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DELAY-DEPENDENT H_∞ CONTROL FOR MARKOVIAN JUMP SYSTEMS WITH DELAYS VARYING IN A RANGE

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ABSTRACT. This paper presents a new method for H_∞ state feedback controller design of Markovian jump systems with delays varying in a range. Firstly, results on delay-dependent stability and H_∞ performance are established by exploiting a new Lyapunov functional making full use of information about time delay and Jensen's inequality. Neither free weighting variables nor model transformation is used in the derivation of less conservative criteria. Secondly, based on the obtained conditions, a new design of state feedback controller is developed in terms of linear matrix inequalities (LMIs). Finally, illustrative examples are presented to show the advantage and validity of the proposed approaches.

Key Words. Markovian jump systems; time delay; H_∞ control

1. INTRODUCTION

Markovian jump systems (MJSs) can be regarded as a special class of hybrid systems with finite operation modes whose structures are subject to random abrupt changes [1, 2]. The studies of MJSs are important in practical applications such as manufacturing systems, aircraft control, target tracking, robotics, solar receiver control, and power systems. Therefore, a great deal of attention has been devoted to the study of this class of systems in recent years such as [3, 4, 5, 6, 7].

On the other hand, time delays are commonly encountered in practical engineering, such as in chemical processes, heating systems, biological systems, network systems, and so on. They are often sources of instability or poor performance. Therefore, various research topics on delay systems have been reported in recent years. The results on time delay systems are classified into two categories: that are delay-independent and delay-dependent criteria. As we know, delay-independent methods in [8, 9] are more conservative than delay-dependent ones in [10, 11, 12, 13, 14], since time delay is not taken into the consideration in the processing of analyzing stability

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or designing controller especially when the size of time delay is small. In order to obtain delay-dependent criteria, the bounding technology in [15, 16, 17] used to evaluate the bounds of some cross terms arising from the analysis of delay-dependent problem and the model transformation technique, such as [18, 19], is employed to transform the original delay models into simpler ones. However, there is still conservativeness through bounding technology or model transformation technique. Therefore, considerable efforts have been made to the development of free weighting matrix methods which are generally classified into two types: that is one with null summing equations added to the Lyapunov functional derivative [20, 21, 22], and one with free matrix items added to the Lyapunov functional combined with the descriptor model transformation [18, 23, 24].

In practice, the range of delay may vary in a range for which the lower bound is not restricted to be 0. A typical example of system with time-varying delay in a range is networked control system (NCS) [22, 25]. From [20, 25], we know that it is more conservative without taking into account the information of the lower bound of delay. In this paper, a new Lyapunov functional is proposed, where both upper bound and lower bound of time delay are contained. In order to further reduce the conservatism of stability criteria and to remove the restriction on the derivative of time-varying delay, the derived term $\int_{t-\tau(t)}^{t} x^T(s)(\sum_{j=1}^{N} \lambda_{ij} Q_j)x(s)ds$ coming from the new Lyapunov functional of MJSs is separated into two parts dealt with respectively. And the derivative of the different Lyapunov functional is estimated via Jensen's inequality without introducing slack matrix variables or model transformation. Based on the developed criteria, a new scheme of H_∞ controller is given in terms of LMIs overcoming the difficulties resulting from the introduction of mode-dependent matrix Q_i in Lyapunov functional. Finally, examples are presented to illustrate the benefits and effectiveness of the proposed approaches in this paper.

The notations in this paper are quite standard. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. $\mathbb{R}^{m\times n}$ is the set of all $m \times n$ real matrices. The superscript "T" denotes matrix transposition. $\mathbb{N}$ is the set of natural numbers. $\mathbb{E}\{\cdot\}$ denotes the expectation operator with respect to some probability measure. In symmetric block matrices, we use " $*$ " as an ellipsis for the terms that are introduced by symmetry, that is

$$\begin{bmatrix} L & N \\ N^T & R \end{bmatrix} = \begin{bmatrix} L & N \\ * & R \end{bmatrix}$$

2. PRELIMINARIES

Consider the following continuous-time MJS described by

$$\begin{cases} \dot{x}(t) = A(\eta(t))x(t) + A_d(\eta(t))x(t-\tau(t)) + B(\eta(t))u(t) + E(\eta(t))\omega(t) \\ y(t) = C(\eta(t))x(t) + C_d(\eta(t))x(t-\tau(t)) + F(\eta(t))\omega(t) \\ x(t) = \phi(t), \ \forall t \in [-\tau_2, 0] \end{cases} \tag{2.1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input, $\omega(t) \in \mathbb{R}^p$ is the disturbance input and $y(t) \in \mathbb{R}^q$ is the control output. The parameter $\eta(t)$ is a continuous-time Markov process with right continuous trajectory taking values in a finite set $\mathbb{S} = \{1, 2, \dots, N\}$ with transition probabilities

$$\Pr(\eta(t+\triangle) = j|\eta(t) = i) = \begin{cases} \lambda_{ij}\triangle + o(\triangle) & i \neq j \\ 1 + \lambda_{ii}\triangle + o(\triangle) & i = j \end{cases} \tag{2.2}$$

where $\triangle > 0$, $\lim_{\triangle \to 0^+}(o(\triangle)/\triangle) = 0$ and the transition probability rate satisfies $\lambda_{ij} \geq 0$, for $i, j \in \mathbb{S}$, $i \neq j$ and

$$\lambda_{ii} = -\sum_{j=1, j\neq i}^{N} \lambda_{ij} \tag{2.3}$$

For notational simplicity, in the sequel, for each possible $\eta(t) = i$, $i \in \mathbb{S}$, the matrix $A(\eta(t))$ will be denoted by A_i, and so on. $\tau(t)$ denotes the time varying delay satisfying

$$0 \leq \tau_1 \leq \tau(t) \leq \tau_2, \ \dot{\tau}(t) \leq \mu \quad \forall \, i \in \mathbb{S} \tag{2.4}$$

Definition 1. System (2.1) is said to be stochastically stable if, when $u(t) = 0$ and $\omega(t) = 0$, there exist a constant $M(\phi(t), \eta_0)$ such that

$$\mathbb{E}\left\{\int_0^\infty x^T(t)x(t)dt|\phi(t), \eta_0\right\} \leq M(\phi(t), \eta_0) \tag{2.5}$$

Definition 2. Given $\gamma > 0$, system (2.1) with $u(t) = 0$ is said to be stochastically stable with γ-disturbance attenuation if it is stochastically stable and satisfies

$$\mathbb{E}\left\{\int_0^\infty y^T(t)y(t)dt\right\} < \gamma^2\mathbb{E}\{\int_0^\infty \omega^T(t)\omega(t)dt\} \tag{2.6}$$

for zero-initial condition and any nonzero $\omega(t) \in \mathcal{L}_2[0, \infty)$.

In this paper, the H_∞ controller such that the resulting closed-loop system is stochastically stable with (2.6) is given as

$$u(t) = K(\eta(t))x(t) \tag{2.7}$$

where $K(\eta(t))$ is to be determined.

3. MAIN RESULTS

In this section, we firstly give a delay-dependent stability condition for system (2.1).

Theorem 3. *Given scalars $0 \leq \tau_1 \leq \tau_2$, system (2.1) with $u(t) = 0$ and $\omega(t) = 0$ is stochastically stable if there exist matrices $P_i > 0$, $Q_i > 0$, $Z_l > 0$, $T_q > 0$ and $R_q > 0$ such that*

$$\begin{bmatrix} \Omega_{i1} & P_iA_{di} & T_1 & 0 & \tau_1 A_i^T T_1 & \tau_{12} A_i^T T_2 \\ * & \Omega_{i2} & T_2 & T_2 & \tau_1 A_{di}^T T_1 & \tau_{12} A_{di}^T T_2 \\ * & * & \Omega_{i3} & 0 & 0 & 0 \\ * & * & * & \Omega_{i4} & 0 & 0 \\ * & * & * & * & -T_1 & 0 \\ * & * & * & * & * & -T_2 \end{bmatrix} < 0 \tag{3.1}$$

$$\sum_{j=1}^{N} \lambda_{ij} Q_j \leq R_q \tag{3.2}$$

where

$$\Omega_{i1} = P_iA_i + A_i^T P_i + \sum_{j=1}^{N} \lambda_{ij} P_j + Q_i + Z_1 + Z_2 + \tau_1 R_1 + \tau_{12} R_2 - T_1$$
$$\Omega_{i2} = -(1-\mu)Q_i - 2T_2$$
$$\Omega_{i3} = -Z_1 - T_1 - T_2 + Z_3, \Omega_{i4} = -Z_2 - T_2 - Z_3, \tau_{12} = \tau_2 - \tau_1$$

hold for all $i \in \mathbb{S}$, $l = 1, 2, 3$, $q = 1, 2$.

Proof. Let $x_t(s) = x(t+s)$, $-\tau(t) \leq s \leq 0$, similar to [10], $\{(x_t, \eta(t)), t \geq 0\}$ is also a Markov process. Choose a stochastic Lyapunov-Krasovskii functional for system (2.1) as

$$V(x_t, \eta(t)) = V_1(x_t, \eta(t)) + V_2(x_t, \eta(t)) + V_3(x_t, \eta(t)) + V_4(x_t, \eta(t)) \tag{3.3}$$

where

$$\begin{aligned} V_1(x_t, \eta(t)) =& x^T(t) P(\eta(t)) x(t) \\ V_2(x_t, \eta(t)) =& \int_{t-\tau(t)}^{t} x^T(s) Q(\eta(t)) x(s) ds + \sum_{l=1}^{2} \int_{t-\tau_l}^{t} x^T(s) Z_l x(s) ds \\ &+ \int_{t-\tau_2}^{t-\tau_1} x^T(s) Z_3 x(s) ds \\ V_3(x_t, \eta(t)) =& \int_{-\tau_1}^{0} \int_{t+\theta}^{t} \tau_1 \dot{x}^T(s) T_1 \dot{x}(s) ds d\theta + \int_{-\tau_2}^{-\tau_1} \int_{t+\theta}^{t} \tau_{12} \dot{x}^T(s) T_2 \dot{x}(s) ds d\theta \\ V_4(x_t, \eta(t)) =& \int_{-\tau_1}^{0} \int_{t+\theta}^{t} x^T(s) R_1 x(s) ds d\theta + \int_{-\tau_2}^{-\tau_1} \int_{t+\theta}^{t} x^T(s) R_2 x(s) ds d\theta \end{aligned}$$

Let $\mathcal{L}$ be the weak infinite generator, when $\omega(t)=0$, we will get

$$\begin{aligned}\mathcal{L}[V(x_t,\eta(t))] \le{}& x^T(t)(2P_iA_i+\sum_{j=1}^{N}\lambda_{ij}P_j+Q_i+\sum_{l=1}^{2}Z_l+\tau_1R_1+\tau_{12}R_2)x(t)\\ &+2x^T(t)P_iA_{di}x(t-\tau(t))-(1-\mu)x^T(t-\tau(t))Q_ix(t-\tau(t))\\ &+\int_{t-\tau(t)}^{t}x^T(s)(\sum_{j=1}^{N}\lambda_{ij}Q_j)x(s)ds-\sum_{l=1}^{2}x^T(t-\tau_l)Z_lx(t-\tau_l)\\ &+x^T(t-\tau_1)Z_3x(t-\tau_1)-x^T(t-\tau_2)Z_3x(t-\tau_2)\\ &+\tau_1^2\dot{x}^T(t)T_1\dot{x}(t)-\tau_1\int_{t-\tau_1}^{t}\dot{x}^T(s)T_1\dot{x}(s)ds\\ &+\tau_{12}^2\dot{x}^T(t)T_2\dot{x}(t)-\tau_{12}\int_{t-\tau_2}^{t-\tau_1}\dot{x}^T(s)T_2\dot{x}(s)ds\\ &-\int_{t-\tau_1}^{t}x^T(s)R_1x(s)ds-\int_{t-\tau_2}^{t-\tau_1}x^T(s)R_2x(s)ds\end{aligned}\tag{3.4}$$

Moreover, we have

$$\int_{t-\tau(t)}^{t}x^T(s)\breve{Q}_ix(s)ds=\int_{t-\tau(t)}^{t-\tau_1}x^T(s)\breve{Q}_ix(s)ds+\int_{t-\tau_1}^{t}x^T(s)\breve{Q}_ix(s)ds\tag{3.5}$$

where $\breve{Q}_i=\sum_{j=1}^{N}\lambda_{ij}Q_j$. Taking into accounting (3.2) and (3.5), it can be verified that

$$\begin{aligned}&\int_{t-\tau(t)}^{t}x^T(s)\breve{Q}_ix(s)ds-\int_{t-\tau_1}^{t}x^T(s)R_1x(s)ds-\int_{t-\tau_2}^{t-\tau_1}x^T(s)R_2x(s)ds\\ &\quad=\int_{t-\tau_1}^{t}x^T(s)(\breve{Q}_i-R_1)x(s)ds+\int_{t-\tau(t)}^{t-\tau_1}x^T(s)\breve{Q}_ix(s)ds-\int_{t-\tau_2}^{t-\tau_1}x^T(s)R_2x(s)ds\\ &\quad\le\int_{t-\tau_1}^{t}x^T(s)(\breve{Q}_i-R_1)x(s)ds+\int_{t-\tau(t)}^{t-\tau_1}x^T(s)(\breve{Q}_i-R_2)x(s)ds\le 0\end{aligned}\tag{3.6}$$

By Jensen's inequality, we derive

$$-\tau_1\int_{t-\tau_1}^{t}\dot{x}^T(s)T_1\dot{x}(s)ds\le-[x(t)-x(t-\tau_1)]^TT_1[x(t)-x(t-\tau_1)]\tag{3.7}$$

and

$$\begin{aligned}&-\tau_{12}\int_{t-\tau_2}^{t-\tau_1}\dot{x}^T(s)T_2\dot{x}(s)ds\\ &\quad=-\tau_{12}\int_{t-\tau_2}^{t-\tau(t)}\dot{x}^T(s)T_2\dot{x}(s)ds-\tau_{12}\int_{t-\tau(t)}^{t-\tau_1}\dot{x}^T(s)T_2\dot{x}(s)ds\\ &\quad\le-[x(t-\tau(t))-x(t-\tau_2)]^TT_2[x(t-\tau(t))-x(t-\tau_2)]\\ &\qquad-[x(t-\tau_1)-x(t-\tau(t))]^TT_2[x(t-\tau_1)-x(t-\tau(t))]\end{aligned}\tag{3.8}$$

Noting from (3.4)-(3.8), we obtain that

$$\mathcal{L}[V(x_t, \eta(t))] \leq \xi^T(t)\Upsilon(\eta(t))\xi(t) < 0 \tag{3.9}$$

where

$$\xi^T(t) = \begin{bmatrix} x^T(t) & x^T(t-\tau(t)) & x^T(t-\tau_1) & x^T(t-\tau_2) \end{bmatrix}$$

$$\Upsilon_i = \begin{bmatrix} \Omega_{i1} & P_iA_{di} & T_1 & 0 \\ * & \Omega_{i2} & T_2 & T_2 \\ * & * & \Omega_{i3} & 0 \\ * & * & * & \Omega_{i4} \end{bmatrix} + \begin{bmatrix} A_i^T \\ A_{di}^T \\ 0 \\ 0 \end{bmatrix} \tilde{T} \begin{bmatrix} A_i^T \\ A_{di}^T \\ 0 \\ 0 \end{bmatrix}^T$$

$$\tilde{T} = \tau_1^2 T_1 + \tau_{12}^2 T_2$$

Via Schur complement for (3.1), it can be shown that (3.9) holds. The next is similar to the existing references such as [10, 12, 14]. This completes the proof. □

Remark 4. Theorem 1 provides the delay-dependent stochastic stability condition for MJSs with time delays varying in a range, where both the lower and upper bounds of $\tau(t)$ are considered. From the above analysis, we see that not only $x^T(t)P(\eta(t))x(t)$ but also $\int_{t-\tau(t)}^t x^T(s)Q(\eta(t))x(s)ds$ are mode-dependent, which are derived from considering the property of MJSs with time varying delays. Compared with unique matrix Q in $V_2(x_t, \eta(t))$, which is $\int_{t-\tau(t)}^t x^T(s)Qx(s)ds$, $Q(\eta(t))$ can reduce the conservatism, which is dependent on system mode $\eta(t)$. Especially, when $Q_1 = Q_2 = \cdots = Q_N$, it will be mode-independent. In order to further reduce conservativeness, terms $\int_{t-\tau(t)}^t x^T(s)\breve{Q}_i x(s)ds$ and $\int_{t-\tau_2}^{t-\tau_1} \dot{x}^T(s)T_2\dot{x}(s)ds$ in Lyapunov functional derivation are separated into two parts and are handled respectively.

Remark 5. In practice, the time-varying delay often lies in an interval, in which the lower bound is not 0. Therefore, the introduction of the lower bound τ_1 compared with neglecting the lower bound will naturally reduce the conservatism. As we know, slack weighting matrices introduced in the derivation of Lyapunov functional can obtain less conservative criteria, see, e.g., [20, 21, 22, 25]. However, too many free matrices will increase the computational complexity especially for MJSs. In this paper, the separated terms $\int_{t-\tau_2}^{t-\tau(t)} \dot{x}^T(s)T_2\dot{x}(s)ds$ and $\int_{t-\tau(t)}^{t-\tau_1} \dot{x}^T(s)T_2\dot{x}(s)ds$ in addition to $\int_{t-\tau_1}^t \dot{x}^T(s)T_1\dot{x}(s)ds$ are estimated via Jensen's inequality to get less conservative results instead of introducing slack variables.

Next, we will analyze the H_∞ disturbance attenuation performance of (2.1).

Theorem 6. *Given scalars $0 \leq \tau_1 \leq \tau_2$ and $\gamma > 0$, system (2.1) with $u(t) = 0$ is stochastically stable with H_∞ performance attenuation if there exist matrices $P_i > 0$,*

$Q_i > 0$, $Z_l > 0$, $T_q > 0$ and $R_q > 0$ such that

$$\begin{bmatrix} \Omega_{i1} & P_iA_{di} & T_1 & 0 & P_iE_i & C_i^T & \tau_1A_i^TT_1 & \tau_{12}A_i^TT_2 \\ * & \Omega_{i2} & T_2 & T_2 & 0 & C_{di}^T & \tau_1A_{di}^TT_1 & \tau_{12}A_{di}^TT_2 \\ * & * & \Omega_{i3} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \Omega_{i4} & 0 & 0 & 0 & 0 \\ * & * & * & * & -\gamma^2I & F_i^T & \tau_1E_i^TT_1 & \tau_{12}E_i^TT_2 \\ * & * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & * & -T_1 & 0 \\ * & * & * & * & * & * & * & -T_2 \end{bmatrix} < 0 \tag{3.10}$$

$$\sum_{j=1}^{N} \lambda_{ij}Q_j \leq R_q \tag{3.11}$$

hold for all $i \in \mathbb{S}$, $l = 1, 2, 3$, $q = 1, 2$.

Proof. From Schur complement, it is shown that (3.10) implies (3.1). Next, for all nonzero $\omega(t)$, we will show that the H_∞ performance of system (2.1) is less than some prescribed value. By computation, we have

$$\begin{aligned} J(T) &= \mathbb{E}\{\int_0^T (y^T(t)y(t) - \gamma^2\omega^T(t)\omega(t) + \mathcal{L}V(x_t, \eta(t)))dt\} \\ &\leq \mathbb{E}\{\int_0^T \zeta^T(t)\Psi(\eta(t))\zeta(t)dt\} < 0 \end{aligned} \tag{3.12}$$

where

$$\zeta^T(t) = \begin{bmatrix} \xi^T(t) & \omega^T(t) \end{bmatrix}$$

$$\Psi_i = \begin{bmatrix} \Omega_{i1} & P_iA_{di} & T_1 & 0 & P_iE_i \\ * & \Omega_{i2} & T_2 & T_2 & 0 \\ * & * & \Omega_{i3} & 0 & 0 \\ * & * & * & \Omega_{i4} & 0 \\ * & * & * & * & -\gamma^2I \end{bmatrix} + \begin{bmatrix} C_i^T \\ C_{di}^T \\ 0 \\ 0 \\ F_i^T \end{bmatrix} \begin{bmatrix} C_i^T \\ C_{di^T} \\ 0 \\ 0 \\ F_i^T \end{bmatrix}^T + \begin{bmatrix} A_i^T \\ A_{di}^T \\ 0 \\ 0 \\ E_i^T \end{bmatrix} \tilde{T} \begin{bmatrix} A_i^T \\ A_{di}^T \\ 0 \\ 0 \\ E_i^T \end{bmatrix}^T$$

From (3.10) and by Schur complement, we conclude (3.12) holds. This completes the proof. $\square$

Now, we will give an LMI approach to solve the H_∞ control problem formulated in the previous section.

Theorem 7. *Given scalars $0 < \tau_1 < \tau_2$ and $\gamma > 0$, system (2.1) is stochastically stable with H_∞ performance attenuation if there exist matrices $X_i > 0$, $\hat{Q}_i > 0$,*

$\hat{Z}_l > 0$, $\hat{T}_q > 0$, $\hat{R}_q > 0$ and Y_i such that

$$\begin{bmatrix} \hat{\Omega}_{i1} & A_{di}X_i & \hat{\Omega}_{i12} & 0 & E_i & X_iC_i^T & \tau_1\hat{\Omega}_{i13} & \tau_{12}\hat{\Omega}_{i13} & \hat{\mathcal{S}}_i(X) \\ * & \hat{\Omega}_{i2} & \hat{\Omega}_{i23} & \hat{\Omega}_{i23} & 0 & X_iC_{di}^T & \tau_1X_iA_{di}^T & \tau_{12}X_iA_{di}^T & 0 \\ * & * & \hat{\Omega}_{i3} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \hat{\Omega}_{i4} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\gamma^2 I & F_i^T & \tau_1E_i^T & \tau_{12}E_i^T & 0 \\ * & * & * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & * & * & -\hat{T}_1 & 0 & 0 \\ * & * & * & * & * & * & * & -\hat{T}_2 & 0 \\ * & * & * & * & * & * & * & * & \hat{\mathcal{R}}_i(X) \end{bmatrix} < 0 \tag{3.13}$$

$$\lambda_{ij}\hat{Q}_j + (N-1)\hat{R}_q \le 2X_j,\ \forall\ i,j \in \mathbb{S},\ j \ne i \tag{3.14}$$

where

$$\hat{\Omega}_{i1} = A_iX_i + B_iY_i + X_iA_i^T + Y_i^TB_i^T + \lambda_{ii}X_i + \hat{Q}_i + \hat{T}_1 - 2X_i$$
$$\hat{\Omega}_{i12} = -\hat{T}_1 + 2X_i, \hat{\Omega}_{i23} = -\hat{T}_2 + 2X_i, \hat{\Omega}_{i13} = X_iA_i^T + Y_i^TB_i^T$$
$$\hat{\Omega}_{i2} = -(1-\mu)\hat{Q}_i + 2\hat{T}_2 - 4X_i, \hat{\Omega}_{i3} = \hat{Z}_1 + \hat{T}_1 + \hat{T}_2 - 6X_i$$
$$\hat{\Omega}_{i4} = \hat{Z}_2 + \hat{Z}_3 + \hat{T}_2 - 6X_i, \hat{\mathcal{S}}_i(X) = \begin{bmatrix} X_i & X_i & X_i & \tau_1X_i & \tau_{12}X_i & \mathcal{S}_i(X) \end{bmatrix}$$
$$\mathcal{S}_i(X) = \begin{bmatrix} \sqrt{\lambda_{i1}}X_i & \cdots & \sqrt{\lambda_{i(i-1)}}X_i & \sqrt{\lambda_{i(i+1)}}X_i & \cdots & \sqrt{\lambda_{iN}}X_i \end{bmatrix}$$
$$\hat{\mathcal{R}}_i(X) = diag(-Z_1, -Z_2, -Z_3, -\tau_1R_1, -\tau_{12}R_2, \mathcal{R}_i(X))$$
$$\mathcal{R}_i(X) = -diag(X_1, \dots, X_{i-1}, X_{i+1}, \dots, X_N)$$

hold for all $i \in \mathbb{S}$, $q = 1,2$, $l = 1,2,3$. Then, a desired stabilizing controller gain is given

$$K_i = Y_iX_i^{-1} \tag{3.15}$$

Proof. Substituting A_i with $\bar{A}_i = A_i + B_iK_i$ in (3.10), then let $X_i = P_i^{-1}$, pre- and post-multiplying both sides of (3.10) with diag$\{X_i, X_i, X_i, X_i, I, I, I, I\}$ and its transpose respectively, we have

$$\begin{bmatrix} \tilde{\Omega}_{i1} & A_{di}X_i & X_iT_1X_i & 0 & E_i & X_iC_i^T & \tau_1X_i\bar{A}_i^T & \tau_{12}X_i\bar{A}_i^T \\ * & \tilde{\Omega}_{i2} & X_iT_2X_i & X_iT_2X_i & 0 & X_iC_{di}^T & \tau_1X_iA_{di}^T & \tau_{12}X_iA_{di}^T \\ * & * & X_i\Omega_{i3}X_i & 0 & 0 & 0 & 0 & 0 \\ * & * & * & X_i\Omega_{i4}X_i & 0 & 0 & 0 & 0 \\ * & * & * & * & -\gamma^2 I & F_i^T & \tau_1E_i^T & \tau_{12}E_i^T \\ * & * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & * & -T_1^{-1} & 0 \\ * & * & * & * & * & * & * & -T_2^{-1} \end{bmatrix} < 0 \tag{3.16}$$

where

$$\begin{aligned}
\tilde{\Omega}_{i1} =& \bar{A}_i X_i + X_i \bar{A}_i^T + \sum_{j=1}^{N} \lambda_{ij} X_i X_j^{-1} X_i + X_i(Q_i + Z_1 + Z_2 + \tau_1 R_1 + \tau_{12} R_2 - T_1) X_i \\
\tilde{\Omega}_{i2} =& -(1-\mu) X_i Q_i X_i - 2 X_i T_2 X_i, \tilde{\Omega}_{i3} = -X_i(Z_1 + T_1 + T_2 - Z_3) X_i \\
\tilde{\Omega}_{i4} =& - X_i(T_2 + Z_2 + Z_3) X_i
\end{aligned}$$

By noticing $\hat{Z}_l = Z_l^{-1} > 0$, $l = 1, 2, 3$, and $\hat{T}_q = T_q^{-1} > 0$, $q = 1, 2$, we have $-X_i Z_l X_i \leq \hat{Z}_l - 2X_i$ and $-X_i T_q X_i \leq \hat{T}_q - 2X_i$. Furthermore, we conclude that

$$\begin{bmatrix} I & 0 & -I & 0 \end{bmatrix}^T (-X_i T_1 X_i - \hat{T}_1 + 2X_i) \begin{bmatrix} I & 0 & -I & 0 \end{bmatrix} \leq 0 \tag{3.17}$$

$$\begin{bmatrix} 0 & I & 0 & -I \end{bmatrix}^T (-X_i T_2 X_i - \hat{T}_2 + 2X_i) \begin{bmatrix} 0 & I & 0 & -I \end{bmatrix} \leq 0 \tag{3.18}$$

$$\begin{bmatrix} 0 & -I & I & 0 \end{bmatrix}^T (-X_i T_2 X_i - \hat{T}_2 + 2X_i) \begin{bmatrix} 0 & -I & I & 0 \end{bmatrix} \leq 0 \tag{3.19}$$

Let $\hat{R}_q = R_q^{-1}$, $q = 1, 2$ and $\hat{Q}_i = X_i Q_i X_i$, it follows from (3.17)-(3.19) with (3.15) that (3.13) implies (3.16) holding. From (3.14), we have

$$\lambda_{ij} \hat{Q}_j = \lambda_{ij} X_j Q_j X_j \leq 2X_j - (N-1)\hat{R}_q \leq X_j \frac{R_q}{N-1} X_j \ \forall\ i, j \in \mathbb{S},\ j \neq i, q = 1, 2 \tag{3.20}$$

Noting $\lambda_{ii} \leq 0$ and $\hat{Q}_i > 0$, we get $\lambda_{ii} \hat{Q}_i \leq 0$. Then, we have the following

$$\begin{aligned}
&\begin{bmatrix} \lambda_{i1} X_1 Q_1 X_1 & & & & \\ & \ddots & & & \\ & & \lambda_{ii} X_i Q_i X_i & & \\ & & & \ddots & \\ & & & & \lambda_{iN} X_N Q_N X_N \end{bmatrix} \\
\leq &\begin{bmatrix} X_1 \frac{R_q}{N-1} X_1 & & & & & & \\ & \ddots & & & & & \\ & & X_{i-1} \frac{R_q}{N-1} X_{i-1} & & & & \\ & & & 0 & & & \\ & & & & X_{i+1} \frac{R_q}{N-1} X_{i+1} & & \\ & & & & & \ddots & \\ & & & & & & X_N \frac{R_q}{N-1} X_N \end{bmatrix}
\end{aligned} \tag{3.21}$$

Pre- and post-multiplying both sides of (3.21) with $\begin{bmatrix} X_1^{-1} & \cdots & X_i^{-1} & \cdots & X_N^{-1} \end{bmatrix}$ and its transpose, respectively, we have (3.11) holding. This completes the proof. □

Remark 8. Though mode-dependent term Q_i introduced in Lyapunov functional is exploited to overcome the shortcoming of mode-independent Q. However, Q_i leads to (3.2) or (3.11), which makes the problems (e.g., stabilization and H_∞ controller design) not easy to be solved in terms of LMIs. Theorem 3 provides a sufficient condition for the solvability of H_∞ controller of form (2.7) for Markovian jump time-varying delay system in terms of LMIs, which could be easily solved via using LMI control toolbox.

The aforementioned criteria can be applied to both slow and fast time-varying delays only if μ is known. However, when time delay $\tau(t)$ is not differentiable or μ is unknown, the obtained criteria are not applied directly and should be revised.

When $\tau(t)$ is not differentiable or μ is unknown, that is

$$0 \le \tau_1 \le \tau(t) \le \tau_2 \tag{3.22}$$

Then, choose a stochastic Lyapunov-Krasovskii functional for system (2.1) as

$$V(x_t, \eta(t)) = V_1(x_t, \eta(t)) + \breve{V}_2(x_t, \eta(t)) + V_3(x_t, \eta(t)) + V_4(x_t, \eta(t)) \tag{3.23}$$

where

$$\begin{aligned}\breve{V}_2(x_t, \eta(t)) = & \int_{t-\tau_2}^{t} x^T(s) Q(\eta(t)) x(s) ds \\ & + \sum_{l=1}^{2} \int_{t-\tau_l}^{t} x^T(s) Z_l x(s) ds + \int_{t-\tau_2}^{t-\tau_1} x^T(s) Z_3 x(s) ds\end{aligned}$$

V_1, V_3, V_4 are given in (3.3). Then, by the similar method, we will have the following theorem.

Theorem 9. *Given scalars $0 < \tau_1 < \tau_2$ and $\gamma > 0$, system (2.1) is stochastically stable with H_∞ performance attenuation if there exist matrices $X_i > 0$, $\hat{Q}_i > 0$, $\hat{Z}_l > 0$, $\hat{T}_q > 0$, $\hat{R}_q > 0$ and Y_i such that*

$$\begin{bmatrix} \hat{\Omega}_{i1} & A_{di}X_i & \hat{\Omega}_{i12} & 0 & E_i & X_iC_i^T & \tau_1\hat{\Omega}_{i13} & \tau_{12}\hat{\Omega}_{i13} & \hat{\mathcal{S}}_i(X) \\ * & \breve{\Omega}_{i2} & \hat{\Omega}_{i23} & \hat{\Omega}_{i23} & 0 & X_iC_{di}^T & \tau_1 X_i A_{di}^T & \tau_{12} X_i A_{di}^T & 0 \\ * & * & \hat{\Omega}_{i3} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \hat{\Omega}_{i4} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\gamma^2 I & F_i^T & \tau_1 E_i^T & \tau_{12} E_i^T & 0 \\ * & * & * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & * & * & -\hat{T}_1 & 0 & 0 \\ * & * & * & * & * & * & * & -\hat{T}_2 & 0 \\ * & * & * & * & * & * & * & * & \hat{\mathcal{R}}_i(X) \end{bmatrix} < 0 \tag{3.24}$$

$$\lambda_{ij}\hat{Q}_j + (N-1)\hat{R}_q \le 2X_j, \ \forall\, i, j \in \mathbb{S},\ j \ne i \tag{3.25}$$

where

$$\breve{\Omega}_{i2} = 2\hat{T}_2 - 4X_i$$

the others are given in Theorem 3, hold for all $i \in \mathbb{S}$, $l = 1, 2, 3$, $q = 1, 2$. Then, a desired stabilizing controller gain of form (2.7) is given by (3.15).

4. NUMERICAL EXAMPLES

In this section, numerical examples are given to demonstrate the benefits and effectiveness of the proposed theories.

Example 10. An example is cited from [14] described as

$$A_1 = \begin{bmatrix} -3.4888 & 0.8057 \\ -0.6451 & -3.2684 \end{bmatrix}, A_{d1} = \begin{bmatrix} -0.8620 & -1.2919 \\ -0.6841 & -2.0729 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -2.4898 & 0.2895 \\ 1.3396 & -0.0211 \end{bmatrix}, A_{d2} = \begin{bmatrix} -2.8306 & 0.4978 \\ -0.8436 & -1.0115 \end{bmatrix}$$

Table 1 presents the comparative results with the assumption that $\lambda_{22} = -0.8$ and $\mu = 0$, while Table 2 gives the comparisons for different μ with $\lambda_{22} = -0.8$.

TABLE 1. Allowable upper bounds of τ_2 with given λ_{22}

λ_{11}	-0.1	-0.5	-0.8	-1
τ_2 [11]	0.1926	0.1900	0.1882	0.1871
τ_2 [10]	0.2224	0.2200	0.2184	0.2174
τ_2 [12, 13]	0.5012	0.4941	0.4915	0.4903
$\tau_2(\tau_1 = 0)$ by Th. 1	0.6797	0.5794	0.5562	0.5465
$\tau_2(\tau_1 = 0.1)$ by Th. 1	0.6803	0.5796	0.5566	0.5471

TABLE 2. Allowable upper bounds of τ_2 with given λ_{22}

λ_{11}	-0.2	-0.7	-1	-1.5
$\tau_2(\mu = 0.8)$ [14]	0.4245	0.4185	0.4161	0.4143
$\tau_2(\mu = 0.8, \tau_1 = 0)$ by Th. 1	0.4389	0.4315	0.4289	0.4273
$\tau_2(\mu = 0.8, \tau_1 = 0.1)$ by Th. 1	0.4527	0.4443	0.4423	0.4412
$\tau_2(\mu = 1.3)$ [14]	0.3860	0.3860	0.3860	0.3859
$\tau_2(\mu = 1.3, \tau_1 = 0)$ by Th. 1	0.4251	0.4246	0.4243	0.4240
$\tau_2(\mu = 1.3, \tau_1 = 0.1)$ by Th. 1	0.4410	0.4401	0.4396	0.4391

Example 11. Consider the following system in the form of (2.1) described as

$$A_1 = \begin{bmatrix} -2.2 & -1.4 \\ -1 & -2.9 \end{bmatrix}, A_{d1} = \begin{bmatrix} -0.7 & 1.1 \\ 0.6 & -3 \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, E_1 = \begin{bmatrix} -0.4 \\ 0.6 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} -0.3 & -0.2 \end{bmatrix}, C_{d1} = \begin{bmatrix} 1 & 0.3 \end{bmatrix}, F_1 = 0.1$$

$$A_2 = \begin{bmatrix} -1.8 & 0.8 \\ 0.6 & -0.7 \end{bmatrix}, A_{d2} = \begin{bmatrix} -1.5 & -1.6 \\ -0.1 & -1.2 \end{bmatrix}, B_2 = \begin{bmatrix} -1 \\ 0 \end{bmatrix}, E_2 = \begin{bmatrix} 0.5 \\ -0.2 \end{bmatrix}$$

$$C_2 = \begin{bmatrix} 1.4 & 0.2 \end{bmatrix}, C_{d2} = \begin{bmatrix} 0 & 1 \end{bmatrix}, F_2 = 0.3$$

Assume the transition probabilities are $\lambda_{11} = -3$ and $\lambda_{22} = -0.6$, when given $\tau_1 = 0.1$, $\tau_2 = 0.3$ and $\mu = 1.4$, we can compute the controller gain with $\gamma^* = 0.547$ as

$$K_1 = \begin{bmatrix} 5.8815 & -5.0136 \end{bmatrix}, K_2 = \begin{bmatrix} 10.2360 & -1.9293 \end{bmatrix}$$

5. CONCLUSIONS

In this paper, we have investigated the problems of delay-dependent stability and H_∞ performance for Markovian jump time-varying delay systems via manipulating a new Lyapunov functional and making full use of information about time delay. Less conservative criteria with reduced computational demand are obtained without introducing slack matrices or model transformation. An easily verifiable condition for the existence of stabilizing H_∞ controller is established into an LMI framework. Finally, numerical examples are provided to illustrate the advantage and applicability of the presented results in this paper.

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