Dynamic Systems and Applications 18 (2009) 179-190

GLOBAL STABILIZATION OF UNCERTAIN NONLINEAR TIME-DELAY SYSTEMS BY OUTPUT FEEDBACK

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ABSTRACT. In this paper, constructive control techniques have been proposed for controlling strict feedback (lower triangular form) nonlinear systems with a time delay in the state. The uncertain nonlinearities are assumed to be bounded by functions of the output multiplied by unmeasured states or delayed states. Based on the using of a linear dynamic high gain observer in combination with a linear dynamic high gain controller, the delay-independent output feedback controller making the closed-loop system globally asymptotically stable (GAS) is explicitly constructed. A simulation example is given to demonstrate the effectiveness of the proposed design procedure.

AMS (MOS) Subject Classification. 93C10

1. INTRODUCTION

Over the years, there have been constant progresses on the problem of global stabilization of triangular structural nonlinear systems by output feedback control. As investigated in [1], some extra growth conditions on the unmeasurable states of the system are usually necessary for the global stabilization of nonlinear systems via output feedback. In [2], under a linear growth assumption, a class of uncertain nonlinear systems were considered by output feedback control. In [3], the problem of robust output feedback control is considered for systems in lower triangular form under the global Lipschitz-like condition on the unmeasurable states with output dependent incremental rate. Global output feedback stabilization for uncertain nonlinear systems with output dependant incremental rate is considered in [4, 5, 6]. Unfortunately, in the literature mentioned above, the stabilization of triangular structural nonlinear time-delay systems has not been fully investigated and remains to be important and challenging.

Received November 20, 2008

The existence of time delays is frequently a source of instability. Hence, the problem of stability analysis of time-delay systems has been one of the main concerns of researchers wishing to inspect the properties of such systems, see [7, 8] and the references therein. The Lyapunov-Krasovskii method and Lyapunov-Razumikhin method are always employed in the stability analysis and robust control problem for linear time-delay systems. The result are often obtained in the form of linear matrix inequalities (LMIs). However little attention has been focused on nonlinear time-delay systems. Based on the backstepping method, strict-feedback nonlinear time-delay systems were considered in [9, 10], but the main results obtained were not quite correct [11, 12, 13, 14].

It was considered the problem of decentralized disturbance attenuation by state feedback for large-scale nonlinear systems with delayed state interconnections in [15] and Guo [15] considered only the systems with delays in parts of states. In [16], the problem of robust output feedback control was considered for time delay systems in lower triangular form under the global Lipschitz-like condition on the unmeasurable states with constant incremental rate, and the delay appears only in the output of the systems. Fu, et al. [17] studied the output feedback stabilization for a class of stochastic systems which only include output or delayed output nonlinearities.

In this paper, we consider nonlinear time-delay systems of the form

$$\begin{aligned} \dot{x}_i(t) &= x_{i+1}(t) + \phi_i(t, x(t), x(t-d), u), \quad i = 1, 2, \ldots, n-1, \\ \dot{x}_n(t) &= u + \phi_n(t, x(t), x(t-d), u), \\ y &= x_1(t), \end{aligned} \tag{1}$$

where $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^{\mathrm{T}}$ is the state, $u \in R$ and $y \in R$ are the system input and output, respectively, the functions $\phi_i : R^{2n+2} \to R$ are continuously uncertain functions, for $i = 1, 2, \ldots, n$. $d \geq 0$ is the time delay of the system.

Throughout this paper, we let $[x_{d,1}(t), \ldots, x_{d,n}(t)]^{\mathrm{T}} = x_d(t) = x(t-d)$. The argument of the functions will be omitted or simplified whenever no confusion can arise from the context. For example, we may denote $x_i(t)$ by x_i. For vector $\xi = (\xi_1, \xi_2, \ldots, \xi_n)$, we denote vector $(|\xi_1|, |\xi_2|, \ldots, |\xi_n|)$ by $|\xi|$. For matrix $B = [b_{ij}]_{m\times n}$, we denote matrix $[|b_{ij}|]_{m\times n}$ by $|B|$. The matrix $B = [b_{ij}]_{m\times n}$ is said to be a nonnegative matrix if $b_{ij} \geq 0$, for $1 \leq i \leq m$, $1 \leq j \leq n$. The property about the nonnegative matrix can be found in [19]. We let $\|\cdot\|$ denote the Euclidean norm for vector, or the induced Euclidean norm for matrix. I is used to represent an identity matrix of appropriate dimension.

In the first part of this paper, the following condition is assumed.

Assumption 1 For $i = 1, 2, \ldots, n$, there exist non-negative constant C and smooth function $L(y)$, such that

$$\begin{array}{rcl} |\phi_1(t,x,x_d,u)| & \leq & C\left(|x_1|+|x_{d,1}|\right) \\ |\phi_i(t,x,x_d,u)| & \leq & L(y)\sum_{j=1}^{i}\left(|x_j|+|x_{d,j}|\right). \end{array} \tag{2}$$

Remark 1 Under a linear growth assumption (e.g. $L(y)=C$), Qian and Lin [2] provide an output feedback controler for the sysytem (1) without delay in the state. Under a linear growth assumption, a class of uncertain nonlinear time-delay systems were considered by both state and output feedback control in [18].

In this note, constructive control techniques are proposed for controlling a class of strict feedback nonlinear systems with a time delay in the state. Comparing our paper with the existed work such as in [2, 4, 21], our paper has three contributions. Firstly, we generalize the results for the systems considered in [2] to the systems with delays in the state. Secondly, the use of a memoryless dynamical high gain observer in combination with a memoryless dynamical high gain controller is proposed to design the output feedback controller, while in [2, 4], it was applied that the use of a linear high gain observer in combination with the backstepping method, which has been widely used to deal with feedback nonlinear systems. Thirdly, we consider the global output stabilization problem for uncertain strict feedback nonlinear systems with *outputs dependent incremental rate*, while Chen and Huang [4] and Lei and Lin [21] only considered the global output stabilization problem for uncertain strict feedback nonlinear systems with *polynomial functions of output incremental rate.*

The following lemma is useful in the proof of our main result.

Lemma 1 There exist real numbers $\alpha>0$, a_k, b_k, $k=2,3,\ldots,n$, and symmetric matrices $P>0$, $Q>0$ satisfying the following inequalities:

$$PA+A^{\mathrm{T}}P\leq -I,\quad QB+B^{\mathrm{T}}Q\leq -I,$$

$$PD+DP-P\geq \alpha I,\quad QD+DQ-Q\geq \alpha I,$$

where

$$A=\begin{pmatrix} -a_2 & 1 & 0 & \cdots & 0 \\ -a_3 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -a_{n-1} & 0 & 0 & \cdots & 1 \\ -a_n & 0 & 0 & \cdots & 0 \end{pmatrix},$$

$$B=\begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -b_2 & -b_3 & -b_3 & \cdots & -b_n \end{pmatrix},$$

$$D=\mathrm{diag}[1,2,\ldots,n-1].$$

This Lemma could be thought as a corollary of Lemma 1 in reference [5], and we omit its proof here.

2. OUTPUT FEEDBACK CONTROLLER

Theorem 1 For a family of uncertain systems (1) satisfying Assumption 1, the following output feedback controller

$$\begin{array}{rcl} \dot{z}_2 &=& z_3 + r^2 a_3 y - \dot{r} a_2 y - r a_2 (z_2 + r a_2 y) \\ \dot{z}_3 &=& z_4 + r^3 a_4 y - 2\dot{r} r a_3 y - r^2 a_3 (z_2 + r a_2 y) \\ &\vdots& \\ \dot{z}_{n-1} &=& z_n + r^{n-1} a_n y - (n-2)\dot{r} r^{n-3} a_{n-1} y - r^{n-2} a_{n-1}(z_2 + r a_2 y) \\ \dot{z}_n &=& u(t) - (n-1)\dot{r} r^{n-2} a_n y - r^{n-1} a_n (z_2 + r a_2 y) \end{array} \tag{3}$$

$$u = -r^n \left(\left(\tfrac{b_2 M}{r} + \textstyle\sum_{i=2}^n (b_i a_i)\right) y + b_2 \tfrac{z_2}{r} + b_3 \tfrac{z_3}{r^2} + \cdots + b_n \tfrac{z_n}{r^{n-1}}, \right) \tag{4}$$

with the observer gain r being dynamically updated (see [4])

$$\begin{array}{rcl} \dot{r} &=& \tfrac{r}{\alpha} \max\left\{ \varpi(y) - \tfrac{r}{4},\ 0 \right\}, \\ r(t) &=& r_0 \geq 1, \quad \text{for } t \in [-d, 0], \end{array} \tag{5}$$

is such that the closed-loop system (1) and (3) is globally asymptotically stable (GAS) at the equilibrium $(x, z) = (0, 0)$, where M is positive constant determined by (24), $\varpi(\cdot)$ is a continuously differentiable positive function determined by (25), α, a_k and $b_k (k = 2, 3, \ldots, n)$ are constants given by Lemma 1.

Remark 2 It is obviously that the state $r(t)$ of the system (5) has the following three properties,

$$i: \qquad \dot{r} \geq 0, \quad t \geq 0, \tag{6}$$

$$ii: \qquad \tfrac{r}{4} + \alpha \tfrac{\dot{r}}{r} \geq \varpi(y), \quad t \geq 0 \tag{7}$$

$$iii: \qquad r(t) \geq r(t-d) \geq 1, \quad t \geq 0. \tag{8}$$

The property (8) shall play a key role in our dealing with the delay in the state of the system (1).

We now give a proof of the main result of this paper. The proof is constructive and carried out by using Lyapunov analysis. For the convenience of the readers, we break up the proof into three parts.

2.1. **The error dynamics and the closed-loop system.** As done in [20], by discarding the uncertain term $\phi_i(t, x(t), x(t-d), u)$ in system (1), we design a high-gain observer (3) with the gain $r(t)$ for the chain of integrators of (1). Let $e_i = z_i + r^{i-1}a_i y - x_i$, $i = 2, 3, \dots, n$ be the estimate errors. Then, the error dynamics is given by

$$\begin{aligned} \dot{e}_2 &= e_3 - ra_2e_2 + ra_2\phi_1 - \phi_2 \\ \dot{e}_3 &= e_4 - r^2a_3e_2 + r^2a_3\phi_1 - \phi_3 \\ &\vdots \\ \dot{e}_{n-1} &= e_n - r^{n-2}a_{n-1}e_2 + r^{n-2}a_{n-1}\phi_1 - \phi_{n-1} \\ \dot{e}_n &= -r^{n-1}a_ne_2 + r^{n-1}a_n\phi_1 - \phi_n, \end{aligned} \tag{9}$$

Letting $\varepsilon_i = \frac{e_i}{r^{i-1}}$, (9) can be converted to the following system

$$\begin{aligned} \dot{\varepsilon}_2 &= r\varepsilon_3 - ra_2\varepsilon_2 - \tfrac{\dot{r}}{r}\varepsilon_2 + (a_2\phi_1 - \tfrac{\phi_2}{r}) \\ \dot{\varepsilon}_3 &= r\varepsilon_4 - ra_3\varepsilon_2 - 2\tfrac{\dot{r}}{r}\varepsilon_3 + (a_3\phi_1 - \tfrac{\phi_3}{r^2}) \\ &\vdots \\ \dot{\varepsilon}_{n-1} &= r\varepsilon_n - ra_{n-1}\varepsilon_2 - (n-2)\tfrac{\dot{r}}{r}\varepsilon_{n-1} + (a_{n-1}\phi_1 - \tfrac{\phi_{n-1}}{r^{n-2}}) \\ \dot{\varepsilon}_n &= -ra_n\varepsilon_2 - (n-1)\tfrac{\dot{r}}{r}\varepsilon_n + (a_n\phi_1 - \tfrac{\phi_n}{r^{n-1}}). \end{aligned} \tag{10}$$

Defining

$$\eta_2 = \frac{z_2 + My}{r} + a_2y,$$

$$\eta_k = \tfrac{z_k}{r^{k-1}} + a_ky, \qquad k = 3, 4, \dots, n,$$

where M is a positive constant to be specified later, (3) can be converted to the following system

$$\begin{aligned} \dot{\eta}_2 &= r\eta_3 - ra_2\varepsilon_2 - \tfrac{\dot{r}}{r}\eta_2 + M(\eta_2 - \varepsilon_2) + a_2\phi_1 + \tfrac{M}{r}(\phi_1 - My) \\ \dot{\eta}_3 &= r\eta_4 - ra_3\varepsilon_2 - 2\tfrac{\dot{r}}{r}\eta_3 + a_3\phi_1 \\ &\vdots \\ \dot{\eta}_{n-1} &= r\eta_n - ra_{n-1}\varepsilon_2 - (n-2)\tfrac{\dot{r}}{r}\eta_{n-1} + a_{n-1}\phi_1 \\ \dot{\eta}_n &= \tfrac{u}{r^{n-1}} - ra_n\varepsilon_2 - (n-1)\tfrac{\dot{r}}{r}\eta_n + a_n\phi_1. \end{aligned} \tag{11}$$

Letting

$$u = -r^n(b_2\eta_2 + b_3\eta_3 + \dots + b_n\eta_n) \ ,$$

where b_k $(k = 2, 3, \dots, n)$ are constants given by Lemma 1, from (1), (10) and (11), a simple calculation gives

$$\dot{y} = r\eta_2 - r\varepsilon_2 - My + \phi_1 \ , \tag{12}$$

$$\dot{\varepsilon} = rA\varepsilon - \tfrac{\dot{r}}{r}D\varepsilon + A_1\phi_1 - \Phi \ , \tag{13}$$

$$\dot{\eta} = rB\eta - \tfrac{\dot{r}}{r}D\eta - rA_1\varepsilon_2 + A_1\phi_1 + \Upsilon \ , \tag{14}$$

where

$$\varepsilon = [\varepsilon_2, \varepsilon_3, \dots, \varepsilon_n]^{\mathrm{T}}, \quad \eta = [\eta_2, \eta_3, \dots, \eta_n]^{\mathrm{T}},$$

$$A_1 = \begin{pmatrix} a_2 \\ a_3 \\ \vdots \\ a_{n-1} \\ a_n \end{pmatrix}, \quad \Phi = \begin{pmatrix} \frac{\phi_2}{r} \\ \frac{\phi_3}{r^2} \\ \vdots \\ \frac{\phi_{n-1}}{r^{n-2}} \\ \frac{\phi_n}{r^{n-1}} \end{pmatrix},$$

$$\Upsilon = \left(M(\eta_2 - \varepsilon_2) + \frac{M}{r}(\phi_1 - My),\ 0,\ 0, \ldots,\ 0 \right)^{\mathrm{T}},$$

and A, B, D as defined in Lemma 1.

In the next two parts, we will choose a positive constant M and a continuously differentiable positive function $\varpi(\cdot)$ such that the all states of the closed-loop system (5), (12), (13), (14) are bounded, and (12), (13), (14), for any observer gain function $r(t)$ generated by (5), is GAS at $(y, \varepsilon, \eta) = (0, 0, 0)$.

2.2. **Lyapunov analysis of the closed-loop system.** Choosing $V_y = \frac{1}{2}y^2$, $V_o = r\varepsilon^{\mathrm{T}}P\varepsilon$ and $V_c = r\eta^{\mathrm{T}}Q\eta$, and using Lemma 1, we have

$$\begin{aligned} \dot{V}_y|_{(12)} &= y(r\eta_2 - r\varepsilon_2 - My + \phi_1) \\ &\le -My^2 + \tfrac{1}{4}r^2\|\eta\|^2 + C_1 r^2\|\varepsilon\|^2 + C_2 y^2 + C_3 y^2(t-d), \end{aligned} \tag{15}$$

where C_i, $i = 1, 2, 3$, are constants independent of M,

$$\begin{aligned} \dot{V}_o|_{(13)} &= \dot{r}\varepsilon^{\mathrm{T}}P\varepsilon + r(rA\varepsilon - \tfrac{\dot{r}}{r}D\varepsilon + A_1\phi_1 - \Phi)^{\mathrm{T}}P\varepsilon \\ &\quad + r\varepsilon^{\mathrm{T}}P(rA\varepsilon - \tfrac{\dot{r}}{r}D\varepsilon + A_1\phi_1 - \Phi) \\ &\le -r^2\|\varepsilon\|^2 - \alpha\dot{r}\|\varepsilon\|^2 + 2r\varepsilon^{\mathrm{T}}PA_1\phi_1 - 2r\varepsilon^{\mathrm{T}}P\Phi, \end{aligned} \tag{16}$$

$$\begin{aligned} \dot{V}_c|_{(14)} &= \dot{r}\eta^{\mathrm{T}}Q\eta + r(rB\eta - \tfrac{\dot{r}}{r}D\eta - rA_1\varepsilon_2 + A_1\phi_1 + \Upsilon)^{\mathrm{T}}Q\eta \\ &\quad + r\eta^{\mathrm{T}}Q(rB\eta - \tfrac{\dot{r}}{r}D\eta - rA_1\varepsilon_2 + A_1\phi_1 + \Upsilon) \\ &\le -r^2\|\eta\|^2 - \alpha\dot{r}\|\eta\|^2 - 2r^2\eta^{\mathrm{T}}QA_1\varepsilon_2 + 2r\eta^{\mathrm{T}}QA_1\phi_1 + 2r\eta^{\mathrm{T}}Q\Upsilon. \end{aligned} \tag{17}$$

Now, let us estimate the last two terms of (16) and last three terms of (17).

From the definitions of ε_i and η_i, we can get

$$\frac{x_2}{r} = \eta_2 - \varepsilon_2 - \frac{My}{r},$$

$$\frac{x_i}{r^{i-1}} = \eta_i - \varepsilon_i. \quad i = 3, 4, \ldots, n$$

From Assumption 1, and (8), we can get

$$\begin{aligned} r|\tfrac{\phi_i(\cdot)}{r^{i-1}}| &\le L(y)\Big((1+M)|y| + r\textstyle\sum_{j=2}^{i}(|\eta_j| + |\varepsilon_j|) + (1+M)|y(t-d)| \\ &\quad + r^{\frac{1}{2}}r^{\frac{1}{2}}(t-d)\textstyle\sum_{j=2}^{i}(|\eta_j(t-d)| + |\varepsilon_j(t-d)|)\Big), \qquad i = 2, \ldots, n. \end{aligned}$$

Using the inequality $2a\,b \le \gamma\, a^2 + \frac{1}{\gamma}\, b^2$ $(a, b \in R, \gamma > 0)$, we can get

$$2r\varepsilon^{\mathrm{T}}PA_1\phi_1 \le \tfrac{1}{2}r^2\|\varepsilon\|^2 + C_4 y^2 + C_4 y^2(t-d), \tag{18}$$

$$2r\varepsilon^{\mathrm{T}}P\Phi \leq rF_1(y,M)\|\varepsilon\|^2 + rF_2(y)\|\eta\|^2 + k_1r(t-d)\|\varepsilon(t-d)\|^2 + k_2r(t-d)\|\eta(t-d)\|^2 + C_5y^2 + C_5y^2(t-d), \quad (19)$$

$$2r^2\eta^{\mathrm{T}}QA_1\varepsilon_2 \leq \tfrac{1}{4}r^2\|\eta\|^2 + C_6r^2\|\varepsilon\|^2, \quad (20)$$

$$2r\eta^{\mathrm{T}}QA_1\phi_1 \leq \tfrac{1}{4}r^2\|\eta\|^2 + C_7y^2 + C_7y^2(t-d), \quad (21)$$

and

$$2r\eta^{\mathrm{T}}Q\Upsilon \leq k_3(M)r\|\eta\|^2 + k_4(M)r\|\varepsilon\|^2 + C_8y^2 + C_9y(t-d)^2, \quad (22)$$

where $C_j > 0$, $j = 4, 5, \ldots, 9$, are constants independent of M, $F_1(\cdot)$ is a function of y and M, $F_2(\cdot)$ is a function of y, k_1 and k_2 are constants, $k_3(\cdot)$ and $k_4(\cdot)$ are functions of M.

Construct the following LKF

$$V = \tilde{V}_y + 4(C_1 + C_6)\tilde{V}_o + \tilde{V}_c,$$

where

$$\tilde{V}_y = V_y + \int_{t-d}^{t} C_3y^2(s)ds,$$

$$\tilde{V}_o = V_o + \int_{t-d}^{t} \left(k_1r(s)\|\varepsilon(s)\|^2 + k_2r(s)\|\eta(s)\|^2 + (C_4 + C_5)y^2(s)\right) ds$$

$$\tilde{V}_c = V_c + \int_{t-d}^{t}(C_7 + C_9)y^2(s)ds.$$

Using (15)–(22), one can get

$$\begin{aligned}
&\dot{V}|_{(12)(13)(14)} \\
&\leq 4(C_1 + C_6)(-\tfrac{1}{4}r^2\|\varepsilon\|^2 - \alpha\dot{r}\|\varepsilon\|^2 + r(F_1(y,M) + k_1)\|\varepsilon\|^2 \\
&\quad + r(F_2(y) + k_2)\|\eta\|^2 + 2(C_4 + C_5)y^2) - \tfrac{1}{4}r^2\|\eta\|^2 - \alpha\dot{r}\|\eta\|^2 + 2C_7y^2 \\
&\quad + k_3(M)r\|\eta\|^2 + k_4(M)r\|\varepsilon\|^2 + (C_8 + C_9)y^2 + (-My^2 + (C_2 + C_3)y^2) \\
&\leq (-M + C_2 + C_3 + 8(C_1 + C_6)(C_4 + C_5) + 2C_7 + C_8 + C_9)y^2 \\
&\quad + 4r(C_1 + C_6)(-\tfrac{1}{4}r - \alpha\tfrac{\dot{r}}{r} + F_1(y,M) + \tfrac{k_4(M)}{4(C_1+C_6)} + k_1)\|\varepsilon\|^2 \\
&\quad + r(-\tfrac{1}{4}r - \alpha\tfrac{\dot{r}}{r} + 4(C_1 + C_6)(F_2(y) + k_2) + k_3(M))\|\eta\|^2
\end{aligned} \quad (23)$$

Now we can find a constant M, such that

$$M > \delta_1 + C_2 + C_3 + 8(C_1 + C_6)(C_4 + C_5) + 2C_7 + C_8 + C_9), \quad (24)$$

where δ_1 is any positive constant, and then we can find a smooth function $\varpi(\cdot)$ such that, for any s, the following inequities hold, simultaneously,

$$\begin{aligned}
\varpi(s) &\geq \delta_2 + F_1(s,M) + \tfrac{k_4(M)}{4(C_1+C_6)} + k_1, \\
\varpi(s) &\geq \delta_3 + 4(C_1 + C_6)(F_2(s) + k_2) + k_3(M)),
\end{aligned} \quad (25)$$

where δ_2 and δ_3 are any positive constants.

By means of (23) and (7), we have

$$\dot{V}|_{(12)(13)(14)} \leq -\delta_1y^2 - 4r(C_1 + C_6)\delta_2\|\varepsilon\|^2 - r\delta_3\|\eta\|^2. \quad (26)$$

Remark 3 The choice of M and $\varpi(\cdot)$ is very important for our controller design. M should be specified firstly, then based on the choice of M, $\varpi(\cdot)$ is specified.

2.3. **Boundedness of the closed-loop system and convergence of the states (y, ε, η).** Next we shall prove that all states of the closed-loop system(5), (12), (13) and (14) are bounded. To this end, defining

$$\bar{V} = \int_0^V S(\tau)\, d\tau,$$

where $S(\cdot)$ is a nondecreasing function satisfying $S(\tau) > 0$, $\forall\, \tau \geq 0$, we have

$$\begin{array}{rcl} \dot{\bar{V}}|_{(12)(13)(14)} & \leq & S(V)\dot{V} \\ & \leq & S(V)\,(-\delta_1 y^2 - 4r(C_1 + C_6)\delta_2\|\varepsilon\|^2 - r\delta_3\|\eta\|^2) \\ & \leq & S(V)\,(-\delta_1 y^2) \\ & \leq & -\delta_1 S\left(\frac{y^2}{2}\right) y^2. \end{array} \tag{27}$$

On the other hand, let $r^* = 4\varpi(0)$, and

$$V_r(r) = \alpha\left[r - r^* - r^* \ln\left(\tfrac{r}{r^*}\right)\right],$$

which is continuously differentiable, proper, and nonnegative in $(0, +\infty)$, see [3] or [4]. As in [4], we can prove that

$$\dot{V}_r(r) \leq [\varpi(y) - \varpi(0)]^2. \tag{28}$$

From (27) and (28), we obtain that

$$\tfrac{d\{\bar{V} + V_r(r)\}}{dt} \leq -\delta_1 S\left(\frac{y^2}{2}\right) y^2 + [\varpi(y) - \varpi(0)]^2 \leq 0$$

by appropriate choice of $S(\cdot)$. As a result, $r(t)$ is bounded, hence all states of the closed-loop system (5), (12), (13) and (14) are bounded.

From (26), we can conclude that the closed-loop (12), (13), (14) is asymptotically stable at $(y, \varepsilon, \eta) = (0, 0, 0)$, and hence the closed-loop (1) and (3) with

$$\begin{array}{rcl} u & = & -r^n(b_2\eta_2 + b_3\eta_3 + \cdots + b_n\eta_n) \\ & = & -r^n(b_2(\frac{z_2 + My}{r} + a_2 y) + b_3(\frac{z_3}{r^{3-1}} + a_3 y) + \cdots + b_n(\frac{z_n}{r^{n-1}} + a_n y)) \\ & = & -r^n\left((\frac{b_2 M}{r} + \sum_{i=2}^n (b_i a_i))y + b_2\frac{z_2}{r} + b_3\frac{z_3}{r^2} + \cdots + b_n\frac{z_n}{r^{n-1}}\right), \end{array}$$

for any observer gain function $r(t)$ generated by (5), is GAS at $(x, z) = (0, 0)$. Therefore, we can conclude that the system (3), (4), (5) is the linear output feedback controller of the system (1). Thus completing the proof of Theorem 1.

Remark 4 It is easy to see that $(y(t), \varepsilon(t), \eta(t), r(t)) \equiv (0, 0, 0, \hat{r})$ for $t \in [-d,\ +\infty)$ satisfies the equations (5)(12)(13)(14), where constant $\hat{r} \geq r^* = 4\varpi(0)$. Hence, it is impossible to prove that the closed-loop (5), (12), (13), (14) is asymptotically stable at solution $(\varepsilon(t), \bar{z}(t), r(t)) \equiv (0, 0, r^*)$. But from the analysis above, the boundedness of states of the closed-loop (5), (12), (13), (14) could be verified. Then, based on (26) and the the boundedness of states of the closed-loop (5), (12), (13), (14), the asymptotical stability of parts of states (i.e. y, ε and η) could be derived.

Remark 5 The time delay $d \geq 0$ of the system (1) could be any known or unknown constant, and it it is not difficult to verified that Theorem 1 is also hold for the system (1) with a known time-varying time delay in the state.

3. AN EXAMPLE

Consider a nonlinear time-delay system of the form:

$$\begin{aligned} \dot{x}_1 &= x_2 + \mu_1 \ln(1 + x_1^2(t-d)), \\ \dot{x}_2 &= u + \mu_2 e^{x_1} x_2^2(t-d), \\ y &= x_1, \end{aligned} \tag{29}$$

where $-\frac{1}{10} \leq \mu_1 \leq \frac{1}{10}$, $-\frac{1}{6} \leq \mu_2 \leq \frac{1}{6}$, $d = 0.5$.

It is easy to verified that Assumption 1 holds for the system (29). We can get the linear output feedback controller for the system (29)

$$u = -\frac{r^2}{2}\left(\frac{z + 1.7781y}{r} + \frac{1}{2}y\right), \tag{30}$$

where z and $r(t)$ are the state of the system

$$\begin{aligned} \dot{z} &= -\tfrac{r^2}{2}\left(\tfrac{z+1.7781y}{r} + \tfrac{1}{2}y\right) - \tfrac{1}{2}\dot{r}y - \tfrac{1}{2}r(z + \tfrac{r}{2}y) \\ &= -rz - 0.8891ry - \tfrac{1}{2}r^2 y - \tfrac{1}{2}\dot{r}y \\ \dot{r} &= r \max\left\{20e^y + 30 - \tfrac{r}{4},\ 0\right\} \\ r(t) &= 10, \quad \text{for } \ t \in [-0.5, 0]. \end{aligned} \tag{31}$$

Fig. 1-Fig. 2 show the state response of the closed-loop system (29), (30) and (31) with $\mu_1 = \frac{1}{10}$ and $\mu_2 = \frac{1}{6}$, for the initial condition, for $t \in [-0.5,\ 0]$,

$$[x_1(t),\ x_2(t),\ z(t),\ r(t)] = [-5,\ 40,\ -3,\ 10].$$

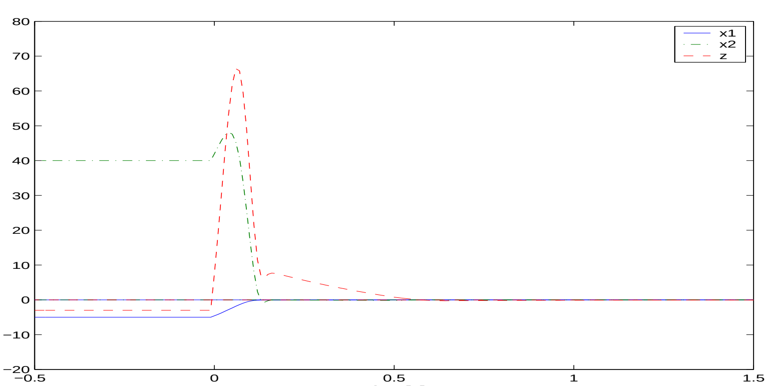

Fig.1: Trajectories of x_1, x_2 and z.

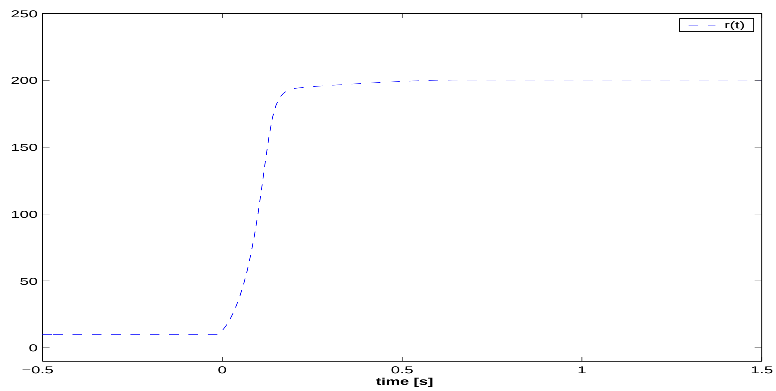

Fig.2: Trajectory of $r(t)$.

4. CONCLUSION

In this paper, a high gain linear output feedback control is proposed for nonlinear time-delay systems in lower triangular form with outputs dependent incremental rate. It is well known that, for the feedback (lower triangular form) systems, the gains of stabilizing controller are, in general, very high, see [2, 3, 4], and the high gains may make the system cause a undesirable transient behavior. Hence, one always wishes to get small gains when designing stabilizing controller for feedback systems. Our $b_i, i = 2, 3, \ldots, n$ in $u = -r^n(b_2\eta_2 + b_3\eta_3 + \cdots + b_n\eta_n)$, are constants given by Lemma 1. While the b_i in [2, 4] are determined by iterative procedure of the backstepping method. Hence, the gains of our controllers are much lower than those in [2, 4]. The method introduced here is maybe feasible for the feedforward (upper triangular form) nonlinear systems.

ACKNOWLEDGEMENT

This work was supported by the National Natural Foundation of China under grant No. 60774004, and the research award foundation for outstanding young and middle-aged scientists in Shandong Province under grant No. 2006BS01022.

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