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EXISTENCE AND STABILITY OF PERIODIC SOLUTIONS FOR STOCHASTIC FUZZY CELLULAR NEURAL NETWORKS WITH TIME-VARYING DELAY ON TIME SCALES

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ABSTRACT: By applying contraction mapping theorem and Gronwall's inequality on time scale, we establish some sufficient conditions on the existence and global exponential stability of periodic solutions for a kind of stochastic fuzzy cellular neural networks on time scales. Moreover an example is presented to illustrate the feasibility of our results obtained.

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Key Words: periodic solutions, stochastic fuzzy cellular neural networks, Gronwall's inequality, time-varying delay, time scales

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1. INTRODUCTION

In recent years, Cellular neural networks [1, 2] have been extensively studied and applied in many different fields such as associative memory, signal and image processing, pattern recognition, and some optimization problems. In such applications, it is very importance to ensure that the designed neural networks are stable. The existence and stability of equilibrium points, periodic solutions or almost periodic solutions for cellular neural networks have been studied by many scholars (see [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15] and references cited therein). For example, in [6], based on Lyapunov functionals, authors obtained sufficient conditions on the stability of Hopfield neural networks on time scales. In [11], by using Mawhins's continuation of coincidence degree theory and constructing suitable Lyapunov functionals, Yang investigated the periodicity and exponential stability for a class of BAM higher-order Hopfield neural networks on time scales.

In practice, Haykin [16] pointed out that in real nervous systems synaptic transmission is noisy process brought on by random fluctuations from the release of neurotransmitters and other probabilistic causes. The neural networks could be stabilized or destabilized by some stochastic inputs. Therefore, it is significant to study stochastic effects on the dynamical behavior of neural networks. And the corresponding neural networks with noise disturbances are called stochastic neural networks. There are many works on the stability and on the synchronization of stochastic neural networks [17, 18, 19, 20, 21, 22]. For example, in [21], Ma studied the synchronization of two classes of stochastic neural networks, respectively. in [22], Li studied the existence and exponential stability of periodic solutions for impulsive stochastic neural networks.

Most results published are on the stochastic neural networks which act in continuous-time manner. When it comes to implementation of continuous-time networks for the sake of computer-based simulation, experimentation or computation, it is usual to discretize the continuous-time networks. Hence, in implementation and applications of neural networks, discrete-time neural networks become more important than their continuous-time counterpart and more suitable to model digitally transmitted signals in a dynamical way. But it is troublesome to study the dynamical properties for continuous and discrete systems, respectively. So it is significant to study dynamical systems on time scales (see [6, 11, 15, 23, 26, 27, 34] and references cited therein), which can unify the continuous and discrete situations.

In this paper, we would like to integrate fuzzy operations into cellular neural networks. Speaking of fuzzy operations, Yang and Yang [28, 29] first introduced fuzzy cellular neural networks (FCNNs) combining those operations with cellular neural networks. So far researchers have founded that FCNNs are useful in image processing, and some results have been reported on stability and periodicity of FCNNs [30, 31, 32, 33]. However, to the best of our knowledge, there are few published papers considering the periodic solutions of stochastic fuzzy cellular neural networks on time scales. Therefore, we consider the following stochastic fuzzy cellular neural networks with time-varying on time scales

$$
\begin{aligned}
\Delta x_i(t) \quad = \quad & \Big[-a_i(t)x_i(t) + \textstyle\sum_{j=1}^{n} c_{ij}(t)f_j(x_j(t)) \\
& + \textstyle\bigwedge_{j=1}^{n} \alpha_{ij}(t)g_j(x_j(t-\tau_{ij}(t))) \\
& + \textstyle\bigvee_{j=1}^{n} \beta_{ij}(t)g_j(x_j(t-\tau_{ij}(t))) + I_i(t)\Big] \Delta t \\
& + \textstyle\sum_{j=1}^{n} \delta_{ij}(x_j(t))\Delta\omega_j(t), t \in \mathbb{T},
\end{aligned} \tag{1}
$$

for $i = 1, 2, \cdots, n$, where $\mathbb{T}$ is an ω-periodic time scale which has the subspace topology inherited from the standard topology on $\mathbb{R}$. n is the number of neurons in layers. $x_i(t)$ denotes the activation of the ith neuron at time t. $f_j(\cdot), g_j(\cdot)$ are signal transmission functions. $\tau_{ij} : \mathbb{T} \to [0, \infty) \bigcap \mathbb{T}$ and satisfies $t - \tau_{ij}(t) \in \mathbb{T}$. $a_i(t) > 0$ represents the rate with which the ith neuron will reset its potential to the resting state in isolation when they are disconnected from the network and the external inputs at time t. $c_{ij}(t)$ is elements of feedback templates at time t. $I_i(t)$ is external input to the $i-$th unit. $\alpha_{ij}(t), \beta_{ij}(t)$ are elements of fuzzy feedback MIN template and fuzzy feedback MAX template, respectively. $\bigwedge$ and $\bigvee$ denote the fuzzy AND and fuzzy OR operation, respectively. $\omega(t) = (\omega_1(t), \omega_2(t), \cdots, \omega_n(t))^T$ is the n-dimensional Brownian motion defined on complete probability space $(\Omega, \mathbb{F}, \mathbb{P})$. where we denote by $\mathbb{F}$ the associated $\sigma-$algebra generated by $\{\omega(t)\}$ with the probability measure $\mathbb{P}$. δ_{ij} are Borel measurable functions, $\delta = (\delta_{ij})_{n\times n}$ is the diffusion coefficient matrix.

Let $x(t) = (x_1(t), x_2(t), \cdots, x_n(t))^T \in BC^b_{\mathbb{F}_\varkappa}(\mathbb{T}, \mathbb{R}^n)$, where $BC^b_{\mathbb{F}_\varkappa}(\mathbb{T}, \mathbb{R}^n)$ is the family of bounded $\mathbb{F}_\varkappa$-measurable $\mathbb{R}^n$ random variables $x(t)$. For convenience,we denote $[a, b]_{\mathbb{T}} = \{t | t \in [a, b] \bigcap \mathbb{T}$. For an ω-periodic rd-continuous function $f : R \to R$, denote $\overline{f} = \sup_{t\in[0,\omega]_{\mathbb{T}}} |f(t)|, \underline{f} = \inf_{t\in[0,\omega]_{\mathbb{T}}} |f(t)|$. The initial conditions associated with system (1) are of the form

$$x_i(s) = \varphi_i(s), \tag{2}$$

where $\varphi_i \in BC^b_{\mathbb{F}_\varkappa}([-\tau_0, 0]_{\mathbb{T}}, \mathbb{R}), i = 1, 2, \cdots, n$, $\tau_0 = \max_{1\le i,j\le n} \sup_{t\in[0,\omega]_{\mathbb{T}}}\{\tau_{ij}(t)\}$.

Throughout this paper, we make the following assumptions

(**A1**) $c_{ij}(t), \alpha_{ij}(t), \beta_{ij}(t), \tau_{ij}(t), I_i(t)$ are all $\omega-$periodic rd-continuous functions for $t \in \mathbb{T}$, $i, j = 1, 2, \cdots, n$.
(**A2**) f_j, g_j, δ_{ij} are all Lipschitz-continuous with Lipschitz constants $l_j > 0, \nu_j > 0, \kappa_{ij} > 0$ respectively, $i, j = 1, 2, \cdots, n$.

The organization of this paper is as follows. In Section 2, we introduce some definitions and lemmas. In Section 3, we establish sufficient conditions for the existence of the periodic solutions of system (1). In Section 4, we prove that the periodic solution obtained is global exponentially stable. In Section 5, an example is given to demonstrate the effectiveness of our results. Conclusions are drawn in Section 6.

2. PRELIMINARIES

In this section, we shall first recall some basic definitions, lemmas which are used in what follows.

Definition 1. (Hilger [35]) Let $\mathbb{T}$ be a nonempty closed subset (time scale) of $\mathbb{R}$. The forward and backward jump operators $\sigma, \rho : \mathbb{T} \to \mathbb{T}$ and the graininess $\mu : \mathbb{T} \to \mathbb{R}^+$ are defined, respectively, by

$$\sigma(t) = \inf\{s \in \mathbb{T} : s > t\},\ \rho(t) = \sup\{s \in \mathbb{T} : s < t\},\ \mu(t) = \sigma(t) - t.$$

Definition 2. [24] A point $t \in \mathbb{T}$ is called left-dense if $t > \inf \mathbb{T}$ and $\rho(t) = t$, left-scattered if $\rho(t) < t$, right-dense if $t < \sup \mathbb{T}$ and $\sigma(t) = t$, and right-scattered if $\sigma(t) > t$. If $\mathbb{T}$ has a left-scattered maximum m, then $\mathbb{T}^k = \mathbb{T} \setminus \{m\}$; otherwise $\mathbb{T}^k = \mathbb{T}$. If $\mathbb{T}$ has a right-scattered minimum m, then $\mathbb{T}_k = \mathbb{T} \setminus \{m\}$. otherwise $\mathbb{T}_k = \mathbb{T}$.

Definition 3. [24] A function $f : \mathbb{T} \to R$ is right-dense continuous provided it is continuous at right-dense point in $\mathbb{T}$ and its left-side limits exist at left-dense points in $\mathbb{T}$. If f is continuous at each right-dense point and each left-dense point, then f is said to be a continuous function on $\mathbb{T}$.

Definition 4. [24] For $y : \mathbb{T} \to \mathbb{R}$ and $t \in \mathbb{T}^k$, we define the delta derivative of $y(t), y^\Delta(t)$, to be the number (if exists) with the property that for given $\varepsilon > 0$, there exists a neighborhood U of t such that $|[y(\sigma(t)) - y(s)] - y^\Delta(t)[\sigma(t) - y(s)]| < \varepsilon|\sigma(t) - s|$ for all $s \in U$. If y is continuous, then y is right-dense continuous, and y is delta differentiable at t, then y is continuous at t. Let y be right-dense continuous. If $Y^\Delta(t) = y(t)$, then we define the delta integral by $\int_a^t y(s)\Delta s = Y(t) - Y(a)$.

Definition 5. [24] We say that a time scale $\mathbb{T}$ is periodic if there exists $p > 0$ such that if $t \in \mathbb{T}$, then $t \pm p \in \mathbb{T}$. For $\mathbb{T} \neq \mathbb{R}$, the least positive p is called the period of the time scale.

Definition 6. [24] Let $\mathbb{T} \neq \mathbb{R}$ be a periodic time scale with periodic p. We say that the function $f : \mathbb{T} \to \mathbb{R}$ is ω periodic if there exists a natural number n such that $\omega = np, f(t + \omega) = f(t)$ for all $t \in \mathbb{T}$ and ω is the least number such that $f(t + \omega) = f(t)$. If $\mathbb{T} = \mathbb{R}$, we say that f is $\omega > 0$ periodic if ω is is the least positive number such that $f(t + \omega) = f(t)$ for all $t \in \mathbb{T}$.

Definition 7. [24] A function $r : \mathbb{T} \to R$ is called regressive if $1 + \mu(t)r(t) \neq 0$, for all $t \in \mathbb{T}^k$. If r is regressive function, then the generalized exponential function e_r is defined by

$$e_r(t, s) = \exp\left\{\int_s^t \xi_{\mu(\tau)}(r(\tau))\Delta\tau\right\},\quad s, t \in \mathbb{T},$$

with the cylinder transformation

$$\xi_{h(z)} = \begin{cases} \frac{\log(1+hz)}{h}, & h \neq 0, \\ z, & h = 0. \end{cases}$$

Let $p, q : \mathbb{T} \to \mathbb{R}$ be two regressive functions, we define

$$p \oplus q := p + q + \mu pq; \; p \ominus q := p \oplus (\ominus q); \; \ominus p := \frac{p}{1+\mu p}.$$

Then the generalized exponential function has the following properties.

Lemma 1. *[35] Let p, q be regressive functions on $\mathbb{T}$. Then*
(i) $e_0(t,s) = 1$ and $e_p(t,t) = 1$; (ii) $e_p(\sigma(t), s) = (1 + \mu(t)p(t))e_p(t,s)$;
(iii) $e_p(t,s)e_p(s,r) = e_p(t,r)$; (iv) $e_p^{\Delta}(\cdot, s) = pe_p(\cdot, s)$.

Lemma 2. *[35] Suppose that $p \in R^+$, then*
(i) $e_p(t,s) > 0$, for all $t, t \in \mathbb{T}$;
(ii) if $p(t) \leq q(t)$ for all $t \geq s, t, s\mathbb{T}$, then $e_p(t,s) \leq e_q(t,s)$ for all $t \geq s$.

Lemma 3. *(Bohner and Peterson [24]). If $p \in R$ and $a, b, c \in \mathbb{T}$, then*

$$[e_p(c,\cdot)]^{\Delta} = -p[e_p(c,\cdot)]^{\sigma}, \text{ and } \int_a^b p(t)e_p(c,\sigma(t))\Delta t = e_p(c,a) - e_p(c,b).$$

Lemma 4. *(Bohner and Peterson [24]). Let $y \in C_{rd}(\mathbb{T}), p \in R^+$ and $\gamma \in \mathbb{R}$. If*

$$y(t) \leq \gamma + \int_{t_0}^{t} y(s)p(s)\Delta s, \; \forall t \in \mathbb{T},$$

then

$$y(t) \leq \gamma e_p(t, t_0), \forall t \in \mathbb{T}.$$

In the following, we give some results on stochastic process on time scale.

Definition 8. [36] A function $X(\cdot) : \Omega \to \mathbb{R}$ is called a random variable if X is a measurable function from (Ω, F) into $(\mathbb{R}, B)$.

Definition 9. [36] A time scale stochastic process is a function $X(\cdot,\cdot) : \mathbb{T} \times \Omega \to \mathbb{R}$, such that $X(t,\cdot) \to \mathbb{R}$ is a random variable for each $t \in \mathbb{T}$. We briefly denote $X(t)$ for $X(t,\omega), t \in \mathbb{T}$.

Definition 10. [36] A time scale stochastic process $X(\cdot,\cdot)$ is said to be regulated (rd-continuous, continuous) if there exists $\Omega_0 \subset \Omega$ with $P(\Omega_0) = 1$ and such that the trajectory $t \mapsto X(t,\omega)$ is a regulated (rd-continuous, continuous) function on $\mathbb{T}$ for each $\omega \in \Omega_0$.

Definition 11. [36] A time scale stochastic process $X(t), t \in \mathbb{T}$ is said to be stochastically bounded if $\lim_{N\to\infty} \sup_{t\in\mathbb{T}} P(|X(t)| > N) = 0$.

Lemma 5. *[25] If $E(\int_a^b f^2(t)\Delta t) < \infty$. Then $E(\int_a^b f(t)\Delta\omega(t)) = 0$ and the Itô-isometry holds*

$$E\left(\left(\int_a^b f(t)\Delta\omega(t)\right)^2\right) = E\left(\int_a^b f^2(t)\Delta t\right).$$

Definition 12. A function $x(t) = (x_1(t), x_2(t), \cdots, x_n(t))^T$ defined on $[-\tau_0, \infty)_{\mathbb{T}}$ is said to be a periodic solution of (1) with initial condition (2) if it is a solution of (1) with initial condition (2) and satisfies $x_i(t+\omega) = x_i(t), i = 1, 2, \cdots, n$.

Definition 13. The periodic solution $x(t, t_0, \varphi)$ with initial value φ of (1) is said to be exponential stable, if there exists a positive constant λ and $M > 1$ such that for any solution $y(t, t_o, \phi)$ with initial value ϕ of (1) satisfies

$$\|x(t) - y(t)\| \leq M\|\varphi - \phi\| e_{\theta\lambda}(t, t_0),\ t \in \mathbb{T},\ t \geq t_0.$$

Lemma 6. *[28] Suppose x and y are two states of system (1), then we have*

$$\left|\bigwedge_{j=1}^n \alpha_{ij}(t)g_j(x) - \bigwedge_{j=1}^n \alpha_{ij}(t)g_j(y)\right| \leq \sum_{j=1}^n |\alpha_{ij}(t)||g_j(x) - g_j(y)|,$$

and

$$\left|\bigvee_{j=1}^n \beta_{ij}(t)g_j(x) - \bigvee_{j=1}^n \beta_{ij}(t)g_j(y)\right| \leq \sum_{j=1}^n |\beta_{ij}(t)||g_j(x) - g_j(y)|.$$

Lemma 7. *[37] For any $x \in R_+^n$ and $p > 0$,*

$$|x|^p \leq n^{(p/2-1)\vee 0} \sum_{i=1}^n x_i^p, \quad \left(\sum_{i=1}^n x_i\right)^p \leq n^{(p-1)\vee 0} \sum_{i=1}^n x_i^p.$$

3. EXISTENCE OF PERIODIC SOLUTION

In this section, we shall prove some sufficient conditions for the existence of periodic solution of (1). Firstly, we give the following theorem.

Theorem 14. *Let assumption $(A1)-(A2)$ hold. Then $x(t) = (x_1(t), x_2(t), \cdots, x_n(t))^T$ is an ω-periodic solution of system (1.1) if and only if $x(t) = (x_1(t), x_2(t), \cdots, x_n(t))^T$ is an ω-periodic solution of the following system*

$$x_i(t) = \int_t^{t+\omega} H_i(t,s)\left[\sum_{j=1}^n c_{ij}(s)f_j(x_j(s)) + \bigwedge_{j=1}^n \alpha_{ij}(t)g_j(x_j(s - \tau_{ij}(s)))\right.$$

$$+\bigvee_{j=1}^{n}\beta_{ij}(t)g_j(x_j(s-\tau_{ij}(s)))+I_i(s)\Bigg]\Delta s$$
$$+\int_t^{t+\omega}H_i(t,s)\sum_{j=1}^{n}\delta_{ij}(x_j(s))\Delta\omega_j(s) \tag{3}$$

where

$$H_i(t,s)=\frac{e_{-a_i}(t+\omega,\sigma(s))}{1-e_{-a_i}(t+\omega,t)}, i=1,2,\cdots,n.$$

Proof. Let $x(t)=(x_1(t),x_2(t),\cdots,x_n(t))^T$ be an ω-periodic solution of system (1). Multiplying both sides of (1) by $e_{-a_i}(\theta,\sigma(t))$, we obtain that

$$\begin{aligned}&\Delta[x_i(t)e_{-a_i}(\theta,\sigma(t))]\\ =\ &\Bigg[\sum_{j=1}^{n}c_{ij}(t)f_j(x_j(t))+\bigwedge_{j=1}^{n}\alpha_{ij}(t)g_j(x_j(t-\tau_{ij}(t)))\\ &+\bigvee_{j=1}^{n}\beta_{ij}(t)g_j(x_j(t-\tau_{ij}(t)))+I_i(t)\Bigg]e_{-a_i}(\theta,\sigma(t))\Delta t\\ &+\sum_{j=1}^{n}\delta_{ij}(x_j(t))e_{-a_i}(\theta,\sigma(t))\Delta\omega_j(t), i=1,2,\cdots,n.\end{aligned} \tag{4}$$

Integrating both sides of (4) from t to $t+\omega$, we have that

$$\begin{aligned}x_i(t+\omega) &= e_{-a_i}(t+\omega,t)x_i(t)\\ &\quad+\int_t^{t+\omega}e_{-a_i}(t+\omega,\sigma(s))\Bigg[\sum_{j=1}^{n}c_{ij}(s)f_j(x_j(s))\\ &\quad+\bigwedge_{j=1}^{n}\alpha_{ij}(s)g_j(x_j(s-\tau_{ij}(s)))\\ &\quad+\bigvee_{j=1}^{n}\beta_{ij}(s)g_j(x_j(s-\tau_{ij}(s)))+I_i(s)\Bigg]\Delta s\\ &\quad+\int_t^{t+\omega}e_{-a_i}(t+\omega,\sigma(s))\sum_{j=1}^{n}\delta_{ij}(x_j(s))\Delta\omega_j(s)\end{aligned}$$

Since $x_i(t+\omega)=x_i(t)$, it follows that $x_i(t)$ satisfies (3). On the other hand, if $(x_1(t),x_2(t),\cdots,x_n(t))^T$ is an ω-periodic solution of (3). Then we have

$$x_i^{\Delta}(t) = H_i(\sigma(t),t+\omega)\Bigg[\sum_{j=1}^{n}c_{ij}(t+\omega)f_j(x_j(t+\omega))$$

$$
\begin{aligned}
& + \bigwedge_{j=1}^{n} \alpha_{ij}(t+\omega) g_j(x_j(t+\omega-\tau_{ij}(t+\omega))) \\
& \left. + \bigvee_{j=1}^{n} \beta_{ij}(t+\omega) g_j(x_j(t+\omega-\tau_{ij}(t+\omega))) + I_i(t+\omega)\right] \\
& + H_i(t,t+\omega) \sum_{j=1}^{n} \delta_{ij}(x_i(t+\omega))\omega_j(t+\omega) \\
& - H_i(\sigma(t),t) \left[\sum_{j=1}^{n} c_{ij}(t) f_j(x_j(t)) + I_i(t) \right. \\
& \left. + \bigwedge_{j=1}^{n} \alpha_{ij}(t) g_j(x_j(t-\tau_{ij}(t))) + \bigvee_{j=1}^{n} \beta_{ij}(t) g_j(x_j(t-\tau_{ij}(t))) \right] \\
& + H_i(t,t) \sum_{j=1}^{n} \delta_{ij}(x_j(t))\omega_j(t) - a_i(t)x_i(t) \\
= & -a_i(t)x_i(t) + \sum_{j=1}^{n} c_{ij}(t) f_j(x_j(t)) + \bigwedge_{j=1}^{n} \alpha_{ij}(t) g_j(x_j(t-\tau_{ij}(t))) \\
& + \bigvee_{j=1}^{n} \beta_{ij}(t) g_j(x_j(t-\tau_{ij}(t))) + I_i(t) + \sum_{j=1}^{n} \delta_{ij}(x_j(t))\omega_j(t)
\end{aligned}
$$

That is

$$
\begin{aligned}
& \Delta x_i(t) \\
= & \left[-a_i(t)x_i(t) + \sum_{j=1}^{n} c_{ij}(t) f_j(x_j(t)) + \bigwedge_{j=1}^{n} \alpha_{ij}(t) g_j(x_j(t-\tau_{ij}(t))) \right. \\
& \left. + \bigvee_{j=1}^{n} \beta_{ij}(t) g_j(x_j(t-\tau_{ij}(t))) + I_i(t) \right] \Delta t + \sum_{j=1}^{n} \delta_{ij}(x_j(t)) \Delta\omega_j(t)
\end{aligned}
$$

Hence $(x_1(t), x_2(t), \cdots, x_n(t))^T$ is also an ω periodic solution of (1). This completes the proof of Theorem 14. □

Theorem 15. *Let* $(A1)-(A2)$ *hold. Suppose that the following assumption hold*

$$
\begin{aligned}
(A3) \quad \Gamma \;:=\; & \max_{1\le i\le n} \left\{ 4D_i^2 \left(\omega^2 \left[\left(\sum_{j=1}^{n} \overline{c}_{ij} l_j \right)^2 + \left(\sum_{j=1}^{n} \overline{\alpha}_{ij} \nu_j \right)^2 \right. \right. \right. \\
& \left. \left. \left. + \left(\sum_{j=1}^{n} \overline{\beta}_{ij} \nu_j \right)^2 \right] + \omega \left(\sum_{j=1}^{n} \kappa_{ij} \right)^2 \right) \right\} < 1. \qquad (5)
\end{aligned}
$$

where

$$D_i = \max\{|H_i(t,s)| : t, s \in [0,\omega]_{\mathbb{T}}, t \le s\}.$$

Then (1.1) has a unique ω *periodic solution.*

Proof. By virtue of Theorem 14, the existence of ω periodic solution of (1) is derived.

Now, set $\mathbb{X} = \{x = (x_1, x_2, \cdots, x_n)^T \in BC^b_{F_0}(\mathbb{T}, \mathbb{R}^n | x_i(t+\omega) = x_i(t)\}$. Then $\mathbb{X}$ is a Banach space with a norm

$$\|x\| = \max_{1\le i\le n} \sup_{t\in[0,\omega]_{\mathbb{T}}} E(|x_i(t)|^2).$$

$E(\cdot)$ denotes the correspond expectation operator.

Define an operator

$$\Psi : \mathbb{X} \to \mathbb{X}; x = (x_1, x_2, \cdots, x_n)^T \to \Psi x = ((\Psi x)_1, (\Psi x)_2, \cdots, (\Psi x)_n)$$

where

$$\begin{aligned}(\Psi x)_i(t) &= \int_t^{t+\omega} H_i(t,s)\left[\sum_{j=1}^n c_{ij}(s) f_j(x_j(s))\right.\\ &\quad + \bigwedge_{j=1}^n \alpha_{ij}(s) g_j(x_j(s-\tau_{ij}(s)))\\ &\quad \left. + \bigvee_{j=1}^n \beta_{ij}(s) g_j(x_j(s-\tau_{ij}(s))) + I_i(s)\right]\Delta s\\ &\quad + \int_t^{t+\omega} H_i(t,s)\sum_{j=1}^n \delta_{ij}(x_j(s))\Delta w_j(s)\end{aligned}$$

It is clear that $H_i(t+\omega, s+\omega) = H_i(t,s)$ for all $(t,s) \in \mathbb{T}\times\mathbb{T}$. Therefore, it is easy to get $(\Psi x)(t+\omega) = (\Psi x)(t)$, namely, $\Psi x \in \mathbb{X}$.

Next we show that Ψ is a contract mapping. For any $x = (x_1, x_2, \cdot, x_n)^T, x^* = (x_1^*, x_2^*, \cdot, x_n^*)^T \in \mathbb{X}$, we have

$$\begin{aligned}(\Psi x &- \Psi x^*)_i(t)\\ &= \int_t^{t+\omega} H_i(t,s)\sum_{j=1}^n c_{ij}(s)[f_j(x_j(s)) - f_j(x_j^*(s))]\Delta s\\ &\quad + \int_t^{t+\omega} H_i(t,s)\left[\bigwedge_{j=1}^n \alpha_{ij}(s) g_j(x_j(s-\tau_{ij}(s)))\right.\\ &\quad \left. - \bigwedge_{j=1}^n \alpha_{ij}(s) g_j(x_j^*(s-\tau_{ij}(s)))\right]\Delta s\end{aligned}$$

$$+\int_t^{t+\omega} H_i(t,s)\left[\bigvee_{j=1}^n \beta_{ij}(s)g_j(x_j(s-\tau_{ij}(s)))\right.$$
$$\left.-\bigvee_{j=1}^n \beta_{ij}(s)g_j(x_j^*(s-\tau_{ij}(s)))\right]\Delta s$$
$$+\int_t^{t+\omega} H_i(t,s)\sum_{j=1}^n[\delta_{ij}(x_j(s))-\delta_{ij}(x_j^*(s))]\Delta w_j(s)$$

Let

$$G_{1i}=\int_t^{t+\omega} H_i(t,s)\sum_{j=1}^n c_{ij}(s)[f_j(x_j(s))-f_j(x_j^*(s))]\Delta s,$$

$$\begin{aligned}G_{2i} &= \int_t^{t+\omega} H_i(t,s)\left[\bigwedge_{j=1}^n \alpha_{ij}(s)g_j(x_j(s-\tau_{ij}(s)))\right.\\ &\qquad \left.-\bigwedge_{j=1}^n \alpha_{ij}(s)g_j(x_j^*(s-\tau_{ij}(s)))\right]\Delta s,\end{aligned}$$

$$\begin{aligned}G_{3i} &= \int_t^{t+\omega} H_i(t,s)\left[\bigvee_{j=1}^n \beta_{ij}(s)g_j(x_j(s-\tau_{ij}(s)))\right.\\ &\qquad \left.-\bigvee_{j=1}^n \beta_{ij}(s)g_j(x_j^*(s-\tau_{ij}(s)))\right]\Delta s,\end{aligned}$$

$$G_{4i}=\int_t^{t+\omega} H_i(t,s)\sum_{j=1}^n[\delta_{ij}(x_j(s))-\delta_{ij}(x_j^*(s))]\Delta w_j(s).$$

Take expectations, by Lemma 7, we have that

$$E(|(\Psi x-\Psi x^*)_i(t)|^2)\le 4(E(|G_{1i}|^2)+E(|G_{2i}|^2)+E(|G_{3i}|^2)+E(|G_{4i}|^2)). \tag{6}$$

We calculate the first term on the right side of (6) as follows

$$\begin{aligned}E(|G_{1i}|^2) &= E\left(\left|\int_t^{t+\omega} H_i(t,s)\sum_{j=1}^n c_{ij}(s)[f_j(x_j(s))-f_j(x_j^*(s))]\Delta s\right|^2\right)\\ &\le D_i^2 E\left(\sum_{j=1}^n \overline{c}_{ij}l_j\int_t^{t+\omega}|x_j(s)-x_j^*(s)|\Delta s\right)^2\\ &\le D_i^2\omega^2\left(\sum_{j=1}^n \overline{c}_{ij}l_j\right)^2\|x-x^*\|^2\end{aligned}$$

For the second term on the right side of (6), we have

$$\begin{aligned} E(|G_{2i}|^2) &= E\left(\left|\int_t^{t+\omega} H_i(t,s)\left[\bigwedge_{j=1}^n \alpha_{ij}(s)g_j(x_j(s-\tau_{ij}(s)))\right.\right.\right.\\ &\quad \left.\left.\left.-\bigwedge_{j=1}^n \alpha_{ij}(s)g_j(x_j^*(s-\tau_{ij}(s)))\right]\Delta s\right|^2\right)\\ &\le D_i^2\omega^2\left(\sum_{j=1}^n \overline{\alpha}_{ij}\nu_j\right)^2 \|x-x^*\|^2 \end{aligned}$$

Similarly, we have

$$\begin{aligned} E(|G_{3i}|^2) &= E\left(\left|\int_t^{t+\omega} H_i(t,s)\left[\bigvee_{j=1}^n \beta_{ij}(s)g_j(x_j(s-\tau_{ij}(s)))\right.\right.\right.\\ &\quad \left.\left.\left.-\bigvee_{j=1}^n \beta_{ij}(s)g_j(x_j^*(s-\tau_{ij}(s)))\right]\Delta s\right|^2\right)\\ &\le D_i^2\omega^2\left(\sum_{j=1}^n \overline{\beta}_{ij}\nu_j\right)^2 \|x-x^*\|^2 \end{aligned}$$

For the last term on the right side of (6), using Ito isometry identity, we obtain

$$\begin{aligned} &E(|G_{4i}|^2)\\ =\ & E\left(\left|\int_t^{t+\omega} H_i(t,s)\sum_{j=1}^n[\delta_{ij}(x_j(s))-\delta_{ij}(x_j^*(s))]\Delta w_j(s)\right|^2\right)\\ =\ & E\left(\int_t^{t+\omega}\left|H_i(t,s)\sum_{j=1}^n[\delta_{ij}(x_j(s))-\delta_{ij}(x_j^*(s))]\right|^2\Delta s\right)\\ \le\ & D_i^2 E\left(\left(\sum_{j=1}^n \kappa_{ij}\right)^2\int_t^{t+\omega}|x_j(s)-x_j^*(s)|^2\Delta s\right)\\ \le\ & D_i^2\omega\left(\sum_{j=1}^n \kappa_{ij}\right)^2 \|x-x^*\|^2 \end{aligned}$$

Therefore, we have

$$E(|(\Psi x-\Psi x^*)_i(t)|^2)$$

$$\leq \; 4D_i^2\left(\omega^2\left[\left(\sum_{j=1}^{n}\overline{c}_{ij}l_j\right)^2+\left(\sum_{j=1}^{n}\overline{\alpha}_{ij}\nu_j\right)^2+\left(\sum_{j=1}^{n}\overline{\beta}_{ij}\nu_j\right)^2\right] +\omega\left(\sum_{j=1}^{n}\kappa_{ij}\right)^2\right)\|x-x^*\|^2$$

It follows that

$$\|\Psi x-\Psi x^*\|\leq \Gamma\|x-x^*\|$$

It is clear that Ψ is a a contraction mapping by assumption (A3). Hence, by the contraction mapping principle, Ψ has a unique fixed point, which implies that (1) has a unique ω-periodic solution. This completes the proof of Theorem 15. □

Remark 3.1 To the best of our knowledge, few papers deal with the existence of periodic solutions to stochastic fuzzy neural networks are done by using fixed point theorems. In [38], authors studied the existence of periodic solutions to stochastic Hopefield neneural networks with time-varying delays. Our results generalized stochastic fuzzy cellular neural networks which complete results existed.

4. GLOBAL EXPONENTIAL STABILITY OF PERIODIC SOLUTIONS

Suppose that $x^*(t)=(x_1^*(t),x_2^*(t),\cdots,x_n^*(t))^T$ is an ω-periodic solution of system (1) with the initial conditions $x_i(s)=\varphi_i^*(s), s\in(-\infty,0]_{\mathbb{T}}, i=1,2,\cdots,n$. We will prove the global exponential stability of this periodic solution.

Theorem 16. *Assume that all conditions of Theorem 15 are satisfied, suppose further that* $\overline{\mu}\underline{a}_i\neq 1$,*where* $\overline{\mu}=\sup_{t\in\mathbb{T}}\mu(t), -a_i\in R$ *and*
(A4) $\gamma<0$ *with* $\gamma\in R^+$, *where* $\gamma=\max_{1\leq i\leq n}\{-\underline{a}_i\oplus\gamma_i\}$

$$\gamma_i=5/(\overline{\mu}\underline{a}_i)\left[(\sum_{j=1}^{n}\overline{c}_{ij}l_j)^2+(\sum_{j=1}^{n}\overline{\alpha}_{ij}\nu_j)^2+(\sum_{j=1}^{n}\overline{\beta}_{ij}\nu_j)^2+(\sum_{j=1}^{n}\kappa_{ij})^2\right].$$

Then the ω*-periodic solution of (1) is globally exponentially stable.*

Proof. According Theorem 15, we know that system (1) has an ω-periodic solution $x^*(t)=(x_1^*(t),x_2^*(t),\cdots,x_n^*(t))^T$ with the initial value $\varphi^*(s)=(\varphi_1^*(s),\varphi_2^*(s),\cdots,\varphi_n^*(s))^T$.

Let $x(t)=(x_1(t),x_2(t),\cdots,x_n(t))^T$ be an arbitrary solution of system (1) with initial value $\varphi(s)=(\varphi_1(s),\varphi_2(s),\cdots,\varphi_n(s))^T$. Set $y(t)=x(t)-x^*(t)$, Multiplying both sides of (1) by $e_{-a_i}(t,\sigma(s))$ and integrating on $[t_0,t]_{\mathbb{T}}$, where $t_0 \in [\tau_0,0]_{\mathbb{T}}$, then we have that

$$\begin{aligned}
&y_i(t) = e_{-a_i(t,t_0)}y_i(t_0)\\
&+\int_{t_0}^{t} e_{-a_i}(t,\sigma(s))\sum_{j=1}^{n} c_{ij}(s)[f_j(x_j(s)) - f_j(x_j*(s))]\Delta s\\
&+\int_{t_0}^{t} e_{-a_i}(t,\sigma(s))\left[\bigwedge_{j=1}^{n}\alpha_{ij}(s)g_j(x_j(s-\tau_{ij}(s)))\right.\\
&\left.-\bigwedge_{j=1}^{n}\alpha_{ij}(s)g_j(x_j^*(s-\tau_{ij}(s)))\right]\Delta s\\
&+\int_{t_0}^{t} e_{-a_i}(t,\sigma(s))\left[\bigvee_{j=1}^{n}\beta_{ij}(s)g_j(x_j(s-\tau_{ij}(s)))\right.\\
&\left.-\bigvee_{j=1}^{n}\beta_{ij}(s)g_j(x_j^*(s-\tau_{ij}(s)))\right]\Delta s\\
&+\int_{t_0}^{t} e_{-a_i}(t,\sigma(s))\sum_{j=1}^{n}[\delta_{ij}(x_j(s)) - \delta_{ij}(x_j^*(s))]\Delta w_j(s) \qquad (7)
\end{aligned}$$

The initial condition of (7) is $\phi_i(s) = \varphi_i(s) - \varphi_i^*(s), i = 1,2,\cdots,n$. Let

$$K_{1i} = e_{-a_i(t,t_0)}y_i(t_0),$$

$$K_{2i} = \int_{t_0}^{t} e_{-a_i}(t,\sigma(s))\sum_{j=1}^{n} c_{ij}(s)[f_j(x_j(s)) - f_j(x_j*(s))]\Delta s,$$

$$\begin{aligned}
K_{3i} \quad = \quad &\int_{t_0}^{t} e_{-a_i}(t,\sigma(s))\left[\bigwedge_{j=1}^{n}\alpha_{ij}(s)g_j(x_j(s-\tau_{ij}(s)))\right.\\
&\left.-\bigwedge_{j=1}^{n}\alpha_{ij}(s)g_j(x_j^*(s-\tau_{ij}(s)))\right]\Delta s,
\end{aligned}$$

$$\begin{aligned}
K_{4i} \quad = \quad &\int_{t_0}^{t} e_{-a_i}(t,\sigma(s))\left[\bigvee_{j=1}^{n}\beta_{ij}(s)g_j(x_j(s-\tau_{ij}(s)))\right.\\
&\left.-\bigvee_{j=1}^{n}\beta_{ij}(s)g_j(x_j^*(s-\tau_{ij}(s)))\right]\Delta s,
\end{aligned}$$

$$K_{5i} = \int_{t_0}^{t} e_{-a_i}(t,\sigma(s)) \sum_{j=1}^{n} [\delta_{ij}(x_j(s)) - \delta_{ij}(x_j^*(s))] \Delta w_j(s).$$

Take expectations, by Lemma 2.7, for $i = 1, 2, \cdots, n$, we have

$$\begin{aligned} E(|y_i(t)|^2) \quad \leq \quad & 5(E(|K_{1i}|^2) + E(|K_{2i}|^2) + E(|K_{3i}|^2) + E(|K_{4i}|^2) \\ & + E(|K_{5i}|^2)). \end{aligned} \tag{8}$$

Calculating the first term on the right side of (8), we have that, for $i = 1, 2, \cdots, n$.

$$E(|K_{1i}|^2) = E(|e_{-a_i(t,t_0)} y_i(t_0)|^2) \leq e_{-\underline{a}_i}(t,t_0) E(|y_i(t_0)|^2).$$

Calculating the second term on the right side of (8), we have that

$$\begin{aligned} & E(|K_{2i}|^2) \\ = \quad & E\left(\left| \int_{t_0}^{t} e_{-a_i}(t,\sigma(s)) \sum_{j=1}^{n} c_{ij}(s)[f_j(x_j(s)) - f_j(x_j * (s))] \Delta s \right|^2 \right) \\ \leq \quad & \left(\sum_{j=1}^{n} \overline{c}_{ij} l_j \right)^2 \int_{t_0}^{t} e_{-\underline{a}_i}(t,\sigma(s)) E(|y_j(s)|^2) \Delta s \end{aligned}$$

Calculating the third term on the right side of (8), we have that

$$\begin{aligned} & E(|K_{3i}|^2) \\ = \quad & E\left(\left| \int_{t_0}^{t} e_{-a_i}(t,\sigma(s)) \left[\bigwedge_{j=1}^{n} \alpha_{ij}(s) g_j(x_j(s - \tau_{ij}(s))) \right.\right.\right. \\ & \left.\left.\left. - \bigwedge_{j=1}^{n} \alpha_{ij}(s) g_j(x_j^*(s - \tau_{ij}(s))) \right] \Delta s \right|^2 \right) \\ \leq \quad & \left(\sum_{j=1}^{n} \overline{\alpha}_{ij} \nu_j \right)^2 \int_{t_0}^{t} e_{-\underline{a}_i}(t,\sigma(s)) E(|y_j(s - \tau_j(s))|^2) \Delta s \end{aligned}$$

Similarly, we have that

$$E(|K_{4i}|^2) \leq \left(\sum_{j=1}^{n} \overline{\beta}_{ij} \nu_j \right)^2 \int_{t_0}^{t} e_{-\underline{a}_i}(t,\sigma(s)) E(|y_j(s - \tau_j(s))|^2) \Delta s$$

Calculating the last term on the right side of (8), applying Ito isometry identity, we have that

$$E(|K_{5i}|^2)$$

$$
\begin{aligned}
&= E\left(\left|\int_{t_0}^{t} e_{-a_i}(t,\sigma(s))\sum_{j=1}^{n}[\delta_{ij}(x_j(s))-\delta_{ij}(x_j^*(s))]\Delta w_j(s)\right|^2\right)\\
&= E\left(\left|\int_{t_0}^{t} e_{-a_i}(t,\sigma(s))\sum_{j=1}^{n}[\delta_{ij}(x_j(s))-\delta_{ij}(x_j^*(s))]^2\Delta s\right|\right)\\
&\leq \left(\sum_{j=1}^{n}\kappa_{ij}\right)^2\int_{t_0}^{t} e_{-\underline{a}_i}(t,\sigma(s))E(|y_j(s)|^2)\Delta s
\end{aligned}
$$

Thus, we obtain that

$$
\begin{aligned}
& E(|y_i(t)|^2)\\
\leq\ & 5\left[\left(\sum_{j=1}^{n}\overline{c}_{ij}l_j\right)^2+\left(\sum_{j=1}^{n}\kappa_{ij}\right)^2\right]\int_{t_0}^{t} e_{-\underline{a}_i}(t,\sigma(s))E(|y_j(s)|^2)\Delta s\\
& +5\left[\left(\sum_{j=1}^{n}\overline{\alpha}_{ij}\nu_j\right)^2+\left(\sum_{j=1}^{n}\overline{\beta}_{ij}\nu_j\right)^2\right]\\
& \times\int_{t_0}^{t} e_{-\underline{a}_i}(t,\sigma(s))E(|y_j(s-\tau_j(s))|^2)\Delta s\\
& +5e_{-\underline{a}_i}(t,t_0)E(|y_i(t_0)|^2)
\end{aligned}
\tag{9}
$$

Let $u_i(t)=E(|y_i(t)|^2)e_{\ominus(-\underline{a}_i}(t,t_0), i=1,2,\cdots,n$. It follows from (9) that

$$
\begin{aligned}
& u_i(t)\leq 5u_i(t_0)\\
+\ & 5\left[\left(\sum_{j=1}^{n}\overline{c}_{ij}l_j\right)^2+\left(\sum_{j=1}^{n}\kappa_{ij}\right)^2+\left(\sum_{j=1}^{n}\overline{\alpha}_{ij}\nu_j\right)^2+\left(\sum_{j=1}^{n}\overline{\beta}_{ij}\nu_j\right)^2\right]\\
& \times\int_{t_0}^{t}(1+\mu(s))(\ominus(-\underline{a}_i)))u_j(s)\Delta s
\end{aligned}
$$

$$
\begin{aligned}
\leq\ & 5u_i(t_0)\\
+\ & \frac{5\left[\left(\sum_{j=1}^{n}\overline{c}_{ij}l_j\right)^2+\left(\sum_{j=1}^{n}\kappa_{ij}\right)^2+\left(\sum_{j=1}^{n}\overline{\alpha}_{ij}\nu_j\right)^2+\left(\sum_{j=1}^{n}\overline{\beta}_{ij}\nu_j\right)^2\right]}{1-\overline{\mu}\underline{a}_i}\\
& \times\int_{t_0}^{t}u_j(s)\Delta s
\end{aligned}
$$

Applying Lemma 4, we have that

$$
u_i(t)\leq 5u_i(t_0)e_{\gamma_i}(t,t_0), i=1,2,\cdots,n.
$$

Hence, it can follows that

$$\|x - x^*\| \leq 5e_\gamma(t, t_0)\|\varphi - \varphi^*\|,$$

that is, the periodic solution $x^*(t)$ of (1) is exponentially stable. This completes the proof of Theorem 16. □

5. AN EXAMPLE

In this section, we give an example to illustrate the feasibility of our results obtained.

Example Consider the following stochastic fuzzy cellular neural networks with time-varying delays on time scales.

$$\begin{aligned}\Delta x_i(t) \quad = \quad & \Bigg[-a_i(t)x_i(t) + \sum_{j=1}^{2} c_{ij}(t)f_j(x_j(t)) + \bigwedge_{j=1}^{2} \alpha_{ij}(t)g_j(x_j(t-\tau_{ij}(t))) \\ & + \bigvee_{j=1}^{2} \beta_{ij}(t)g_j(x_j(t-\tau_{ij}(t))) + I_i(t)\Bigg]\Delta t \\ & + \sum_{j=1}^{2} \delta_{ij}(x_j(t))\Delta w_j(t), \end{aligned} \tag{10}$$

$t \in \mathbb{T}, t > 0, i = 1, 2,$ where $\mathbb{T} = \bigcup_{k\in Z}[\frac{1}{4}k, \frac{1}{4}(k+1)]$ is a $\frac{1}{4}$-periodic time scale, where the coefficients are as follows:

$$a_1(t) = 1.05 + 0.03\cos(8\pi t), a_2(t) = 1.06 + 0.01\sin(8\pi t),$$

$$c_{11}(t) = 0.03\cos(8\pi t), c_{12}(t) = 0.05\sin(8\pi t),$$

$$c_{21}(t) = 0.05\cos(8\pi t), c_{22}(t) = 0.04\sin(8\pi t),$$

$$f_i(u) = g_i(u) = \frac{1}{2}(|u+1| - |u-1|), i = 1, 2,$$

$$\alpha_{11}(t) = 0.02\cos(8\pi t), \alpha_{12}(t) = 0.04\sin(8\pi t),$$

$$\alpha_{21}(t) = 0.06\sin(8\pi t), \alpha_{22}(t) = 0.08\cos(8\pi t),$$

$$\beta_{11}(t) = 0.04\sin(8\pi t), \beta_{12}(t) = 0.05\cos(8\pi t),$$

$$\beta_{21}(t) = 0.07\cos(8\pi t), \beta_{22}(t) = 0.05\sin(8\pi t),$$

$$\delta_{11}(t) = 0.01\sin(8\pi t), \delta_{12}(t) = 0.03\sin(8\pi t),$$

$$\delta_{21}(t) = 0.05\sin(8\pi t), \delta_{22}(t) = 0.07\sin(8\pi t),$$

$$I_1(t) = I_2(t) = 0.6\cos(8\pi t), \tau_{ij}(t) = \sin(8\pi t), i, j = 1, 2.$$

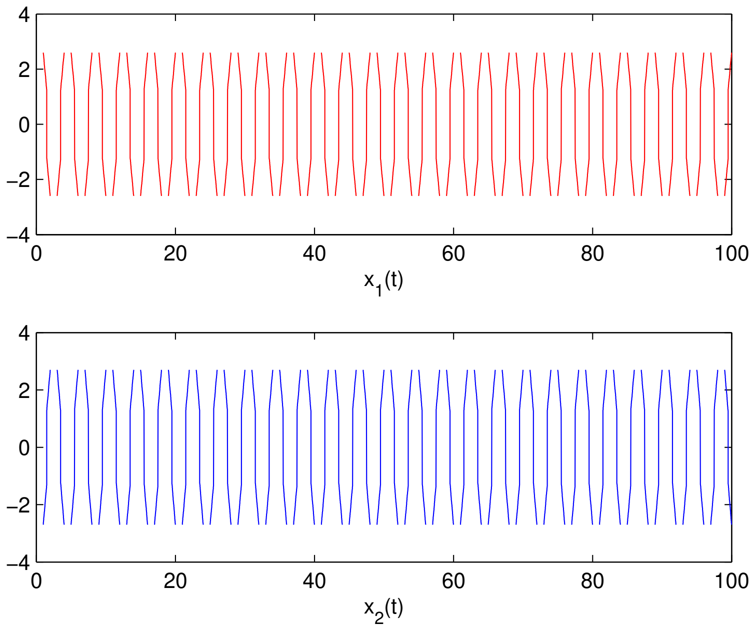

Figure 1: Numerical solution $x(t) = (x_1(t), x_2(t))^T$ of systems (10) for initial value $\varphi(s) = (2.6, -2.7)^T$,$s \in [-1, 0]$.

We have

$$\mu(t) = \begin{cases} 0, & t \in \bigcup_{k\in Z}[\frac{1}{4}k, \frac{1}{4}(k+1)) \\ \frac{1}{8}, & t \in \{\frac{1}{4}(k+1)\} \end{cases}$$

By calculating, we have

$$\omega = \frac{1}{4}, \underline{a}_1 = 1.02, \underline{a}_2 = 1.05, \overline{c}_{11} = 0.03, \overline{c}_{12} = 0.05, \overline{c}_{21} = 0.05, \overline{c}_{22} = 0.04,$$

$$\overline{\alpha}_{11} = 0.02, \overline{\alpha}_{12} = 0.04, \overline{\alpha}_{21} = 0.06, \overline{\alpha}_{22} = 0.08, l_j = \nu_j = 1, j = 1, 2,$$

$$\overline{\beta}_{11} = 0.04, \overline{\beta}_{12} = 0.05, \overline{\beta}_{21} = 0.07, \overline{\beta}_{22} = 0.05, \kappa_{11} = 0.01,$$

$$\kappa_{12} = 0.03, \kappa_{21} = 0.05, \kappa_{22} = 0.07, \Gamma \approx 0.02695 < 1, \gamma \approx -0.7375 < 0.$$

Therefore, all conditions of Theorem 15 and Theorem 16 are satisfied. Hence (10) has a $\frac{1}{4}$-periodic solution which is exponentially stable. (see Figure 1).

6. CONCLUSION

In this paper, using the time scale calculus theory, we have studied the existence, globally exponential stability of the periodic solution for stochastic fuzzy cellular neural networks with time-varying delays on time scales. Some sufficient conditions set up here are easily verified and these conditions are correlated with parameters and time delays of the system (1). To the best of our knowledge,the results presented here have been not appeared in the related literature. The method used in this paper can be applied to to some other stochastic fuzzy neural networks such as the stochastic fuzzy Cohen- Grossberg neural networks and stochastic fuzzy BAM neural networks.

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