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ON INITIAL VALUE PROBLEMS FOR FIRST-ORDER IMPLICIT IMPULSIVE FUZZY DIFFERENTIAL EQUATIONS

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ABSTRACT. In this paper, by using Banach contraction mapping principle theorem, we obtain some new existence and uniqueness theorems of solutions for a new class of initial value problems of first-order implicit impulsive fuzzy differential equations in the metric space of normal fuzzy convex sets with distance given by maximum of the Hausdorff distance between level sets.

Keywords: First-order implicit impulsive fuzzy differential equation, initial value problem, normal fuzzy convex sets, fixed point, existence and uniqueness

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1. INTRODUCTION

Let E^n be the set of real fuzzy numbers, $J = [t_0, t_0 + a] \subset R = (-\infty, +\infty)$ be a compact interval, $f : J \times E^n \times E^n \to E^n$ be continuous and $I_k \in C[E^n, E^n]$ for all $k = 1, 2, \ldots, m$. For each fixed $x_0 \in E^n$ and any constant $\lambda \geq 0$, we consider the following initial value problem of first order implicit impulsive fuzzy differential equation:

Find $x : J \to E^n$ such that

$$(1.1)\qquad \begin{cases} x'(t) = f(t, x(t), \lambda x'(t)), & t \neq t_k, \\ \triangle x|_{t=t_k} = I_k(x(t_k)), & (k = 1, 2, \ldots, m), \\ x(t_0) = x_0. \end{cases}$$

Some special cases of the problem (1.1) are as follows:

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(1) If $\lambda = 1$, then problem (1.1) reduces to finding $x : J \to E^n$ such that

$$\begin{cases} x'(t) = f(t, x(t), x'(t)), & t \neq t_k, \\ \triangle x|_{t=t_k} = I_k(x(t_k)), & (k = 1, 2, \ldots, m), \\ x(t_0) = x_0. \end{cases} \tag{1.2}$$

The problem (1.2) was introduced and studied by Huang and Lan [1] in real Banach spaces.

(2) If $\lambda = 0$, then problem (1.1) becomes to finding $x : J \to E^n$ such that

$$\begin{cases} x'(t) = f(t, x(t)), & t \neq t_k, \\ \triangle x|_{t=t_k} = I_k(x(t_k)), & (k = 1, 2, \ldots, m), \\ x(t_0) = x_0, \end{cases} \tag{1.3}$$

The study of such types of problems is motivated by an increasing interest in the differential equations with applications in Banach spaces and fuzzy differential equations under various initial and boundary conditions. In 1972, Chang and Zadeh [2] first introduced the concept of fuzzy derivative. Afterwards, the framework for the study of fuzzy differential equations has also been developed and the basic properties of solutions of fuzzy differential equations and applications are available (see, for example, [3–15] and the references therein).

Recently, Nieto [11] proved a version of the classical Peano existence theorem for initial value problems for a fuzzy differential equation in the metric space of normal fuzzy convex sets with the distance given by the maximum of the Hausdorff distance between level sets. The results of Nieto [11] complements the existence and uniqueness result of Kaleva [8]. Very recently, Nieto and Rodríguez-López [12] found sufficient conditions for the boundedness of every solution of first-order fuzzy differential equations as well as certain fuzzy integral equations. Georgiou et al. [16] considered nth-order fuzzy differential equations with initial value conditions and proved the existence and uniqueness of solution for nonlinearities satisfying a Lipschitz condition.

Further, by using Banach fixed point theorem, Lan and Huang [17] obtained some new existence and uniqueness theorems of solutions for a class of initial value problems of nonlinear first order implicit fuzzy differential equations in the metric space of normal fuzzy convex sets E^n with distance given by maximum of the Hausdorff distance between level sets:

$$\begin{cases} x'(t) = f(t, x(t), \lambda x'(t)), \\ x(t_0) = x_0. \end{cases}$$

On the other hand, the theory of impulsive differential equations or implicit impulsive integro-differential equations has been emerging as an important area of investigation in recent years and has been developed very rapidly due to the fact that such equations find a wide range of applications modeling adequately many real processes observed in physics, chemistry, biology and engineering (see, for example, [1, 18] and the references therein). Correspondingly, applications of the theory of impulsive differential equations to different areas were considered by many authors and some basic results on impulsive differential equations have been obtained (see, for example, [19–23], and, in particular [21] and the references therein). Furthermore, some basic results on impulsive fuzzy differential equations have also been studied by several authors, see [24, 25], but the theory still remains to be developed.

Motivated and inspired by the above works, in this paper, by using Banach fixed point theorem, we obtain some new existence and uniqueness theorems of solutions for the initial value problem (1.1) of nonlinear first order implicit impulsive fuzzy differential equations in the metric space of normal fuzzy convex sets with distance given by maximum of the Hausdorff distance between level sets.

2. PRELIMINARIES

Let $\mathcal{P}_k(R^n)$ denote the family of non-empty compact, convex subsets of R^n. If $\alpha, \beta \in R$ and $A, B \in \mathcal{P}_k(R^n)$

$$\alpha(A+B) = \alpha A + \alpha B,$$
$$\alpha(\beta A) = (\alpha\beta)A, \quad 1 \cdot A = A$$

and if $\alpha, \beta \geq 0$, then $(\alpha+\beta)A = \alpha A + \beta A$. For $A, B \in \mathcal{P}_k(R^n)$, the Hausdorff metric is defined as

$$d(A,B) = \max\left\{ \sup_{x\in A}\inf_{y\in B} \|x-y\|, \ \sup_{y\in B}\inf_{x\in A} \|x-y\| \right\}.$$

A fuzzy set in R^n is a function with domain R^n and values in $[0,1]$, i.e., an element of $[0,1]^{R^n}$ (see [26]). Let $u, v \in [0,1]^{R^n}$. Then we have (see [26])

(a) u is contained in v denoted by $u \leq v$ if and only if $u(x) \leq v(x)$ for all $x \in R^n$;
(b) $u \wedge v \in [0,1]^{R^n}$ by $(u \wedge v)(x) = \min\{u(x), v(x)\}$ for all $x \in R^n$ (intersection);
(c) $u \vee v \in [0,1]^{R^n}$ by $(u \vee v)(x) = \max\{u(x), v(x)\}$ for all $x \in R^n$ (union);
(d) $u^c \in [0,1]^{R^n}$ by $u^c(x) = 1 - u(x)$ for all $x \in R^n$.

Denote by $E^n = \{u : R^n \to [0,1]$ such that u satisfies (i) to (iv) mentioned below$\}$:

(i) u is normal, that is, there exists an $x_0 \in R$ such that $u(x_0) = 1$;
(ii) u is fuzzy convex, that is, for $x, y \in R^n$ and $0 \leq \nu \leq 1$,

$$u(\nu x + (1-\nu)y) \geq \min\{u(x), u(y)\};$$

(iii) u is upper semicontinuous;
(iv) $[u]^0 = \overline{\{x \in R^n : u(x) > 0\}}$ is compact.

Thus, if $u \in E^n$, then it follows from (i)-(iv) that, for each $\alpha \in (0,1]$, the α-level set

$$[u]^\alpha = \{x \in R^n : u(x) \geq \alpha\}$$

is a nonempty compact convex subset of R^n, that is, $[u]^\alpha \in \mathcal{P}_k(R^n)$ for all $0 \leq \alpha \leq 1$. Further, define $D : E^n \times E^n \to [0, +\infty)$ as

$$D(u,v) = \sup\{d([u]^\alpha, [v]^\alpha) : \alpha \in [0,1]\}.$$

It is well known that D is a metric in E^n and (E^n, D) is a complete metric space (see [27]). Moreover, if $u, v, w \in E^n$ and $\lambda > 0$, then the addition and (positive) scalar multiplication in E^n are defined in terms of the α-level sets by

$$[u+v]^\alpha = [u]^\alpha + [v]^\alpha, \quad [\lambda \cdot u]^\alpha = \lambda[u]^\alpha, \quad \forall \alpha \in [0,1]$$

and D has a linear structure in the sense that

$$D(u+w, v+w) = D(u,v), \quad D(\lambda u, \lambda v) = \lambda D(u,v).$$

Note that (E^n, D) is not a vector space but it can be embedded isomorphically as a cone in a Banach space (see [9]).

Let $J = [t_0, t_0 + a]$ with $a > 0$ and $x, y \in E^n$. A mapping $F : J \to E^n$ is differentiable at $t \in J$ if there exists a $F'(t) \in E^n$ such that the limits

$$\lim_{h \to 0^+} \frac{F(t+h) - F(t)}{h}$$

and

$$\lim_{h \to 0^+} \frac{F(t) - F(t-h)}{h}$$

exist and are equal to $F'(t)$. Here the limits are taken in the metric space (E^n, D). At the endpoints of J, we consider the one-sided derivatives.

Let $F : J \to E^n$. Then the integral of F over J denoted by $\int_J F(t)dt$, is defined levelwise by the equation

$$\begin{aligned} [\int_J F(t)dt]^\alpha &= \int_J F_\alpha(t)dt \\ &= \left\{ \int_J F(t)dt | \, F : J \to R^n \text{is a measurable selection for } F_\alpha \right\}. \end{aligned}$$

We say that a mapping $F : J \to E^n$ is strongly measurable if, for all $\alpha \in [0,1]$, the set-valued mapping $F_\alpha : J \to \mathcal{P}_k(R^n)$ is defined by $F_\alpha(t) = [F(t)]^\alpha$. Moreover, the following results (see [7]) will be useful in what follows.

Lemma 2.1. *If $F : J \to E^n$ is continuous, then it is integrable and the function*

$$G(t) = \int_{t_0}^t F(s)ds, \quad t \in J$$

is differentiable and $G'(t) = F(t)$*. Furthermore,*

$$F(t) - F(t_0) = \int_{t_0}^{t} F'(s)ds.$$

3. MAIN RESULTS

In this section, we are in a position to prove our main results concerning with the solutions of first order fuzzy differential equation problems (1.1)–(1.3).

Throughout this paper, let $J = [t_0, t_0 + a]$ (where $a > 0$), $t_0 < t_1 < \cdots < t_m < t_0 + a < +\infty$, $J_0 = [t_0, t_1]$, $J_1 = (t_1, t_2], \ldots, J_k = (t_k, t_{k+1}], \ldots, J_m = (t_m, t_0 + a]$ and

$$\begin{aligned} PC^1(J, E^n) \;=\; \{x : x \text{ is a map from } J \text{ into } E^n \text{ such that } x(t) \text{ is} \\ \text{continuously differentiable on } (t_k, t_{k+1}), \text{ left continuous at } t_k, \\ \text{and } x(t_k^+), x'(t_k^-), x'(t_k^+) \text{ exists }, k = 1, 2, \ldots, m\}. \end{aligned}$$

where $x(t_k^+)$ represents the right limits of $x(t)$ at $t = t_k$, and $x'(t_k^-)$ and $x'(t_k^+)$ represent the left and right derivatives of $x(t)$ at $t = t_k$, respectively. For $x \in PC^1(J, E^n)$, by virtue of the mean value theorem

$$x(t_k) - x(t_k - h) \in h\overline{co}\{x'(t) : t_k - h < t < t_k\} \quad (h > 0),$$

it is easy to see that the left derivative $x'_-(t_k)$ exists and

$$x'_-(t_k) = \lim_{h \to 0^+} h^{-1}[x(t_k) - x(t_k - h)] = x'(t_k^-).$$

In the sequel, $x'(t_k)$ is understood as $x'_-(t_k)$. Further, we define $H(x, y)$ by

$$H(x, y) = \sup_{t \in J}\{D(x(t), y(t)) + D(x'(t), y'(t))\} \tag{3.1}$$

for all $x, y \in PC^1(J, E^n)$, where $\Gamma > 0$ is a constant. Then, by using the same method as in [7], it is clear that $(PC^1(J, E^n), H)$ is a complete metric space.

By using Lemma 1 and Lemma 2.1 of [28], it is easy to prove the following lemma.

Lemma 3.1. *Assume that* $f : J \times E^n \times E^n \to E^n$ *is continuous. Then a mapping* $x : J \to E^n$ *is a solution of problem* (1.1) *in* $PC^1(J, E^n)$ *if and only if* x *satisfies the following impulsive integral equation*

$$x(t) = x_0 e^{-M(t-t_0)} + \int_{t_0}^{t} e^{-M(t-s)}[f(s, x(s), \lambda x'(s)) + Mx(s)]ds + \sum_{t_0 < t_k < t} e^{-M(t-t_k)} I_k(x(t_k)),$$

where $M > 0$ *is a constant.*

Theorem 3.1. *Suppose that $f : J \times E^n \times E^n \to E^n$ and $g : J \times E^n \to E^n$ is continuous. If for all $x_i, y_i : J \to E^n$ $(i = 1, 2)$, there exist nonnegative constants ρ, ϱ and b such that for all $t \in J$,*

$$D(f(t, x_1(t), y_1(t)), f(t, x_2(t), y_2(t))) \leq \rho D(x_1(t), x_2(t)) + \varrho D(y_1(t), y_2(t)), \tag{3.2}$$

$$D(I_k(x_1(t)), I_k(x_2(t))) \leq bD(x_1(t), x_2(t)),\ \forall k = 1, 2, \ldots, m. \tag{3.3}$$

Then problem (1.1) *has a unique solution on J.*

Proof. Let $(\tau_1, \bar{x}(t)) \in J \times E^n$ be arbitrary and $\delta > 0$ be a constant such that $\sigma = \max\{\rho + mb + h(\rho + M) + M(1 - mb - h(\rho + M)),\ h\lambda\varrho + \lambda\varrho(1 - hM)\} < 1$, where $h = \frac{e^{M\delta}-1}{M}$ and $\lambda > 0$ is a constant. We will first show that the initial value problem

$$\begin{cases} x'(t) = f(t, x(t), \lambda x'(t)), & \forall t \in J_1 \quad \text{and} \quad t \neq t_k, \\ \triangle x|_{t=t_k} = I_k(x(t_k)), & (k = 1, 2, \ldots, m), \\ x(\tau_1) = \bar{x}, \end{cases} \tag{3.4}$$

has a unique solution on $J_1 = [\tau_1, \tau_1 + \delta]$. For any $x \in PC^1(J, E^n)$, define Fx on J_1 by the equation

$$\begin{aligned} F(x(t)) = \bar{x}e^{-M(t-\tau_1)} &+ \int_{\tau_1}^{t} e^{-M(t-s)}[f(s, x(s), \lambda x'(s)) + Mx(s)]ds \\ &+ \sum_{\tau_1 < t_k < t} e^{-M(t-t_k)} I_k(x(t_k)), \end{aligned} \tag{3.5}$$

In the sequel, we prove that $F : PC^1(J, E^n) \to PC^1(J, E^n)$ is a contraction mapping. Indeed, for any given $x \in PC^1(J, E^n)$ and $t \neq t_k$, $k = 1, 2 \ldots, m$, it follows from (3.5) that

$$(Fx)'(t) = -MF(x(t)) + Mx(t) + f(t, x(t), \lambda x'(t)), \tag{3.6}$$

and so $Fx \in PC^1(J, E^n)$, i.e., F is a mapping from $PC^1(J, E^n)$ into $PC^1(J, E^n)$. By virtue of (3.2), (3.3) and (3.5), for any $x_1, x_2 \in PC^1(J, E^n)$,

$$\begin{aligned} &D(F(x_1(t)), F(x_2(t))) \\ &\quad \leq \int_{\tau_1}^{\tau_1+\delta} e^{-M(t-s)}[D(f(s, x_1(s), \lambda x_1'(s)), f(s, x_2(s), \lambda x_2'(s))) \\ &\qquad + MD(x_1(s), x_2(s))]ds + \sum_{\tau_1 < t_k < t} e^{-M(t-t_k)} D(I_k(x_1(t_k)), I_k(x_2(t_k))) \\ &\quad = e^{-Mt} \int_{\tau_1}^{\tau_1+\delta} e^{Ms}[D(f(s, x_1(s), \lambda x_1'(s)), f(s, x_2(s), \lambda x_2'(s))) \\ &\qquad + MD(x_1(s), x_2(s))]ds + \sum_{\tau_1 < t_k < t} e^{-M(t-t_k)} D(I_k(x_1(t_k)), I_k(x_2(t_k))) \end{aligned}$$

$$\leq e^{-Mt}\int_{\tau_1}^{\tau_1+\delta} e^{Ms}[(\rho+M)D(x_1(s),x_2(s)) + \lambda\varrho D(x_1'(s),x_2'(s))]ds + \sum_{\tau_1<t_k<t} e^{-M(t-t_k)}bD(x_1(t_k),x_2(t_k))$$

$$= e^{-Mt}\int_{\tau_1}^{\tau_1+\delta} e^{Ms}[(\rho+M)D(x_1(s),x_2(s)) + \lambda\varrho D(x_1'(s),x_2'(s))]ds + B(t), \tag{3.7}$$

where

$$B(t) = \sum_{\tau_1<t_k<t} be^{-M(t-t_k)}D(x_1(t_k),x_2(t_k)) \leq bD(x_1(t),x_2(t)) \sum_{t_0<t_k<t} e^{-M(t-t_k)},$$

and so

$$\sup_{t\in J} e^{-\Gamma t}B(t) \leq bD(x_1(t),x_2(t)) \max_{1\leq k\leq m}\{C_k\}, \tag{3.8}$$

where

$$C_k = \sup_{t\in J_k}\sum_{t_0<t_j<t} e^{-M(t-t_j)}, \quad \forall 1\leq k\leq m.$$

Since

$$C_k = \sup_{t\in J_k}[\sum_{j=1}^{k-1} e^{-M(t-t_j)} + e^{-M(t-t_k)}] \leq \sup_{t\in J_k}\sum_{j=1}^{k-1} e^{-M(t-t_j)} + \sup_{t\in J_k} e^{-M(t-t_k)} = D_k + E_k$$

for all $1 \leq k \leq m$, and

$$D_k = \sup_{t\in J_k}\sum_{j=1}^{k-1} e^{-M(t-t_j)}, \quad E_k = \sup_{t\in J_k} e^{-M(t-t_k)}.$$

For all $1 \leq j \leq k-1$, setting $\nu_j(t) = e^{-M(t-t_j)}$ and $\delta_* = \min\{t_{k+1}-t_k|1 \leq k \leq m\}$, we have

$$\nu_j(t) \leq e^{-M\delta_*}, \quad \forall t \in J_k = (t_k, t_{k+1}],$$

and so

$$D_k \leq \sum_{j=1}^{k-1} e^{-M\delta_*} \leq \sum_{j=1}^{m-1} e^{-M\delta_*} = (m-1)e^{-M\delta_*} < m-1, \quad \forall 1 \leq k \leq m.$$

Now we consider E_k: take $\nu_k(t) = e^{-M(t-t_k)}$, $t \in J_k$. Since $t - t_k \geq 0$ and $-M(t-t_k) \leq 0$, we know that $\nu_k(t) \leq 1$, i.e., $E_k \leq 1$.

Hence, $E_k \leq 1$ and $C_k \leq m$ for all $1 \leq k \leq m$. It follows from (3.8) that

$$\sup_{t\in J} B(t) \leq mbD(x_1(t),x_2(t)). \tag{3.9}$$

It follows from (3.7) and (3.9) that

$$D(F(x_1(t)),F(x_2(t))) \leq e^{-Mt}\int_{\tau_1}^{\tau_1+\delta} e^{Ms}[(\rho+M)D(x_1(s),x_2(s)) + \lambda\varrho D(x_1'(s),x_2'(s))]ds$$

$$
\begin{aligned}
&+ mbD(x_1(t), x_2(t))\\
&\le \frac{e^{-M(t-\tau_1)}}{M}(e^{M\delta}-1)[(\rho+M)D(x_1(t), x_2(t)) + \lambda\varrho D(x_1'(t), x_2'(t))]\\
&\quad + mbD(x_1(t), x_2(t))\\
&\le [mb + h(\rho+M)]D(x_1(t), x_2(t)) + h\lambda\varrho D(x_1'(t), x_2'(t)), \qquad (3.10)
\end{aligned}
$$

where $h = \frac{e^{M\delta}-1}{M}$.

Further, by (3.6) and the above proof, now we know

$$
\begin{aligned}
D((Fx_1)'(t), (Fx_2)'(t)) &= D(-MF(x_1(t)) + Mx_1(t) + f(t, x_1(t), \lambda x_1'(t)),\\
&\qquad - MF(x_2(t)) + Mx_2(t) + f(t, x_2(t), \lambda x_2'(t)))\\
&= -MD(F(x_1(t)), F(x_2(t)))\\
&\qquad + MD(x_1(t), x_2(t)) + D(f(t, x_1(t), \lambda x_1'(t)), f(t, x_2(t), \lambda x_2'(t)))\\
&\le -M\{[mb + h(\rho+M)]D(x_1(t), x_2(t)) + h\lambda\varrho D(x_1'(t), x_2'(t))\}\\
&\qquad + MD(x_1(t), x_2(t)) + \rho D(x_1(t), x_2(t)) + \lambda\varrho D(x_1'(t), x_2'(t))\\
&= [M(1 - mb - h(\rho+M)) + \rho]D(x_1(t), x_2(t))\\
&\qquad + \lambda\varrho(1 - hM)D(x_1'(t), x_2'(t)). \qquad (3.11)
\end{aligned}
$$

From (3.1), (3.10) and (3.11), we have

$$
\begin{aligned}
H(Fx_1, Fx_2) &= \sup_{t\in J_1}\{D(F(x_1(t)), F(x_2(t))) + D((Fx_1)'(t), (Fx_2)'(t))\}\\
&\le D(F(x_1(t)), F(x_2(t))) + D((Fx_1)'(t), (Fx_2)'(t))\\
&\le [mb + h(\rho+M)]D(x_1(t), x_2(t)) + h\lambda\varrho D(x_1'(t), x_2'(t))\\
&\quad + [M(1 - mb - h(\rho+M)) + \rho]D(x_1(t), x_2(t))\\
&\quad + \lambda\varrho(1 - hM)D(x_1'(t), x_2'(t))\\
&\le \sigma H(x_1, x_2),
\end{aligned}
$$

where $\sigma = \max\{\rho + mb + h(\rho+M) + M(1 - mb - h(\rho+M)),\ h\lambda\varrho + \lambda\varrho(1 - hM)\}$. Therefore, by Banach fixed point theorem, F has a unique fixed point, which by Lemma 2 is the desired solution to problem (3.4).

Express J as a union of a finite family of intervals J_k with the length of each interval less than δ. The preceding paragraph guarantees the existence of a unique solution to problem (1.1) on each interval J_k. Piecing these solutions together gives us the unique solution on the whole interval J. This completes the proof. □

Remark 3.1. If $\varrho = 0$ in (3.2), then we can obtain the corresponding result for problem (1.1).

Theorem 3.2. *Let $f : J \times E^n \times E^n \to E^n$ be a continuous mapping. Assume that for all $x_i, y_i : J \to E^n$ $(i = 1, 2)$, there exist nonnegative constants ρ, ϱ and b such that for all $t \in J$, $\alpha \in [0, 1]$ and $k = 1, 2, \ldots, m$,*

$$d([f(t, x_1(t), y_1(t))]^\alpha, [f(t, x_2(t), y_2(t))]^\alpha) \leq \rho d([x_1(t)]^\alpha, [x_2(t)]^\alpha) + \varrho d([y_1(t)]^\alpha, [y_2(t)]^\alpha)$$

and

$$d([I_k(x_1(t))]^\alpha, [I_k(x_2(t))]^\alpha) \leq b d([x_1(t)]^\alpha, [x_2(t)]^\alpha).$$

Then problem (1.1) *has a unique solution on J.*

Proof. In fact, we have

$$\begin{aligned}
&D(f(t, x_1(t), y_1(t)), f(t, x_2(t), y_2(t))) \\
&= \sup\{d([f(t, x_1(t), y_1(t))]^\alpha, [f(t, x_2(t), y_2(t))]^\alpha) : \alpha \in [0, 1]\} \\
&\leq \rho \sup\{d([x_1(t)]^\alpha, [x_2(t)]^\alpha) : \alpha \in [0, 1]\} + \varrho \sup\{d([y_1(t)]^\alpha, [y_2(t)]^\alpha) : \alpha \in [0, 1]\} \\
&= \rho D(x_1(t), x_2(t)) + \varrho D(y_1(t), y_2(t))
\end{aligned}$$

and

$$\begin{aligned}
D(I_k(x_1(t)), I_k(x_2(t))) &= \sup\{d([I_k(x_1(t))]^\alpha, [I_k(x_2(t))]^\alpha) : \alpha \in [0, 1]\} \\
&\leq b \sup\{d([x_1(t)]^\alpha, [x_2(t)]^\alpha) : \alpha \in [0, 1]\} \\
&= b D(x_1(t), x_2(t)).
\end{aligned}$$

Thus, by Theorem 3.1 we know that problem (1.1) has a unique solution on J. □

Remark 3.2. Using the same method as Theorems 3.1–3.2, we can consider initial value problems (1.2)–(1.3) and get the corresponding conclusions.

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