Dynamic Systems and Applications 16 (2007) 481-496

NONLINEAR IMPULSIVE INTEGRO-DIFFERENTIAL EQUATIONS OF MIXED TYPE WITH TIME-VARYING GENERATING OPERATORS AND OPTIMAL CONTROLS

Y. PENG, X. XIANG, AND W. WEI

Department of Mathematics, Guizhou University
Guiyang, Guizhou, 550025, P.R. China
(pengyf0803@163.com, xxl3621070@yahoo.com.cn, wei66@yahoo.com)

ABSTRACT. Nonlinear impulsive integro-differential equations of mixed type with time-varying generating operators is considered. Existence of $PC-\alpha$-mild solutions is proved. Existence of optimal pairs of systems governed by impulsive integro-differential equations of mixed type is also presented. An example is given for demonstration.

Keywords. Semigroup, Integral operator of mixed type, Fractional power space, $PC-\alpha-$mind solution, Compactness, Existence, Optimal control.

AMS (MOS) Subject Classification. 34G20, 34K30, 35B62, 93C25.

1. INTRODUCTION

The theory of impulsive differential equations in the field of modern applied mathematics has made considerable headway in recent years, because the structure of its emergence has deeply physical background and realistic mathematical models. For the basic theory on impulsive differential equations on finite dimensional Banach spaces, the reader can refer to Lakshmikantham's book (see [9]). There are some papers discussing impulsive integro-differential equations on finite dimensional spaces, and most of authors used the method of upper and lower solutions (see [5], [7]).

In recent years impulsive evolution equations on infinite dimensional Banach spaces have been investigated by many authors including us (see [1], [2], [4], [13], [16]). Particularly, Ahmed considered optimal control problems of systems governed by impulsive evolution equations in infinite dimensional spaces (see [1], [2]). To our knowledge, there are few papers concerned with impulsive integro-differential equations on infinite dimensional Banach spaces. In 2005, Guo use the monotone

Received November 29, 2005 1056-2176 \$15.00 ©Dynamic Publishers, Inc.
This work is supported by the National Nature Science Foundation of China under Grant: 10661004.

iterative method to study the following impulsive integro-differential equation

$$\text{(1.1)} \qquad \begin{cases} \ddot{x}(t) = f(t, x(t), \dot{x}(t), (Wx)(t), (Hx)(t)), & t \in (0, a],\ t \neq t_k, \\ x(0) = x_0, \quad \Delta x(t_k) = I_k(x(t_k), \dot{x}(t_k)), & k = 1, 2, \cdots, m, \\ \dot{x}(0) = x_1, \quad \Delta \dot{x}(t_k) = \overline{I}_k(x(t_k), \dot{x}(t_k)), & k = 1, 2, \cdots, m, \end{cases}$$

where

$$(Wx)(t) = \int_0^t b(t, s)x(s)ds, \quad (Hx)(t) = \int_0^a h(t, s)x(s)ds, \quad t \in [0, a]$$

which are linear integral operators (see [6], [10]). Recently, using semigroup theory we considered the following impulsive integro-differential equation

$$\text{(1.2)} \qquad \begin{cases} \dot{x}(t) + A(t)x(t) = F(t, x(t), (Gx)(t)), & t \in (0, T] \setminus \Theta, \\ x(0) = x_0, \quad \Delta x(t_i) = J_i(x(t_i)), & i = 1,\ 2,\ \cdots,\ n, \end{cases}$$

and impulsive integro-differential equation of mixed type

$$\text{(1.3)} \qquad \begin{cases} \dot{x}(t) + Ax(t) = F(t, x(t), (Gx)(t), (Sx)(t)), & t \in (0, T] \setminus \Theta, \\ x(0) = x_0, \quad \Delta x(t_i) = J_i(x(t_i)), & i = 1,\ 2,\ \cdots,\ n, \end{cases}$$

where A is the infinitesimal generator of a C_0-semigroup. At the same time the optimal control problems of systems governed by (1.2) or (1.3) is also discussed (see [13], [16]). In this paper, we continue to study impulsive integro-differential equations and optimal control problems. Here we consider impulsive integro-differential equations of mixed type with time-varying generating operators

$$\text{(1.4)} \qquad \begin{cases} \dot{x}(t) + A(t)x(t) = F(t, x(t), (Gx)(t), (Sx)(t)), & t \in (0, T] \setminus \Theta, \\ x(0) = x_0, \quad \Delta x(t_i) = J_i(x(t_i)), & i = 1,\ 2,\ \cdots,\ n, \end{cases}$$

where $\Theta = \{t_1,\ t_2,\ \cdots,\ t_n\} \subset (0, T)$, $0 < t_1 < t_2 < \cdots < t_n < T$, $\{A(t) | t \in [0, T]\}$ is a family of closed densely defined linear operators, and G, S are nonlinear integral operators given by

$$(Gx)(t) = \int_0^t k(t, \tau)g(\tau, x(\tau))d\tau, \quad (Sx)(t) = \int_0^T h(t, \tau)s(\tau, x(\tau))d\tau.$$

The operator S is much different from G. $J_i (i = 1,\ 2,\ \cdots,\ n)$ is a nonlinear map and $\Delta x(t_i) = x(t_i + 0) - x(t_i - 0) = x(t_i + 0) - x(t_i)$. This represents the jump in the state x at time t_i, with J_i determining the size of the jump at time t_i.

The integral operator S makes the problem (1.4) much more difficult. In addition, the time-varying generating operators system (1.4) concerns the fractional power operator. In order to get a priori estimate of mild solutions of system (1.4) we have to look for another generalized Gronwall Lemma which is suitable for (1.4), that is, we can use it to deal with such a class of inequalities including singularity and integral of mixed type (see Lemma 2.1). Of course, this Lemma is useful for other problems. Next, due to the operator S, we can not use the step by step method coping with

(1.2) to obtain global existence (see [14], [16]). Here we use the Leray-Schauder fixed point theory to prove the existence of a solution of equation (1.4).

The paper is organized as follows. In section 2, we give some associated notations and important lemmas. In section 3, the existence of $PC-\alpha$-mild solution for impulsive integro-differential equations of mixed type with time-varying generating operators is presented. In section 4, we introduce a class of admissible controls and an existence result of optimal controls for a Lagrange problem (P) is proved. In last section, an example demonstrates the applicability of our results.

2. PRELIMINARIES

Let X, Y denote a pair of Banach spaces, if X is continuously embedded in Y, we write $X \hookrightarrow Y$, if X is compactly embedded in Y, we write $X \hookrightarrow\hookrightarrow Y$. $L(X)$ is the class of (not necessary bounded) linear operators in X, $L_b(X)$ is the class of bounded linear operators in X. For $A \in L(X)$, let $\rho(A)$ denote the resolvent set and $R(\lambda, A)$ the resolvent corresponding to $\lambda \in \rho(A)$. Let T be a fixed positive number, $\{A(t)|t \in [0,T]\}$ is a family of closed densely defined linear operators in X satisfying the following assumptions.

Assumption [A]:

(1) The domain $D(A(t))$ of $A(t)$ is independent of t;

(2) For $t \in [0,T]$, the resolvent $R(\lambda, A(t))$ exists for all λ with $\lambda \geq 0$ and

$$|R(\lambda, A(t))| \leq C(1+|\lambda|)^{-1}$$

where C is some constant independent of λ and t.

(3) The map $A(\cdot) : [0,T] \longrightarrow L_b(X_1, X)$ is Hölder continuous where $X_1 = (D(A), \|\cdot\|_1)$, $\|x\|_1 = \|Ax\|$. It is clearly that X_1 is a Banach space and $X_1 \hookrightarrow X$. More generally, in a usual way we introduce the fractional power operator $A^\alpha(t)$ ($\alpha \in [0,1]$), having dense domain $D(A^\alpha(t))$, which we also assume to be independent of t. We denote $D(A) = D(A(t))$ and $D(A^\alpha) = D(A^\alpha(t))$. Let $\|x\|_\alpha = \|A^\alpha x\|$ for $x \in D(A^\alpha)$ and denote the Banach space $(D(A^\alpha), \|\cdot\|_\alpha)$ by X_α. Then it is clear that $X_\beta \hookrightarrow X_\alpha$ for $0 \leq \alpha \leq \beta \leq 1$.

For the initial value problem

$$\begin{cases} \dot{x}(t) + A(t)x(t) = h(t), \quad t \in (0,T], \\ x(0) = x_0, \end{cases} \tag{2.1}$$

it is well known that (2.1) has unique classical solution x for every Hölder continuous right-hand side h. Moreover, $x \in C^1([0,T], X)$ provided $x_0 \in D(A)$ (see [3], [12], [16]). Further, there exists a unique evolution operator $U(t,\tau) \in L_b(X)$, $0 \leq \tau \leq t \leq T$, such that every solution of (2.1) can be represented in the form

$$x(t) = U(t,0)x_0 + \int_0^t U(t,\tau)h(\tau)d\tau, \quad t \in [0,T].$$

In order to derive a priori estimates on solutions of integro-differential equation, we need the following generalized Gronwall lemma.

Lemma 2.1: Let $x \in C([0,T],\, X_\alpha)$ and satisfy the following inequality

$$\|x(t)\|_\alpha \le a + b\int_0^t (t-s)^{-\gamma}\|x(s)\|_\alpha ds + c\int_0^t (t-s)^{-\gamma}\|x_s\|_B ds + e\int_0^T \|x(s)\|_\alpha^\lambda ds,$$

where $1 > \lambda > 0$, a, b, c, $d \ge 0$ are constants and $\|x_s\|_B = \sup\limits_{0\le\xi\le s}\|x(\xi)\|_\alpha$. Then there exists a constant $M > 0$ independent of α such that

$$\|x(t)\|_\alpha \le m^{-1}(0) \text{ for all } t \in [0,T],$$

where

$$m(s) = (2s - Ma)^{1-\lambda} - s^{1-\lambda} - (1-\lambda)MeT.$$

Proof. By Lemma 4.1 of [15] and Lemma 2.1 of [16], there exists constant $M > 0$ such that

$$\|x(t)\|_\alpha \le M\left(a + e\int_0^T \|x(s)\|_\alpha^\lambda ds\right).$$

Define

$$p(t) \equiv M\left(a + e\int_0^t \|x(s)\|_\alpha^\lambda ds + e\int_0^T \|x(s)\|_\alpha^\lambda ds\right),$$

we get

$$p(0) = Ma + Me\int_0^T \|x(s)\|_\alpha^\lambda ds\ ,\ \ p'(t) \le Mep^\lambda(t).$$

Integrating from 0 to t, we obtain

$$p^{1-\lambda}(t) - p^{1-\lambda}(0) \le (1-\lambda)Met,$$

that is

$$p(t) \le [p^{1-\lambda}(0) + (1-\lambda)et]^{\frac{1}{1-\lambda}},\ 0 \le t \le T.$$

Now, observe that

$$2p(0) - Ma = p(T) \le \left[p^{1-\lambda}(0) + (1-\lambda)MeT\right]^{\frac{1}{1-\lambda}}.$$

As a result, we get

$$(2p(0) - Ma)^{1-\lambda} - p^{1-\lambda}(0) \le (1-\lambda)MeT.$$

Letting

$$m(s) = (2s - Ma)^{1-\lambda} - s^{1-\lambda} - (1-\lambda)MeT,$$

we have $m \in C\left(\left[\frac{Ma}{2},\infty\right), R\right)$, and

$$m\left(\frac{Ma}{2}\right) = -\left(\frac{Ma}{2}\right)^{1-\lambda} - (1-\lambda)MeT < 0, \quad \lim_{s\to 0}\frac{m(s)}{s^{1-\lambda}} = 2^{1-\lambda} - 1 > 0.$$

It is clear that there exists a s_0 such that $m(s_0) = 0$ and therefore, $p(0) \leq s_0$. Thus,

$$\|x(t)\|_\alpha \leq m^{-1}(0) \text{ for all } t \in [0, T].$$

The proof is completed. □

Let $\Theta \subset (0, T)$, define $PC([0, T], X_\alpha) = \{x : [0, T] \longrightarrow X_\alpha | x$ is continuous at $t \in [0, T] \backslash \Theta$, x is continuous from left and has right hand limits at $t \in \Theta\}$. We see that $PC([0, T], X_\alpha)$ is a Banach space with the norm

$$\|x\|_{PC} = \max\left\{\sup_{t\in[0,T]} \|x(t+0)\|_\alpha, \quad \sup_{t\in[0,T]} \|x(t-0)\|_\alpha\right\}.$$

We need the Ascoli-Arzela lemma on $PC([0, T], X)$ where X is a Banach space.

Lemma 2.2: Let $W \subset PC([0, T], X)$. If the following conditions are satisfied

(1) W is uniformly bounded subset of $PC([0, T], X)$,

(2) W is equicontinuous in (t_i, t_{i+1}), $i = 0, 1, 2, \cdots, n$, where $t_0 = 0$, $t_{n+1} = T$,

(3) Its t-sections $W(t) \equiv \{x(t)|x \in W, t \in [0, T]\backslash\Theta\}$, $W(t_i+0) \equiv \{x(t_i+0)|x \in W\}$ and $W(t_i - 0) \equiv \{x(t_i - 0)\|x \in W\}$ are relatively compact subsets of X,

W is a relatively compact subset of $PC([0, T], X)$.

The proof is given by us in [13].

For the integral operators, we assume that:

[G]: (1) $g : [0, T] \times X_\alpha \longrightarrow X_\alpha$ is measurable in t on $[0, T]$ and locally Lipschitz continuous, that is, let $\rho > 0$, $\forall x_1, x_2 \in X_\alpha$ satisfying $\|x_1\|_\alpha$, $\|x_2\|_\alpha \leq \rho$, then there exists a constant $L_g(\rho)$ dependent only on ρ such that

$$\|g(t, x_1) - g(t, x_2)\|_\alpha \leq L_g(\rho)\|x_1 - x_2\|_\alpha.$$

(2) There exists a constant a_g such that

$$\|g(t, x)\|_\alpha \leq a_g(1 + \|x\|_\alpha) \text{ for all } t \in [0, T], \quad x \in X_\alpha.$$

(3) $k \in C([0, T]^2, R)$.

[S]: (1) $s : [0, T] \times X_\alpha \longrightarrow X_\alpha$ is measurable in t on $[0, T]$ and locally Lipschitz continuous, that is, let $\rho > 0$, $\forall x_1, x_2 \in X_\alpha$ satisfying $\|x_1\|_\alpha$, $\|x_2\|_\alpha \leq \rho$, then there exists a constant $L_s(\rho)$ dependent only on ρ such that

$$\|s(t, x_1) - s(t, x_2)\|_\alpha \leq L_s(\rho)\|x_1 - x_2\|_\alpha$$

(2) There exist constant a_s and $0 < \lambda < 1$ such that

$$\|s(t, x)\|_\alpha \leq a_s\left(1 + \|x\|_\alpha^\lambda\right) \text{ for all } t \in [0, T], \quad x \in X_\alpha.$$

(3) $h \in C([0, T]^2, R)$.

Using moving norm $\|\cdot\|_B$, one can verify that integral operators G and S have the following important properties (ref. see Lemma 2.3 and Lemma 2.4 of [13], Lemma 2.2 of [16]).

Lemma 2.3: Under assumption (G), operator G has the following properties:

(1) $G : C([0,T], X_\alpha) \longrightarrow C([0,T], X_\alpha)$.

(2) Let x_1, $x_2 \in C([0,T], X_\alpha)$ and $\|x_1\|_{C([0,T],X_\alpha)}$, $\|x_1\|_{C([0,T],X_\alpha)} \le \rho$, then

$$\|(Gx_1)(t) - (Gx_2)(t)\|_\alpha \le TL_g(\rho)\|k\|\|(x_1)_t - (x_2)_t\|_B,$$

(3) For $x \in C([0,T], X_\alpha)$, we have

$$\|Gx(t)\|_\alpha \le Ta_g\|k\|(1 + \|x_t\|_B).$$

Lemma 2.4: Under assumption (S), operator S has the following properties:

(1) $S : C([0,T], X_\alpha) \longrightarrow C([0,T], X_\alpha)$.

(2) Let $x_1, x_2 \in C([0,T], X_\alpha)$ and $\|x_1\|_{C([0,T],X_\alpha)}$, $\|x_1\|_{C([0,T],X_\alpha)} \le \rho$, then

$$\|(Sx_1)(t) - (Sx_2)(t)\|_\alpha \le TL_s(\rho)\|h\|\|x_1 - x_2\|_{C([0,T],X_\alpha)}$$

(3)For $x \in C([0,T], X_\alpha)$, we have

$$\|(Sx)(t)\| \le Ta_s\|h\| \left(1 + \|x\|^\lambda_{C([0,T],X_\alpha)}\right).$$

Remark 2.1: One can easily verify that these properties of integral operators G and S also hold on the Banach space $PC([0,T], X_\alpha)$.

3. EXISTENCE OF SOLUTIONS OF IMPULSIVE INTEGRO-DIFFERENTIAL EQUATIONS OF MIXED TYPE

In this section, we consider the existence of $PC-\alpha$-mild solution for the following nonlinear impulsive integro-differential equation of mixed type with time-varying generator operators

$$\begin{cases} \dot{x}(t) + A(t)x(t) = F(t, x(t), (Gx)(t), (Sx)(t)), & t \in (0,T] \setminus \Theta \\ x(0) = x_0, \quad \Delta x(t_i) = J_i(x(t_i)), & i = 1, 2, \cdots, n. \end{cases} \tag{3.1}$$

Definition 3.1. A function $x \in PC([0,T], X_\alpha)$ is said to be a $PC-\alpha$-mild solution of the problem (3.1) if x satisfies the following equation

$$\begin{aligned} x(t) = U(t,0)x_0 + \int_0^t U(t,\tau)F(\tau, x(\tau), (Gx)(\tau), (Sx)(\tau))d\tau \\ + \sum_{0<t_i<t} U(t,t_i)J_i(x(t_i)) \quad \text{for } t \in [0,T]. \end{aligned}$$

In addition to assumptions [A], [G] and [S], we introduce the following assumptions.

[F]: (1) $F : [0,T] \times X_\alpha \times X_\alpha \times X_\alpha \longrightarrow X$ is measurable in $t \in [0,T]$ and locally Lipschitz continuous with respect to last variables, i.e, $\forall x_1$, x_2, y_1, y_2, z_1, $z_2 \in X_\alpha$

satisfying $\|x_1\|_\alpha,\ \|x_2\|_\alpha,\ \|y_1\|_\alpha,\ \|y_2\|_\alpha,\ \|z_1\|_\alpha,\ \|z_2\|_\alpha \le \rho$, there exists a constant $L(\rho) > 0$ such that

$$\|F(t, x_1, y_1, z_1) - F(t, x_2, y_2, z_2)\| \le L(\rho)\left(\|x_1 - x_2\|_\alpha + \|y_1 - y_2\|_\alpha + \|z_1 - z_2\|_\alpha\right).$$

(2) There exists constant $a \ge 0$ such that

$$\|F(t, x, y, z)\| \le a\left(1 + \|x\|_\alpha + \|y\|_\alpha + \|z\|_\alpha\right) \text{ for all } t \in [0, T], \quad \text{for all } x, y, z \in X_\alpha.$$

(3) $x_0 \in X_\beta$ $(0 < \alpha < \beta < 1)$.

[J]: There exists a constant $e_i \ge 0$ such that map $J_i : X_{\beta_i} \longrightarrow X_{\beta_i}$ $(0 < \alpha < \beta_i < 1)$ satisfies

$$\|J_i(x(t)) - J_i(y(t))\|_{\beta_i} \le e_i \|x(t) - y(t)\|_{\beta_i} \ (i = 1,\ 2,\ \cdots,\ n).$$

Now we can give the existence of $PC - \alpha$-mild solution of (3.1).

Theorem 3.A: Suppose that $A^{-1}(0)$ is compact. Under the assumptions [A], [F], [G], [S] and [J], the equation (3.1) has a $PC - \alpha$-mild solution on $[0, T]$.

Proof. Let $x_0 \in X_\beta$ be fixed, set $f(x(t)) = F(t, x(t), (Gx)(t), (Sx)(t))$, define the operator P on $PC([0, T], X_\alpha)$ given by

$$(Px)(t) = U(t, 0)x_0 + \int_0^t U(t, \tau) f(x(\tau)) d\tau + \sum_{0 < t_i < t} U(t, t_i) J_i(x(t_i)).$$

By virtue properties of $U(\cdot, \cdot)$ (see P150 Theorem 6.1 of [11]), one easily verify that $Px \in PC([0, T], X_\alpha)$ for $x \in PC([0, T], X_\alpha)$.

(1) The operator P is continuous on $PC([0, T], X_\alpha)$.

In fact, let $x_1,\ x_2 \in PC([0, T], X_\alpha)$ and $\|x_1\|_{PC([0,T],X_\alpha)},\ \|x_2\|_{PC([0,T],X_\alpha)} \le \rho$ which ρ is a constant. Using assumptions [G](1), [S](1), [F](1), [J] and the properties of evolution operators (see [3], [12]), we obtain

$$\begin{aligned}
&\|(Px_1)(t) - (Px_2)(t)\|_\alpha \\
&\le C(\alpha, \gamma) L(\rho) \int_0^t (t - s)^{-\gamma} \big(\|x_1(s) - x_2(s)\|_\alpha + \|(Gx_1)(s) - (Gx_2)(s)\|_\alpha \\
&\quad + \|(Sx_1)(s) - (Sx_2)(s)\|_\alpha\big) ds \\
&\quad + \sum_{0 < t_i < t} C(\alpha, \beta_i) e_i \|x_1(t_i) - x_2(t_i)\|_\alpha \\
&\le C(\alpha, \gamma) L(\rho) \int_0^t (t - s)^{-\gamma} \left(1 + T L_g(\rho)\|k\| + T L_s(\rho)\|h\|\right) ds \|x_1 - x_2\|_{PC([0,T],X_\alpha))} \\
&\le \left(L + \sum_{i=1}^n C(\alpha, \beta_i) e_i\right) \|x_1 - x_2\|_{PC([0,T],X_\alpha)}
\end{aligned}$$

where $L = L(\rho) C(\alpha, \gamma) \frac{T^{1-\gamma}}{1-\gamma}\left[1 + T L_g(\rho)\|k\| + T L_s(\rho)\|h\|\right]$.

(2) The P is a compact operator.

Let B be bounded subset of $PC\left([0,T],X_\alpha\right)$, there exists a constant μ such that $\|x\|_{PC([0,T],X_\alpha)} \leq \mu$ for all $x \in B$. Using assumption [J], there exists a constant N such that $\|J_i(x(t))\|_{\beta_i} \leq N$ for all $x \in B$, $t \in [0,T]$, $i = 1,2,\cdots,n$. Also by Lemma 2.3 and Lemma 2.4, assumption [F](2), there exists a constant ω such that $\|F(t,x(t),(Gx)(t),(Sx)(t))\|_X \leq \omega$ for all $x \in B$. Further PB is a bounded subset of $PC\left([0,T],X_\alpha\right)$. In fact, let $x \in B$, we obtain

$$\|(Px)(t)\|_\alpha \leq C(\alpha,\beta)\|x_0\|_\beta + N\sum_{i=1}^{n} C(\alpha,\beta_i) + T\omega C(\alpha,\gamma)\frac{T^{1-\gamma}}{1-\gamma} \text{ for all } t \in [0,T].$$

Define $\mathcal{K} = PB$ and $\mathcal{K}(t) = \{(Px)(t)|x \in B\}$ for $t \in [0,T]$. Clearly, $\mathcal{K}(0) = \{x_0\}$ is compact, and hence, it is only necessary to consider $t > 0$. For $0 < \varepsilon < t \leq T$, define

$$\mathcal{K}_\varepsilon(t) = (P_\varepsilon B)(t) = \left\{U(t,t-\varepsilon)(Px)(t-\varepsilon)|x \in B\right\}. \tag{3.2}$$

Since $A^{-1}(0)$ is compact, $X_\beta \hookrightarrow\hookrightarrow X_\alpha$ for $0 \leq \alpha < \beta \leq 1$. Further, by $\mathcal{K}(t)$ is a bounded subset of $PC\left([0,T],X_\alpha\right)$, it follows from the above expression that $\mathcal{K}_\varepsilon(t)$ is relatively compact in X_α for $t \in (\varepsilon,T]$.

For interval $(0,t_1]$, (3.2) reduces to

$$\mathcal{K}_\varepsilon(t) = \left\{(U(t,t-\varepsilon)(Px)(t-\varepsilon)|x \in B\right\},$$

furthermore

$$\begin{aligned}\sup\left\{\|(Px)(t)-(P_\varepsilon x)(t)\|_\alpha|x \in B\right\} &\leq \sup\left\{\int_{t-\varepsilon}^{t}\|U(t,\tau)f(x(\tau))\|_\alpha d\tau\middle| x \in B\right\}\\ &\leq \omega C(\alpha,\gamma)\frac{\varepsilon^{1-\gamma}}{1-\gamma} \text{ for } t \in (\varepsilon,t_1],\end{aligned}$$

showing that the set $\mathcal{K}(t)$ can be approximated to an arbitrary degree of accuracy by a relatively compact set for $t \in (0,t_1]$. Hence $\mathcal{K}(t)$ itself is relatively compact in X_α for $t \in (0,t_1]$.

Consider interval $(t_1, t_2]$, we define

$$\mathcal{K}(t_1+0) = \left\{(Px)(t_1) + J_1((Px)(t_1))|x \in B\right\}.$$

By the assumption [J], one can verify that $\mathcal{K}(t_1+0)$ is a bounded subset of X_{β_1}. Since $A^{-1}(0)$ is compact, $X_{\beta_1} \hookrightarrow\hookrightarrow X_\alpha(\alpha < \beta_1)$, so, $\mathcal{K}(t_1+0)$ is relatively compact in X_α. Also since (3.2) reduces to

$$\mathcal{K}_\varepsilon(t) = \left\{U(t,t-\varepsilon)(Px)(t-\varepsilon)|x \in B\right\},$$

furthermore

$$\sup\left\{\|(Px)(t)-(P_\varepsilon x)(t)\|_\alpha\middle| x \in B\right\} \leq C(\alpha,\gamma)\frac{\omega\varepsilon^{1-\gamma}}{1-\gamma} \text{ for } t \in [t_1+\varepsilon,t_2],$$

note that the set $\mathcal{K}(t)$ can be approximated to an arbitrary degree of accuracy by a relatively compact set $\mathcal{K}_\varepsilon(t)$ for $t \in (t_1, t_2]$. Thus $\mathcal{K}(t)$ is relatively compact in X_α for $t \in (t_1, t_2]$.

In general, given any $t_i \in \Theta$, $i = 1, 2, \cdots, n$, we define $\mathcal{K}(t_i + 0) = \big\{(Px)(t_i) + J_i((Px)(t_i))|x \in B\big\}$, using assumption [J], $X_{\beta_i} \hookrightarrow\hookrightarrow X_\alpha (\alpha < \beta_i)$, we known $\mathcal{K}(t_i+0)$ is relatively compact in X_α. And the associated $\mathcal{K}_\varepsilon(t)$ over the interval $(t_i, t_{i+1}]$ is given by

$$\mathcal{K}_\varepsilon(t) = \{U(t, t-\varepsilon)(Px)(t-\varepsilon)|x \in B\} \, (i = 1, 2, \cdots, n),$$

furthermore

$$\sup\left\{\|(Px)(t) - (P_\varepsilon x)(t)\|_\alpha \middle| x \in B\right\} \leq C(\alpha,\gamma)\frac{\omega\varepsilon^{1-\gamma}}{1-\gamma} \text{ for } t \in [t_i + \varepsilon, t_{i+1}],$$

Similarly, we can know $\mathcal{K}(t)$ itself is relatively compact in X_α for $t \in (t_i, t_{i+1}]$.

Now, we repeat the procedures till the time interval which is expanded. Thus we obtain that the set $\mathcal{K}(t)$ itself is relatively compact for $t \in [0, T]\backslash\Theta$ and $\mathcal{K}(t_i + 0)$ is relatively compact for $t_i \in \Theta$.

For piece wise equicontinuity, we show that the $\mathcal{K}$ is equicontinuity in interval (t_i, t_{i+1}), $i = 0, 1, \cdots, n$.

For interval $(0, t_1)$, we note that for $t_1 > h \geq 0$, $x \in B$, have

$$\|(Px)(h) - (Px)(0)\|_\alpha \leq C(\alpha,\beta,\theta)h^\theta\|x_0\|_\beta + C(\alpha,\gamma)\omega\frac{h^{1-\gamma}}{1-\gamma},$$

and, for $t_1 \geq t + h \geq t \geq \zeta > 0$ and $x \in B$,

$$\begin{aligned}(Px)(t+h) - (Px)(t) &= U(t+h,0)x_0 - U(t,0)x_0 + \int_t^{t+h} U(t+h,s)f(x(s))ds \\ &\quad + \int_{t-\zeta}^{t} [U(t+h,s) - U(t,s)]f(x(s))ds \\ &\quad + \int_0^{t-\zeta} [U(t+h,s) - U(t,s)]f(x(s))ds.\end{aligned}$$

By

$$\lim_{h\to 0^+} \|U(t+h,s) - U(t,s)\|_{0,\alpha} = 0 \text{ for all } 0 \leq s \leq t \leq T,$$

we obtain

$$\begin{aligned}&\|(Px)(t+h) - (Px)(t)\|_\alpha \leq C(\alpha,\beta,\theta)h^\theta\|x_0\|_\beta + 2\omega C(\alpha,0)\zeta + \omega C(\alpha,\gamma)\frac{h^{1-\gamma}}{1-\gamma} \\ &\quad + \omega\int_0^{t-\zeta} \|U(t+h,s) - U(t,s)\|_{0,\alpha}ds \\ &\leq C(\alpha,\beta,\theta)h^\theta\|x_0\|_\beta + 3\omega\widetilde{C}h^{1-\gamma} + \omega\int_0^{t-\zeta} \|U(t+h,s) - U(t,s)\|_{0,\alpha}ds\end{aligned}$$

for $\zeta \le h < 1$, where $\widetilde{C} = C(\alpha, 0) + \frac{C(\alpha,\gamma)}{1-\gamma}$. Thus right hand side of this expression can be made as small as desired by choosing h sufficiently small. That is, $\mathcal{K}$ is equicontinuity in interval $(0, t_1)$.

In general, for time interval (t_i, t_{i+1}) $(i = 1, 2, \cdots, n)$, we similarly obtain following inequalities

$$\|(Px)(t+h) - (Px)(t)\|_\alpha \le C(\alpha, \beta_i, \theta)h^\theta \|x_i\|_{\beta_i} + 3\omega \widetilde{C} h^{1-\gamma} + \omega \int_{t_i}^{t-\zeta} \|U(t+h, s) - U(t, s)\|_{0,\alpha} ds$$

same show that the $\mathcal{K}$ is equicontinuity in interval (t_i, t_{i+1}).

Hence, by the generalize Ascoli-Arzela theorem (see Lemma 2.2), this justifies that PB is a relatively compact subset of $PC([0, T], X_\alpha)$. Furthermore, we show that P is a compact operator.

(3) The P has a fixed point in $PC([0, T], X_\alpha)$.

According to Leray-Schauder fixed point theory it suffices to show that $\coprod \equiv \{x \in PC([0, T], X_\alpha) \,|x = \sigma Px, \sigma \in [0, 1]\}$ is a bounded subset of $PC([0, T], X_\alpha)$. In fact, let $x \in \coprod$, we have

$$\begin{aligned}\|x(t)\|_\alpha \le\ & C(\alpha, \beta)\|x_0\|_\beta + a_0 T \left(\frac{1}{T} + a_g\|k\| + a_s\|h\|\right) \\ & + \sigma^\lambda a_F a_s \|h\| T C(\alpha, \gamma) \int_0^T \|x(\tau)\|_\alpha^\lambda d\tau \\ & + b_0 \int_0^t (t-\tau)^{-\gamma} \|x(\tau)\|_\alpha d\tau + b_0 a_g \|k\| \int_0^t (t-\tau)^{-\gamma} \|x_\tau\|_B d\tau \\ & + \sum_{0<t_i<t} C(\alpha, \beta_i) \|J_i(x(t_i))\|_{\beta_i}\end{aligned}$$

where $a_0 = aC(\alpha, \gamma)\frac{T^{1-\gamma}}{1-\gamma}$, $b_0 = \sigma a C(\alpha, \gamma)$.

Let $t \in [0, t_1]$, by Lemma 2.1, we know that there exists a constant $M_1^* > 0$ such that

$$\|x\|_{C([0,t_1],X_\alpha)} \le M_1^*.$$

Using assumption [J], there exists a $\overline{M}_1^*$ such that

$$\left\|x\left(t_1^+\right)\right\|_\alpha \le \overline{M}_1^*.$$

Consider time interval $(t_1, t_2]$, we have

$$\begin{aligned}\|x(t)\|_\alpha \le\ & C(\alpha, \beta_1)\left\|x\left(t_1^+\right)\right\|_{\beta_1} + a_0 T \left(\frac{1}{T} + a_g\|k\| + a_s\|h\|\right) \\ & + \sigma^\lambda a_F a_s \|h\| T C(\alpha, \gamma) \int_0^T \|x(\tau)\|_\alpha^\lambda d\tau \\ & + b_0 \int_0^t (t-\tau)^{-\gamma} \|x(\tau)\|_\alpha d\tau + b_0 a_g \|k\| \int_0^t (t-\tau)^{-\gamma} \|x_\tau\|_B d\tau.\end{aligned}$$

By Lemma 2.1 and assumption [J], there exist constants M_2^*, $\overline{M}_2^* > 0$ such that

$$\|x\|_{C((t_1,t_2],X_\alpha)} \leq M_2^*, \quad \left\|x\left(t_2^+\right)\right\|_\alpha \leq \overline{M}_2^*.$$

In general, for time interval $(t_i, t_{i+1}]$ $(i = 1, 2, \cdots, n-1)$, similarly, there exist constants M_{i+1}^*, $\overline{M}_{i+1}^* > 0$ such that

$$\|x\|_{C((t_i,t_{i+1}],X_\alpha)} \leq M_{i+1}^*, \quad \left\|x\left(t_{i+1}^+\right)\right\|_\alpha \leq \overline{M}_{i+1}^*.$$

For interval $(t_n, T]$, we know that

$$\begin{aligned}\|x(t)\|_\alpha \leq{}& C(\alpha,\beta_n)\left\|x\left(t_n^+\right)\right\|_{\beta_n} + a_0 T\left(\frac{1}{T} + a_g\|k\| + a_s\|h\|\right)\\ &+ \sigma^\lambda a_F a_s \|h\| T C(\alpha,\gamma)\int_0^T \|x(\tau)\|_\alpha^\lambda d\tau\\ &+ b_0\int_0^t (t-\tau)^{-\gamma}\|x(\tau)\|_\alpha d\tau + b_0 a_g\|k\|\int_0^t (t-\tau)^{-\gamma}\|x_\tau\|_B d\tau.\end{aligned}$$

By Lemma 2.1, there exists a constant $M_{n+1}^* > 0$ such that

$$\|x\|_{C((t_n,T],X_\alpha)} \leq M_{n+1}^*.$$

Define

$$M^* = \sum_{i=1}^{n}\left(M_i^* + \overline{M}_i^*\right) + M_{n+1}^*,$$

then we obtain

$$\|x\|_{PC([0,T],X_\alpha)} \leq M^*.$$

Thus $\coprod$ is a bounded subset of $PC([0,T], X_\alpha)$.

Thus, by Leray-Schauder fixed point theory, we obtain P has a fixed point in $PC([0,T], X_\alpha)$. This implies that the equation (3.1) have a $PC-\alpha$-mild solution on $PC([0,T], X_\alpha)$. □

The above theorem tells us that solvability of impulsive integro-differential equation of mixed type. The next result gives the existence of $PC-\alpha$-mild solution of integro-differential equation without impulsive.

Corollary 3.1: Suppose $A^{-1}(0)$ is compact. Under the assumptions [F], [G], [S], the equation

$$\begin{cases} \dot{x}(t) + A(t)x(t) = F(t, x(t), (Gx)(t), (Sx)(t)), & t \in (0,T],\\ x(0) = x_0, \end{cases}$$

has a α-mild solution on $[0,T]$.

Remark 3.1: The uniqueness of solution of equation (3.1) cannot be obtained.

4. EXISTENCE OF OPTIMAL CONTROLS

In this section, we discuss the existence of optimal controls of systems governed by general equation (1.4).

We suppose that $A(0)$ has a compact resolvent, Y is another separable reflexive Banach space from which the controls u take the value. We denote a class of nonempty closed and convex subsets of Y by $P_f(Y)$. The multifunction $\omega : [0,T] \longrightarrow P_f(Y)$ is measurable and $\omega(\cdot) \subset E$ where E is bounded set of Y, the admissible control set $U_{ad} = S^p_\omega = \{u \in L^p(E) | u(t) \in \omega(t)\ a.e\}$. $U_{ad} \neq \emptyset$ (see P142 Proposition 1.7 and P174 Lemma 3.2 of [8]).

Consider the following controlled system

$$(4.1) \quad \begin{cases} \dot{x}(t) + A(t)x(t) = F(t, x(t), (Gx)(t), (Sx)(t)) + B(t)u(t), & t \in (0,T] \setminus \Theta, \\ u \in U_{ad}, \quad x(0) = x_0, \quad \Delta x(t_i) = J_i(x(t_i)), & t_i \in \Theta. \end{cases}$$

Suppose [B]: $B \in L_\infty ([0,T],\ \pounds\,(Y,\ X_\alpha))$.

It is easy to see that $Bu \in L_p([0,T],\ X_\alpha)$ for all $u \in U_{ad}$. Define $\widetilde{F}(t,x) = F(t, x(t), (Gx)(t), (Sx)(t)) + B(t)u(t)$. It is obvious that $\widetilde{F}$ satisfies the assumption [F].

Theorem 4.A: Suppose $A(0)$ has a compact resolvent. Under assumptions [F], [G], [S], [J] and [B], for every $u \in U_{ad}$, system (4.1) has a $PC-\alpha$-mild solution corresponding to u.

We consider the Lagrange problem (P):

Find $(x^0, u^0) \in PC([0,T], X_\alpha) \times U_{ad}$ such that

$$J\left(x^0, u^0\right) \leq J\left(x^u, u\right) \text{ for all } u \in U_{ad},$$

where

$$J\left(x^u, u\right) = \int_0^T l\left(t,\ x^u(t),\ u(t)\right) dt,$$

x^u denotes the $PC-\alpha$-mild solution of system (4.1) corresponding to the control $u \in U_{ad}$.

We introduce some assumptions on l.

[L]: (1) The functional $l : [0,T] \times X_\alpha \times Y \longrightarrow R \bigcup \{\infty\}$ is Borel measurable.

(2) $l(t,\cdot,\cdot)$ is sequentially lower semicontinuous on $X_\alpha \times Y$ for almost all $t \in [0,T]$.

(3) $l(t,x,\cdot)$ is convex on Y for each $x \in X_\alpha$ and almost all $t \in [0,T]$.

(4) There exist constants $b \geq 0$, $c > 0$ and $\varphi \in L_1([0,T],\ R)$ such that

$$l(t,\ x,\ u) \geq \varphi(t) + b\|x\|_\alpha + c\|u\|^p_Y \quad \text{for all } x \in X_\alpha, \quad u \in Y.$$

By the Lemma 4.1 of [16] and Lemma 4.1 of [13], it's easy to get following important lemma.

Lemma 4.1: In addition to [A] and [B], we assume that $A(0)$ has a compact resolvent, then the operator

$$Q : L_p([0,T],X) \longrightarrow C([0,T],X_\alpha) \left(0 < \alpha < \frac{p-1}{p}\right)$$

given by

$$(Qf)(t) = \int_0^t U(t,s)f(s)ds$$

is strongly continuous.

Now we can give the another main result of this paper again, existence of optimal controls for problem.

Theorem 4.B: Suppose the assumption [L] and the assumptions of Theorem 4.A holds, problem (P) has admits at least one optimal pair.

Proof. If $\inf\{J(x^u,u)\,|u \in U_{ad}\} = +\infty$, there is nothing to prove.

Assume that $\inf\{J(x^u,u)\,|u \in U_{ad}\} = m < +\infty$. Using assumption [L], we have $m > -\infty$. By definition of infimum there exists a minimizing sequence feasible pair $\{(x^n,u^n)\} \subset A_{ad} \equiv \{(x,u)|x$ is a $PC-\alpha$-mild solution of equation (4.1) corresponding to $u \in U_{ad}\}$, such that $J(x^n,u^n) \longrightarrow m$ as $n \to +\infty$. Since $\{u^n\} \subseteq U_{ad}$, $\{u^n\}$ is bounded in $L_p([0,T],Y)$, there exists a subsequence, relabeled as $\{u^n\}$, and $u^0 \in L_p([0,T],Y)$ such that

$$u^n \xrightarrow{w} u^0 \text{ in } L_p([0,T],Y).$$

U_{ad} is closed and convex, thanks to Marzur Lemma, $u^0 \in U_{ad}$.

Suppose x^n is the $PC-\alpha$-mild solution of system (4.1) corresponding to u^n $(n = 0,\,1,\,2,\,\cdots)$, x^n satisfies the following integral equation

$$\begin{aligned} x^n(t) = U(t,0)x_0 &+ \int_0^t U(t,\tau)\left[F(\tau,x^n(\tau),(Gx^n)(\tau),(Sx^n)(\tau)) + B(\tau)u^n(\tau)\right]d\tau \\ &+ \sum_{0<t_i<t} U(t,t_i)J_i(x^n(t_i)). \end{aligned}$$

Let $F_n(\tau) \equiv F(\tau,x^n(\tau),(Gx^n)(\tau),(Sx^n)(\tau))$, by assumptions [G](2), [S](2), [F](2), [J], [B] and Lemma 2.1, we obtain that $F_n \in L_b([0,T],X)$, hence $F_n(\cdot) \in L_p([0,T],X)$, furthermore, $\{F_n(\cdot)\} \subseteq X$, $\{F_n(\cdot)\}$ is bounded in $L_p([0,T],X)$, there exists a subsequence, relabeled as $\{F_n(\cdot)\}$, and $\bar{F}(\cdot) \in L_p([0,T],X)$ such that

$$F_n(\cdot) \xrightarrow{w} \bar{F}(\cdot) \text{ in } L_p([0,T],X).$$

By Lemma 4.1, we have

$$QF_n \longrightarrow Q\bar{F} \text{ in } PC([0,T],X_\alpha).$$

We consider the following impulsive differential equation

$$\begin{cases} \dot{x}(t) + A(t)x(t) = \overline{F}(t) + B(t)u^0(t), & t \in (0,T) \setminus \Theta, \\ x(0) = x_0, \quad \Delta x(t_i) = J_i(x(t_i)), & t_i \in \Theta. \end{cases} \tag{4.2}$$

By Theorem 3.A, we know that equation (4.2) has a $PC - \alpha$-mild solution

$$\overline{x}(t) = U(t,0)x_0 + \int_0^t U(t,\tau)\left[\overline{F}(\tau) + B(\tau)u^0(\tau)\right] d\tau + \sum_{0<t_i<t} U(t,t_i)J_i(\overline{x}(t_i)).$$

Define

$$\eta_n(t) = \left\| \int_0^t U(t,\tau)\left[\left(F_n(\tau) - \overline{F}(\tau)\right) + B(\tau)\left(u^n(\tau) - u^0(\tau)\right)\right] d\tau \right\|_\alpha .$$

Using Lemma 4.1, we have

$$\eta_n \longrightarrow 0 \text{ in } C([0,T],R) \text{ as } n \longrightarrow \infty.$$

By assumptions [J](2), we obtain

$$\|x^n(t) - \overline{x}(t)\|_\alpha \leq \eta_n(t) + \sum_{0<t_i<t} C(\alpha,\beta_i)e_i \|x^n(t_i) - \overline{x}(t_i)\|_\alpha .$$

Using Gronwall inequality with impulse, we obtain that

$$x^n \longrightarrow \overline{x} \text{ in } PC\left([0,T],X_\alpha\right),$$

furthermore, using assumption [F](1), we also obtain

$$F_n(\cdot) \longrightarrow F(\cdot,\overline{x}(\cdot),(G\overline{x})(\cdot),(S\overline{x})(\cdot)) \text{ in } PC([0,T],X).$$

Using the uniqueness of limit, we have

$$\overline{F}(t) = F(t,\overline{x}(t),(G\overline{x})(t),(S\overline{x})(t)),$$

furthermore,

$$\overline{x}(t) = U(t,0)x_0 + \int_0^t U(t,\tau)\left[F(t,\overline{x}(t),(G\overline{x})(t),(S\overline{x})(t)) + B(\tau)u^0(\tau)\right] d\tau + \sum_{0<t_i<t} U(t,t_i)J_i(\overline{x}(t_i)).$$

Thus, $\overline{x}$ is a $PC - \alpha$-mild solution of equation (4.1) corresponding to u^0.

Since $PC\left([0,T],X_\alpha\right) \hookrightarrow L_1\left([0,T]I,X_\alpha\right)$, using the assumption [L] and Balder's theorem, we can obtain

$$m = \lim_{n\to\infty} \int_0^T l\left(t,x^n(t),u^n(t)\right) dt \geq \int_0^T l\left(t,\bar{x}\,(t),u^0(t)\right) dt = J\left(\bar{x},u^0\right) \geq m.$$

This show that J attains its minimum at $u^0 \in U_{ad}$. □

5. EXAMPLE

In this section, an example is given to illustrate our theory, we consider the following problem:

$$\text{(5.1)}\quad \begin{cases} \frac{\partial}{\partial t}x(t,y) = (t+1)\Delta x(t,y) + x(t,y) + u(t,y) + \int_0^t \sqrt{x^2(\tau,y)+1}d\tau \\ \qquad\qquad + \int_0^1 (t+\tau)^2\sqrt{x^2(\tau,y)+1}d\tau, \qquad y\in\Omega, t\in(0,1]\setminus\Lambda, \\ x(t,y)|_{[0,1]\times\partial\Omega} = 0, \quad x(0,y)=0, \quad x(t_i+0,y)-x(t_i-0,y) = -x(t_i,y), \\ \qquad\qquad\qquad\qquad\qquad\qquad y\in\Omega,\ t_i\in\Lambda, \end{cases}$$

where $\Lambda = \left\{\frac{i}{9} | i = 1,\cdots,8\right\}$, $\Omega\subset R^3$ is bounded domain with C^3-boundary $\partial\Omega$.

Let $X = L_2(\Omega)$, $D(A) = H^2(\Omega)\bigcap H_0^1(\Omega)$, and $A(t)x = -(t+1)\Delta x$ for $x\in D(A)$, $U_{ad} = \{u\in X : \|u\|_X \le 1\}$ is closed and convex. By Nirenberg-Gagliardo inequality and the condition of Theorem 4.A, we can choose $\alpha = \frac{11}{24}$, such that $X_{\frac{11}{24}} \hookrightarrow C^1(\overline{\Omega})$. Let

$$J(x,u) = \int_0^T \int_\Omega |x(t,\xi)|^2 d\xi dt + \int_0^T \int_\Omega |u(t,\xi)|^2 d\xi dt.$$

Define $[x(\cdot)](y) = x(\cdot,y)$, $[(Gx)(\cdot)](y) = \int_0^{\cdot} \sqrt{x^2(\tau,y)+1}d\tau$, $[(Sx)(\cdot)](y) = \int_0^1(\cdot+\tau)^2\sqrt{x^2(\tau,y)+1}d\tau$, $F(\cdot,x(\cdot),(Gx)(\cdot),(Sx)(\cdot))(y) = x(\cdot,y)+Gx(\cdot,y)+Sx(\cdot,y)$, $B(\cdot)[u(\cdot)](y) = u(\cdot,y)$, then F satisfying assumption [F]. The problem (5.1) can be rewritten as

$$\text{(5.2)}\quad \begin{cases} \dot{x}(t) + A(t)x(t) = F(t,x(t),(Gx)(t),(Sx)(t)) + B(t)u(t), & t\in(0,1]\setminus\Lambda, \\ x(0) = x_0, \quad x\left(\frac{i}{9}+0\right) - x\left(\frac{i}{9}-0\right) = -x\left(\frac{i}{9}\right), & i = 1,\ \cdots,\ 8, \end{cases}$$

with the cost function

$$J(u) = \int_0^T \left(\|x(t)\|_X^2 + \|u(t)\|_X^2\right) dt.$$

Obviously, it satisfies all the assumptions given in our former theorems, our results can be used to (5.1).

Acknowledgment : We wish to thank the referee very much.

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