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EXISTENCE OF POSITIVE SOLUTIONS TO SEMIPOSITONE DIRICHLET BVPS ON TIME SCALES

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ABSTRACT. In this paper, we are concerned with the following semipositone Dirichlet boundary value problem on a time scale $\mathbb{T}$

$$\begin{cases} -u^{\Delta\Delta}(t) = g(t, u(t)), \ t \in [0, T]_{\mathbb{T}}, \\ u(0) = 0 = u(\sigma^2(T)), \end{cases}$$

where $g : [0, T]_{\mathbb{T}} \times [0, +\infty) \to [-M, +\infty)$ is continuous and $M > 0$ is a constant. Some existence criteria for at least one positive solution are established by using well-known results from fixed point index theory.

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1. INTRODUCTION

Let $\mathbb{T}$ be a time scale (arbitrary nonempty closed subset of the real numbers $\mathbb{R}$). For each interval $\mathbf{I}$ of $\mathbb{R}$, we denote by $\mathbf{I}_{\mathbb{T}} = \mathbf{I} \cap \mathbb{T}$. For more details on time scales, one can refer to [1, 4, 8, 9]. In this paper, we are interested in the nonlinear dynamic equation on a time scale $\mathbb{T}$

$$-u^{\Delta\Delta}(t) = g(t, u(t)), \ t \in [0, T]_{\mathbb{T}}, \tag{1.1}$$

satisfying Dirichlet boundary conditions

$$u(0) = 0 = u(\sigma^2(T)), \tag{1.2}$$

where $T > 0$ is fixed, $0, T \in \mathbb{T}$, $g : [0, T]_{\mathbb{T}} \times [0, +\infty) \to [-M, +\infty)$ is continuous and $M > 0$ is a constant.

Equations of form (1.1) have been discussed extensively when $M = 0$ (i.e., positone problems); see [2, 5, 6, 11] and the references therein. However, to the best of our knowledge, few papers can be found in the literature for (1.1) when $M > 0$ (i.e., semipositone problems). The purpose of this paper is to study the existence of at least one positive solution for the semipositone Dirichlet boundary value problem

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(BVP for short) (1.1) and (1.2). Our main idea comes from [3, 10, 13], and our main tool is the well-known results from fixed point index theory which we state here.

Theorem 1.1 ([7]). *Let $\mathbb{X}$ be a Banach space and K be a cone in $\mathbb{X}$. Assume that Ω is a bounded open subset of $\mathbb{X}$ with $\theta \in \Omega$ and let $\Phi : K \cap \overline{\Omega} \to K$ be a completely continuous operator. Then,*

(i) if $\Phi z \neq \lambda z$, $\forall z \in K \cap \partial\Omega$, $\lambda \geq 1$, then $i(\Phi,\ K \cap \Omega,\ K) = 1$;

(ii) if $\Phi z \nleq z$, $\forall z \in K \cap \partial\Omega$, then $i(\Phi,\ K \cap \Omega,\ K) = 0$.

For the continuous function $g : [0,T]_{\mathbb{T}} \times [0,+\infty) \to [-M,+\infty)$, we list the following conditions which we need later:

(C1) $g(t,\ 1) + M > 0$, $\forall t \in [0,T]_{\mathbb{T}}$;

(C2) There exist constants λ_1 and λ_2 with $\lambda_1 \geq \lambda_2 > 1$ such that for any $(t,\ y) \in [0,T]_{\mathbb{T}} \times [0,+\infty)$ and any $c \in [0,1]$,

$$c^{\lambda_1}\left[g(t,\ y) + M\right] \leq g(t,\ cy) + M \leq c^{\lambda_2}\left[g(t,\ y) + M\right]; \tag{1.3}$$

(C3) $\int_0^{\sigma(T)} [g(\tau,1) + M]\,\Delta\tau < \frac{M\sigma^2(T)}{(M\sigma^2(T)\sigma(T)+1)^{\lambda_1}}$.

We can obtain the following useful remarks easily.

Remark 1.2. Assume that (C2) is satisfied. Then for any $t \in [0,T]_{\mathbb{T}}$, $g(t,\ y)$ is increasing for $y \in [0,+\infty)$, and for any $(t,\ y) \in [0,T]_{\mathbb{T}} \times [0,+\infty)$ and $c \in [1,+\infty)$,

$$c^{\lambda_2}\left[g(t,\ y) + M\right] \leq g(t,\ cy) + M \leq c^{\lambda_1}\left[g(t,\ y) + M\right]. \tag{1.4}$$

Remark 1.3. Assume that (C1) and (C2) are satisfied. Then

$$\lim_{y\to+\infty} \min_{t\in[0,T]_{\mathbb{T}}} \frac{g(t,\ y)}{y} = +\infty. \tag{1.5}$$

2. PRELIMINARIES

Let

$$\mathbb{X} = \left\{u \,\middle|\, u : [0,\sigma^2(T)]_{\mathbb{T}} \to \mathbb{R} \text{ is continuous}\right\}$$

be equipped with the norm

$$\|u\| = \max_{t\in[0,\sigma^2(T)]_{\mathbb{T}}} |u(t)|.$$

Then, $\mathbb{X}$ is a Banach space.

Define

$$P = \left\{u \in \mathbb{X} :\ u(t) \geq 0,\ t \in [0,\sigma^2(T)]_{\mathbb{T}}\right\}$$

and

$$K = \left\{u \in P :\ u(t) \geq q(t)\,\|u\|,\ t \in [0,\sigma^2(T)]_{\mathbb{T}}\right\},$$

where $q(t) = \frac{t\left(\sigma^2(T)-t\right)}{\left(\sigma^2(T)\right)^2}$, $t \in [0,\sigma^2(T)]_{\mathbb{T}}$. Then, it is easy to see that P and K are cones of $\mathbb{X}$ and $K \subset P$.

To obtain a solution of the BVP (1.1) and (1.2), we require a mapping whose kernel $G(t,s)$ is the Green's function of the BVP

$$\begin{cases} -u^{\Delta\Delta}(t) = 0, \ t \in [0,T]_{\mathbb{T}}, \\ u(0) = 0 = u(\sigma^2(T)). \end{cases} \tag{2.1}$$

It is known that [4]

$$G(t,s) = \frac{1}{\sigma^2(T)} \begin{cases} t\left(\sigma^2(T) - \sigma(s)\right), \ t \le s, \\ \sigma(s)\left(\sigma^2(T) - t\right), \ t \ge \sigma(s). \end{cases} \tag{2.2}$$

For $G(t,s)$, we have the following two simple but important lemmas.

Lemma 2.1. *For any $t \in [0,\sigma^2(T)]_{\mathbb{T}}$ and $s \in [0,\sigma(T)]_{\mathbb{T}}$,*

$$0 \le G(t,s) \le \frac{t\left(\sigma^2(T) - t\right)}{\sigma^2(T)}. \tag{2.3}$$

Lemma 2.2. *Let*

$$x(t) = M \int_0^{\sigma(T)} G(t,s)\Delta s, \ t \in [0,\sigma^2(T)]_{\mathbb{T}}.$$

Then, $x \in P$ and

$$\begin{cases} x^{\Delta\Delta}(t) = -M, \ t \in [0,T]_{\mathbb{T}}, \\ x(0) = 0 = x(\sigma^2(T)). \end{cases} \tag{2.4}$$

For $u \in \mathbb{X}$, we define the function $[u(t)]^*$ by

$$[u(t)]^* = \begin{cases} u(t), \ u(t) \ge 0, \\ 0, \ u(t) < 0 \end{cases}$$

and for $z \in P$, we define the operator $\Phi : P \to P$ by

$$(\Phi z)(t) = \int_0^{\sigma(T)} G(t,s)\left(g\left(s, [z(s) - x(s)]^*\right) + M\right)\Delta s, \ t \in [0,\sigma^2(T)]_{\mathbb{T}},$$

where x is defined in Lemma 2.2.

Lemma 2.3. *If z is a fixed point of the operator Φ and $z(t) \ge x(t)$, $t \in [0,\sigma^2(T)]_{\mathbb{T}}$, then $u = z - x$ is a solution of the BVP (1.1) and (1.2).*

Proof. Since z is a fixed point of the operator Φ, we have

$$\begin{cases} -z^{\Delta\Delta}(t) = g\left(t, [z(t) - x(t)]^*\right) + M, \ t \in [0,T]_{\mathbb{T}}, \\ z(0) = 0 = z(\sigma^2(T)). \end{cases} \tag{2.5}$$

In view of the fact that $z(t) \ge x(t)$, $t \in [0,\sigma^2(T)]_{\mathbb{T}}$, we know that

$$[z(t) - x(t)]^* = z(t) - x(t), \ t \in [0,\sigma^2(T)]_{\mathbb{T}}. \tag{2.6}$$

It follows from (2.5), (2.6) and Lemma 2.2 that

$$\begin{cases} -u^{\Delta\Delta}(t) = g\left(t, u(t)\right), \ t \in [0,T]_{\mathbb{T}}, \\ u(0) = 0 = u(\sigma^2(T)), \end{cases} \tag{2.7}$$

which shows that u is a solution of the BVP (1.1) and (1.2). □

Lemma 2.4. *Suppose that* $g : [0,T]_{\mathbb{T}} \times [0,+\infty) \to [-M,+\infty)$ *is continuous. Then,* $\Phi : K \to K$ *is completely continuous.*

Proof. Let $z \in K$. By the definition of Φ, we know that $(\Phi z)(0) = 0 = (\Phi z)(\sigma^2(T))$. So, there exists a $t_0 \in (0, \sigma^2(T))_{\mathbb{T}}$ such that $\|\Phi z\| = (\Phi z)(t_0)$. Since

$$\frac{G(t,s)}{G(t_0,s)} = \begin{cases} \frac{t}{t_0}, \ t, \ t_0 \le s, \\ \frac{t\left(\sigma^2(T)-\sigma(s)\right)}{\sigma(s)(\sigma^2(T)-t_0)}, \ t \le s < t_0, \\ \frac{\sigma(s)\left(\sigma^2(T)-t\right)}{t_0(\sigma^2(T)-\sigma(s))}, \ t_0 \le s < t, \\ \frac{\sigma^2(T)-t}{\sigma^2(T)-t_0}, \ t, \ t_0 \ge \sigma(s), \end{cases}$$

we obtain that

$$\frac{G(t,s)}{G(t_0,s)} \ge q(t), \ t \in [0,\sigma^2(T)]_{\mathbb{T}} \text{ and } s \in [0,\sigma(T)]_{\mathbb{T}}. \tag{2.8}$$

So,

$$\begin{aligned} (\Phi z)(t) &= \int_0^{\sigma(T)} G(t,s)\left(g\left(s,[z(s)-x(s)]^*\right)+M\right)\Delta s \\ &= \int_0^{\sigma(T)} \frac{G(t,s)}{G(t_0,s)} G(t_0,s)\left(g\left(s,[z(s)-x(s)]^*\right)+M\right)\Delta s \\ &\ge q(t)\int_0^{\sigma(T)} G(t_0,s)\left(g\left(s,[z(s)-x(s)]^*\right)+M\right)\Delta s \\ &= q(t)(\Phi z)(t_0) \\ &= q(t)\left\|\Phi z\right\|, \ t \in [0,\sigma^2(T)]_{\mathbb{T}}, \end{aligned}$$

which shows that $\Phi z \in K$. Furthermore, by using similar arguments to those in [12], we can prove that $\Phi : K \to K$ is completely continuous. □

3. MAIN RESULTS

Our main result is the following theorem.

Theorem 3.1. *Assume that* $g : [0,T]_{\mathbb{T}} \times [0,+\infty) \to [-M,+\infty)$ *is continuous and (C1)-(C3) are satisfied. Then, the BVP (1.1) and (1.2) has at least one positive solution.*

Proof. First, let $r = M\sigma^2(T)\sigma(T)$ and $\Omega_r = \{z \in \mathbb{X} : \|z\| < r\}$. Then, we may assert that

$$\Phi z \neq \lambda z, \ \forall z \in K \cap \partial\Omega_r, \ \lambda \geq 1. \tag{3.1}$$

Suppose on the contrary that there exist $\lambda_0 \geq 1$ and $z_0 \in K \cap \partial\Omega_r$ such that $\Phi z_0 = \lambda_0 z_0$. By Lemma 2.1, we get

$$x(t) = M\int_0^{\sigma(T)} G(t,s)\Delta s \leq \frac{M\sigma(T)t\left(\sigma^2(T)-t\right)}{\sigma^2(T)}, \ t \in [0,\sigma^2(T)]_{\mathbb{T}}, \tag{3.2}$$

which together with $z_0(t) \geq q(t)\|z_0\| = rq(t)$, $t \in [0,\sigma^2(T)]_{\mathbb{T}}$ imply that for any $t \in [0,\sigma^2(T)]_{\mathbb{T}}$,

$$z_0(t) - x(t) \geq rq(t) - \frac{M\sigma(T)t\left(\sigma^2(T)-t\right)}{\sigma^2(T)} = 0. \tag{3.3}$$

It follows from $z_0 = \frac{1}{\lambda_0}\Phi z_0$ that

$$\begin{cases} -z_0^{\Delta\Delta}(t) = \frac{1}{\lambda_0}\left(g\left(t,[z_0(t)-x(t)]^*\right)+M\right), \ t \in [0,T]_{\mathbb{T}}, \\ z_0(0) = 0 = z_0(\sigma^2(T)). \end{cases} \tag{3.4}$$

In view of (3.3) and (3.4), we have

$$\begin{cases} -z_0^{\Delta\Delta}(t) = \frac{1}{\lambda_0}\left(g\left(t,z_0(t)-x(t)\right)+M\right), \ t \in [0,T]_{\mathbb{T}}, \\ z_0(0) = 0 = z_0(\sigma^2(T)), \end{cases} \tag{3.5}$$

which shows that there exists a $t_0 \in (0,\sigma^2(T))_{\mathbb{T}}$ such that

$$z_0(t_0) = \|z_0\| = r \text{ and } z_0^{\Delta}(t_0) \leq 0. \tag{3.6}$$

Let $t \in [0,\ t_0]_{\mathbb{T}}$. Since $0 \leq z_0(t) - x(t) \leq z_0(t) \leq \|z_0\| = r < r+1$, integrating the equation in (3.5) from t to t_0, we know by Remark 1.2 that

$$\begin{aligned} z_0^{\Delta}(t) - z_0^{\Delta}(t_0) &= \int_t^{t_0} -z_0^{\Delta\Delta}(s)\Delta s = \int_t^{t_0} \frac{1}{\lambda_0}\left[g\left(s,z_0(s)-x(s)\right)+M\right]\Delta s \\ &\leq \int_t^{t_0}\left[g\left(s,z_0(s)-x(s)\right)+M\right]\Delta s \leq \int_t^{t_0}\left[g\left(s,r+1\right)+M\right]\Delta s \\ &\leq (r+1)^{\lambda_1}\int_t^{t_0}\left[g(s,1)+M\right]\Delta s, \ t \in [0,t_0]_{\mathbb{T}}, \end{aligned}$$

and so,

$$z_0^{\Delta}(t) \leq (r+1)^{\lambda_1}\int_t^{t_0}\left[g(s,1)+M\right]\Delta s, \ t \in [0,t_0]_{\mathbb{T}}. \tag{3.7}$$

Integrating (3.7) from 0 to t_0, we get

$$\begin{aligned} r &= \int_0^{t_0} z_0^{\Delta}(s)\Delta s \leq (r+1)^{\lambda_1}\int_0^{t_0}\int_s^{t_0}\left[g(\tau,1)+M\right]\Delta\tau\Delta s \\ &\leq (r+1)^{\lambda_1}\sigma(T)\int_0^{\sigma(T)}\left[g(\tau,1)+M\right]\Delta\tau, \end{aligned}$$

i.e.,

$$\frac{M\sigma^2(T)}{(M\sigma^2(T)\sigma(T)+1)^{\lambda_1}} \le \int_0^{\sigma(T)} [g(\tau,1)+M]\,\Delta\tau, \tag{3.8}$$

which contradicts (C3). Therefore, (3.1) is true. So, it follows from (i) of Theorem 1.1 that

$$i(\Phi, K\cap\Omega_r, K) = 1. \tag{3.9}$$

Next, choose constants α, β and L such that $[\alpha,\beta]_{\mathbb{T}} \subset (0,T)_{\mathbb{T}}$ and

$$L > \frac{2\left(\sigma^2(T)\right)^2}{\alpha\left(\sigma^2(T)-\beta\right)}\left[\max_{t\in[0,\sigma^2(T)]_{\mathbb{T}}} \int_\alpha^\beta G(t,s)\Delta s\right]^{-1}. \tag{3.10}$$

By (1.5), there exists a $R_1 > 2r$ such that

$$g(t,y)+M \ge Ly,\ t\in[\alpha,\beta]_{\mathbb{T}} \text{ and } y\in[R_1,+\infty). \tag{3.11}$$

Let $R = \frac{2R_1\left(\sigma^2(T)\right)^2}{\alpha(\sigma^2(T)-\beta)}$ and $\Omega_R = \{z\in\mathbb{X} : \|z\| < R\}$. Then, we may assert that

$$\Phi z \nleq z,\ \forall z\in K\cap\partial\Omega_R. \tag{3.12}$$

Suppose on the contrary that there exists $z_1 \in K\cap\partial\Omega_R$ such that $\Phi z_1 \le z_1$. Then, for $t\in[\alpha,\beta]_{\mathbb{T}}$,

$$\begin{aligned} z_1(t)-x(t) &\ge z_1(t) - rq(t) \ge z_1(t) - r\frac{z_1(t)}{\|z_1\|} = z_1(t) - \frac{r}{R}z_1(t) \\ &\ge \frac{1}{2}z_1(t) \ge \frac{1}{2}q(t)\|z_1\| = \frac{Rt\left(\sigma^2(T)-t\right)}{2\left(\sigma^2(T)\right)^2} \\ &\ge \frac{R\alpha\left(\sigma^2(T)-\beta\right)}{2\left(\sigma^2(T)\right)^2} = R_1 > 0. \end{aligned} \tag{3.13}$$

In view of (3.11) and (3.13), for $t\in[0,\sigma^2(T)]_{\mathbb{T}}$, we have

$$\begin{aligned} R \ge z_1(t) \ge (\Phi z_1)(t) &= \int_0^{\sigma(T)} G(t,s)\left(g\left(s,[z_1(s)-x(s)]^*\right)+M\right)\Delta s \\ &\ge \int_\alpha^\beta G(t,s)\left(g\left(s,[z_1(s)-x(s)]^*\right)+M\right)\Delta s \\ &= \int_\alpha^\beta G(t,s)\left(g\left(s,z_1(s)-x(s)\right)+M\right)\Delta s \\ &\ge L\int_\alpha^\beta G(t,s)\left(z_1(s)-x(s)\right)\Delta s \ge \frac{LR\alpha\left(\sigma^2(T)-\beta\right)}{2\left(\sigma^2(T)\right)^2}\int_\alpha^\beta G(t,s)\Delta s. \end{aligned}$$

So,

$$R \ge \frac{LR\alpha\left(\sigma^2(T)-\beta\right)}{2\left(\sigma^2(T)\right)^2}\max_{t\in[0,\sigma^2(T)]_{\mathbb{T}}}\int_\alpha^\beta G(t,s)\Delta s,$$

i.e.,

$$L \le \frac{2\left(\sigma^2(T)\right)^2}{\alpha\left(\sigma^2(T)-\beta\right)}\left[\max_{t\in[0,\sigma^2(T)]_{\mathbb{T}}} \int_\alpha^\beta G(t,s)\Delta s\right]^{-1}, \tag{3.14}$$

which contradicts (3.10). Therefore, (3.12) is true. So, by (ii) of Theorem 1.1, we have

$$i(\Phi, K \cap \Omega_R, K) = 0. \tag{3.15}$$

It follows from (3.9), (3.15) and the property of the fixed point index that

$$i(\Phi, K \cap \left(\Omega_R \backslash \overline{\Omega_r}\right), K) = -1, \tag{3.16}$$

which implies that Φ has a fixed point z in K with $r < \|z\| < R$. Since

$$z(t) - x(t) \geq q(t)\,\|z\| - rq(t) = (\|z\| - r)\,q(t) \geq 0,\ t \in [0, \sigma^2(T)]_{\mathbb{T}}, \tag{3.17}$$

by Lemma 2.3, we know that $u = z - x$ is a positive solution of the BVP (1.1) and (1.2). □

Corollary 3.2. *Assume that $f : [0,T]_{\mathbb{T}} \times [0,+\infty) \to [0,+\infty)$ is continuous and satisfies the following conditions:*

(C1)′ $f(t,\ 1) > 0$, $\forall t \in [0,T]_{\mathbb{T}}$;

(C2)′ There exist constants λ_1 and λ_2 with $\lambda_1 \geq \lambda_2 > 1$ such that for any $(t, y) \in [0,T]_{\mathbb{T}} \times [0,+\infty)$ and any $c \in [0,1]$

$$c^{\lambda_1} f(t,\ y) \leq f(t,\ cy) \leq c^{\lambda_2} f(t,\ y); \tag{3.18}$$

(C3)′ $\int_0^{\sigma(T)} f(\tau,\ 1)\Delta\tau < \frac{M\sigma^2(T)}{(M\sigma^2(T)\sigma(T)+1)^{\lambda_1}}$.
Then, the BVP

$$\begin{cases} -u^{\Delta\Delta}(t) = f(t, u(t)) - M,\ t \in [0,T]_{\mathbb{T}}, \\ u(0) = 0 = u(\sigma^2(T)), \end{cases} \tag{3.19}$$

has at least one positive solution.

Proof. If we let $g(t,u) = f(t,u) - M$, then all the conditions of Theorem 3.1 are fulfilled. So, the BVP (3.19) has at least one positive solution. □

Example 3.3. Let $\mathbb{T} = [0, \frac{1}{8}] \cup \left\{\frac{1}{4}\right\} \cup [\frac{1}{2}, 1]$. We consider the following BVP on $\mathbb{T}$

$$\begin{cases} -u^{\Delta\Delta}(t) = \frac{u^2(t)}{8(t+1)} - 1,\ t \in [0,1]_{\mathbb{T}}, \\ u(0) = 0 = u(1). \end{cases} \tag{3.20}$$

It is easy to verify that if we let $T = 1$, $g(t,u) = \frac{u^2}{8(t+1)} - 1$, $(t,u) \in [0,1] \times [0,+\infty)$, $M = 1$, $\lambda_1 = 3$ and $\lambda_2 = 2$, then all the conditions of Theorem 3.1 are satisfied. So, the BVP (3.20) has at least one positive solution.

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