Dynamic Systems and Applications 22 (2013) 33-48

UTILITY INDIFFERENCE PRICING OF REVERSE MORTGAGE AND ASSOCIATED INSURANCE CONTRACTS UNDER STOCHASTIC INTEREST AND VOLATILITY RATES

LINA MA[a], JINGXIAO ZHANG[b], AND D. KANNAN[c]

[a]School of Science, Tianjin University of Commerce, Tianjin 300134, China
[b]Center for Applied Statistics, School of Statistics, Renmin University of China, Beijing 100872, China
[c]Department of Mathematics, University of Georgia, Athens, Georgia 30606, USA

ABSTRACT. This article continues our earlier work, [16] and [17], in pricing the indifference continuous annuity benefits for the insurance contract with reference to a home reversion plan that involvs a single insured and a pair of insureds using the principle of equivalent utility. Whereas in [16] we analyzed this problem with fixed interest and volatility rates, we extended the analysis to a case of stochastic interest rates in [17], while keeping the volatility rate fixed. In this work, we look at the more general case of stochastic interest and volatility rates. We assume that the dynamics of the stochastic interest and volatility rates are governed by two diffusion processes. We show that the systems of partial differential equations under the stochastic interest and volatility rates parallel in form with those under the constant interest and volatility rates. However, the ramifications show the existence of connections as well as the differences between them.

Keywords: Stochastic interest rate; Stochastic volatility rate; Home reversion plan; Long-term care; Indifference continuous annuity; HJB equation

AMS Classification: Primary: 60H15, 60J65; Secondary: 91D20

1. INTRODUCTION

A reverse mortgage, in its broadest sense, is a special type of financial tool. It allows the elderly home-owners to tap their home equity but still maintain ownership and residence in their homes. With a reverse mortgage, the house-rich but cash-poor seniors can obtain much needed cash for their living expenses, premiums for long-term care, and/or other necessary expenditures, deferring the final disposition of their houses.

With the increase in the aging population, the pressure on social security and medicare entitlements is on the rise. Since the reverse mortgage will help the seniors from the inadequate fixed income, several countries, such as America, Great Britain,

[a]Research of this author was supported in part by the CNSSF for Young Scholars (09CTJ003), and in part by the Key Subject of Applied Mathematics in TJUC (X0803).

[c]Corresponding Author: kannan@uga.edu

Received July 13, 2012 1056-2176 $15.00

and France, have introduced the different types of reverse mortgage systems, such as the American *home equity conversion mortgage*, the French *rente viager*, and the British *lifetime mortgage*, and *home reversion plan*. Since the advent of the reverse mortgage in 1980s, several researchers and practitioners have analyzed the reverse mortgage system putting emphasis on qualitative research, particularly involving:

(i) the feasibility and the effectiveness of the reverse mortgage, see [19], [20], [23], [24], [21], [5];
(ii) the risk of the reverse mortgage, see [4], [28], [29], [14] [26];
(iii) the pricing of annuity of the reverse mortgage, see [7], [25], [28], [4].
(iv) the law and regulation relevant to the reverse mortgage, see [2], [3];
(v) and the financing of long-term care insurance using reverse mortgage, see [9], [10], [1], [11], [27].

The pricing of insurance risk is a classical problem in actuarial mathematics. The presence of mortality risk associated with insurance contracts causes the economic market to be incomplete. The equivalent martingale measure is not unique for incomplete markets. Hence, the standard no-arbitrage arguments do not provide unique prices any more. Therefore, the utility indifference pricing is applied increasingly in order to price the insurance risk in incomplete markets; for the equity-indexed life insurance see [22], [15], [12], [32], [31], for the life insurance liabilities see [6], for the catastrophe bonds see [8], [33]. Several of the recent literatures for utility indifference pricing of insurance risk have relaxed some restrictive assumptions, mainly including:

(i) The price process of risky asset or the claim process: from Geometric Brownian motion (see [22], [32], [31]) to the jump-diffusion processes (see [6], [12]);
(ii) The interest rate: from the constant interest rate (see [22], [32], [31]) to the stochastic interest rate (see [15], [33]);
(iii) The mortality: from the deterministic mortality model (see [22], [12], [32], [31]) to the stochastic mortality model (see [6], [15]).

However, research work on pricing the reverse mortgage and the insurance contract linking the reverse mortgage to the long-term care is still lagging behind. The recent work [30] designs first a special insurance contract linking the home reversion plan to the long-term care involving a single insured, and then prices the contract with the equivalent utility principle. Ma-Zhang-Kannan [16] follows similar ideas and methods to design and price the continuous annuities of home reversion plan and the insurance contract linking home reversion plan to long-term care for a pair of insureds ('pair' meaning husband and wife). In [16] we assume that (a) the insurer can choose investment proportion dynamically between the risky assets and the riskless bonds, (b) that the instantaneous yield of the risky assets is governed by a geometric

Brownian motion, and (c) the riskless bonds accumulate with constant interest rate. Following that, [18] generalizes the dynamics of the risky assets (i.e. home price) to follow a Lévy process, while the riskless bonds still accumulate with the constant interest rate. Our recent article [17] moves from the assumption of constant interest rate to the stochastic interest rate that is modeled by a diffusion process, while the instantaneous yield of the risky assets continues to be governed by a geometric Brownian motion. In this current article, we further extend the above by assuming that the volatility rate is also stochastic. In this, we use different diffusion processes to model the stochastic interest rates and stochastic volatility rates.

The remainder of the paper is organized as follows: In section 2, we present the results of the optimal investment without the insurance risk under the stochastic interest and volatility rate. In Section 3, as for the home reversion plan involving a couple presented by [16], we derive the partial differential equation system that the indifferent annuities satisfy under the exponential utility function. In Section 4, as for the contract linking home reversion plan and Long-term care insurance for a single insured, which are researched in [30], we derive the partial differential equation system that the indifferent annuities satisfy under the stochastic interest and volatility rates. It is of interest to note that the partial differential equation system that the indifference annuities satisfy under the stochastic interest and volatility rates are the same in form with that under the constant interest and volatility rates. However, the ramifications provide different meanings. The final Section 5 presents the conclusion.

2. THE OPTIMAL INVESTMENT UNDER STOCHASTIC INTEREST AND VOLATILITY RATES

The goal of this section is to obtain the results for the optimal portfolio investment without the insurance risk, where we assume that both the interest rate and the volatility rate to be stochastic. We start by setting up the basic notations.

- We have a complete probability space $(\Omega, \mathcal{F}, P)$ with a filtration $\mathcal{F}^o = (\mathcal{F}_s)_{t\leq s\leq T}$, where T denotes the term of the trading horizon.
- The filtration $\mathcal{F}^o = (\mathcal{F}_s)_{t\leq s\leq T}$ satisfies the usual conditions: *viz,* it is right continuous, increasing, and complete (*i.e.,* $\mathcal{F}_0$ contains all the sets of P-measure 0).
- Since we deal with risk-free and risky assets, and stochastic interest and volatility rates, we further assume that the filtration $\mathcal{F}^o$ consists of three subfiltrations:

$$\mathcal{F}^o = \mathcal{F}^H \vee \mathcal{F}^\sigma \vee \mathcal{F}^r.$$

  Here, $\mathcal{F}^H = (\mathcal{F}^H_s)_{t\leq s\leq T}$ and $\mathcal{F}^\sigma = (\mathcal{F}^\sigma_s)_{t\leq s\leq T}$ contain the information about risky asset, and $\mathcal{F}^r = (\mathcal{F}^r_s)_{t\leq s\leq T}$ contains the information about the stochastic interest rate.

- We assume that $\mathcal{F}^H, \mathcal{F}^\sigma$ and $\mathcal{F}^r$ are independent.
- The instantaneous yield H_s of the risky asset is governed by a geometric Brownian motion:

$$dH_s = H_s(\mu ds + \sigma_s dB_s^H),\ H_t = H > 0, \quad t \le s \le T, \tag{1}$$

where B_s^H is a standard Brownian motion on $(\Omega, \mathcal{F}, P)$, and adapted to the filtration $\mathcal{F}_s^H = \sigma(B_u^H : t \le u \le s)$, $s \ge t$. Here, μ is the constant return rate.

- The σ_s in Equation (1) represents the stochastic volatility rate of risky asset, and its dynamics is described by the following diffusion process:

$$d\sigma_s = c(s, \sigma_s)ds + d(s, \sigma_s)dB_s^\sigma, \quad \sigma_t = \sigma > 0, \quad t \le s \le T, \tag{2}$$

where B_s^σ is a standard Brownian motion defined on the complete probability space $(\Omega, \mathcal{F}, P)$, and adapted to the filtration $\mathcal{F}_s^\sigma = \sigma(B_u^\sigma : t \le u \le s)$, $s \ge t$. As mentioned above, the filtration $\{\mathcal{F}_s^\sigma\}$ independent of the Brownian motion $\{B_s^H\}_{\ 0\le s\le T}$. We assume Equation (2) has a unique strong solution.

- The value of risk-free asset grows subject to the stochastic interest rate $r_s > 0$, for $t < s \le T$. The dynamics of the stochastic interest rate is governed by the SDE

$$dr_s = a(s, r_s)ds + b(s, r_s)dB_s^r,\ r_t = r > 0,\ t \le s \le T, \tag{3}$$

where B_s^r is a standard Brownian motion defined on the probability space $(\Omega, \mathcal{F}, P)$ and adapted to the filtration $\mathcal{F}_s^r = \sigma(B_u^r : 0 \le u \le s)$. As noted above, the filtration $\{\mathcal{F}_s^r\}$ is independent of the Brownian motions $\{B_s^H\}_{\ 0\le s\le T}$ and $\{B_s^\sigma\}_{\ 0\le s\le T}$. We assume that $a(s, r_s)$ and $b(s, r_s) \ge 0$ are so that $r_s \ge 0$ almost surely and so that Equation (3) has a unique strong solution, see Section 5.2 of [13]. Now, the dynamics of risk-free asset M_s $(t \le s \le T)$ is given by

$$dM_s = r_s M_s ds, \quad t \le s \le T.$$

- Let W_s denote the wealth of the insurer at time s, with initial wealth w. The insurer can adjust the dynamic proportion of risky to risk-free asset; in particular, the insurer invests a portion π_s into the risky asset (the real estate, in our context) at time s $(t \le s \le T)$, and the remainder $W_s - \pi_s$ of the wealth into the riskless asset. Then, the wealth process of the insurer W_s associated with π_s is a solution to the following SDE

$$\begin{aligned} dW_s &= \pi_s \frac{dH_s}{H_s} + (W_s - \pi_s)\frac{dM_s}{M_s} \\ &= [r_s W_s + (\mu - r_s)\pi_s]\ ds + \sigma_s \pi_s\ dB_s^H,\ (t \le s \le T), \end{aligned}$$

with the initial condition $W_t = w$.

- Without the insurance risk, the *value function of the insurer* is defined by

$$U^{(0)}(w,\sigma,r,t) = \sup_{\{\pi_s\in\mathcal{A}\}} E[u(W_T)|W_t = w, \sigma_t = \sigma, r_t = r], \tag{4}$$

where the utility function $u : R \to R$ is assumed to be strictly increasing and concave. Let $\mathcal{A}$ denote the set of all admissible strategies π_s that are $\mathcal{F}_s$-adapted, self-financing and square integrable (i.e. $E\left(\int_t^T \pi_s^2 ds\right) < \infty$).

- Throughout this paper, we will always assume that the utility function u is given by the exponential utility function

$$u(w) = -\frac{1}{\alpha}e^{-\alpha w} \ (\alpha > 0), \tag{5}$$

where the parameter α measures the *absolute risk aversion of the insurer.*

We introduce the following partial differential operators for the notational brevity and quick reference.

Definition 1. Let ${}_{\mathbf{6}}\mathcal{A}_b^\pi$ and ${}_{\mathbf{7}}\mathcal{A}_b^\pi$ denote, respectively, the following partial differential operator

$$\begin{aligned}{}_{\mathbf{6}}\mathcal{A}_b^\pi f(w,\sigma,r,H,t) \ &\doteq \ \frac{\partial f}{\partial t} + (rw + (\mu - r)\pi - b)\frac{\partial f}{\partial w} + c(t,\sigma)\frac{\partial f}{\partial \sigma} \\ &+a(t,r)\frac{\partial f}{\partial r} + \mu H\frac{\partial f}{\partial H} + \frac{1}{2}\sigma^2\pi^2\frac{\partial^2 f}{\partial w^2} + \frac{1}{2}d^2(t,\sigma)\frac{\partial^2 f}{\partial \sigma^2} \\ &+\frac{1}{2}b^2(t,r)\frac{\partial^2 f}{\partial r^2} + \frac{1}{2}\sigma^2 H^2\frac{\partial^2 f}{\partial H^2} + \pi\sigma^2 H\frac{\partial^2 f}{\partial w\partial H}\end{aligned} \tag{6}$$

$$\begin{aligned}{}_{\mathbf{7}}\mathcal{A}_b^\pi f(w,\sigma,r,t) \ &\doteq \ \frac{\partial f}{\partial t} + (rw + (\mu - r)\pi - b)\frac{\partial f}{\partial w} + c(t,\sigma)\frac{\partial f}{\partial \sigma} + a(t,r)\frac{\partial f}{\partial r} \\ &+\frac{1}{2}\sigma^2\pi^2\frac{\partial^2 f}{\partial w^2} + \frac{1}{2}d^2(t,\sigma)\frac{\partial^2 f}{\partial \sigma^2} + \frac{1}{2}b^2(t,r)\frac{\partial^2 f}{\partial r^2}.\end{aligned} \tag{7}$$

$${}_{\mathbf{0}}\mathcal{L}_b^{r,\sigma} f(H,t) \doteq \frac{\partial f}{\partial t} + rH\frac{\partial f}{\partial H} + \frac{1}{2}\sigma^2 H^2\frac{\partial^2 f}{\partial H^2} + b\alpha e^{r(T-t)}. \tag{8}$$

Here the partial derivatives in Equations (6) and (7) are defined as a function of (w,σ,r,H,t) and (w,σ,r,t), respectively. For instance, $\frac{\partial f}{\partial t}$ in (6) means that $\frac{\partial f}{\partial t} \equiv \frac{\partial f}{\partial t}(w,\sigma,r,H,t)$, and $\frac{\partial f}{\partial t}$ in (7) means that $\frac{\partial f}{\partial t} \equiv \frac{\partial f}{\partial t}(w,\sigma,r,t)$. The parameter α is the same as in the utility function (5). T, μ, σ denote, respectively, the term of trading horizon, the mean return rate, and the volatility of the risky asset given by the Equation (1). Let r denote the initial value of stochastic interest rate given by (3).

Lemma 1. *Assume that the exponential utility function* $u(w) = -\frac{1}{\alpha}e^{-\alpha w}$ *is given.*

1. The value function $U^{(0)}(w,\sigma,r,t)$ solves the following HJB equation

$$\frac{\partial U^{(0)}}{\partial t}+rw\frac{\partial U^{(0)}}{\partial w}+c(t,\sigma)\frac{\partial U^{(0)}}{\partial \sigma}+\frac{1}{2}d^2(t,\sigma)\frac{\partial^2 U^{(0)}}{\partial \sigma^2}+a(t,r)\frac{\partial U^{(0)}}{\partial r}$$
$$+\frac{1}{2}b^2(t,r)\frac{\partial^2 U^{(0)}}{\partial r^2}+\max_{\pi}\left\{(\mu-r)\pi\frac{\partial U^{(0)}}{\partial w}+\frac{1}{2}\sigma^2\pi^2\frac{\partial^2 U^{(0)}}{\partial w^2}\right\}=0, \tag{9}$$

satisfying the terminal condition $U^{(0)}(w,\sigma,r,T)=-\frac{1}{\alpha}e^{-\alpha w}$.

2. Let $\bar{\alpha}=\alpha e^{r(T-t)}$. If the value function in the form

$$U^{(0)}(w,\sigma,r,t)=-\frac{1}{\alpha}\exp\left(-\alpha w e^{r(T-t)}\right)g(\sigma,r,t), \tag{10}$$

then $g(\sigma,r,t)$ solves the following PDE with the terminal condition $g(\sigma,r,T)=0$:

$$\frac{\partial g}{\partial t}+c(t,\sigma)\frac{\partial g}{\partial \sigma}+\frac{1}{2}d^2(t,\sigma)\frac{\partial^2 g}{\partial \sigma^2}+\frac{1}{2}b^2(t,r)\frac{\partial^2 g}{\partial r^2}$$
$$+\left[a(t,r)-b^2(t,r)\bar{\alpha}w(T-t)\right]\frac{\partial g}{\partial r}-\frac{(\mu-r)^2}{2\sigma^2}g(\sigma,r,t)$$
$$+\bar{\alpha}w(T-t)\left[-a(t,r)+\frac{1}{2}b^2(t,r)(T-t)(\bar{\alpha}w-1)\right]g(\sigma,r,t)=0. \tag{11}$$

3. Moreover, the optimal investment strategy π_t^* is given by

$$\pi_t^*=\frac{\mu-r_t}{\sigma_t^2\alpha e^{r_t(T-t)}}. \tag{12}$$

Proof. Applying the Itô formula to $U^{(0)}(w,\sigma,r,t)$, we obtain

$$dU^{(0)}(w,\sigma,r,t)=\frac{\partial U^{(0)}}{\partial t}+(r_tW_t+(\mu-r_t)\pi_t)\frac{\partial U^{(0)}}{\partial w}+c(t,\sigma_t)\frac{\partial U^{(0)}}{\partial \sigma}+a(t,r_t)\frac{\partial U^{(0)}}{\partial r}$$
$$+\frac{1}{2}\sigma_t^2\pi_t^2\frac{\partial^2 U^{(0)}}{\partial w^2}+\frac{1}{2}d^2(t,\sigma_t)\frac{\partial^2 U^{(0)}}{\partial \sigma^2}+\frac{1}{2}b^2(t,r_t)\frac{\partial^2 U^{(0)}}{\partial r^2}$$
$$+\sigma_t\pi_t\frac{\partial U^{(0)}}{\partial w}dB_t^H+d(t,\sigma_t)\frac{\partial U^{(0)}}{\partial \sigma}dB_t^\sigma+b(t,r_t)\frac{\partial U^{(0)}}{\partial r}dB_t^r. \tag{13}$$

To obtain the HJB equation, we fix the investment strategy $\{\pi_s\}$ as π in the time interval $[t,t+h]$ $(h\ll 1)$, and after the time $t+h$, we invest with the optimal investment strategy $\{\pi_s^*\}$. From the definition of $U^{(0)}(w,\sigma,r,t)$, it easily follows that

(14)

$$U^{(0)}(w,\sigma,r,t)\geq E[U^{(0)}(W_{t+h},\sigma_{t+h},r_{t+h},t+h)|\;W_t=w,\sigma_t=\sigma,r_t=r]$$
$$=U^{(0)}(w,\sigma,r,t)+E\left[\int_t^{t+h}dU^{(0)}(W_s,\sigma_s,r_s,s)|W_t=w,\sigma_t=\sigma,r_t=r\right]. \tag{15}$$

Let $b_0=0$. Applying the operator ${}_{\mathbf{7}}\mathcal{A}_{b_0}^{\pi}$, defined by (7), to $U^{(0)}(w,\sigma,r,t)$, inserting (13) into (15), dividing both sides of the inequality by h, and finally letting $h\to 0$, we obtain

$$0\geq {}_{\mathbf{7}}\mathcal{A}_{b_0}^{\pi}U^{(0)}(w,\sigma,r,t). \tag{16}$$

When we invest with the optimal investment strategy $\{\pi_s^*\}$ in the time interval $[t, t+h]$, $h \ll 1$, the equality holds true in (14). This yields the equality in (16) and hence (9) holds.

In order to achieve the explicit expression for $U^{(0)}(w,\sigma,r,t)$, we make an ansatz of the form

$$U^{(0)}(w,\sigma,r,t) = -\frac{1}{\alpha}\exp(-\alpha w e^{r(T-t)})g(\sigma,r,t). \tag{17}$$

Let $\bar{w} = we^{r(T-t)}$. Note the definition of utility function $u(x)$ given by Equation (5), then $u(\bar{w}) \equiv -\frac{1}{\alpha}\exp(-\alpha w e^{r(T-t)})$. We can obtain, from (17), the corresponding partial derivatives of $U^{(0)}(w,\sigma,r,t)$ in (9):

$$\begin{aligned}
\frac{\partial U^{(0)}}{\partial \sigma} &= u(\bar{w})\frac{\partial g}{\partial \sigma}, & \frac{\partial U^{(0)}}{\partial t} &= u(\bar{w})\left[\bar{\alpha} w r g(\sigma,r,t) + \frac{\partial g}{\partial t}\right],\\
\frac{\partial^2 U^{(0)}}{\partial \sigma^2} &= u(\bar{w})\frac{\partial^2 g}{\partial \sigma^2}, & \frac{\partial U^{(0)}}{\partial r} &= u(\bar{w})\left[-\bar{\alpha} w (T-t) g(\sigma,r,t) + \frac{\partial g}{\partial r}\right],\\
\frac{\partial U^{(0)}}{\partial w} &= -\bar{\alpha} u(\bar{w}) g(\sigma,r,t), & \frac{\partial^2 U^{(0)}}{\partial w^2} &= \bar{\alpha}^2 u(\bar{w}) g(\sigma,r,t),
\end{aligned}$$

$$\frac{\partial^2 U^{(0)}}{\partial r^2} = u(\bar{w})\left[\bar{\alpha} w (T-t)^2(\bar{\alpha} w - 1) g(\sigma,r,t) - 2\bar{\alpha} w (T-t)\frac{\partial g}{\partial r} + \frac{\partial^2 g}{\partial r^2}\right].$$

Inserting the above partial derivatives into (9), we can obtain $g(\sigma,r,t)$ solves the partial differential equation (11) with the boundary condition $g(\sigma,r,T) = 0$.

If we take

$$\pi = -\frac{(\mu - r)\frac{\partial U^{(0)}}{\partial w}}{\sigma^2 \frac{\partial^2 U^{(0)}}{\partial w^2}} = \frac{\mu - r}{\sigma^2 \alpha e^{r(T-t)}},$$

the term $\max_\pi\{\cdots\}$ in the Equation (9) attains the maximum value, thus we observe that the optimal investment strategy is given by (12). □

3. HOME REVERSION PLAN FOR A PAIR OF INSUREDS

Under the assumption that the interest rate and volatility are both fixed, our earlier work [16] prices the home reversion plan for a pair of insureds under the equivalent utility principle. In this section, we shall first extend the fixed interest and volatility rates case to the stochastic interest and volatility rates case, and then we continue to explore the utility indifference pricing of home reversion plan.

For the states and transitions of the corresponding policy we refer to Figure 1(a). Figure 1(a) implicitly assume that the couple of insureds cannot die simultaneously. Without the implicit assumption, we can employ Figure 1(b) to describe the actuarial structure of policy. We introduce the policy states for the couple as follows:

A. State 3 represents that both (x) and (y) are alive;
B. State 2 represents that (x) is dead and (y) is alive;

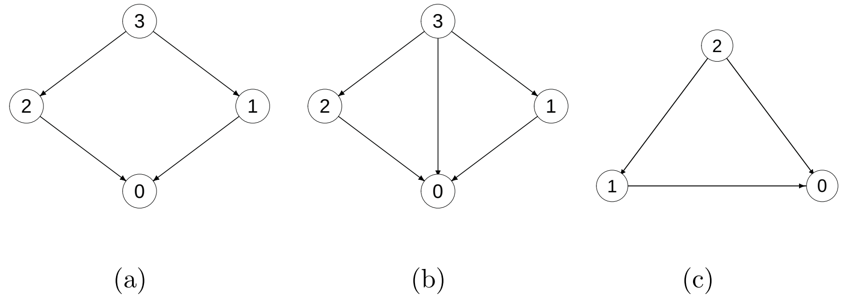

FIGURE 1. Markov models for home reversion plan for a pair of insureds

C. State 1 represents that (x) is alive and (y) is dead;

D. State 0 represents that both (x) and (y) are dead.

Here, as in [16], (x) and (y) denote the x-year old husband and y-year old wife, respectively. Let $\tau_i = \inf\{t; Z_t = i\}$ denote the stopping times of entrance into state i, $(i = 0, 1, 2)$.

The home reversion plan applied jointly by a couple is designed as follows:

- As for the benefits are concerned, we assume that a continuous annuity benefit is paid at an instantaneous constant rate b_i when the insureds are in state i, $i = 1, 2, 3$.
- In return, the insurer will be repaid with $\mathbf{g}(H_{\tau_0}, \tau_0)$ at the time the system enters into state 0, where $0 \le \mathbf{g}(H_{\tau_0}, \tau_0) \le H_{\tau_0}$. In other words, the insureds agree, at the beginning of entering the contract, that the whole or part of the cash generated from the sale of the house will automatically go to the insurer at time τ_0.

When the insurer underwrites the home reversion plan for the couple, the dynamics of wealth, based on the financial market in Section 2, becomes

$$\begin{cases} W_t = w, & \\ W_{\tau_0^+} = W_{\tau_0^-} + \mathbf{g}(H_{\tau_0}, \tau_0), & t < \tau_0 < T, \\ dW_s = [r_s W_s + (\mu - r_s)\pi_s - b_3]ds + \sigma_s \pi_s dB_s^H, & t < s < \min(\tau_1, \tau_2) < T, \\ dW_s = [r_s W_s + (\mu - r_s)\pi_s - b_2]ds + \sigma_s \pi_s dB_s^H, & t < \tau_2 < s < \tau_0 < T,\ \tau_1 = \infty, \\ dW_s = [r_s W_s + (\mu - r_s)\pi_s - b_1]ds + \sigma_s \pi_s dB_s^H, & t < \tau_1 < s < \tau_0 < T,\ \tau_2 = \infty, \\ dW_s = [r_s W_s + (\mu - r_s)\pi_s]ds + \sigma_s \pi_s dB_s^H, & t < \tau_0 < s < T. \end{cases}$$

For the above home reversion plan for a pair of insureds, the corresponding value function $U^{(i)}(w, \sigma, r, H, t)$ is given by

$$U^{(i)}(w, \sigma, r, H, t) = \sup_{\{\pi_s \in \mathcal{A}\}} E[u(W_T) | W_t = w, \sigma_s = \sigma, r_t = r, H_t = H, Z_t = i]. \tag{18}$$

The following lemma presents the HJB equation that the value function $U^{(i)}(w, \sigma, r, H, t)$ solves.

Lemma 2. *The value function* $U^{(3)}(w, \sigma, r, H, t)$ *solve the HJB equation*

(19)
$$\max_{\pi}[{}_{\mathbf{6}}\mathcal{A}^{\pi}_{b_3} U^{(3)}(w, \sigma, r, H, t)] + \sum_{i=1,2} \lambda_{3i}(t)[U^{(i)}(w, \sigma, r, H, t) - U^{(3)}(w, \sigma, r, H, t)] = 0,$$

where $U^{(i)}(w, \sigma, r, H, t)$, $(i = 1, 2)$, *satisfy the following HJB equation*

(20)
$$\max_{\pi}\left[\,{}_{\mathbf{6}}\mathcal{A}^{\pi}_{b_i} U^{(i)}(w, \sigma, r, H, t)\right] + \lambda_{i0}(t)[U^{(0)}(w + \mathbf{g}(H, t), \sigma, r, t) - U^{(i)}(w, \sigma, r, H, t)] = 0.$$

Here the Equations (19) *and* (20) *are subject to the boundary conditions*

$$U^{(i)}(w, \sigma, r, H, T) = u(w), \quad i = 1, 2, 3.$$

Proof. Assume that the insurer fixes the investment strategy $\{\pi_s\}$ as $\{\pi\}$ in time interval $[t, t + h]$, which may not be the optimal investment strategy. From the time $t+h$ to the end of horizon, the insurer follows the optimal investment strategy $\{\pi^*_s\}$. Then, it follows from the definition of $U^{(3)}(w, \sigma, r, H, t)$ that

(21)
$$U^{(3)}(w, \sigma, r, H, t) \geq \sum_{i=1,2,3} P_{3i}(t, t + h)E^{w,\sigma,r,H,t}[U^{(i)}(W_{t+h}, \sigma_{t+h}, r_{t+h}, H_{t+h}, t + h)],$$

where the notation $E^{w,\sigma,r,H,t}$ stands for the conditional expectation given $\{W_t = w, \sigma_t = \sigma, r_t = r, H_t = H\}$.

Assume that $U^{(i)}(W_t, \sigma_t, r_t, H_t, t)$ $(i = 1, 2, 3)$ is sufficiently smooth. Apply the Itô formula to obtain

$$\begin{aligned} &U^{(i)}(W_{t+h}, \sigma_{t+h}, r_{t+h}, H_{t+h}, t + h) \\ &= U^{(i)}(w, \sigma, r, H, t) + \int_t^{t+h} \left[\,{}_{\mathbf{6}}\mathcal{A}^{\pi}_{b_i} U^{(i)}(W_s, \sigma_s, r_s, H_s, s)\right] ds \\ &\quad + \int_t^{t+h} \left[\sigma_s \pi \,\frac{\partial U^{(i)}}{\partial w} + \sigma_s H_s \,\frac{\partial U^{(i)}}{\partial H}\right] dB^H_s \\ &\quad + \int_t^{t+h} \left[b(r_s, s)\frac{\partial U^{(i)}}{\partial r}\right] dB^r_s + \int_t^{t+h} \left[d(\sigma_s, s)\frac{\partial U^{(i)}}{\partial \sigma}\right] dB^{\sigma}_s, \end{aligned}$$

where let $b_0 \equiv 0$, and $\frac{\partial U^{(i)}}{\partial w}$, $\frac{\partial U^{(i)}}{\partial H}$, $\frac{\partial U^{(i)}}{\partial r}$ and $\frac{\partial U^{(i)}}{\partial \sigma}$ are the partial derivatives at the point $(W_s, \sigma_s, r_s, H_s, s)$.

Insert these equations into (21), reorganize the terms, divide both sides of the equation by h, and finally let $h \to 0$ to obtain

(22)
$$0 \geq \,{}_{\mathbf{6}}\mathcal{A}^{\pi}_{b_3} U^{(3)}(w, \sigma, r, H, t) + \sum_{i=1,2} \lambda_{3i}(t)[U^{(i)}(w, \sigma, r, H, t) - U^{(3)}(w, \sigma, r, H, t)].$$

Finally, we follow the optimal strategy $\pi = \pi^*$ in the time interval $[t, t+h]$. Now the equality part holds in each of (21) and (22). Thus we obtain (19).

Considering all transitions and the next arrival states, and applying the corresponding method that we used to derive Equation (19), we obtain the Equation (20). □

When the insurer pays out the annuity so that the optimal investment with the insurance risk and paying the continuous annuity coincide with the optimal investment without insurance risk and not paying the annuity, the insurer is indifferent with and without underwriting the insurance risk. In such a case, the annuity rates are known as the *indifference annuity rates.* Thus, when the annuity rate is the same with the indifference annuity rate, we have

$$U^{(0)}(w, \sigma, r, t) = U^{(3)}(w, \sigma, r, H, t; b_1, b_2, b_3).$$

In order to solve the indifference continuous annuity rates, we need to reduce the dimension of HJB equation of Lemma 2. The following Theorem 3 provides the HJB pricing equation for the indifference annuity rates b_1, b_2, b_3. The associated functions there are dependent on two variables: *viz.*, the time t and the house price at time t.

Theorem 3. *Assume that the exponential utility function* $u(w) = -\frac{1}{\alpha}e^{-\alpha w}$ *is given. Then, the indifference continuous annuity rates* b_i $(i = 1, 2, 3)$ *solve the following equation*

$$\phi_3(H, t; b_1, b_2, b_3) = 0, \tag{23}$$

where $\phi_3(H, t)$ *satisfies the equation*

$$\mathbf{0}\mathcal{L}^{r,\sigma}_{b_3}\phi_3(H, t) + \sum_{i=1,2} \lambda_{3i}(t) \left[e^{\phi_i(H,t)-\phi_3(H,t)} - 1\right] = 0, \tag{24}$$

in which $\phi_i(H, t)$ $(i = 1, 2)$ *solves the HJB equation*

$$\mathbf{0}\mathcal{L}^{r,\sigma}_{b_i}\phi_i(H, t) + \lambda_{i0}(t) \left(e^{-(\bar{\alpha}\mathbf{g}(H,t)+\phi_i(H,t))} - 1\right) = 0. \tag{25}$$

Here, the Equataions (24) *and* (25) *are subject to the terminal conditions* $\phi_i(H, T) = 0$, $i = 1, 2, 3$.

Proof. Let

$$U^{(i)}(w, \sigma, r, H, t) = U^{(0)}(w, \sigma, r, t)e^{\phi_i(H,t)}, \quad i = 1, 2, 3. \tag{26}$$

From (26), we obtain the following partial derivatives of $U^{(i)}(w, \sigma, r, H, t)$:

$$\frac{\partial U^{(i)}}{\partial r} = \frac{\partial U^{(0)}}{\partial r}e^{\phi_i(H,t)}, \qquad \frac{\partial U^{(i)}}{\partial H} = U^{(0)}e^{\phi_i(H,t)}\frac{\partial \phi_i}{\partial H},$$
$$\frac{\partial U^{(i)}}{\partial w} = \frac{\partial U^{(0)}}{\partial w}e^{\phi_i(H,t)}, \qquad \frac{\partial^2 U^{(i)}}{\partial w^2} = \frac{\partial^2 U^{(0)}}{\partial w^2}e^{\phi_i(H,t)},$$

$$
\begin{aligned}
\frac{\partial U^{(i)}}{\partial \sigma} &= \frac{\partial U^{(0)}}{\partial \sigma} e^{\phi_i(H,t)}, \qquad \frac{\partial U^{(i)}}{\partial t} = e^{\phi_i(H,t)} \left[\frac{\partial U^{(0)}}{\partial t} + U^{(0)} \frac{\partial \phi_i}{\partial t} \right], \\
\frac{\partial^2 U^{(i)}}{\partial r^2} &= \frac{\partial^2 U^{(0)}}{\partial r^2} e^{\phi_i(H,t)}, \qquad \frac{\partial^2 U^{(i)}}{\partial H^2} = U^{(0)} e^{\phi_i(H,t)} \left[\frac{\partial^2 \phi_i}{\partial H^2} + \left(\frac{\partial \phi_i}{\partial H} \right)^2 \right], \\
\frac{\partial^2 U^{(i)}}{\partial \sigma^2} &= \frac{\partial^2 U^{(0)}}{\partial \sigma^2} e^{\phi_i(H,t)}, \qquad \frac{\partial^2 U^{(i)}}{\partial w \partial H} = \frac{\partial U^{(0)}}{\partial w} e^{\phi_i(H,t)} \frac{\partial \phi_i}{\partial H}.
\end{aligned}
$$

Substituting these partial derivatives in ${}_{\mathbf{6}}\mathcal{A}_{b_i}^{\pi} U^{(i)}(w,\sigma,r,H,t)$, (see Definition 1), and noting (9), we can further simplify $\max_\pi \left\{ {}_{\mathbf{6}}\mathcal{A}_{b_i}^{\pi} U^{(i)}(w,\sigma,r,H,t) \right\}$ $(i = 1,2,3)$ to

$$
\begin{aligned}
&\max_{\pi} \left\{ {}_{\mathbf{6}}\mathcal{A}_{b_i}^{\pi} U^{(i)}(w,\sigma,r,H,t) \right\} \\
&= U^{(0)}(w,\sigma,r,t) e^{\phi_i(H,t)} \left[\frac{\partial \phi_i}{\partial t} + b_i \alpha e^{r(T-t)} + \frac{1}{2}\sigma^2 H^2 \frac{\partial^2 \phi_i}{\partial H^2} + rH \frac{\partial \phi_i}{\partial H} \right] \\
&= U^{(0)}(w,\sigma,r,t) e^{\phi_i(H,t)} \ {}_{\mathbf{0}}\mathcal{L}_{b_i}^{r,\sigma} \phi_i(H,t), \qquad (27)
\end{aligned}
$$

where the definitions of the operators ${}_{\mathbf{6}}\mathcal{A}_{b}^{\pi} U^{(i)}(w,\sigma,r,H,t)$ and ${}_{\mathbf{0}}\mathcal{L}_{b_i}^{r,\sigma} \phi_i(H,t)$ are given by (6) and (8), respectively.

Noting that $\bar{\alpha} = \alpha e^{r(T-t)}$ and Equation (26), we get

$$
\begin{aligned}
&\sum_{i=1,2} \lambda_{3i}(t) [U^{(i)}(w,\sigma,r,H,t) - U^{(3)}(w,\sigma,r,H,t)] \\
[1.0ex] \quad &= U^{(0)}(w,\sigma,r,t) e^{\phi_3(H,t)} \sum_{i=1,2} \lambda_{3i}(t) \left[e^{\phi_i(H,t) - \phi_3(H,t)} - 1 \right], \qquad (28)
\end{aligned}
$$

$$
\begin{aligned}
&U^{(0)}(w + \mathbf{g}(H,t), \sigma, r, t) - U^{(i)}(w,\sigma,r,H,t) \\
[1.0ex] \quad &= U^{(0)}(w,\sigma,r,t) e^{\phi_i(H,t)} \left[e^{-\bar{\alpha}\mathbf{g}(H,t) - \phi_i(H,t)} - 1 \right], \quad (i = 1,2). \qquad (29)
\end{aligned}
$$

Substituting (27) and (28) into (19), we note that (24) obtains.

In the same way, substituting (27) and (29) into (20), we see that (25) holds. □

Remark 1. The definition of the differential operator ${}_{\mathbf{0}}\mathcal{L}_{b_i}^{r,\sigma} \phi_i(H,t)$ in Theorem 3 is given by the Relation (8). The differential operator ${}_{\mathbf{0}}\mathcal{L}_{b_i}^{r,\sigma} \phi_i(H,t)$, $(i = 1,2,3)$, can be expanded as follows

$$
{}_{\mathbf{0}}\mathcal{L}_{b_i}^{r,\sigma} \phi_i(H,t) = \frac{\partial \phi_i}{\partial t} + rH \frac{\partial \phi_i}{\partial H} + \frac{1}{2}\sigma^2 H^2 \frac{\partial^2 \phi_i}{\partial H^2} + b_i \alpha e^{r(T-t)}.
$$

Here, $r = r_t$ represents the initial value of stochastic interest rate r_s, $(t \le s \le T)$, (i.e., at the time of writing the home reversion plan for a pair of insureds); while r in [16] means the constant interest rate. In this paper, σ represents the stochastic volatility σ_s, $(t \le s \le T)$, at initial time t; while σ in [16] means the constant volatility of the value of the risky asset.

Remark 2. The reader will notice that Theorem 3 coincides with Theorem 4.3 in [16] in form, but with different notations. However, there exist some similarities and differences between them in inferences.

- The main differences are due to the different meanings of r, σ in these two theorems. r, σ in Theorem 3 represents, respectively, the initial value of stochastic interest rate and stochastic volatility whose dynamics are modeled by the diffusion process (2) and (3), respectively. Thus, Theorem 3 indicates that the indifference annuity rates only relate with the initial value of stochastic interest and volatility rates at the beginning of underwriting the insurance contract, and have nothing in common with how these two diffusion processes evolve; while r, σ in Theorem 4.3 in [16] denotes, respectively, the constant interest rate and volatility rate during the whole insurance period.
- There exist mutual connections between them. In case that the initial value of stochastic interest and volatility rates in Theorem 3 coincide with the constant interest and volatility rates in Theorem 4.3 in [16], the indifference annuity rates under stochastic interest and volatility rates are the same with that under the constant interest rates and volatility rates.

4. THE INSURANCE CONTRACT LINKING HOME REVERSION PLAN AND LONG TERM CARE FOR A SINGLE INSURED

Under the hypothesis that the interest rate and volatility rate are both constants, Xiao's work [30] prices the insurance contract linking home reversion plan and long-term care insurance using the principle of equivalent utility. In this section, we will continue to price this contract presented by [30] now under the assumption of stochastic interest and stochastic volatility rates. The results show that the nonlinear PDE system that the indifference annuity rates satisfy coincides in form with those in Theorem 2 of [30]. Since the discussions of this section parallels that of Section 3 above, we use correspondingly similar notations, but omit the proofs.

This section will continue to apply the three-state Markov model (see Figure 1(c)) present in [30] to describe the actuarial structure of the linking contract for a single insured. We work with the financial market described in Section 2.

The stopping time $\tau_i = \inf\{t; Z_t = i\}$ represent the time of entering the State i, $i = 0, 1$. The linked insurance contract designed by Xiao (2010) is characterized by the following clauses:

(I) *When the insured stays at state i ($i = 1, 2$), the insurer pays the continuous annuities with the constant rate b_i, $i = 1, 2$, and $b_2 < b_1$.*

(II) *At the stopping time $\tau = \min\{\tau_0, \tau_1\}$, the insureds repay the insurer with the money from the sale of the house.*

Now, when the insurer signs the contract linking the home reversion plan to the long term care, the dynamics of the wealth process for the insurer follows the system:

$$\begin{cases} W_t = w, & \\ W_{\tau_1^+} = W_{\tau_1^-} + H_{\tau_1}, & \tau_1 < \tau_0 < T, \\ W_{\tau_0^+} = W_{\tau_0^-} + H_{\tau_0}, & \tau_1 = \infty, \tau_0 < T, \\ dW_s = [r_s W_s + (\mu - r_s)\pi_s - b_2]\ ds + \sigma\pi_s\ dB_s^H, & t < s < \min(\tau_0, \tau_1), \\ dW_s = [r_s W_s + (\mu - r_s)\pi_s - b_1]\ ds + \sigma\pi_s\ dB_s^H, & \tau_1 < s < \tau_0, \\ dW_s = [r_s W_s + (\mu - r_s)\pi_s]\ ds + \sigma\pi_s\ dB_s^H, & \tau_0 < s < T. \end{cases}$$

The value functions describe the goal of the insurer, where the goal is to maximize the expected utility of terminal wealth. The value functions for the insurance contract linking the home reversion plan to the long term care are defined as follows:

(30)
$$U^{(2)}(w, \sigma, r, H, t; b_1, b_2) = \sup_{\{\pi_s \in \mathcal{A}\}} E[u(W_T)|W_t = w, \sigma_t = \sigma, r_t = r, H_t = H, Z_t = 2],$$

(31)
$$U^{(1)}(w, \sigma, r, t; b_1) = \sup_{\{\pi_s \in \mathcal{A}\}} E[u(W_T)|W_t = w, \sigma_t = \sigma, r_t = r, Z_t = 1].$$

Mimicing the method used in establishing Lemma 2, we can obtain the following HJB equation system that the value functions defined by (30) and (31) solve.

Lemma 4. *The value function* $U^{(2)}(w, \sigma, r, H, t)$ *solves the HJB equation*

(32)
$$\max_{\pi}[{}_6\mathcal{A}_{b_2}^{\pi} U^{(2)}(w, \sigma, r, H, t)] + \sum_{i=0,1} \lambda_{2i}(t)[U^{(i)}(w + H, \sigma, r, t) - U^{(2)}(w, \sigma, r, H, t)] = 0,$$

where $U^{(1)}(w, \sigma, r, t)$ *satisfies the following HJB equation*

(33)
$$\max_{\pi} \left[{}_7\mathcal{A}_{b_1}^{\pi} U^{(1)}(w, \sigma, r, t)\right] + \lambda_{10}(t)[U^{(0)}(w, \sigma, r, t) - U^{(1)}(w, \sigma, r, t)] = 0.$$

Here, the Equations (32) *and* (33) *are subject to the boundary conditions*

$$U^{(2)}(w, \sigma, r, H, T) = u(w),\ U^{(1)}(w, \sigma, r, T) = u(w).$$

When the continuous annuity rates b_1 and b_2 paid by the insurer cause the optimal investment involving the insurance risk to be the same with that of without the insurance risk, we get

$$U^{(0)}(w, \sigma, r, t) = U^{(2)}(w, \sigma, r, H, t; b_1, b_2).$$

Such annuity rates are the indifference annuity rates. With the above equation, we can obtain the indifference annuity rates b_1 and b_2 that satisfy the HJB system given in the following theorem.

Theorem 5. *Assume that the exponential utility function* $u(w) = -\frac{1}{\alpha}e^{-\alpha w}$ *is given. Then, the indifference annuity rates* b_i $(i = 1, 2)$ *satisfy the following equation*

$$\phi_2(H, t; b_1, b_2) = 0, \tag{34}$$

where $\phi_2(H, t)$ *solves the following HJB equation*

$$_{\mathbf{0}}\mathcal{L}^{r,\sigma}_{b_2}\phi_2(H, t) + \sum_{j=0,1} \lambda_{2j}(t)\left[e^{-\bar{\alpha}H-\phi_2(H,t)}(\phi_1(t)\mathbf{1}_{\{j=1\}} + \mathbf{1}_{\{j=0\}}) - 1\right] = 0, \tag{35}$$

in which $\phi_1(t)$ *satisfies the HJB equation*

$$\frac{d\phi_1(t)}{dt} + [b_1\bar{\alpha} - \lambda_{10}(t)]\,\phi_1(t) + \lambda_{10}(t) = 0. \tag{36}$$

Here, the Equations (35) *and* (36) *are subject to the terminal conditions* $\phi_2(H, T) = 0$ *and* $\phi_1(T) = 1$.

Proof. Let

$$\begin{aligned} U^{(2)}(w, \sigma, r, H, t) &= U^{(0)}(w, \sigma, r, t)e^{\phi_2(H,t)}, \\ U^{(1)}(w, \sigma, r, t) &= U^{(0)}(w, \sigma, r, t)\phi_1(t). \end{aligned} \tag{37}$$

Suitably modifying the derivation of Equation (24) to suit the present case, we obtain the Equation(35) from the Equation (32).

Substituting the partial derivatives of $U^{(1)}(w, \sigma, r, t)$ in $_{\mathbf{7}}\mathcal{A}^{\pi}_{b_1}U^{(1)}(w, \sigma, r, t)$, (see Definition 1), and using (9), we can further simplify $\max_\pi\left[_{\mathbf{7}}\mathcal{A}^{\pi}_{b_1}U^{(1)}(w, \sigma, r, t)\right]$ to

$$\max_{\pi}\left[\,_{\mathbf{7}}\mathcal{A}^{\pi}_{b_1}U^{(1)}(w, \sigma, r, t)\right] = U^{(0)}(w, \sigma, r, t)\left[\frac{d\phi_1}{dt} + b_1\alpha e^{r(T-t)}\phi_1(t)\right], \tag{38}$$

From Equation (37), we get

$$U^{(0)}(w, \sigma, r, t) - U^{(1)}(w, \sigma, r, t) = U^{(0)}(w, \sigma, r, t)\left[1 - \phi_1(t)\right]. \tag{39}$$

Inserting Equations (38) and (39) into (33), we obtain (36). □

5. CONCLUSION

In this article we assume that the dynamics of the stochastic interest rate and stochastic volatility rate are governed by two diffusion processes. Under such an assumption, we applied the utility indifference pricing arguments to derive the partial differential equation system that the indifferent annuity benefits satisfy under the exponential utility function. Interestingly, the PDE systems under stochastic interest and stochastic volatility rates are of the same form with those under the constant interest rate and volatility rate. However, the implications provide us with some similarities as well as differences between them. In case that the value of stochastic interest and volatility rates at the beginning of signing the insurance contract are the same with the constant interest and volatility rates in [30] and [16], the indifference annuity benefits under the stochastic interest and volatility rates coincide with those under the constant interest and volatility rates. The indifference annuity rates under the stochastic interest and volatility rates relate only with the initial value of stochastic interest and volatility rates at the start of writing the insurance contract, and have nothing to do with the specific changing path of these two diffusion processes that drive the dynamics of stochastic interest and volatility rates.

REFERENCES

[1] A. A. Benejam: Home equity conversions as alternatives to health care financing; *Medicine and Law*, **6**(4), 1987, 329–348.

[2] British Columbia Law Institute: Report on reverse mortgages; Available at: http://www.bcli.org/sites/default/files/Reverse-Mortgages-Rep.pdf.

[3] B. Case and A. B. Schnare: Preliminary evaluation of the HECM reverse mortgage program; *Journal of the American Real Estate and Urban Economics Association*, **22**(2), 1994, 301–346.

[4] P. Chinloy and I. F. Megbolugbe: Reverse mortgages: contracting and crossover risk; *Journal of the American Real Estate and Urban Economics Association*, **22**(2), 1994, 367–386.

[5] K. L. Chou, N. Chow, and I. Chi: Willingness to consider applying for reverse mortgage in Hong Kong Chinese middle-aged homeowners; *Habitat International*, **30**(4), 2006, 716–727.

[6] L. Delong: Indifference pricing of a life insurance portfolio with systematic mortality risk in a market with an asset driven by a Lévy process; *Scandinavian Actuarial Journal*, **1**, 2009, 1–26.

[7] T. R. DiVenti and T. N. Herzog: Modeling home equity conversion mortgages; *Actuarial research clearing houses*, **2**, 1990, 1–24.

[8] M. Egamia and V. R. Young: Indifference prices of structured catastrophe (CAT) bonds; *Insurance: Mathematics and Economics*, **42**, 2008, 771–778.

[9] J. Fireman: Reforming Community Care for the Elderly and the Disabled; *Health Affairs* **2**(1), 1983, 66–82.

[10] J. Fireman: Health Care Cooperatives: Innovations for Older People; *Health Affairs* **4**(4), 1985, 50–61.

[11] B. Jacobs and W. Weissert: Using Home Equity to Finance Long-term Care; *J of Health Politics, Policy, and Law*, **12**, 1987, 77–95.

[12] S. Jaimungal and V. R. Young: Pricing equity-linked pure endowments with risky assets that follow Levy processes; *Insurance: Mathematics and Economics*, **36**, 2005, 329–346.

[13] I. Karatzas and S. E. Shreve: *Brownian Motion and Stochastic Calculus*, second ed. Springer, New York, 1991.

[14] L. S. Kleinand C. F. Sirmans: Reverse Mortgages and Prepayment Risk; *Journal of the American Real Estate and Urban Economics Association*, **22**(2), 1994, 409–431.

[15] M. Ludkovski and V. R. Young: Indifference pricing of pure endowments and life annuities under stochastic hazard and interest rates; *Insurance: Mathematics and Economics*, **42**, 2008, 14–30.

[16] L. Ma, J. Zhang, and D. Kannan: A Markov Process Modeling and Analysis of Indifference Pricing of Insurance Contracts for Home Reversion Plan for a Pair of Insureds; *Stochastic Analysis and Applications*, **29**, 860–880.

[17] L. Ma, J. Zhang, and D. Kannan: Utility Indifference Pricing of Insurance Contracts for Home Reversion Plan under Stochastic Interest Rate; To appear in *Dynamic Systems and Applications, 2012*.

[18] L. Ma, Y. Xiao, and J. Zhang: Indifference pricing of a continuous annuity relevant to Home Reversion Plan in a Levy process financial market; To appear.

[19] S. R. Merrill, M. Finkel, and N. K. Kutty: Potential Beneficiaries from Reverse Mortgage Products for Elderly Homeowners: An Analysis of AHS Data; *Journal of the American Real Estate and Urban Economics Association*, **22**(2), 1994, 257–299.

[20] S. R. Merrill, M. Finkel, and N. K. Kutty: Reverse Mortgages and the Liquidity of Housing Wealth; *Journal of the American Real Estate and Urban Economics Association*, **22**(2), 1994, 235–255.

[21] O. S. Mitchell and J. Piggott: Unlocking housing equity in Japan; *Journal of the Japanese and international economies*, **18**(4), 2004, 466–505.

[22] K. S. Moore and V. R. Young: Pricing equity-linked pure endowments via the principle of equivalent utility; *Insurance: Mathematics and Economics*, **33**, 2003, 497–516.

[23] D. W. Rasmussen, I. F. Megbolugbe, and B. A. Morgan: Using the 1990 Public Use Microdata Sample to Estimate Potential Demand for Reverse Mortgage Products; *Journal of Housing Research*, **6**(1), 1995, 1–23.

[24] D. W. Rasmussen, I. F. Megbolugbe, and B. A. Morgan: The Reverse Mortgage as an Asset Management Tool; *Housing Policy Debate*, **8**(1), 1997, 173–194.

[25] S. Schapira: An insurance plan to guarantee reverse mortgages:comment; *Journal of Risk and Insuranc*, **57**(4), 1990, 712–714.

[26] R. J. Shiller and A. N. Weiss: Moral Hazard in Home Equity Conversion; *Real Estate Economics*, **28**(1), 2000, 1–31.

[27] B. R. Stucki: Using reverse mortgages to manage the financial risk of long-term care; *North American Actuarial Journal*, **10**(4), 2006, 90–102.

[28] E. J. Szymanoski: Risk and the home equity conversion Mortgage; *Journal of the American Real Estate and Urban Economics Association*, **22**(2), 1994, 347–366.

[29] P. B. Thomas and M. C. Ehrhardt: Reverse mortgages and interest rate risk; *Journal of the American Real Estate and Urban Economics Association*, **22**(2), 1994, 387–408.

[30] Y. G. Xiao: Pricing a Contract of Linking Home Reversion Plan and Long-Term Care Insurance via the Principle of Equivalent Utility; *Quality and Quantity*, **45**(2), 2010, 465–475.

[31] V. R. Young, and T. Zariphopoulou: Pricing dynamic insurance risks using the principle of equivalent utility; *Scandinavian Actuarial Journal*, **4**, 2002, 246–279.

[32] V. R. Young: Equity-indexed life insurance: pricing and reserving using the principle of equivalent utility; *North American Actuarial Journal*, **17**(1), 2003, 68–86.

[33] V. R. Young: Pricing in an incomplete market with an affine term structure. *Mathematical Finance*, **14**(3), 2004, 359–381.