Dynamic Systems and Applications 22 (2013) 1-14

PERMANENCE, GLOBAL ASYMPTOTIC STABILITY AND ALMOST PERIODIC SOLUTIONS IN A NON-AUTONOMOUS PREDATOR-PREY SYSTEM WITH HASSELL-VARLEY TYPE FUNCTIONAL RESPONSE

WEIPENG ZHANG[a*], LAN YIN[a], AND XIANYI LI[b]

[a]School of Mathematics and Statistics, Northeast Normal University, Changchun, Jilin 130024, P. R. China
wpzhang808@163.com
[b]College of Mathematics and Computational Science, Shenzhen University, Shenzhen, Guangdong 518060, P. R. China

ABSTRACT. In this paper, we investigate the positive invariance, permanence and global asymptotic stability of a non-autonomous predator-prey model with Hassell-Varley type functional response. Meanwhile, we also obtain criteria for the existence, uniqueness and uniformly asymptotic stability of the positive almost periodic solutions for the associated almost periodic system.

AMS (MOS) Subject Classification. 34C27; 34D23

1. INTRODUCTION

In population dynamics, interactions between populations determine the stability of ecology system. The predator-prey relationship is one of the main problems that has been extensively studied in both ecology and mathematical ecology. A functional response of the predator to the prey density refers to the change in the density of prey per unit time per predator as a function of the prey density. The prey-dependent functional response depends solely on the density of prey. The most popular prey-dependent functional responses are Holling I, II, III type, especially the Holling II type which has been studied extensively. The Lotka-Volterra predator-prey system with Holling II type functional response [10] takes the form

$$\begin{cases} x'(t) = x(t)[a - bx(t)] - \dfrac{cx(t)y(t)}{m + x(t)}, \\ y'(t) = -dy(t) + \dfrac{fx(t)y(t)}{m + x(t)}, \end{cases} \tag{1.1}$$

This work was supported by the National Natural Science Foundation of PR China (No. 11001041 and No. 10926105), SRFDP (No.200802001008) and the Foundation for Innovation Group of Shenzhen University (N0.000133).

*Corresponding author

Received December 9, 2011

where $x(t)$ and $y(t)$ represent population densities of prey and predator at time t respectively, a/b is the carrying capacity of the prey, and a, c, f, d, m stand for prey intrinsic growth rate, capturing rate, conversion rate, death rate and half saturation constant, respectively. The model exhibits the well known "paradox of enrichment", observed by Hairston et al. [13] and by Rosenzweig [22], which has been highly controversial.

Recently, there is a growing explicit biological and physiological evidences [3, 5, 6] that a more suitable functional response in a predator-prey model should be ratio-dependent or predator-dependent. Roughly speaking, the per capita predator growth rate should be a function of the ratio of prey to predator abundance, which has been strongly supported by laboratory experiments and observations. Generally, a ratio-dependent predator-prey model first proposed by Arditi and Ginzburg [4] in 1989 behaves as the following

$$\begin{cases} x'(t) = x(t)[a - bx(t)] - \dfrac{cx(t)y(t)}{my(t) + x(t)}, \\ y'(t) = -dy(t) + \dfrac{fx(t)y(t)}{my(t) + x(t)}. \end{cases} \tag{1.2}$$

Furthermore, (1.2) has been studied by several authors and much richer dynamics has been obtained in Fan et al. [9], Freedman and Mathsen [11], Hsu et al. [16], Jost et al. [18], Kuang and Beretta [19], and Xiao and Ruan [23]. If the functional response $\dfrac{x}{my + x}$ in system (1.2) is replaced by $\dfrac{x}{my^{\gamma} + x}$, $\gamma \in (0, 1)$, one obtains the following predator-prey model with Hassell-Varley type functional response [14]

$$\begin{cases} x'(t) = x(t)[a - bx(t)] - \dfrac{cx(t)y(t)}{my^{\gamma}(t) + x(t)}, \\ y'(t) = -dy(t) + \dfrac{fx(t)y(t)}{my^{\gamma}(t) + x(t)}, \qquad \gamma \in (0, 1), \end{cases} \tag{1.3}$$

which is appropriate for interactions where predators form groups. Here γ stands for the Hassell-Varley constant [14]. Mathematically, when one chooses $\gamma = 0$ or $\gamma = 1$, (1.1) or (1.2) can be viewed as extreme cases of system (1.3). In [8], Cosner et al. provided a unified mechanistic approach to derive the functional response in (1.3). Hsu et al. [17] give a systematic global qualitative analysis for (1.3).

However, it is well known that many biological and environmental parameters vary in time in reality. Therefore, one may consider its non-autonomous case. In this paper, we will study a non-autonomous predator-prey system with Hassell-Varley type functional response which takes the following form

$$\begin{cases} x'(t) = x(t)[a(t) - b(t)x(t)] - \dfrac{c(t)x(t)y(t)}{m(t)y^{\gamma}(t) + x(t)}, \\ y'(t) = -d(t)y(t) + \dfrac{f(t)x(t)y(t)}{m(t)y^{\gamma}(t) + x(t)}, \qquad \gamma \in (0, 1). \end{cases} \tag{1.4}$$

In addition, in most natural and social science branches, such as electrical systems, ecological systems, economics, engineering etc, almost periodic phenomena are more easily seen. So it is more realistic to seek for the almost periodic solutions of the associated differential equations as mathematical models. Recently, many researchers [1, 2, 12, 15, 20, 21, 24, 25, 27] have made progress on the study of properties of almost periodic functions and almost periodic phenomena emerging in some biological models. In this paper, we will explore the almost periodicity of system (1.4).

The organization of this paper is as follows. In Section 2, we mainly show the permanence and global asymptotic stability by using Comparison Theorem and Liapunov function. In Section 3, we show the existence, uniqueness and uniformly asymptotic stability of positive almost periodic solution for almost periodic system. We close with a conclusion in Section 4 on our mathematical results.

2. PERMANENCE AND GLOBAL ASYMPTOTIC STABILITY

The objective of this section is mainly to investigate the permanence and global asymptotic stability of system (1.4). Throughout this paper, we always assume that the parameters in (1.4) are all continuous and bounded above and below by positive constants. For biological reasons, we only consider solutions $(x(t),\ y(t))$ with the initial conditions $x(t_0) > 0$, $y(t_0) > 0$.

2.1. Preliminary.

Definition 2.1. The solution of system (1.4) is said to be ultimately bounded if there exist $B > 0$ such that for every solution $(x(t),\ y(t))$ of (1.4), there exists $T > 0$ such that $\|(x(t), y(t))\| \leq B$ for all $t \geq t_0 + T$, where B is independent of particular solution while T may depend on the solution.

Definition 2.2. System (1.4) is said to be permanent if there are positive constants θ and Λ with $0 < \theta < \Lambda$ such that all solutions of (1.4) with positive initial values satisfy

$$\min\left\{\lim_{t\to+\infty}\inf x(t),\ \lim_{t\to+\infty}\inf y(t)\right\} \geq \theta,$$

$$\max\left\{\lim_{t\to+\infty}\sup x(t),\ \lim_{t\to+\infty}\sup y(t)\right\} \leq \Lambda.$$

Definition 2.3. Let $(\hat{x}(t), \hat{y}(t))$ be a bounded non-negative solution of system (1.4). If for any other solution $(x(t),\ y(t))$ of (1.4) with positive initial values, it holds that

$$\lim_{t\to+\infty}(|x(t) - \hat{x}(t)| + |y(t) - \hat{y}(t)|) = 0,$$

then $(\hat{x}(t), \hat{y}(t))$ is said to be globally asymptotically stable.

2.2. **Permanence.** For convenience and simplicity, in the sequel we will adopt the notations

$$g^u \triangleq \sup_{t\in R} g(t), \qquad g^l \triangleq \inf_{t\in R} g(t),$$

where $g(t)$ is any bounded continuous function in the model (1.4).

Lemma 2.4. *$R^2_+ \triangleq \{(x,y)\in R^2 \mid x\geq 0, y\geq 0\}$ is positively invariant set of system* (1.4).

Proof. System (1.4) is equivalent to the following one

$$\begin{cases} x(t) = x(t_0)\exp\left\{\displaystyle\int_{t_0}^{t}\left[a(s)-b(s)x(s)-\frac{c(s)y(s)}{m(s)y^\gamma(s)+x(s)}\right]ds\right\}, \\ y(t) = y(t_0)\exp\left\{\displaystyle\int_{t_0}^{t}\left[-d(s)+\frac{f(s)x(s)}{m(s)y^\gamma(s)+x(s)}\right]ds\right\}. \end{cases}$$

For $t\geq t_0$, the correctness of the conclusion is obvious according to the above system. □

Theorem 2.5. *If $f^l > d^u$, $m^l a^l > c^u(M_2^\varepsilon)^{1-\gamma}$, then the set*

$$\Gamma_\varepsilon \triangleq \{(x,y)\in R^2 \mid m_1^\varepsilon \leq x \leq M_1^\varepsilon, m_2^\varepsilon \leq y \leq M_2^\varepsilon\} \tag{2.1}$$

is a positively invariant set of system (1.4)*, where*

$$M_1^\varepsilon \triangleq \frac{a^u}{b^l}+\varepsilon, \qquad M_2^\varepsilon \triangleq \left[\frac{(f^u-d^l)M_1^\varepsilon}{d^l m^l}\right]^{\frac{1}{\gamma}},$$

$$m_1^\varepsilon \triangleq \frac{a^l m^l - c^u(M_2^\varepsilon)^{1-\gamma}}{b^u m^l}-\varepsilon, \qquad m_2^\varepsilon \triangleq \left[\frac{(f^l-d^u)m_1^\varepsilon}{d^u m^u}\right]^{\frac{1}{\gamma}}, \tag{2.2}$$

and $\varepsilon \geq 0$ is assumed to be sufficiently small such that $m_1^\varepsilon > 0$.

Proof. Let $(x(t), y(t))$ be a solution of system (1.4) satisfying the initial condition $m_1^\varepsilon \leq x(t_0) \leq M_1^\varepsilon$, $m_2^\varepsilon \leq y(t_0) \leq M_2^\varepsilon$. It can be derived from the first equation of (1.4) that

$$x'(t) \leq x(t)[a^u - b^l x(t)] \leq b^l x(t)[\frac{a^u}{b^l}+\varepsilon - x(t)] = b^l x(t)[M_1^\varepsilon - x(t)]. \tag{2.3}$$

Since $0 < x(t_0) \leq M_1^\varepsilon$, it follows by Comparison Theorem that $x(t)\leq M_1^\varepsilon$, $t\geq t_0$.

According to the second equation of (1.4), one can see

$$\begin{aligned} y'(t) &\leq y(t)\left[-d^l+\frac{f^u M_1^\varepsilon}{m^l y^\gamma(t)+M_1^\varepsilon}\right] \\ &\leq \frac{y(t)}{m^l y^\gamma(t)+M_1^\varepsilon}[-d^l(m^l y^\gamma(t)+M_1^\varepsilon)+f^u M_1^\varepsilon] \\ &= \frac{d^l m^l y^\gamma(t)}{m^l y^\gamma(t)+M_1^\varepsilon}\left[\frac{M_1^\varepsilon(f^u-d^l)y^{1-\gamma}(t)}{d^l m^l}-y(t)\right] \end{aligned} \tag{2.4}$$

$$= \frac{d^l m^l y^\gamma(t)}{m^l y^\gamma(t) + M_1^\varepsilon}[(M_2^\varepsilon)^\gamma y^{1-\gamma}(t) - y(t)].$$

In view of $0 < y(t_0) \le M_2^\varepsilon$, we have $y(t) \le M_2^\varepsilon$, $t \ge t_0$.

Again by the first equation of (1.4), one has

$$\begin{aligned} x'(t) &\ge x(t)\left[a^l - b^u x(t) - \frac{c^u y(t)}{m^l y^\gamma(t) + x(t)}\right] \\ &\ge x(t)[a^l - b^u x(t) - \frac{c^u}{m^l}(M_2^\varepsilon)^{1-\gamma}] \\ &= b^u x(t)\left[\frac{m^l a^l - c^u (M_2^\varepsilon)^{1-\gamma}}{b^u m^l} - x(t)\right] \\ &\ge b^u x(t)[m_1^\varepsilon - x(t)]. \end{aligned} \tag{2.5}$$

Together with $x(t_0) \ge m_1^\varepsilon$, we obtain $x(t) \ge m_1^\varepsilon$, $t \ge t_0$.

By the second equation of (1.4), one derives

$$\begin{aligned} y'(t) &\ge y(t)\left[-d^u + \frac{f^l x(t)}{m^u y^\gamma(t) + x(t)}\right] \\ &\ge y(t)\left[-d^u + \frac{f^l m_1^\varepsilon}{m^u y^\gamma(t) + m_1^\varepsilon}\right] \\ &= \frac{y(t)}{m^u y^\gamma(t) + m_1^\varepsilon}[-d^u m^u y^\gamma(t) - d^u m_1^\varepsilon + f^l m_1^\varepsilon] \\ &= \frac{d^u m^u y^\gamma(t)}{m^u y^\gamma(t) + m_1^\varepsilon}\left[\frac{m_1^\varepsilon(f^l - d^u)}{d^u m^u} y^{1-\gamma}(t) - y(t)\right] \\ &= \frac{d^u m^u y^\gamma(t)}{m^u y^\gamma(t) + m_1^\varepsilon}[(m_2^\varepsilon)^\gamma y^{1-\gamma}(t) - y(t)]. \end{aligned} \tag{2.6}$$

In light of $y(t_0) \ge m_2^\varepsilon$, it is easy to claim that $y(t) \ge m_2^\varepsilon$, $t \ge t_0$. □

Theorem 2.6. *Assume that $f^l > d^u$ and $m^l a^l > c^u (M_2^\varepsilon)^{1-\gamma}$. Then system* (1.4) *is permanent.*

Proof. Suppose that $(x(t), y(t))$ is a solution of system (1.4) with the initial condition $x(t_0) > 0$, $y(t_0) > 0$. If $m^l a^l > c^u (M_2^\varepsilon)^{1-\gamma}$, then from (2.3) and (2.5) and Comparison Theorem one can easily derive $\lim_{t\to+\infty} \sup x(t) \le M_1^0$ and $\lim_{t\to+\infty} \inf x(t) \ge m_1^0$;

Based on $\lim_{t\to+\infty} \sup x(t) \le M_1^0$, for any sufficiently small $\varepsilon > 0$, there exists a $t_1 > t_0$ such that $x(t) < M_1^0 + \varepsilon$ for $t > t_1$. Using the second equation of system (1.4), one finds that

$$\begin{aligned} y'(t) &\le y(t)\left[-d^l + \frac{f^u M_1^\varepsilon}{m^l y^\gamma(t) + M_1^\varepsilon}\right] \\ &= \frac{m^l d^l y^\gamma(t)}{m^l y^\gamma(t) + M_1^\varepsilon}\left[\frac{M_1^\varepsilon(f^u - d^l)}{m^l d^l} y^{1-\gamma}(t) - y(t)\right] \end{aligned}$$

$$= \frac{m^l d^l y(t)}{m^l y^\gamma(t) + M_1^\varepsilon} \left[\frac{M_1^\varepsilon (f^u - d^l)}{m^l d^l} - y^\gamma(t) \right], \qquad \forall\, t \geq t_1.$$

Since $f^l > d^u$, $m^l a^l > c^u (M_2^\varepsilon)^{1-\gamma}$, letting $\varepsilon \to 0$, one has

$$\lim_{t \to +\infty} \sup y(t) \leq M_2^0.$$

Similarly, one obtains

$$\lim_{t \to +\infty} \inf y(t) \geq m_2^0.$$

The proof is complete. □

From the proof of Theorem 2.6, one can get the following theorem directly.

Theorem 2.7. *Assume $f^l > d^u$ and $m^l a^l > c^u (M_2^\varepsilon)^{1-\gamma}$. Then $\Gamma_\varepsilon (\varepsilon > 0)$ is the ultimately bounded region of system* (1.4).

2.3. **Global asymptotical stability.** The following lemma, which can be easily derived in [7], will be used in the proof of the main result of this section.

Lemma 2.8. [7] *Suppose that $h \in R$ and f is a non-negative function defined on $[h, +\infty)$. If f is integrable and uniformly continuous on $[h, +\infty)$, then $\lim_{t \to +\infty} f(t) = 0$.*

Theorem 2.9. *Let $(\hat{x}(t), \hat{y}(t))$ be a positive bounded solution of system* (1.4). *If $f^l > d^u$, $m^l a^l > c^u (M_2^\varepsilon)^{1-\gamma}$, and*

$$\begin{aligned} &\inf_{t \in R} \left\{ b(t) - \frac{\hat{y}(t) \left[c(t) + m(t) f(t) (M_2^\varepsilon)^\gamma \right]}{\left[m(t) (m_2^\varepsilon)^\gamma + m_1^\varepsilon \right] \left[m(t) \hat{y}^\gamma(t) + \hat{x}(t) \right]} \right\} > 0, \\ &\inf_{t \in R} \left\{ d(t) - \frac{c(t) + f(t) M_1^\varepsilon}{m(t) (m_2^\varepsilon)^\gamma + m_1^\varepsilon} \right\} > 0, \\ &\inf_{t \in R} \left\{ f(t) m_1^\varepsilon - c(t) \right\} > 0, \end{aligned} \tag{2.7}$$

where m_i^ε, M_i^ε, $(i = 1, 2)$ are defined in (2.2), *then the solution $(\hat{x}(t), \hat{y}(t))$ is globally asymptotically stable.*

Proof. Suppose that $(x(t), y(t))$ is any solution of system (1.4) with positive initial condition. Based on Theorem 2.7, there is a $T_1 > 0$ such that $(x(t), y(t)) \in \Gamma_\varepsilon$ and $(\hat{x}(t), \hat{y}(t)) \in \Gamma_\varepsilon$ for $\forall t \geq t_0 + T_1$.

Let

$$V(t) = |\ln\{x(t)\} - \ln\{\hat{x}(t)\}| + |y(t) - \hat{y}(t)|, \tag{2.8}$$

then the derivative of $V(t)$ along the solution of system (1.4) has the following expression

$$D^+ V(t) = sgn\{x(t) - \hat{x}(t)\} \left[-b(t)(x(t) - \hat{x}(t)) - \left(\frac{c(t) y(t)}{m(t) y^\gamma(t) + x(t)} \right. \right.$$

$$
\begin{aligned}
&\quad -\frac{c(t)\hat{y}(t)}{m(t)\hat{y}^{\gamma}(t)+\hat{x}(t)}\Big)\Big] + sgn\{y(t)-\hat{y}(t)\}\Big[-d(t)(y(t)-\hat{y}(t))\\
&\quad + \frac{f(t)x(t)y(t)}{m(t)y^{\gamma}(t)+x(t)} - \frac{f(t)\hat{x}(t)\hat{y}(t)}{m(t)\hat{y}^{\gamma}(t)+\hat{x}(t)}\Big]\\
&= -b(t)|x(t)-\hat{x}(t)| - d(t)|y(t)-\hat{y}(t)|\\
&\quad - sgn\{x(t)-\hat{x}(t)\}\left[\frac{c(t)m(t)\hat{y}^{\gamma}(t)[y(t)-\hat{y}(t)]}{(m(t)y^{\gamma}(t)+x(t))(m(t)\hat{y}^{\gamma}(t)+\hat{x}(t))}\right.\\
&\quad \left.+\frac{c(t)m(t)\hat{y}(t)[y^{\gamma}(t)-y^{\gamma}(t)]}{(m(t)y^{\gamma}(t)+x(t))(m(t)\hat{y}^{\gamma}(t)+\hat{x}(t))}\right]\\
&\quad - sgn\{x(t)-\hat{x}(t)\}\left[\frac{-c(t)\hat{y}(t)[x(t)-\hat{x}(t)]+c(t)\hat{x}(t)[y(t)-\hat{y}(t)]}{(m(t)y^{\gamma}(t)+x(t))(m(t)\hat{y}^{\gamma}(t)+\hat{x}(t))}\right]\\
&\quad + sgn\{y(t)-\hat{y}(t)\}\left\{\frac{f(t)m(t)\left[x(t)\hat{y}^{\gamma}(t)(y(t)-\hat{y}(t))+x(t)\hat{y}(t)(\hat{y}^{\gamma}(t)-y^{\gamma}(t))\right]}{(m(t)y^{\gamma}(t)+x(t))(m(t)\hat{y}^{\gamma}(t)+\hat{x}(t))}\right.\\
&\quad \left.+\frac{f(t)m(t)\left[\hat{y}(t)y^{\gamma}(t)(x(t)-\hat{x}(t))\right]}{(m(t)y^{\gamma}(t)+x(t))(m(t)\hat{y}^{\gamma}(t)+\hat{x}(t))}\right\}\\
&\quad + sgn\{y(t)-\hat{y}(t)\}\left[\frac{f(t)x(t)\hat{x}(t)(y(t)-\hat{y}(t))}{(m(t)y^{\gamma}(t)+x(t))(m(t)\hat{y}^{\gamma}(t)+\hat{x}(t))}\right]\\
&\le -b(t)|x(t)-\hat{x}(t)| - d(t)|y(t)-\hat{y}(t)|\\
&\quad + \frac{c(t)m(t)\hat{y}^{\gamma}(t)}{(m(t)y^{\gamma}(t)+x(t))(m(t)\hat{y}^{\gamma}(t)+\hat{x}(t))}|y(t)-\hat{y}(t)|\\
&\quad + \frac{c(t)m(t)\hat{y}(t)}{(m(t)y^{\gamma}(t)+x(t))(m(t)\hat{y}^{\gamma}(t)+\hat{x}(t))}|y^{\gamma}(t)-\hat{y}^{\gamma}(t)|\\
&\quad + \frac{c(t)\hat{x}(t)}{(m(t)y^{\gamma}(t)+x(t))(m(t)\hat{y}^{\gamma}(t)+\hat{x}(t))}|y(t)-\hat{y}(t)|\\
&\quad + \frac{c(t)\hat{y}(t)}{(m(t)y^{\gamma}(t)+x(t))(m(t)\hat{y}^{\gamma}(t)+\hat{x}(t))}|x(t)-\hat{x}(t)|\\
&\quad + \frac{f(t)m(t)x(t)\hat{y}^{\gamma}(t)}{(m(t)y^{\gamma}(t)+x(t))(m(t)\hat{y}^{\gamma}(t)+\hat{x}(t))}|y(t)-\hat{y}(t)|\\
&\quad - \frac{f(t)m(t)x(t)\hat{y}(t)}{(m(t)y^{\gamma}(t)+x(t))(m(t)\hat{y}^{\gamma}(t)+\hat{x}(t))}|y^{\gamma}(t)-\hat{y}^{\gamma}(t)|\\
&\quad + \frac{f(t)m(t)\hat{y}(t)y^{\gamma}(t)}{(m(t)y^{\gamma}(t)+x(t))(m(t)\hat{y}^{\gamma}(t)+\hat{x}(t))}|x(t)-\hat{x}(t)|\\
&\quad + \frac{f(t)x(t)\hat{x}(t)}{(m(t)y^{\gamma}(t)+x(t))(m(t)\hat{y}^{\gamma}(t)+\hat{x}(t))}|y(t)-\hat{y}(t)|\\
&= -\left[b(t)-\frac{c(t)\hat{y}(t)+f(t)m(t)\hat{y}(t)y^{\gamma}(t)}{(m(t)y^{\gamma}(t)+x(t))(m(t)\hat{y}^{\gamma}(t)+\hat{x}(t))}\right]|x(t)-\hat{x}(t)|\\
&\quad - \left[d(t)-\frac{c(t)m(t)\hat{y}^{\gamma}(t)+c(t)\hat{x}(t)+f(t)m(t)x(t)\hat{y}^{\gamma}(t)+f(t)x(t)\hat{x}(t)}{(m(t)y^{\gamma}(t)+x(t))(m(t)\hat{y}^{\gamma}(t)+\hat{x}(t))}\right]|y(t)-\hat{y}(t)|
\end{aligned}
$$

$$
\begin{aligned}
&- \left[\frac{f(t)m(t)x(t)\hat{y}(t) - c(t)m(t)\hat{y}(t)}{(m(t)y^{\gamma}(t) + x(t))(m(t)\hat{y}^{\gamma}(t) + \hat{x}(t))} \right] |y^{\gamma}(t) - \hat{y}^{\gamma}(t)| \\
\leq &- \left[b(t) - \frac{\hat{y}(t)(c(t) + f(t)m(t)(M_2^{\varepsilon})^{\gamma})}{(m(t)(m_2^{\varepsilon})^{\gamma} + m_1^{\varepsilon})(m(t)\hat{y}^{\gamma}(t) + \hat{x}(t))} \right] |x(t) - \hat{x}(t)| \\
&- \left[d(t) - \frac{c(t) + f(t)M_1^{\varepsilon}}{m(t)(m_2^{\varepsilon})^{\gamma} + m_1^{\varepsilon}} \right] |y(t) - \hat{y}(t)| \\
&- \left[\frac{(f(t)m_1^{\varepsilon} - c(t))m(t)\hat{y}(t)}{(m(t)(M_2^{\varepsilon})^{\gamma} + M_1^{\varepsilon})(m(t)\hat{y}^{\gamma}(t) + \hat{x}(t))} \right] |y^{\gamma}(t) - \hat{y}^{\gamma}(t)|.
\end{aligned}
$$

According to the assumption (2.7), one concludes that there is a constant $\mu > 0$ such that

$$
\begin{aligned}
D^+V(t) \leq -\mu \, [&|x(t) - \hat{x}(t)| + |y(t) - \hat{y}(t)| \\
&+ |y^{\gamma}(t) - \hat{y}^{\gamma}(t)|] \, , t \geq t_0 + T_1.
\end{aligned} \tag{2.9}
$$

Integrating both sides of (2.9) from $t_0 + T_1$ to t yields

$$
\begin{aligned}
&V(t) + \mu \int_{t_0+T_1}^{t} [|x(s) - \hat{x}(s)| \\
&\quad + |y(s) - \hat{y}(s)| + |y^{\gamma}(s) - \hat{y}^{\gamma}(s)|] ds \leq V(t_0 + T_1) < +\infty, \quad t \geq t_0 + T_1,
\end{aligned}
$$

which indicates that

$$
\begin{aligned}
&\int_{t_0+T_1}^{t} [|x(s) - \hat{x}(s)| + |y(s) - \hat{y}(s)| + |y^{\gamma}(s) - \hat{y}^{\gamma}(s)|] ds \\
&\quad \leq \mu^{-1} V(t_0 + T_1) < +\infty, \forall t \geq t_0 + T_1,
\end{aligned}
$$

and hence $|x(t) - \hat{x}(t)| + |y(t) - \hat{y}(t)| + |y^{\gamma}(t) - \hat{y}^{\gamma}(t)| \in L^1([t_0 + T_1, +\infty))$. From the boundedness of $\hat{x}(t)$ and $\hat{y}(t)$ and the ultimate boundedness of $x(t)$ and $y(t)$, it follows that $\hat{x}(t)$, $\hat{y}(t)$, $x(t)$, $y(t)$, $y^{\gamma}(t)$, $\hat{y}^{\gamma}(t)$ and their derivatives remain bounded for $t \geq t_0 + T_1$. So $|x(t) - \hat{x}(t)| + |y(t) - \hat{y}(t)| + |y^{\gamma}(t) - \hat{y}^{\gamma}(t)|$ is uniformly continuous on $[t_0 + T_1, +\infty)$. By Lemma 2.8, we have

$$
\lim_{t \to +\infty} [|x(t) - \hat{x}(t)| + |y(t) - \hat{y}(t)| + |y^{\gamma}(t) - \hat{y}^{\gamma}(t)|] = 0,
$$

which is equivalent to

$$
\lim_{t \to +\infty} [|x(t) - \hat{x}(t)| + |y(t) - \hat{y}(t)|] = 0,
$$

as desired. □

3. **ALMOST PERIODIC SYSTEM**

In this section, we will assume further that all parameters in system (1.4) are almost periodic.

Consider the following system

$$
x' = f(t, x), \quad f(t, x) \in C(R \times D, R^n) \tag{3.1}
$$

and its product system

$$x' = f(t,x), \qquad y' = f(t,y), \tag{3.2}$$

where D is an open set in R^n and $f(t,x)$ is a uniformly almost periodic function in t for $x \in D$.

Lemma 3.1 ([26]). *Assume that there is a Liapunov function $V(t,x,y)$ defined on $[0,+\infty) \times D \times D$ which satisfies the following conditions*

(i) $\alpha(\|x-y\|) \leq V(t,x,y) \leq \beta(\|x-y\|)$, *where $\alpha(\delta)$, $\beta(\delta)$ are continuous, increasing and positive definite functions;*
(ii) $\|V(t,x_1,y_1) - V(t,x_2,y_2)\| \leq K(\|x_1-x_2\| + \|y_1-y_2\|)$, *and K is a positive constant;*
(iii) $V'_{(3.2)}(t,x,y) \leq -\mu V(\|x-y\|)$, *and μ is a positive constant.*

Moreover, assume that system (3.1) *has a solution in a compact set $S \subset D$ for $t \geq t_0 > 0$. Then system* (3.1) *has a unique almost periodic solution in S, which is uniformly asymptotically stable in D.*

Theorem 3.2. *Assume that $f^l > d^u$ and $m^l a^l > c^u (M_2^\varepsilon)^{1-\gamma}$, then*

$$\Gamma_\varepsilon^* \triangleq \left\{(x,y) \in R^2 \mid \ln\{m_1^\varepsilon\} \leq x \leq \ln\{M_1^\varepsilon\}, \ln\{m_2^\varepsilon\} \leq y \leq \ln\{M_2^\varepsilon\}\right\}$$

is an ultimately bounded positive invariant set of the following system

$$\begin{cases} \tilde{x}'(t) = a(t) - b(t)\exp\{\tilde{x}(t)\} - \frac{c(t)\exp\{\tilde{y}(t)\}}{m(t)\exp\{\gamma\tilde{y}(t)\} + \exp\{\tilde{x}(t)\}}, \\ \tilde{y}'(t) = -d(t) + \frac{f(t)\exp\{\tilde{x}(t)\}}{m(t)\exp\{\gamma\tilde{y}(t)\} + \exp\{\tilde{x}(t)\}}, \qquad \gamma \in (0,1). \end{cases} \tag{3.3}$$

Here, $m_i^\varepsilon, M_i^\varepsilon$, $(i=1,2)$ are defined in (2.2).

Proof. Let $x(t) = \exp\{\tilde{x}(t)\}, y(t) = \exp\{\tilde{y}(t)\}$, then system (1.4) is reduced into system (3.3). Then the conclusion is easy to be derived from Theorem 2.7 and (3.3). □

By virtue of Theorem 3.2, we obtain the following corollary.

Corollary 3.3. *System* (1.4) *has a unique positive almost periodic solution which is uniformly asymptotically stable in Γ_ε if and only if system* (3.3) *has a unique almost periodic solution which is uniformly asymptotically stable in Γ_ε^*.*

Theorem 3.4. *Assume that $f^l > d^u, m^l a^l > c^u (M_2^\varepsilon)^{1-\gamma}$, and*

$$\inf_{t\in R} \left\{ b(t) - \frac{c(t)M_2^\varepsilon + f(t)m(t)(M_2^\varepsilon)^\gamma}{(m(t)(m_2^\varepsilon)^\gamma + m_1^\varepsilon)^2} \right\} > 0,$$
$$\inf_{t\in R} \left\{\gamma m(t)(m_2^\varepsilon)^\gamma \left[f(t)m_1^\varepsilon - c(t)M_2^\varepsilon\right] - c(t)M_2^\varepsilon[m(t)(M_2^\varepsilon)^\gamma + M_1^\varepsilon]\right\} > 0 \tag{3.4}$$

hold, then there is a unique positive almost periodic solution of system (1.4), *which is uniformly asymptotically stable in Γ_ε. Here, $m_i^\varepsilon, M_i^\varepsilon$, $(i=1,2)$ are defined in* (2.2).

Proof. For $(u(t), v(t)) \in R^2$, define $\|(u(t), v(t))\| \triangleq |u(t)| + |v(t)|$.

Consider

$$\begin{cases} \tilde{x}_1'(t) = a(t) - b(t)\exp\{\tilde{x}_1(t)\} - \frac{c(t)\exp\{\tilde{y}_1(t)\}}{m(t)\exp\{\gamma\tilde{y}_1(t)\}+\exp\{\tilde{x}_1(t)\}}, \\ \tilde{y}_1'(t) = -d(t) + \frac{f(t)\exp\{\tilde{x}_1(t)\}}{m(t)\exp\{\gamma\tilde{y}_1(t)\}+\exp\{\tilde{x}_1(t)\}}, \\ \tilde{x}_2'(t) = a(t) - b(t)\exp\{\tilde{x}_2(t)\} - \frac{c(t)\exp\{\tilde{y}_2(t)\}}{m(t)\exp\{\gamma\tilde{y}_2(t)\}+\exp\{\tilde{x}_2(t)\}}, \\ \tilde{y}_2'(t) = -d(t) + \frac{f(t)\exp\{\tilde{x}_2(t)\}}{m(t)\exp\{\gamma\tilde{y}_2(t)\}+\exp\{\tilde{x}_2(t)\}}. \end{cases} \tag{3.5}$$

Define a Liapunov function on $[0, +\infty) \times \Gamma_\varepsilon^* \times \Gamma_\varepsilon^*$ as follows

$$V(t, \tilde{x}_1, \tilde{y}_1, \widetilde{x}_2, \tilde{y}_2) = |\tilde{x}_1(t) - \tilde{x}_2(t)| + |\tilde{y}_1(t) - \tilde{y}_2(t)|.$$

Choose $\alpha(\delta) = \beta(\delta) = \delta$ $(\delta \geq 0)$. Then it is easy to see that $V(t, \tilde{x}_1, \tilde{y}_1, \tilde{x}_2, \tilde{y}_2)$ satisfies condition (i) of Lemma 3.1. Furthermore,

$$\begin{aligned} &|V(t, \tilde{x}_1, \tilde{y}_1, \tilde{x}_2, \tilde{y}_2) - V(t, \tilde{x}_3, \tilde{y}_3, \tilde{x}_4, \tilde{y}_4)| \\ &\quad = |(|\tilde{x}_1(t) - \tilde{x}_2(t)| + |\tilde{y}_1(t) - \tilde{y}_2(t)|) - (|\tilde{x}_3(t) - \tilde{x}_4(t)| + |\tilde{y}_3(t) - \tilde{y}_4(t)|)| \\ &\quad \leq |\tilde{x}_1(t) - \tilde{x}_3(t)| + |\tilde{y}_1(t) - \tilde{y}_3(t)| + |\tilde{x}_2(t) - \tilde{x}_4(t)| + |\tilde{y}_2(t) - \tilde{y}_4(t)| \\ &\quad = \|(\tilde{x}_1(t), \tilde{y}_1(t)) - (\tilde{x}_3(t), \tilde{y}_3(t))\| + \|(\tilde{x}_2(t), \tilde{y}_2(t)) - (\tilde{x}_4(t), \tilde{y}_4(t))\|, \end{aligned} \tag{3.6}$$

which implies that $V(t, \tilde{x}_1, \tilde{y}_1, \tilde{x}_2, \tilde{y}_2)$ satisfies condition (ii) of Lemma 3.1. □

Let $(\tilde{x}_i(t), \tilde{y}_i(t))$, $(i = 1, 2)$ be any two solutions of system (3.3), which are defined on $[0, +\infty) \times \Gamma_\varepsilon^* \times \Gamma_\varepsilon^*$. Then

$$\begin{aligned} D^+V(t) = {} & sgn\{\tilde{x}_1(t) - \tilde{x}_2(t)\}\Big[-b(t)(\exp\{\tilde{x}_1(t)\} - \exp\{\tilde{x}_2(t)\}) \\ & - \frac{c(t)\exp\{\tilde{y}_1(t)\}}{m(t)\exp\{\gamma\tilde{y}_1(t)\} + \exp\{\tilde{x}_1(t)\}} + \frac{c(t)\exp\{\tilde{y}_2(t)\}}{m(t)\exp\{\gamma\tilde{y}_2(t)\} + \exp\{\tilde{x}_2(t)\}}\Big] \\ & + sgn\{\tilde{y}_1(t) - \tilde{y}_2(t)\}\Big[\frac{f(t)\exp\{\tilde{x}_1(t)\}}{m(t)\exp\{\gamma\tilde{y}_1(t)\} + \exp\{\tilde{x}_1(t)\}} \\ & - \frac{f(t)\exp\{\tilde{x}_2(t)\}}{m(t)\exp\{\gamma\tilde{y}_2(t)\} + \exp\{\tilde{x}_2(t)\}}\Big] \\ = {} & sgn\{\tilde{x}_1(t) - \tilde{x}_2(t)\}\Big[-b(t)(\exp\{\tilde{x}_1(t)\} - \exp\{\tilde{x}_2(t)\}) \\ & - \frac{c(t)m(t)[\exp\{\tilde{y}_1(t) + \gamma\tilde{y}_2(t)\} - \exp\{\tilde{y}_2(t) + \gamma\tilde{y}_1(t)\}]}{[m(t)\exp\{\gamma\tilde{y}_1(t)\} + \exp\{\tilde{x}_1(t)\}][m(t)\exp\{\gamma\tilde{y}_2(t)\} + \exp\{\tilde{x}_2(t)\}]} \\ & - \frac{c(t)[\exp\{\tilde{y}_1(t) + \tilde{x}_2(t)\} - \exp\{\tilde{y}_2(t) + \tilde{x}_1(t)\}]}{[m(t)\exp\{\gamma\tilde{y}_1(t)\} + \exp\{\tilde{x}_1(t)\}][m(t)\exp\{\gamma\tilde{y}_2(t)\} + \exp\{\tilde{x}_2(t)\}]}\Big] \\ & + sgn\{\tilde{y}_1(t) - \tilde{y}_2(t)\} \\ & \times \frac{f(t)m(t)[\exp\{\tilde{x}_1(t) + \gamma\tilde{y}_2(t)\} - \exp\{\tilde{x}_2(t) + \gamma\tilde{y}_1(t)\}]}{[m(t)\exp\{\gamma\tilde{y}_1(t)\} + \exp\{\tilde{x}_1(t)\}][m(t)\exp\{\gamma\tilde{y}_2(t)\} + \exp\{\tilde{x}_2(t)\}]} \\ \leq {} & -b(t)|\exp\{\tilde{x}_1(t)\} - \exp\{\tilde{x}_2(t)\}| + |\exp\{\gamma\tilde{y}_1(t)\} - \exp\{\gamma\tilde{y}_2(t)\}| \end{aligned}$$

$$
\begin{aligned}
&\times \frac{c(t)m(t)\exp\{\tilde{y}_1(t)\}}{[m(t)\exp\{\gamma\tilde{y}_1(t)\}+\exp\{\tilde{x}_1(t)\}][m(t)\exp\{\gamma\tilde{y}_2(t)\}+\exp\{\tilde{x}_2(t)\}]}\\
&+|\exp\{\tilde{y}_1(t)\}-\exp\{\tilde{y}_2(t)\}|\\
&\times \frac{c(t)m(t)\exp\{\gamma\tilde{y}_1(t)\}}{[m(t)\exp\{\gamma\tilde{y}_1(t)\}+\exp\{\tilde{x}_1(t)\}][m(t)\exp\{\gamma\tilde{y}_2(t)\}+\exp\{\tilde{x}_2(t)\}]}\\
&+|\exp\{\tilde{x}_1(t)\}-\exp\{\tilde{x}_2(t)\}|\\
&\times \frac{c(t)\exp\{\tilde{y}_1(t)\}}{[m(t)\exp\{\gamma\tilde{y}_1(t)\}+\exp\{\tilde{x}_1(t)\}][m(t)\exp\{\gamma\tilde{y}_2(t)\}+\exp\{\tilde{x}_2(t)\}]}\\
&+|\exp\{\tilde{y}_1(t)\}-\exp\{\tilde{y}_2(t)\}|\\
&\times \frac{c(t)\exp\{\tilde{x}_1(t)\}}{[m(t)\exp\{\gamma\tilde{y}_1(t)\}+\exp\{\tilde{x}_1(t)\}][m(t)\exp\{\gamma\tilde{y}_2(t)\}+\exp\{\tilde{x}_2(t)\}]}\\
&-|\exp\{\gamma\tilde{y}_1(t)\}-\exp\{\gamma\tilde{y}_2(t)\}|\\
&\times \frac{f(t)m(t)\exp\{\tilde{x}_1(t)\}}{[m(t)\exp\{\gamma\tilde{y}_1(t)\}+\exp\{\tilde{x}_1(t)\}][m(t)\exp\{\gamma\tilde{y}_2(t)\}+\exp\{\tilde{x}_2(t)\}]}\\
&+\exp\{\tilde{x}_1(t)\}-\exp\{\tilde{x}_2(t)\}|\\
&\times \frac{f(t)m(t)\exp\{\gamma\tilde{y}_1(t)\}}{[m(t)\exp\{\gamma\tilde{y}_1(t)\}+\exp\{\tilde{x}_1(t)\}][m(t)\exp\{\gamma\tilde{y}_2(t)\}+\exp\{\tilde{x}_2(t)\}]}\\
&\le -|\exp\{\tilde{x}_1(t)\}-\exp\{\tilde{x}_2(t)\}|\\
&\times \left[b(t)-\frac{c(t)\exp\{\tilde{y}_1(t)\}+f(t)m(t)\exp\{\gamma\tilde{y}_1(t)\}}{[m(t)\exp\{\gamma\tilde{y}_1(t)\}+\exp\{\tilde{x}_1(t)\}][m(t)\exp\{\gamma\tilde{y}_2(t)\}+\exp\{\tilde{x}_2(t)\}]}\right]\\
&+|\exp\{\tilde{y}_1(t)\}-\exp\{\tilde{y}_2(t)\}|\\
&\times \frac{[c(t)m(t)\exp\{\gamma\tilde{y}_1(t)\}+c(t)\exp\{\tilde{x}_1(t)\}]}{[m(t)\exp\{\gamma\tilde{y}_1(t)\}+\exp\{\tilde{x}_1(t)\}][m(t)\exp\{\tilde{y}_2(t)\}+\exp\{\tilde{x}_2(t)\}]}\\
&+|\exp\{\gamma\tilde{y}_1(t)\}-\exp\{\gamma\tilde{y}_2(t)\}|\\
&\times \frac{c(t)m(t)\exp\{\tilde{y}_1(t)\}-f(t)m(t)\exp\{\tilde{x}_1(t)\}}{[m(t)\exp\{\gamma\tilde{y}_1(t)\}+\exp\{\tilde{x}_1(t)\}][m(t)\exp\{\gamma\tilde{y}_2(t)\}+\exp\{\tilde{x}_2(t)\}]}.
\end{aligned}
$$

In addition,

$$\exp\{\tilde{x}_1(t)\}-\exp\{\tilde{x}_2(t)\}=\exp\{\xi(t)\}[\tilde{x}_1(t)-\tilde{x}_2(t)],$$

$$\exp\{\tilde{y}_1(t)\}-\exp\{\tilde{y}_2(t)\}=\exp\{\eta(t)\}[\tilde{y}_1(t)-\tilde{y}_2(t)],$$

$$\exp\{\gamma\tilde{y}_1(t)\}-\exp\{\gamma\tilde{y}_2(t)\}=\gamma\exp\{\psi(t)\}[\tilde{y}_1(t)-\tilde{y}_2(t)],$$

where $\xi(t)$, $\eta(t)$, $\psi(t)$ lie between $\tilde{x}_1(t)$ and $\tilde{x}_2(t)$, $\tilde{y}_1(t)$ and $\tilde{y}_2(t)$, $\gamma\tilde{y}_1(t)$ and $\gamma\tilde{y}_2(t)$ respectively. Hence

$$
\begin{aligned}
D^+V(t) \le &\left[-b(t)+\frac{c(t)M_2^\varepsilon+f(t)m(t)(M_2^\varepsilon)^\gamma}{[m(t)(m_2^\varepsilon)^\gamma+m_1^\varepsilon]^2}\right]\exp\{\xi(t)\}|\tilde{x}_1(t)-\tilde{x}_2(t)|\\
&+\frac{c(t)m(t)(M_2^\varepsilon)^\gamma+c(t)M_1^\varepsilon}{[m(t)(m_2^\varepsilon)^\gamma+m_1^\varepsilon]^2}\exp\{\eta(t)\}|\tilde{y}_1(t)-\tilde{y}_2(t)|
\end{aligned}
$$

$$
\begin{aligned}
&+\gamma\frac{m(t)[c(t)M_2^\varepsilon - f(t)m_1^\varepsilon]}{[m(t)(m_2^\varepsilon)^\gamma + m_1^\varepsilon]^2}\exp\{\psi(t)\}|\tilde{y}_1(t)-\tilde{y}_2(t)| \\
\le& -\left[b(t) - \frac{c(t)M_2^\varepsilon + f(t)m(t)(M_2^\varepsilon)^\gamma}{[m(t)(m_2^\varepsilon)^\gamma + m_1^\varepsilon]^2}\right] m_1^\varepsilon|\tilde{x}_1(t)-\tilde{x}_2(t)| \\
&- \frac{\gamma m(t)(m_2^\varepsilon)^\gamma[f(t)m_1^\varepsilon - c(t)M_2^\varepsilon] - c(t)M_2^\varepsilon[m(t)(M_2^\varepsilon)^\gamma + M_1^\varepsilon]}{[m(t)(m_2^\varepsilon)^\gamma + m_1^\varepsilon]^2}|\tilde{y}_1(t)-\tilde{y}_2(t)| \\
\le& -\mu(|\tilde{x}_1(t)-\tilde{x}_2(t)| + |\tilde{y}_1(t)-\tilde{y}_2(t)|) \\
=& -\mu\|(\tilde{x}_1(t),\tilde{y}_1(t)) - (\tilde{x}_2(t),\tilde{y}_2(t))\|,
\end{aligned}
$$

where

$$
\mu = \min\left\{\inf_{t\in R}\left[\left(b(t) - \frac{c(t)M_2^\varepsilon + f(t)m(t)(M_2^\varepsilon)^\gamma}{[m(t)(m_2^\varepsilon)^\gamma + m_1^\varepsilon]^2}\right)m_1^\varepsilon\right]\right\},
$$

$$
\inf_{t\in R}\left\{\frac{\gamma m(t)(m_2^\varepsilon)^\gamma[f(t)m_1^\varepsilon - c(t)M_2^\varepsilon] - c(t)M_2^\varepsilon[m(t)(M_2^\varepsilon)^\gamma + M_1^\varepsilon]}{[m(t)(m_2^\varepsilon)^\gamma + m_1^\varepsilon]^2}\right\}.
$$

Thus, condition (iii) of Lemma 3.1 is also satisfied. Applying Lemma 3.1 and Theorem 3.2, we conclude that there is a unique almost periodic solution $(\tilde{x}^*(t), \tilde{y}^*(t))$ of system (3.3) which is uniformly asymptotically stable in Γ_ε^*. Set $x^*(t) = \exp\{\tilde{x}^*(t)\}$ and $y^*(t) = \exp\{\tilde{y}^*(t)\}$. Then, from Corollary 3.3, it follows that system (1.4) has a unique uniformly asymptotically stable positive almost periodic solution $(x^*(t), y^*(t))$ in Γ_ε.

4. CONCLUSION

In this paper, we investigate the dynamics of a non-autonomous predator-prey system with Hassell-Varley type functional response. When $\gamma = 1$, the non-autonomous predator-prey system with Hassell-Varley type functional response is reduced into a non-autonomous ratio-dependent predator-prey system which has been studied by Fan et al. [9]. In [9], sufficient conditions are obtained for the permanence if $f^l > d^u$ and $m^l a^l > c^u$ which coincide with Theorem 2.6 when $\gamma = 1$ in this paper. When $\gamma = 0$, it is a non-autonomous predator-prey system with Holling II type functional response. Theorem 2.6 is still valid in this case.

REFERENCES

[1] R. P. Agarwal, D. O′Regan and P. J. Y. Wong, Constant-sign periodic and almostperiodic solutions for a system of integral equations, Acta Appl. Math. 89 (2005) 177–216.

[2] J. O. Alzabut, J. J. Nieto and G. Tr. Stamov, Existence and exponential stability of positive almost periodic solutions for a model of hematopoiesis, Bound. Value Probl. 2009 (2009) Article ID 127510.

[3] R. Arditi and H. Saiah, Empirical evidence of the role of heterogeneity in ratio-dependent consumption, Ecology 73 (1992) 1544–1551.

[4] R. Arditi and L. R. Ginzburg, Coupling in predator-prey dynamics: Ratio-dependence, J. Theoret. Biol. 139 (1989) 311–326.

[5] R. Arditi, L. R. Ginzburg and H. R. Akcakaya, Variation in plankton densities among lakes: a case for ratio-dependent models, Am. Nat. 138 (1991) 1287–1296.

[6] R. Arditi, N. Perrin and H. Saiah, Functional response and heterogeneities: An experimental test with cladocerans, OIKOS 60 (1991) 69–75.

[7] I. Barbălat, Systems d'equations differential d'oscillations nonlinearies, Rev. Roumaine Math. Pure Appl. 4 (2) (1959) 267–270.

[8] C. Cosner, D. L. DeAngelis, J. S. Ault and D. B. Olson, Effects of spatial grouping on the functional response of predators, Theor. Pop. Biol. 56 (1999) 65–75.

[9] M. Fan, Q. Wang and X. F. Zou, Dynamics of a nonautonomous ratio-dependent predator-prey system, Proc. Roy. Soc. Edinb. Sect. A 133 (1) (2003) 97–118.

[10] H. I. Freedman, Deterministic Mathematical Models in Population Ecology, Marcel Dekker, New York, 1980.

[11] H. I. Freedman and R. M. Mathsen, Persistence in predator-prey systems with ratio-dependent predator influence, Bull. Math. Biol. 55 (1993) 817–827.

[12] J. S. Geng, Almost periodic solutions for a class of higher dimensional Schrödinger equations, Front. Math. China 4 (3) (2009) 463–482.

[13] N. G. Hairston, F. E. Smith and L. B. Slobodkin, Community structure, population control and competition, Am. Nat. 94 (1960) 421–425.

[14] M. P. Hassell and G. C. Varley, New inductive population model for insect parasites and its bearing on biological control, Nature 223 (1969) 1133–1136.

[15] C. Y. He, Almost Periodic Differential Equation, Beijing, Higher Education Publishing House, 1992 (in Chinese).

[16] S. B. Hsu and T. W. Huang, Global stability for a class of predator-prey systems, SIAM J. Appl. Math. 55 (1995) 763–783.

[17] S. B. Hsu, T. W. Hwang and Y. Kuang, Global dynamics of a predator-prey model with Hassell-Varley type functional response, Discrete Contin. Dyn. Syst. Sect. B 10 (4) (2008) 857–871.

[18] C. Jost, O. Arino and R. Arditi, About deterministic extinction in ratio-dependent predator-prey models, Bull. Math. Biol. 61 (1999) 19–32.

[19] Y. Kuang and E. Beretta, Global qualitative analysis of a ratio-dependent predator-prey systems, J. Math. Biol. 36 (1998) 389–406.

[20] Y. K. Li and K. H. Zhao, Positive almost periodic solutions of non-autonomous delay competitive systems with weak Allee effect, Electron. J. Differential Eqs. 2009 (100) (2009) 1–11.

[21] X. Lin and F. D. Chen, Almost periodic solution for a Volterra model with mutual interference and Beddington-DeAngelis functional response, Appl. Math. Comput. 214 (2) (2009) 548–556.

[22] M. L. Rosenzweig, Paradox of enrichment: destabilization of exploitation ecosystems in ecological time, Science, 171 (1969) 385–387.

[23] D. Xiao and S. Ruan, Global dynamics of a ratio-dependent predator-prey system, J. Math. Biol. 43 (2001) 268–290.

[24] B. Xu and R. Yuan, On the positive almost periodic type solutions for some nonlinear delay integral equations, J. Math. Anal. Appl. 304 (1) (2005) 249–268.

[25] X. T. Yang and R. Yuan, Global attractivity and positive almost periodic solution for delay logistic differential equation, Nonlinear Anal. Theory Methods Appl. 68 (1) (2008) 54–72.

[26] T. Yoshizawa, Stability theory and the existence of periodic solutions and almost periodic solutions, Springer-Verlag, New York Inc., 1975, 210–223.

[27] C. Y. Zhang, Almost periodic type functions and ergodicity, Science Press, Beijing; Kluwer Academic Publishers, Dordrecht, 2003.