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NEW OSCILLATION RESULTS FOR LINEAR MATRIX HAMILTONIAN SYSTEMS

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ABSTRACT. Some new criteria have been established for the oscillation of the linear matrix Hamiltonian system $X' = A(t)X + B(t)Y$, $Y' = C(t)X - A^*(t)Y$ under the hypothesis: $A(t)$, $B(t) = B^*(t) > 0$ and $C(t) = C^*(t)$ are $n \times n$ real continuous matrix functions on the interval $[t_0, \infty)$ $(t_0 > -\infty)$. Our results are different from most known ones in the sense that they are given in the form $\limsup_{t\to\infty} g[\cdot] > const.$ rather than $\limsup_{t\to\infty} \lambda_1[\cdot] = \infty$, where g is a positive linear functional on the linear space of $n \times n$ matrices with real entries. Our results improve some previous ones. Two examples are worked out to illustrate the effectiveness of our results.

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1. INTRODUCTION

Consider the linear matrix Hamiltonian system

$$\begin{cases} X' = A(t)X + B(t)Y \\ Y' = C(t)X - A^*(t)Y, \end{cases} \tag{1.1}$$

where $A(t)$, $B(t) = B^*(t) > 0$, $C(t) = C^*(t)$ are $n \times n$-matrices of real valued continuous functions on $[t_0, \infty)$. By M^* we mean the conjugate transpose of the matrix M.

For any solution (X, Y) of (1.1), $X^*(t)Y(t) - Y^*(t)X(t)$ is a constant matrix. The solution (X, Y) of (1.1) is said to be *nontrivial* if $\det X(t) \neq 0$ for at least one $t \in [t_0, \infty)$. A nontrivial solution (X, Y) of (1.1) is said to be *prepared* if $X^*(t)Y(t) - Y^*(t)X(t) = 0$ for every $t \in [t_0, \infty)$. A prepared solution (X, Y) of (1.1) is said to be oscillatory on $[t_0, \infty)$ if $\det X(t)$ has arbitrarily large zeros. System (1.1) is said to be *oscillatory* on $[t_0, \infty)$ if each nontrivial prepared solution of (1.1) is oscillatory.

In the case when $A(t) \equiv 0$, $B(t) > 0$, system (1.1) reduces to the second order self-adjoint matrix differential system

$$(P(t)X')' + Q(t)X = 0 \tag{1.2}$$

with $P(t) = B^{-1}(t)$, $Q(t) = -C(t)$. Oscillation and nonoscillation of system (1.1) or (1.2) have been extensively studied by many authors [1-16, 19]. However, all the

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results in [1-16, 19] are given in the form $\limsup_{t\to\infty} \lambda_1[\cdot] = \infty$, where $\lambda_1[P]$ denotes the largest eigenvalue of an $n \times n$ Hermitian matrix P. In our recent papers [18, 20, 21], we gave several oscillation criteria in the form $\limsup_{t\to\infty} \lambda_1[\cdot] > const.$. In this paper, we will further the investigation and establish some new oscillation criteria that are presented in the form $\limsup_{t\to\infty} g[\cdot] > const.$ for the system (1.1) by using a class of particular functions $\Phi(t, s, r)$ defined by:

$$\Phi(t,s,r) = (t-s)^{\alpha}(s-r)^{\beta}, \quad \text{for } t \geq s \geq r \geq t_0 \text{ and } \alpha,\ \beta > 1/2, \tag{1.3}$$

where g is a positive linear functional on the linear space of $n \times n$ matrices with real entries. Our results improve many known oscillation results even for the self-adjoint differential system (1.2), which can be illustrated by the examples given at the end of this paper.

In the sequel, Let $\mathbb{R}^{n\times n}$ be the linear space of $n \times n$ matrices with real entries, $\wp \subset \mathbb{R}^{n\times n}$ be the subspace of $n \times n$ symmetric matrices, and g be a linear functional on $\mathbb{R}^{n\times n}$. g is said to be *positive* if $g(A) > 0$ whenever $A \in \wp$ and $A > 0$.

2. MAIN RESULTS

The following lemma will be used to prove the main results of this paper.

Lemma 2.1. *([17]) If g is a positive linear functional on $\mathbb{R}^{n\times n}$ then for all $A,\ B \in \mathbb{R}^{n\times n}$, we have $|g[A^*B]|^2 \leq g[A^*A]g[B^*B]$.*

Now we give the main results of this paper.

Theorem 2.2. *Let $\Phi(t, s, r)$ be defined by (1.3). If there exist a positive linear functional g on $\Re$ and a function $f(t) \in C^1[0, \infty)$ such that for each $r \geq t_0$*

$$\limsup_{t\to\infty} \int_r^t g\left[M_1(t,s,r)\right] ds > 0, \tag{2.1}$$

where

$$\begin{aligned} M_1(t,s,r) \;=\; & -\Phi^2(t,s,r)(C_1 + A^*B_1^{-1}A)(s) \\ & +2\Phi(t,s,r)\Phi'_s(t,s,r)(B_1^{-1}A)(s) \\ & -\Phi'^{\,2}_s(t,s,r)B_1^{-1}(s), \end{aligned} \tag{2.2}$$

$$B_1(t) = a^{-1}(t)B(t), \quad a(t) = \exp\left\{-2\int^t f(s)ds\right\}, \tag{2.3}$$

and

$$C_1(t) = a(t)\left\{C(t) + f(t)[B^{-1}A + A^*B^{-1}](t) + [f(t)B^{-1}(t)]' - f^2(t)B^{-1}(t)\right\}, \tag{2.4}$$

then the system (1.1) is oscillatory.

Proof. Assume to the contrary that (1.1) is nonoscillatory. Then there exists a nontrivial prepared solution $(X(t), Y(t))$ of (1.1) such that $X(t)$ is nonsingular for all sufficiently large t, say $t \geq T \geq t_0$. This allows us to make a transformation

$$W(t) = -a(t)\left[Y(t)X^{-1}(t) + f(t)B^{-1}(t)\right],\ t \geq T. \tag{2.5}$$

From (1.1) and (2.3)-(2.5) we have

$$W'(t) + A^*(t)W(t) + W^*(t)A(t) - W(t)^*B_1(t)W(t) + C_1(t) = 0, \tag{2.6}$$

Multiplying (2.6), with t replaced by s, by $\Phi^2(t,s,T)$ and integrating from T to t, we obtain

$$\begin{aligned}\int_T^t \Phi^2(t,s,T)(-C_1(s))ds &= -2\int_T^t \Phi(t,s,T)\Phi'_s(t,s,T)W(s)ds \\ &\quad - \int_T^t \Phi^2(t,s,T)(W^*B_1W - A^*W - W^*A)(s)ds.\end{aligned} \tag{2.7}$$

Now the substitution

$$P(t) = W(t) - B_1^{-1}(t)A(t)$$

in the above equation (2.7) gives us

$$\begin{aligned}\int_T^t M_0(t,s,T)ds &= -2\int_T^t \Phi(t,s,T)\Phi'_s(t,s,T)P(s)ds \\ &\quad - \int_T^t \Phi^2(t,s,T)(P^*B_1P)(s)ds,\end{aligned} \tag{2.8}$$

where

$$M_0(t,s,T) = -\Phi^2(t,s,T)(C_1 + A^*B_1^{-1}A)(s) + 2\Phi(t,s,T)\Phi'_s(t,s,T)(B_1^{-1}A)(s).$$

Applying the linear functional g on both sides of (2.8), we obtain

$$\begin{aligned}\int_T^t g[M_0(t,s,T)]ds &= -\int_T^t 2\Phi(t,s,T)\Phi'_s(t,s,T)g[P(s)]ds \\ &\quad - \int_T^t \Phi^2(t,s,T)g[(P^*B_1P)(s)]ds\end{aligned} \tag{2.9}$$

We now claim that for $t \in [T, \infty)$,

$$g\left[(P^*B_1P)(t)\right] \geq \left\{g\left[B_1^{-1}(t)\right]\right\}^{-1}\left\{g\left[P(t)\right]\right\}^2. \tag{2.10}$$

In fact, by the lemma, for all $t \in [T, \infty)$

$$\begin{aligned}g[B_1^{-1}(t)]g[(P^*B_1P)(t)] &= g[(B_1^{-1/2^*}B_1^{-1/2})(t)]g[(B_1^{1/2}P)^*(B_1^{1/2}P)(t)] \\ &\geq \left\{g[(B_1^{-1/2}B_1^{1/2}P)(t)]\right\}^2 \\ &= \left\{g[P(t)]\right\}^2.\end{aligned}$$

Hence the claim is true. By (2.9) and (2.10), we have that

(2.11)
$$\begin{aligned}\int_T^t g[M_0(t,s,T)]ds &\leq -\int_T^t 2\Phi(t,s,T)\Phi_s'(t,s,T)g[P(s)]ds \\ &\quad -\int_T^t \Phi^2(t,s,T)\left\{g\left[B_1^{-1}(s)\right]\right\}^{-1}\{g[(P(s)]\}^2 ds \\ &= \int_T^t {\Phi_s'}^2(t,s,T)g[B_1^{-1}(s)]ds \\ &\quad -\int_T^t \left[\frac{\Phi(t,s,T)}{\sqrt{g[B_1^{-1}(s)]}}g[P(s)]+\Phi_s'(t,s,T)\sqrt{g[B_1^{-1}(s)]}\right]^2 \\ &\leq \int_T^t {\Phi_s'}^2(t,s,T)g[B_1^{-1}(s)]ds.\end{aligned}$$

From (2.2) and (2.11) we have

$$\int_T^t g\left[M_1(t,s,T)\right]ds = \int_T^t g\left[M_0(t,s,T)-{\Phi_s'}^2(t,s,T)B_1^{-1}(s)\right]ds \leq 0,$$

which implies a contradiction to the hypothesis (2.1). This completes the proof of Theorem 2.2. □

If we choose an appropriate function $f(t)$ and a positive linear functional g in Theorem 2.2 such that $g[B_1^{-1}(t)] = g[a(t)B^{-1}(t)] \leq m$ for $t \geq t_0$, where $m > 0$ is a constant, and let $\Phi(t,s,r) = (t-s)(s-r)^\alpha$ for $\alpha > 1/2$, then we have the following theorem from Theorem 2.2:

Theorem 2.3. *System (1.1) is oscillatory provided that for some $\alpha > 1/2$ and for each $r \geq t_0$,*

$$\limsup_{t\to\infty} \frac{1}{t^{2\alpha+1}}\int_r^t g\left[M_2(t,s,r)\right]ds > \frac{m\alpha}{(2\alpha-1)(2\alpha+1)}, \tag{2.12}$$

where

$$\begin{aligned}M_2(t,s,r) &= -(t-s)^2(s-r)^{2\alpha}g[(C_1+A^*B_1^{-1}A)(s)] \\ &\quad +2(t-s)(s-r)^{2\alpha-1}[\alpha t-(\alpha+1)s+r]g[(B_1^{-1}A)(s)],\end{aligned} \tag{2.13}$$

and $B_1(t)$, $C_1(t)$ are the same as in Theorem 2.2.

Proof. Assume to the contrary that (1.1) exists a nontrivial prepared solution $(X(t), Y(t))$ such that $X(t)$ is nonsingular for $t \geq T \geq t_0$. Similar to the proof of Theorem 2.2, and noting that $g[B_1^{-1}(t)] \leq m$ for $t \geq t_0$, we have for $t \geq T$

$$\begin{aligned}\int_T^t g\left[M_2(t,s,r)\right]ds &\leq \int_T^t [\alpha(t-s)(s-T)^{\alpha-1}-(s-T)^\alpha]^2 g[B_1^{-1}(s)]ds \\ &\leq m\int_T^t [\alpha(t-s)(s-T)^{\alpha-1}-(s-T)^\alpha]^2 ds.\end{aligned} \tag{2.14}$$

Integrating by parts, we can easily obtain,

$$\int_T^t [\alpha(t-s)(s-T)^{\alpha-1} - (s-T)^\alpha]^2 ds = \frac{\alpha}{(2\alpha-1)(2\alpha+1)}(t-T)^{2\alpha+1}. \tag{2.15}$$

Thus, from (2.13), (2.14) and (2.15) we have

$$\lim_{t\to\infty}\sup \frac{1}{t^{2\alpha+1}} \int_r^t g\left[M_2(t,s,r)\right] ds \le \frac{m\alpha}{(2\alpha-1)(2\alpha+1)},$$

which contradicts the hypothesis (2.12). This completes the proof of Theorem 2.3. □

If we choose an appropriate function $f(t)$ and a positive linear functional g in Theorem 2.2 such that $g[B_1^{-1}(t)] = g[a(t)B^{-1}(t)] \le m$ for $t \ge t_0$, where $m > 0$ is a constant, and let $\Phi(t,s,r) = (t-s)^\alpha(s-r)$ for $\alpha > 1/2$, then, similar to the proof of Theorem 2.3, we have the following theorem:

Theorem 2.4. *System (1.1) is oscillatory provided that for some $\alpha > 1/2$ and for each $r \ge t_0$,*

$$\lim_{t\to\infty}\sup \frac{1}{t^{2\alpha+1}} \int_r^t g\left[M_3(t,s,r)\right] ds > \frac{m\alpha}{(2\alpha-1)(2\alpha+1)}, \tag{2.16}$$

where

$$\begin{aligned} M_3(t,s,r) &= -(t-s)^{2\alpha}(s-r)^2 g[(C_1 + A^* B_1^{-1} A)(s)] \\ &\quad +2(t-s)^{2\alpha-1}(s-r)[t-(\alpha+1)s+\alpha r] g[(B_1^{-1}A)(s)], \end{aligned}$$

and $B_1(t)$, $C_1(t)$ are the same as in Theorem 2.2.

In the sequel, we generalize Theorems 2.3 and 2.4 as the following theorem:

Theorem 2.5. *If there exist a positive linear functional g on $\Re$, a function $f(t) \in C^1[0,\infty)$ and a constant $m > 0$ such that $g[B_1^{-1}(t)] = g[a(t)B^{-1}(t)] \le m$ for $t \ge t_0$, and for each $r \ge t_0$,*

$$\lim_{t\to\infty}\sup \frac{1}{t^{2\alpha+2\beta-1}} \int_r^t g[M_1(t,s,r)] ds > 2m\alpha\beta(\alpha+\beta-1)\frac{\Gamma(2\alpha-1)\Gamma(2\beta-1)}{\Gamma(2\alpha+2\beta)}, \tag{2.17}$$

where $M_1(t,s,r)$ is defined as in Theorem 2.2, then system (1.1) is oscillatory.

Proof. Noting that

$$\begin{aligned} \int_T^t {\Phi_s'}^2(t,s,T) ds &= \int_T^t [\beta(t-s)^\alpha(s-T)^{\beta-1} - \alpha(t-s)^{\alpha-1}(s-r)^\beta]^2 ds \\ &= \int_T^t [\beta^2(t-s)^{2\alpha}(s-T)^{2(\beta-1)} - 2\alpha\beta(t-s)^{2\alpha-1}(s-r)^{2\beta-1} \\ &\quad +\alpha^2(t-s)^{2(\alpha-1)}(s-r)^{2\beta}] ds. \end{aligned}$$

Integrating the above equality by parts and setting $u = s - t$ and $w = t - T$, we have eventually

$$\begin{aligned}\int_T^t {\Phi'_s}^2(t,s,T)ds &= \int_T^t (t-s)^{2(\alpha-1)}(s-T)^{2(\beta-1)}[\beta(t-s)-\alpha(s-T)] \\ &= \int_0^{t-T}(t-T-u)^{2(\alpha-1)}u^{2(\beta-1)}[\beta(t-T-u)-\alpha u]^2du \\ &= \int_0^w (w-u)^{2(\alpha-1)}u^{2(\beta-1)}[\beta(w-u)-\alpha u]^2du.\end{aligned} \tag{2.18}$$

We now evaluate this integral using Euler's Beta function

$$\int_0^1 x^a(1-x)^b dx = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}, \quad Re(a,b)>0.$$

Since

$$\begin{aligned}\int_0^w (w-u)^{2(\alpha-1)}u^{2(\beta-1)}[\beta(w-u)-\alpha u]^2du = \int_0^w [\beta^2(w-u)^{2\alpha}u^{2(\beta-1)} \\ -2\alpha\beta(w-u)^{2\alpha-1}u^{2\beta-1}+\alpha^2(w-u)^{2(\alpha-1)}u^{2\beta}]du,\end{aligned} \tag{2.19}$$

by evaluating the first integral by Euler's Beta function and setting $u = wx$, we obtain

$$\begin{aligned}\int_0^w (w-u)^{2\alpha}u^{2(\beta-1)}du &= w\int_0^1 (w-wx)^{2\alpha}(wx)^{2(\beta-1)}dx \\ &= w^{2(\alpha+\beta)-1}\int_0^1 (1-x)^{2\alpha}x^{2(\beta-1)}dx \\ &= w^{2\alpha+2\beta-1}\frac{\Gamma(2\alpha+1)\Gamma(2\beta-1)}{\Gamma(2\alpha+2\beta)}.\end{aligned}$$

Similarly, we can evaluate the second and the third integral as

$$\int_0^w (w-u)^{2\alpha-1}u^{2\beta-1}du = w^{2\alpha+2\beta-1}\frac{\Gamma(2\alpha)\Gamma(2\beta)}{\Gamma(2\alpha+2\beta)}$$

and

$$\int_0^w (w-u)^{2(\alpha-1)}u^{2\beta}du = w^{2\alpha+2\beta-1}\frac{\Gamma(2\alpha-1)\Gamma(2\beta+1)}{\Gamma(2\alpha+2\beta)}.$$

Thus, we have

$$\begin{aligned}&\int_0^w (w-u)^{2(\alpha-1)}u^{2(\beta-1)}[\beta(w-u)-\alpha u]^2du \\ &= \frac{w^{2\alpha+2\beta-1}}{\Gamma(2\alpha+2\beta)}[\beta^2\Gamma(2\alpha+1)\Gamma(2\beta-1)-2\alpha\beta\Gamma(2\alpha)\Gamma(2\beta) \\ &\quad +\alpha^2\Gamma(2\alpha-1)\Gamma(2\beta+1)] \\ &= 2\alpha\beta(\alpha+\beta-1)\frac{\Gamma(2\alpha-1)\Gamma(2\beta-1)}{\Gamma(2\alpha+2\beta)}w^{2\alpha+2\beta-1}.\end{aligned} \tag{2.20}$$

From (2.18)-(2.20), we get

$$\int_T^t {\Phi'_s}^2(t,s,T)ds = 2\alpha\beta(\alpha+\beta-1)\frac{\Gamma(2\alpha-1)\Gamma(2\beta-1)}{\Gamma(2\alpha+2\beta)}(t-T)^{2\alpha+2\beta-1}.$$

The following proof is similar to that of Theorem 2.2, and hence is omitted. This completes the proof of Theorem 2.5. □

The following two examples illustrate our results.

Example 2.6. Consider the Euler differential system

$$Y'' + \operatorname{diag}\left(\frac{\gamma}{t^2}, \frac{\beta}{t^2}\right) Y = 0, \quad t \geq 1, \tag{2.21}$$

where $\gamma \geq \beta > 0$ are constants. Our Theorem 2.3 can be applied to (2.21) and easily reveal the well-known fact that (2.21) is oscillatory for $\gamma > 1/4$. In fact, if we choose $f(t) = 0$, $\Phi(t,s,r) = (t-s)(s-r)^\alpha$ for $\alpha > 1/2$, then we have

$$M_2(t,s,r) = (t-s)^2(s-r)^{2\alpha} g\left[\operatorname{diag}\left(\frac{\gamma}{t^2}, \frac{\beta}{t^2}\right)\right].$$

Let the positive linear functional $g[A] = a_{11}$, where $A = (a_{ij})$ is a 2×2 matrix. Note that $g[B_1^{-1}(t)] = 1$ and for each $r \geq 1$

$$\lim_{t\to\infty} \frac{1}{t^{2\alpha+1}} \int_r^t (t-s)^2(s-r)^{2\alpha} \frac{\gamma}{s^2} ds = \frac{\gamma}{\alpha(2\alpha-1)(2\alpha+1)}.$$

For any $\gamma > 1/4$, there exists an constant $\alpha > 1/2$ such that

$$\frac{\gamma}{\alpha(2\alpha-1)(2\alpha+1)} > \frac{\alpha}{(2\alpha-1)(2\alpha+1)},$$

i.e., $\gamma > \alpha^2$. This means that (2.12) holds. By Theorem 2.3, we find that (2.21) is oscillatory for $\gamma > 1/4$. However, we can easily see that criteria in [3, 6, 8] fail to reveal this fact.

Example 2.7. Consider the 4−dimensional matrix Hamiltonian system (2.22) with system parameters

$$A(t) = \begin{bmatrix} 0 & -1/t \\ 2/t & 0 \end{bmatrix}, \quad B(t) = \begin{bmatrix} t & 0 \\ 0 & 2t \end{bmatrix}, \quad C(t) = -\begin{bmatrix} \theta/t^3 & 0 \\ 0 & \eta/t^3 \end{bmatrix}, \tag{2.22}$$

where $t \geq 1$, $\eta \geq \theta > 0$ are constants. If we let $f(t) = -\frac{1}{2t}$, $\Phi(t,s,r) = (t-s)(s-r)^\alpha$ for $\alpha > 1/2$, and the positive linear functional $g[A] = a_{22}$, where $A = (a_{ij})$ is a 2×2 matrix, then we have

$$a(t) = t, \quad g\left[M_2(t,s)\right] = (t-s)^2(s-r)^{2\alpha}(\eta - 11/8)/s^2,$$

and $g[B_1^{-1}(t)] = g[a(t)B^{-1}(t)] = 1/2$ for $t \geq 1$. Similar to the proof of Example 2.6, we can obtain that system (2.22) is oscillatory for $\eta > 3/2$ by Theorem 2.3.

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