, T]_{\mathbb{T}} \times [-\sigma(T)\sigma^2(T)M, +\infty) \to [-M, +\infty)$ is continuous and $M > 0$ is a constant; this implies that the BVP (1.1) is semipositone.

The BVP (1.1) has been discussed extensively when $M = 0$ (i.e., positone problem); see [2, 4, 5, 10] and the references therein. Recently, by using fixed point index theory, we [12] established some existence criteria for at least one positive solution to the BVP (1.1) assuming $M > 0$ (i.e., semipositone problem) and *global* conditions

Received June 10, 2007 1056-2176 \$15.00 ©Dynamic Publishers, Inc.

on g (that is to say, these conditions are concerned with the growth of g on its whole domain). This paper is a continuation of our study in [12]. Our results show that the BVP (1.1) has at least n solutions and/or positive solutions provided that the "heights" of g on some bounded sets of its domain are appropriate, i.e., such existence results do not concern the growth of g outside these bounded sets. In other words, our conditions imposed on g are *local*. Our main idea comes from [9, 13, 14], and our main tool is the well-known Guo-Krasnosel'skii fixed point theorem, which we state here for the convenience of the reader.

Theorem 1.1 ([6]). *Let $\mathbb{X}$ be a Banach space and K be a cone in $\mathbb{X}$. Assume that Ω_1 and Ω_2 are bounded open subsets of $\mathbb{X}$ with $0 \in \Omega_1$, $\overline{\Omega_1} \subset \Omega_2$, and let $\Phi : K \cap (\overline{\Omega_2} \backslash \Omega_1) \to K$ be a completely continuous operator such that either*

$$\text{(i)}\ \|\Phi u\| \leq \|u\|,\ \forall u \in K \cap \partial\Omega_1 \text{ and } \|\Phi u\| \geq \|u\|,\ \forall u \in K \cap \partial\Omega_2,$$

or

$$\text{(ii)}\ \|\Phi u\| \geq \|u\|,\ \forall u \in K \cap \partial\Omega_1 \text{ and } \|\Phi u\| \leq \|u\|,\ \forall u \in K \cap \partial\Omega_2.$$

Then Φ has a fixed point in $K \cap (\overline{\Omega_2} \backslash \Omega_1)$.

2. MAIN RESULTS

Let

$$\mathbb{X} = \left\{ u \,\middle|\, u : [0, \sigma^2(T)]_{\mathbb{T}} \to \mathbb{R} \text{ is continuous} \right\}$$

be equipped with the norm

$$\|u\| = \max_{t \in [0, \sigma^2(T)]_{\mathbb{T}}} |u(t)|.$$

Then, $\mathbb{X}$ is a Banach space.

Define

$$K = \left\{ u \in \mathbb{X} \,\middle|\, u(t) \geq q(t) \|u\|,\ t \in [0, \sigma^2(T)]_{\mathbb{T}} \right\},$$

where $q(t) = \frac{t\left(\sigma^2(T) - t\right)}{\left(\sigma^2(T)\right)^2}$, $t \in [0, \sigma^2(T)]_{\mathbb{T}}$. Then, it is easy to see that K is a cone of $\mathbb{X}$.

To obtain a solution of the BVP (1.1), we require a mapping whose kernel $G(t, s)$ is the Green's function of the BVP

$$\left\{ \begin{array}{c} -u^{\Delta\Delta}(t) = 0,\ t \in [0, T]_{\mathbb{T}}, \\ u(0) = 0 = u(\sigma^2(T)). \end{array} \right. \tag{2.1}$$

It is known that [3]

$$G(t, s) = \frac{1}{\sigma^2(T)} \left\{ \begin{array}{c} t\left(\sigma^2(T) - \sigma(s)\right),\ t \leq s, \\ \sigma(s)\left(\sigma^2(T) - t\right),\ t \geq \sigma(s). \end{array} \right. \tag{2.2}$$

For $G(t, s)$, we have the following simple but important lemma.

Lemma 2.1. *For any* $t \in [0, \sigma^2(T)]_{\mathbb{T}}$ *and* $s \in [0, \sigma(T)]_{\mathbb{T}}$,

$$0 \le G(t,s) \le \frac{t\,(\sigma^2(T) - t)}{\sigma^2(T)}. \tag{2.3}$$

Lemma 2.2. *Let* $p(t)$ *be the solution of the BVP*

$$\begin{cases} -p^{\Delta\Delta}(t) = 1, \ t \in [0, T]_{\mathbb{T}}, \\ p(0) = 0 = p(\sigma^2(T)). \end{cases} \tag{2.4}$$

Then,

$$0 \le p(t) \le q(t)\sigma(T)\sigma^2(T), \ t \in [0, \sigma^2(T)]_{\mathbb{T}}. \tag{2.5}$$

In particular,

$$0 \le p(t) \le \sigma(T)\sigma^2(T), \ t \in [0, \sigma^2(T)]_{\mathbb{T}}. \tag{2.6}$$

Proof. Since $p(t)$ is the solution of the BVP (2.4), we know that

$$p(t) = \int_0^{\sigma(T)} G(t,s)\Delta s, \ t \in [0, \sigma^2(T)]_{\mathbb{T}}.$$

In view of Lemma 2.1, we have

$$0 \le p(t) = \int_0^{\sigma(T)} G(t,s)\Delta s \le \frac{t\,(\sigma^2(T) - t)\,\sigma(T)}{\sigma^2(T)} = q(t)\sigma(T)\sigma^2(T), \ t \in [0, \sigma^2(T)]_{\mathbb{T}}.$$

□

Let $u_0(t) = Mp(t)$, $t \in [0, \sigma^2(T)]_{\mathbb{T}}$. We consider the following BVP

$$\begin{cases} -u^{\Delta\Delta}(t) = g(t, u(t) - u_0(t)) + M, \ t \in [0, T]_{\mathbb{T}}, \\ u(0) = 0 = u(\sigma^2(T)). \end{cases} \tag{2.7}$$

It is easy to verify that if $u(t)$ is a solution of the BVP (2.7), then $u(t) - u_0(t)$ is a solution of the BVP (1.1). So, we will focus our attention on the BVP (2.7).

Since the BVP (2.7) is equivalent to the integral equation

$$u(t) = \int_0^{\sigma(T)} G(t,s)[g(s, u(s) - u_0(s)) + M]\Delta s, \ t \in [0, \sigma^2(T)]_{\mathbb{T}}, \tag{2.8}$$

we define the operator $\Phi : K \to \mathbb{X}$ as follows

$$(\Phi u)(t) = \int_0^{\sigma(T)} G(t,s)[g(s, u(s) - u_0(s)) + M]\Delta s, \ t \in [0, \sigma^2(T)]_{\mathbb{T}}. \tag{2.9}$$

Noticing that

$$-\sigma(T)\sigma^2(T)M \le u(t) - u_0(t) < +\infty \text{ for } u \in K \text{ and } t \in [0, \sigma^2(T)]_{\mathbb{T}}, \tag{2.10}$$

we know that $\Phi : K \to \mathbb{X}$ is well-defined.

Lemma 2.3. $\Phi : K \to K$ *is completely continuous.*

Proof. Let $u \in K$. By the definition of Φ, we know that $(\Phi u)(0) = 0 = (\Phi u)(\sigma^2(T))$. So, there exists a $t_0 \in (0, \sigma^2(T))_{\mathbb{T}}$ such that $\|\Phi u\| = (\Phi u)(t_0)$. Since

$$\frac{G(t,s)}{G(t_0,s)} = \begin{cases} \frac{t}{t_0}, \ t, \ t_0 \le s, \\ \frac{t\left(\sigma^2(T)-\sigma(s)\right)}{\sigma(s)(\sigma^2(T)-t_0)}, \ t \le s < t_0, \\ \frac{\sigma(s)\left(\sigma^2(T)-t\right)}{t_0(\sigma^2(T)-\sigma(s))}, \ t_0 \le s < t, \\ \frac{\sigma^2(T)-t}{\sigma^2(T)-t_0}, \ t, \ t_0 \ge \sigma(s), \end{cases}$$

we obtain that

$$\frac{G(t,s)}{G(t_0,s)} \ge q(t), \ t \in [0, \sigma^2(T)]_{\mathbb{T}} \text{ and } s \in [0, \sigma(T)]_{\mathbb{T}}. \tag{2.11}$$

So,

$$\begin{aligned} (\Phi u)(t) &= \int_0^{\sigma(T)} G(t,s)[g(s, u(s) - u_0(s)) + M]\Delta s \\ &= \int_0^{\sigma(T)} \frac{G(t,s)}{G(t_0,s)} G(t_0,s)[g(s, u(s) - u_0(s)) + M]\Delta s \\ &\ge q(t) \int_0^{\sigma(T)} G(t_0,s)[g(s, u(s) - u_0(s)) + M]\Delta s \\ &= q(t)(\Phi u)(t_0) \\ &= q(t) \|\Phi u\|, \ t \in [0, \sigma^2(T)]_{\mathbb{T}}, \end{aligned}$$

which shows that $\Phi u \in K$. Furthermore, by using similar arguments to those in [11], we can prove that $\Phi : K \to K$ is completely continuous. □

In the remainder of this paper, we let $\xi, \eta \in \mathbb{T}$ be such that $0 < \xi < \eta < T$ and denote

$$A = \left[\max_{t \in [0,\sigma^2(T)]_{\mathbb{T}}} \int_0^{\sigma(T)} G(t,s)\Delta s \right]^{-1},$$

$$B = \left[\max_{t \in [0,\sigma^2(T)]_{\mathbb{T}}} \int_\xi^{\eta} G(t,s)\Delta s \right]^{-1},$$

$$\varphi(r) = \max\left\{ g(t,u) + M \,\middle|\, t \in [0,T]_{\mathbb{T}}, \ u \in [-\sigma(T)\sigma^2(T)M, r] \right\}$$

and

$$\psi(r) = \min\left\{ g(t,u) + M \,\middle|\, t \in [\xi,\eta]_{\mathbb{T}}, \ u \in \left[\frac{\xi(\sigma^2(T)-\eta)r}{(\sigma^2(T))^2} - \sigma(T)\sigma^2(T)M, r \right] \right\}.$$

It is obvious that $0 < A < B$.

Now, we state and prove a basic existence criterion as follows:

Theorem 2.4. *Assume that there exist two positive numbers* r_1 *and* r_2 *such that* $\varphi(r_1) \leq r_1 A$ *and* $\psi(r_2) \geq r_2 B$. *Then, the BVP (1.1) has at least one solution* u^* *satisfying* $u^* + u_0 \in K$ *and*

$$\min\{r_1, r_2\} \leq \|u^* + u_0\| \leq \max\{r_1, r_2\}.$$

Moreover, if $\min\{r_1, r_2\} > \sigma(T)\sigma^2(T)M$, *then* u^* *is a positive solution of the BVP (1.1).*

Proof. Since $0 < A < B$, it is easy to see that $r_1 \neq r_2$. Without loss of generality, we assume that $r_1 < r_2$. Let

$$\Omega_i = \{u \in \mathbb{X} \mid \ \|u\| < r_i\}, \ i = 1, 2.$$

If $u \in K \cap \partial\Omega_1$, i.e., $u \in K$ and $\|u\| = r_1$, then $0 \leq u(t) \leq r_1$, $t \in [0, \sigma^2(T)]_{\mathbb{T}}$. So,

$$-\sigma(T)\sigma^2(T)M \leq u(t) - u_0(t) \leq r_1, \ t \in [0, \sigma^2(T)]_{\mathbb{T}}.$$

And so,

$$g(t, u(t) - u_0(t)) + M \leq \varphi(r_1) \leq r_1 A, \ t \in [0, T]_{\mathbb{T}}. \tag{2.12}$$

It follows that

$$\begin{aligned}
(\Phi u)(t) &= \int_0^{\sigma(T)} G(t,s)[g(s, u(s) - u_0(s)) + M]\Delta s \\
&\leq r_1 A \int_0^{\sigma(T)} G(t,s)\Delta s \\
&\leq r_1 A \max_{t \in [0,\sigma^2(T)]_{\mathbb{T}}} \int_0^{\sigma(T)} G(t,s)\Delta s \\
&= r_1, \ t \in [0, \sigma^2(T)]_{\mathbb{T}},
\end{aligned}$$

which shows that

$$\|\Phi u\| \leq \|u\| \ \text{ for } u \in K \cap \partial\Omega_1. \tag{2.13}$$

If $u \in K \cap \partial\Omega_2$, i.e., $u \in K$ and $\|u\| = r_2$, then for $t \in [\xi, \eta]_{\mathbb{T}}$, we have

$$\frac{\xi\left(\sigma^2(T) - \eta\right) r_2}{\left(\sigma^2(T)\right)^2} \leq q(t) r_2 \leq u(t) \leq r_2$$

and

$$\frac{\xi(\sigma^2(T) - \eta) r_2}{(\sigma^2(T))^2} - \sigma(T)\sigma^2(T)M \leq u(t) - u_0(t) \leq r_2.$$

So,

$$g(t, u(t) - u_0(t)) + M \geq \psi(r_2) \geq r_2 B, \ t \in [\xi, \eta]_{\mathbb{T}}. \tag{2.14}$$

It follows that

$$\begin{aligned}\|\Phi u\| &= \max_{t\in[0,\sigma^2(T)]_{\mathbb{T}}}\int_0^{\sigma(T)} G(t,s)[g(s,u(s)-u_0(s))+M]\Delta s\\ &\geq \max_{t\in[0,\sigma^2(T)]_{\mathbb{T}}}\int_\xi^{\eta} G(t,s)[g(s,u(s)-u_0(s))+M]\Delta s\\ &\geq r_2 B\max_{t\in[0,\sigma^2(T)]_{\mathbb{T}}}\int_\xi^{\eta} G(t,s)\Delta s\\ &= r_2,\end{aligned}$$

i.e.,

$$\|\Phi u\| \geq \|u\| \text{ for } u\in K\cap\partial\Omega_2. \tag{2.15}$$

In view of (2.13), (2.15), Lemma 2.3, and Theorem 1.1, we know that the operator Φ has at least one fixed point $u\in K\cap(\overline{\Omega_2}\backslash\Omega_1)$, which implies that the BVP (2.7) has at least one solution $u\in K$ such that $r_1\leq\|u\|\leq r_2$. Therefore, $u^*=u-u_0$ is a solution of the BVP (1.1) such that

$$u^*+u_0\in K \text{ and } r_1\leq\|u^*+u_0\|\leq r_2. \tag{2.16}$$

Moreover, if $r_1>\sigma(T)\sigma^2(T)M$, then for any $t\in(0,\sigma^2(T))_{\mathbb{T}}$, by (2.16) and Lemma 2.2, we have

$$\begin{aligned}u^*(t) &= [u^*(t)+u_0(t)]-u_0(t)=[u^*(t)+u_0(t)]-Mp(t)\\ &\geq q(t)\,\|u^*+u_0\|-q(t)\sigma(T)\sigma^2(T)M\\ &\geq q(t)r_1-q(t)\sigma(T)\sigma^2(T)M\\ &= \left[r_1-\sigma(T)\sigma^2(T)M\right]q(t)\\ &> 0,\end{aligned}$$

which shows that u^* is a positive solution of the BVP (1.1). □

Next, based on Theorem 2.4, we establish some criteria which ensure the existence of n solutions and/or positive solutions to the BVP (1.1); here n is an arbitrary positive integer.

Corollary 2.5. *Suppose that there exist three positive numbers r_1, r_2 and r_3 with $r_1<r_2<r_3$ such that one of the following conditions is satisfied:*

(a) $\varphi(r_1)\leq r_1A,\ \psi(r_2)>r_2B,\ \varphi(r_3)\leq r_3A,$

or

(b) $\psi(r_1)\geq r_1B,\ \varphi(r_2)<r_2A,\ \psi(r_3)\geq r_3B.$

Then the BVP (1.1) has at least two solutions u_1^*, u_2^* satisfying $u_1^*+u_0$, $u_2^*+u_0\in K$ and

$$r_1\le\|u_1^*+u_0\|<r_2<\|u_2^*+u_0\|\le r_3.$$

Moreover, if $r_2>\sigma(T)\sigma^2(T)M$, then u_2^* is a positive solution of the BVP (1.1), and if $r_1>\sigma(T)\sigma^2(T)M$, then u_1^*, u_2^* are both positive solutions of the BVP (1.1).

Proof. It is enough to prove case (a). Since $\frac{\psi(r)}{r}:(0,+\infty)\to[0,+\infty)$ is continuous and $\frac{\psi(r_2)}{r_2}>B$, there exist two positive numbers $\widetilde{r_2}$ and $\overline{r_2}$ with $r_1<\widetilde{r_2}<r_2<\overline{r_2}<r_3$ such that $\psi(\widetilde{r_2})\ge\widetilde{r_2}B$ and $\psi(\overline{r_2})\ge\overline{r_2}B$. It follows from Theorem 2.4 that the BVP (1.1) has at least two solutions u_1^*, u_2^* satisfying $u_1^*+u_0$, $u_2^*+u_0\in K$ and

$$r_1\le\|u_1^*+u_0\|\le\widetilde{r_2}<r_2<\overline{r_2}\le\|u_2^*+u_0\|\le r_3.$$

□

Corollary 2.6. *Suppose that there exist four positive numbers r_1, r_2, r_3 and r_4 with $r_1<r_2<r_3<r_4$ such that one of the following conditions is satisfied:*

$$\text{(a) } \varphi(r_1)\le r_1A,\ \psi(r_2)>r_2B,\ \varphi(r_3)<r_3A,\ \psi(r_4)\ge r_4B,$$

or

$$\text{(b) } \psi(r_1)\ge r_1B,\ \varphi(r_2)<r_2A,\ \psi(r_3)>r_3B,\ \varphi(r_4)\le r_4A.$$

Then the BVP (1.1) has at least three solutions u_1^*, u_2^*, u_3^* satisfying $u_1^*+u_0$, $u_2^*+u_0$, $u_3^*+u_0\in K$ and

$$r_1\le\|u_1^*+u_0\|<r_2<\|u_2^*+u_0\|<r_3<\|u_3^*+u_0\|\le r_4.$$

Moreover, if $r_3>\sigma(T)\sigma^2(T)M$, then u_3^* is a positive solution of the BVP (1.1), if $r_2>\sigma(T)\sigma^2(T)M$, then u_2^*, u_3^* are both positive solutions of the BVP (1.1), and if $r_1>\sigma(T)\sigma^2(T)M$, then u_1^*, u_2^*, u_3^* are all positive solutions of the BVP (1.1).

Proof. We only prove case (a). Since $\frac{\psi(r)}{r}:(0,+\infty)\to[0,+\infty)$, $\frac{\varphi(r)}{r}:(0,+\infty)\to[0,+\infty)$ are continuous and $\frac{\psi(r_2)}{r_2}>B$, $\frac{\varphi(r_3)}{r_3}<A$, there exist four positive numbers $\widetilde{r_2}$, $\overline{r_2}$, $\widetilde{r_3}$, $\overline{r_3}$ with $r_1<\widetilde{r_2}<r_2<\overline{r_2}<\widetilde{r_3}<r_3<\overline{r_3}<r_4$ such that $\psi(\widetilde{r_2})\ge\widetilde{r_2}B$, $\psi(\overline{r_2})\ge\overline{r_2}B$, $\varphi(\widetilde{r_3})\le\widetilde{r_3}A$, $\varphi(\overline{r_3})\le\overline{r_3}A$. It follows from Theorem 2.4 that the BVP (1.1) has at least three solutions u_1^*, u_2^*, u_3^* satisfying $u_1^*+u_0$, $u_2^*+u_0$, $u_3^*+u_0\in K$ and

$$r_1\le\|u_1^*+u_0\|\le\widetilde{r_2}<r_2<\overline{r_2}\le\|u_2^*+u_0\|\le\widetilde{r_3}<r_3<\overline{r_3}\le\|u_3^*+u_0\|\le r_4.$$

□

Similarly, for arbitrary positive integer n, the existence results of n solutions and/or positive solutions to the BVP (1.1) still hold.

Example 2.7. Consider the following BVP

$$\left\{\begin{array}{c} -u^{\Delta\Delta}(t) = 128\sqrt{t(u(t)+1)} - 1,\ t \in [0,1]_{\mathbb{T}}, \\ u(0) = 0 = u(1), \end{array}\right. \tag{2.17}$$

where $\mathbb{T} = \left\{0, \frac{1}{4}\right\} \cup [\frac{1}{2}, 1]$.

Let $T = 1$, $\xi = \frac{1}{4}$ and $\eta = \frac{1}{2}$. We first compute the values of A and B. In view of

$$\int_0^{\frac{1}{2}} G(t,s)\Delta s = \sum_{s \in [0, \frac{1}{2})_{\mathbb{T}}} \mu(s) G(t,s) = \left\{\begin{array}{c} 0,\ t = 0, \\ \frac{5}{64},\ t = \frac{1}{4}, \\ \frac{3(1-t)}{16},\ t \geq \frac{1}{2}, \end{array}\right.$$

and

$$\int_{\frac{1}{2}}^1 G(t,s)\Delta s = \left\{\begin{array}{c} \frac{t}{8},\ t \leq \frac{1}{2}, \\ -\frac{t^2}{2} + \frac{5t}{8} - \frac{1}{8},\ t \geq \frac{1}{2}, \end{array}\right.$$

we have

$$\int_0^1 G(t,s)\Delta s = \left\{\begin{array}{c} 0,\ t = 0, \\ \frac{7}{64},\ t = \frac{1}{4}, \\ -\frac{t^2}{2} + \frac{7t}{16} + \frac{1}{16},\ t \geq \frac{1}{2}. \end{array}\right.$$

So,

$$A = \left[\max_{t \in [0,1]_{\mathbb{T}}} \int_0^1 G(t,s)\Delta s\right]^{-1} = \frac{32}{5}.$$

Since

$$\int_{\frac{1}{4}}^{\frac{1}{2}} G(t,s)\Delta s = \sum_{s \in [\frac{1}{4}, \frac{1}{2})_{\mathbb{T}}} \mu(s) G(t,s) = \left\{\begin{array}{c} \frac{t}{8},\ t \leq \frac{1}{4}, \\ \frac{1-t}{8},\ t \geq \frac{1}{2}, \end{array}\right.$$

we get

$$B = \left[\max_{t \in [0,1]_{\mathbb{T}}} \int_{\frac{1}{4}}^{\frac{1}{2}} G(t,s)\Delta s\right]^{-1} = 16.$$

Then, it is easy to verify that all the conditions of Theorem 2.4 are satisfied if we let $g(t,u) = 128\sqrt{t(u+1)} - 1$, $(t,u) \in [0,1]_{\mathbb{T}} \times [-1, +\infty)$, $M = 1$, $r_1 = 10^4$ and $r_2 = 2$. So, the BVP (2.17) has at least one positive solution.

3. **ACKNOWLEDGEMENT**

JIAN-PING SUN was supported by the NSF of Gansu Province of China and WAN-TONG LI was supported by the NNSF of China (10571078).

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