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EXISTENCE, UNIQUENESS AND QUENCHING OF THE SOLUTION FOR A NONLOCAL DEGENERATE SEMILINEAR PARABOLIC PROBLEM

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ABSTRACT. Let a and T be positive constants, $D = (0, a)$, $\bar{D} = [0, a]$, $\Omega = D \times (0, T]$, and $Lu = x^q u_t - u_{xx}$, where q is a nonnegative number. This article studies the following problem,

$$Lu(x,t) = \int_0^x k(y) f(u(y,t))dy \text{ in } \Omega,$$

where k is a positive function on $\bar{D}$, $f > 0$, $f' \geq 0$, $f'' \geq 0$, and $\lim_{u\to 1^-} f(u) = \infty$, subject to the initial condition $u(x, 0) = 0$ on $\bar{D}$, and the boundary conditions $u(0, t) = 0 = u(a, t)$ for $0 < t \leq T$. Existence of a unique solution, the critical length, and the quenching behavior of the solution are studied.

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1. INTRODUCTION

Let a and T be constants, $D = (0, a)$, $\bar{D} = [0, a]$, $\Omega = D \times (0, T]$, $\partial\Omega$ be the parabolic boundary, and $Lu = x^q u_t - u_{xx}$, where q is a nonnegative number. We consider the following nonlocal initial-boundary value problem,

$$Lu(x,t) = \int_0^x k(y) f(u(y,t))dy \; in \; \Omega, \tag{1.1}$$

$$u(x,0) = 0 \; on \; \bar{D}, \; u(0,t) = 0 = u(a,0) \; for \; 0 < t \leq T, \tag{1.2}$$

where k is a positive function on $\bar{D}$, $f > 0$, $f' > 0$, $f'' \geq 0$, and $\lim_{u\to 1^-} f(u) = \infty$. Chan and Kong [2], and Chan and Liu [3] studied existence, uniqueness and quenching behavior of the solution u in the case $\int_0^x k(y)f(u(y,t))dy$ being replaced by $f(u)$. We show that the problem (1.1)-(1.2) has a unique classical solution, and give a criterion for quenching to occur and for global existence.

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2. EXISTENCE AND UNIQUENESS

Since $k(x) > 0$ on $\bar{D}$, we have $\int_0^x k(y)f(u(y,t))dy > 0$ for $x > 0$. From the strong maximum principle (cf. Friedman [5]), $u > 0$ in Ω.

We now prove the comparison results.

Lemma 2.1. *Let w be a function such that*

$$Lw > \int_0^x g(y,t)w(y,t)dy \text{ in } \Omega,$$

where $g(x,t)$ is a bounded nonnegative function on $\bar{\Omega}$, and $w > 0$ on $\partial\Omega$, then $w > 0$ on $\bar{\Omega}$.

Proof. Suppose that $w \le 0$ somewhere on $\bar{\Omega}$. Let

$$\tilde{t} = \inf\{t : w(x,t) \le 0 \text{ for some } x \in \bar{D}\}.$$

Since $w > 0$ on $\partial\Omega$, we have $\tilde{t} > 0$, and there exists some $\tilde{x} \in D$ such that $w(\tilde{x},\tilde{t}) = 0$, $w(x,\tilde{t}) \ge 0$ on $\bar{D}$, $w_t(\tilde{x},\tilde{t}) \le 0$, and $w_{xx}(\tilde{x},\tilde{t}) \ge 0$. This implies $0 \ge \tilde{x}^q w_t(\tilde{x},\tilde{t}) > w_{xx}(\tilde{x},\tilde{t}) + \int_0^{\tilde{x}} g(y,\tilde{t})w(y,\tilde{t})dy \ge 0$. We have a contradiction. Thus, $w > 0$ on $\bar{\Omega}$. □

Theorem 2.2. *If w satisfies the inequality*

$$Lw \ge \int_0^x g(y,t)w(y,t)dy \text{ in } \Omega,$$

where $g(x,t)$ is a bounded nonnegative function on $\bar{\Omega}$, and $w \ge 0$ on $\partial\Omega$, then $w \ge 0$ on $\bar{\Omega}$.

Proof. For a fixed positive number η, let

$$V(x,t) = w(x,t) + \eta(1 + x^{\frac{1}{2}})e^{ct},$$

where c is some positive constant to be determined. Since $g(x,t)$ is bounded on $\bar{\Omega}$, let $\bar{M} = \sup_{t\in[0,T]} \left\{\int_0^a g(x,t)(1 + x^{1/2})dx\right\}$. We have $V_{xx} = w_{xx} - \eta x^{-3/2}e^{ct}/4$. Let s be the first positive zero of $x^{-3/2}/4 - \bar{M}$. If $s \ge a$, then for any $x \in \bar{D}$,

$$cx^q(1 + x^{\frac{1}{2}}) + \frac{1}{4}x^{-\frac{3}{2}} - \bar{M} > 0. \tag{2.1}$$

If $s < a$, then for any $x \in (0,s)$, the inequality (2.1) holds. For $x \in [s,a]$, let us choose c such that $c > \bar{M}/s^q$. Then on $[s,a]$, $cx^q(1+x^{1/2}) > \bar{M}(1+x^{1/2}) > \bar{M} > \bar{M} - x^{-3/2}/4$, and the inequality (2.1) holds. Thus for any $x \in \bar{D}$,

$$\begin{aligned} x^q c\eta(1 + x^{\frac{1}{2}})e^{ct} &> \eta e^{ct}\left(-\tfrac{1}{4}x^{-\frac{3}{2}} + \bar{M}\right) \\ &> -\tfrac{1}{4}\eta x^{-\frac{3}{2}}e^{ct} + \eta e^{ct}\int_0^a g(x,t)(1 + x^{\frac{1}{2}})dx \\ &\ge -\tfrac{1}{4}\eta x^{-\frac{3}{2}}e^{ct} + \eta e^{ct}\int_0^x g(y,t)(1 + y^{\frac{1}{2}})dy. \end{aligned}$$

This gives

$$\begin{aligned} x^q V_t &> w_{xx} + \int_0^x g(y,t)w(y,t)dy \\ &\quad -\tfrac{1}{4}\eta x^{-\frac{3}{2}}e^{ct} + \eta e^{ct}\int_0^x g(y,t)(1+y^{\frac{1}{2}})dy \\ &= w_{xx} - \frac{1}{4}\eta x^{-\frac{3}{2}}e^{ct} + \int_0^x g(y,t)\left[w(y,t) + \eta(1+y^{\frac{1}{2}})e^{ct}\right]dy \\ &= V_{xx}(x,t) + \int_0^x g(y,t)V(y,t)\,dy. \end{aligned}$$

Since $w \geq 0$ on $\partial\Omega$, we have $V > 0$ on $\partial\Omega$. By Lemma 2.1, we have $V > 0$ on $\bar{\Omega}$. As $\eta \to 0$, we obtain $w \geq 0$ on $\bar{\Omega}$. □

Theorem 2.3. *Suppose that u is a solution of the problem (1.1)-(1.2), and v satisfies*

$$Lv \geq \int_0^x k(y)f(v(y,t))dy \text{ in } \Omega,\ v \geq 0 \text{ on } \partial\Omega,$$

then $v \geq u$ on $\bar{\Omega}$.

Proof. We have $L(v-u) \geq \int_0^x k(y)(f(v)-f(u))dy$. By the mean value theorem, $L(v-u) \geq \int_0^x k(y)(f'(\xi)(v(y,t)-u(y,t))dy$ for some ξ between u and v. By Theorem 2.2, $v-u \geq 0$ on $\bar{\Omega}$. □

As a consequence of the comparison theorem, we have the following results.

Theorem 2.4. *The problem (1.1)-(1.2) has at most one solution.*

Let u^a denote the solution of the problem (1.1)-(1.2).

Theorem 2.5. *If $a_1 > a_2$, then $u^{a_1}(x,t) \geq u^{a_2}(x,t)$ for $(x,t) \in [0,a_2] \times [0,T]$.*

Proof. We have $u^{a_1}(0,t) = 0$ and $u^{a_1}(a_2,t) \geq 0$. Since $u^{a_2}(0,t) = 0 = u^{a_2}(a_2,t)$, it follows from Theorem 2.3 that $u^{a_1}(x,t) \geq u^{a_2}(x,t)$. □

We now show existence of the solution. Let $\Omega_{t_0} = D \times (0,t_0]$, and $\bar{\Omega}_{t_0}$ be its closure.

Theorem 2.6. *There exists some $t_0(>0)$ such that the problem (1.1)-(1.2) has a unique nonnegative solution $u \in C(\bar{\Omega}_{t_0}) \cap C^{2,1}(\Omega_{t_0})$.*

Proof. Let δ and t_0 be positive constants with $\delta < a$, $\Omega_\delta = (\delta,a) \times (0,t_0]$, $S_\delta = \{\delta,a\} \times (0,t_0]$, $\bar{\Omega}_\delta$ be the closures of Ω_δ, and u_δ be the solution of the problem,

$$\text{(2.2)} \qquad \begin{cases} Lu_\delta = \displaystyle\int_\delta^x k(y)f(u_\delta(y,t))dy \text{ in } \Omega_\delta, \\ u_\delta(x,0) = 0 \text{ on } [\delta,a];\ u_\delta(x,t) = 0 \text{ on } S_\delta. \end{cases}$$

Let us construct an upper solution $h(x,t)$ for all u_δ as follows:

(i) Let $\theta(x) = x^\gamma(a-x)^\gamma$, where $\gamma \in (0,1)$; also let k_1 be a positive constant such that $1 > k_1\theta(x)$.

(ii) Let ϵ be a positive number such that $k_1\theta(x) < 1-\epsilon < 1$.

(iii) Since $\theta''(x)$ tends to $-\infty$ as x tends to 0 or a, there exists a positive number $k_2 \in D$ such that $k_1\theta''(x) + f(1-\epsilon)\int_0^a k(y)dy \le 0$ for $x \in (0,k_2) \cup (a-k_2,a)$.

(iv) Let $g(t)$ be the solution of the initial value problem,

$$k_2^q g'(t)\theta(k_2) = \left(\int_0^a k(y)dy\right) f\left(\left(\frac{a}{2}\right)^{2\gamma} g(t)\right),\ g(0) = k_1.$$

(v) Since $(a/2)^{2\gamma}k_1 < 1-\epsilon$ and $g'(t) > 0$, we can choose $t_0 > 0$ such that $(a/2)^{2\gamma}g(t_0) = 1-\epsilon$.

To show that $h(x,t) = \theta(x)g(t)$ is an upper solution of u_δ, let $J = Lh - \int_0^a k(y)f(h(y,t))dy$. By a direct computation,

$$J(x,t) = x^q\theta(x)g'(t) - \theta''(x)g(t) - \int_0^a k(y)f\left(\theta(y)g(t)\right)dy.$$

For $x \in (0,k_2)$ and $t \in (0,t_0]$,

$$\begin{aligned} J(x,t) &\ge -\theta''(x)g(t) - \int_0^a k(y)f(\theta(y)g(t))dy \\ &\ge -k_1\theta''(x) - f(1-\epsilon)\int_0^a k(y)dy \\ &\ge 0. \end{aligned}$$

For $x \in [k_2, a-k_2]$ and $t \in (0,t_0]$,

$$\begin{aligned} J(x,t) &\ge x^q\theta(x)g'(t) - \int_0^a k(y)f(\theta(y)g(t))dy \\ &\ge k_2^q\theta(k_2)g'(t) - \left(\int_0^a k(y)dy\right) f\left(\left(\frac{a}{2}\right)^{2\gamma} g(t)\right) \\ &= 0. \end{aligned}$$

For $x \in (a-k_2,a)$ and $t \in (0,t_0]$,

$$\begin{aligned} J(x,t) &\ge -\theta''(x)g(t) - \int_0^a k(y)f(\theta(y)g(t))dy \\ &\ge -k_1\theta''(x) - f(1-\epsilon)\int_0^a k(y)dy \\ &\ge 0. \end{aligned}$$

Now, $h(x,0) = k_1\theta(x) \ge 0$. Since $h(\delta,t) > 0$, and $h(a,t) = 0$, it follows from Theorem 2.3 that h is an upper solution.

We note that $x^{-q} \in C^{\alpha,\alpha/2}(\bar{\Omega}_\delta)$,

$$\left| x^{-q} \int_\delta^x k(y) f(u_\delta(y,t)) dy \right| \leq \delta^{-q} \int_\delta^x k(y) f(u_\delta(y,t)) dy \text{ for } (x,t,u) \in \bar{\Omega}_\delta \times R.$$

Since $v \equiv 0$ is a lower solution, it follows from Theorem 4.2.2 of Ladde, Lakshmikantham and Vatsala [6, p.143] that the problem (2.2) has a unique solution $u_\delta \in C^{2+\alpha,1+\alpha/2}(\bar{\Omega}_\delta)$. Since $u_{\delta_1} < u_{\delta_2}$ in Ω_{δ_1} for $\delta_1 > \delta_2$, $\lim_{\delta \to 0} u_\delta$ exists for all $(x,t) \in \bar{\Omega}_{t_0}$.

For any $(x_1,t_1) \in \Omega_{t_0}$, there is a set $Q = [b_1, b_2] \times [t_2, t_3] \subset \Omega_{t_0}$, where b_1, b_2, t_2 and t_3 are positive numbers such that $b_1 < x_1 < b_2 < a$ and $t_2 < t_1 \leq t_3$. Since $u_\delta \leq h(x,t)$ in Q and $h(x,t) < 1$, we have for some constant $p_1 > 1$, and some positive constants k_3 and k_4,

$$||u_\delta||_{L^{p_1}(Q)} \leq ||h(x,t)||_{L^{p_1}(Q)} \leq k_3,$$

$$\left\| x^{-q} \int_\delta^x k(y) f(u_\delta(y,t)) dy \right\|_{L^{p_1}(Q)} \leq b_1^{-q} \left\| \int_0^x k(y) f(h(y,t)) dy \right\|_{L^{p_1}(Q)} \leq k_4.$$

By Ladyženskaja, Solonnikov and Ural′ceva [7, pp. 341-342], $u_\delta \in W^{2,1}_{p_1}(Q)$. By the embedding theorems there [7, pp. 61 and 80], $W^{2,1}_{p_1}(Q) \hookrightarrow H^{\alpha,\alpha/2}(Q)$ by choosing $p_1 > 2/(1-\alpha)$ with $\alpha \in (0,1)$. Then, $||u_\delta||_{H^{\alpha,\alpha/2}(Q)} \leq k_5$ for some constant k_5. For $x_1 < x_2$,

$$\begin{aligned} & \left\| x^{-q} \textstyle\int_\delta^x k(y) f(u_\delta(y,t)) dy \right\|_{H^{\alpha,\alpha/2}(Q)} \\ \leq \; & b_1^{-q} \left\| \textstyle\int_0^x k(y) f(h(y,t)) dy \right\|_\infty \\ & + \sup_{\substack{(x_1,t) \in Q \\ (x_2,t) \in Q}} \frac{\left| x_1^{-q} \int_\delta^{x_1} k(y) f(u_\delta(y,t)) dy - x_2^{-q} \int_\delta^{x_2} k(y) f(u_\delta(y,t)) dy \right|}{|x_1 - x_2|^\alpha} \\ & + \sup_{\substack{(x,t_1) \in Q \\ (x,t_2) \in Q}} \frac{x^{-q} \left| \int_\delta^x k(y) f(u_\delta(y,t_1)) dy - \int_\delta^x k(y) f(u_\delta(y,t_2)) dy \right|}{|t_1 - t_2|^{\alpha/2}}, \end{aligned}$$

the first term of which is bounded while the second term,

$$\begin{aligned} & \sup_{\substack{(x_1,t) \in Q \\ (x_2,t) \in Q}} \frac{\left| x_1^{-q} \int_\delta^{x_1} k(y) f(u_\delta(y,t)) dy - x_2^{-q} \int_\delta^{x_2} k(y) f(u_\delta(y,t)) dy \right|}{|x_1 - x_2|^\alpha} \\ \leq \; & \sup_{\substack{(x_1,t) \in Q \\ (x_2,t) \in Q}} \frac{\left| x_1^{-q} \int_\delta^{x_1} k(y) f(u_\delta(y,t)) dy - x_2^{-q} \int_\delta^{x_1} k(y) f(u_\delta(y,t)) dy \right|}{|x_1 - x_2|^\alpha} \end{aligned}$$

$$+ \sup_{\substack{(x_1,t)\in Q\\(x_2,t)\in Q}} \frac{\left|x_2^{-q}\int_\delta^{x_1} k(y)f(u_\delta(y,t))dy - x_2^{-q}\int_\delta^{x_2} k(y)f(u_\delta(y,t))dy\right|}{|x_1-x_2|^\alpha}$$

$$\leq \sup_{\substack{(x_1,t)\in Q\\(x_2,t)\in Q}} \frac{\left|\int_\delta^{x_1} k(y)f(u_\delta(y,t))dy\right|\left|x_1^{-q}-x_2^{-q}\right|}{|x_1-x_2|^\alpha}$$

$$+ \sup_{\substack{(x_1,t)\in Q\\(x_2,t)\in Q}} \frac{\left|x_2^{-q}\right|\left|\int_{x_2}^{x_1} k(y)f(u_\delta(y,t))dy\right|}{|x_1-x_2|^\alpha}$$

$$\leq \left|\sup_{\bar{D}} k(x)\right|\left|\sup_{\bar{D}\times[0,t_0]} f(u_\delta(x,t))\right| \sup_{\substack{(x_1,t)\in Q\\(x_2,t)\in Q}} \frac{|x_1-\delta|\left|x_1^{-q}-x_2^{-q}\right|}{|x_1-x_2|^\alpha}$$

$$+ b_1^{-q}\left|\sup_{\bar{D}} k(x)\right|\left|\sup_{\bar{D}\times[0,t_0]} f(u_\delta(x,t))\right| \sup_{\substack{(x_1,t)\in Q\\(x_2,t)\in Q}} \frac{|x_1-x_2|}{|x_1-x_2|^\alpha}$$

$$\leq a\left|\sup_{\bar{D}} k(x)\right|\left|\sup_{\bar{D}\times[0,t_0]} f(h(x,t))\right| \left|\left|x^{-q}\right|\right|_{H^{\alpha,\alpha/2}(Q)}$$

$$+ b_1^{-q}\left|\sup_{\bar{D}} k(x)\right|\left|\sup_{\bar{D}\times[0,t_0]} f(h(x,t))\right| \sup_{\substack{(x_1,t)\in Q\\(x_2,t)\in Q}} |x_1-x_2|^{1-\alpha}$$

$$\leq k_6 \text{ for some constant } k_6,$$

and the last term,

$$\sup_{\substack{(x,t_1)\in Q\\(x,t_2)\in Q}} \frac{x^{-q}\left|\int_\delta^{x} k(y)f(u_\delta(y,t_1))dy - \int_\delta^{x} k(y)f(u_\delta(y,t_2))dy\right|}{|t_1-t_2|^{\alpha/2}}$$

$$\leq b_1^{-q}\left\|\int_0^a k(y)f'(h(y,t))dy\right\|_\infty \sup_{\substack{(x,t_1)\in Q\\(x,t_2)\in Q}} \frac{|u_\delta(x,t_1)-u_\delta(x,t_2)|}{|t_1-t_2|^{\alpha/2}}$$

$$\leq k_7 \text{ for some constant } k_7.$$

Hence, $\left\|x^{-q}\int_\delta^x k(y)f(u_\delta(y,t))dy\right\|_{H^{\alpha,\alpha/2}(Q)} \leq k_8$ for some constant k_8 which is independent of δ. By Theorem 4.10.1 of Ladyženskaja, Solonnikov and Ural′ceva [7, pp. 351-352], we have

$$\|u_\delta\|_{H^{2+\alpha,1+\alpha/2}(Q)} \leq K$$

for some constant K, which is independent of δ. This implies that u_δ, $(u_\delta)_t$, $(u_\delta)_x$ and $(u_\delta)_{xx}$ are equicontinuous in Q. By the Ascoli-Arzela theorem,

$$\|u\|_{H^{2+\alpha,1+\alpha/2}(Q)} \leq K,$$

and the partial derivatives of u are the limits of the corresponding partial derivatives of u_δ. Thus, $u \in C(\bar{\Omega}_{t_0}) \cap C^{2,1}(\Omega_{t_0})$. □

Let $T = \sup\{\bar{t} :$ the problem (1.1)-(1.2) has a solution on $\bar{D} \times [0,\bar{t})\}$. A proof similar to that of Theorem 2.5 of Floater [4] gives the following result.

Theorem 2.7. *There is a unique solution $u \in C(\bar{D} \times [0,T)) \cap C^{2,1}(D \times (0,T))$. If $T < \infty$, then $\sup u$ tends to 1 as $t \to T$.*

Existence of an upper solution guarantees a global existence result of the solution.

Theorem 2.8. *For a sufficiently small, the solution u exists globally.*

Proof. Let $w(x) = \varepsilon(a^2 - x^2)$, where ε is a positive number such that $\varepsilon a^2/4 < 1$. Then

$$\begin{aligned} w_{xx} + \int_0^x k(y)f(w(y))dy &= -2\varepsilon + \int_0^x k(y)f(w(y))dy \\ &\leq -2\varepsilon + f\left(\frac{\varepsilon a^2}{4}\right)\int_0^a k(y)dy. \end{aligned}$$

Since $\int_0^a k(y)dy \to 0$ as $a \to 0$, the right-hand side of the above inequality is negative when a is small. On the other hand, $w(0) > 0$, and $w(a) = 0$. This implies w is an upper solution which is bounded away from 1 when a is small. □

3. QUENCHING

Let us consider the Sturm-Liouville problem,

$$\varphi'' + \lambda x^q \varphi = 0,\ \varphi(0) = 0 = \varphi(a).$$

For $q = 0$, the eigenfunctions exist. For $q > 0$, Chan and Chan [1] showed that the eigenfunctions are given by

$$\tilde{\phi}_i(z) = 2^{1/2} z^{1/(q+2)} J_{1/(q+2)}\left(\frac{2\lambda_i^{1/2}}{q+2} z\right) \Big/ \left| J_{[1/(q+2)]+1}\left(\frac{2\lambda_i^{1/2}}{q+2}\right)\right|,$$

where $z = x^{(q+2)/2}$, $i = 1, 2, 3, \cdots$, and $J_{1/(q+2)}$ is the Bessel function of the first kind of order $1/(q+2)$. Since $\{\tilde{\phi}_i(z)\}$ forms an orthonormal set with the weight function $z^{q/(q+2)}$, we have $\{\phi_i(x)\}$ forms an orthonormal set with the weight function x^q. Let

β denote the first positive zero of $J_{1/(q+2)}$, $\varphi(x)$ be the fundamental eigenfunction with $\int_0^a x^q\varphi(x)dx = 1$, and μ be the fundamental eigenvalue. By $\varphi(a) = 0$, we have $(2\mu^{1/2}a^{(q+2)/2})/(q+2) = \beta$. This gives $\mu a^{q-1} = a^{-3}\left[\beta(q+2)/2\right]^2$. Hence, μa^{q-1} decreases when a increases. We now give a condition for the solution u to quench in a finite time.

Theorem 3.1. *If $f(0)\inf k(x) > \mu a^{q-1}$, then the solution u of the problem (1.1)-(1.2) quenches in a finite time.*

Proof. Let $F(t) = \int_0^a x^q\varphi(x)u(x,t)dx$. Then,

$$\begin{aligned} F'(t) &= \int_0^a \left(u_{xx}(x,t) + \int_0^x k(y)\, f(u(y,t))\, dy\right)\varphi(x)dx \\ &= \int_0^a u(x,t)\varphi''(x)dx + \int_0^a \varphi(x)\int_0^x k(y)\, f(u(y,t))\, dydx \\ &\geq -\mu F(t) + f(0)\inf k(x)\int_0^a \varphi(x)xdx \\ &\geq -\mu F(t) + \frac{f(0)}{a^{q-1}}\inf k(x). \end{aligned}$$

By a direct calculation,

$$F(t) \geq f(0)\frac{\inf k(x)}{\mu a^{q-1}}(1 - e^{-\mu t}).$$

Since $f(0)\inf k(x) > \mu a^{q-1}$, there exists $t_0 > 0$ such that $F(t_0) \geq 1$. Hence, u quenches somewhere in a finite time. □

We note that the condition $f(0)\inf k(x) > \mu a^{q-1}$ can be satisfied if a is large enough. Therefore, by combining Theorems 2.5, 2.8, and 3.1 we get the following result.

Theorem 3.2. *There exists $a^* > 0$ such that the solution u quenches in a finite time for $a < a^*$ and the solution exists globally for $a > a^*$.*

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