

ON SOME NONSTANDARD SOFTWARE RELIABILITY MODELS

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ABSTRACT: The Hausdorff approximation of the shifted Heaviside function $h_{t_0}(t)$ by sigmoidal functions based on the Song–Chang–Pham [2] cumulative functions is investigated and an expression for the error of the best approximation is obtained in this paper. The results of numerical examples confirm theoretical conclusions and they are obtained using programming environment Mathematica.

We give also real-world examples with data provided in [4] using Song–Chang–Pham [2] software reliability model. Dataset included [5] Year 2000 compatibility modifications, operating system upgrade, and signaling message processing.

AMS Subject Classification: 41A46, 68N30

Key Words: three–parameter Song–Chang–Pham fault–detection rate model, five–parameter Song–Chang–Pham fault–detection rate model, shifted Heaviside function $h_{t_0}(t)$, Hausdorff approximation, confidence intervals, upper and lower bounds

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1. INTRODUCTION

There are two main approaches to testing: structured and functional. Very good description of all elements in the area of debugging theory may be found in the following books [7]–[9]. For some degradation models with applications to reliability and survival analysis, see [10]. Practical treatment of similar topics is given in [11], [37]–[68]. In the book [11], we pay particular attention to both deterministic approaches

and probability models for debugging theories. A Hausdorff metric was chosen to evaluate the test data which are fitted to the sigmoid models proposed in this book. Some software reliability models, can be found in [12]–[36]. In this article we study the Hausdorff approximation of the shifted Heaviside function $h_{t_0}(t)$ by sigmoidal functions based on the Song–Chang–Pham [2] cumulative functions.

Definition 1. Chang, Pham, Lee and Song [1] developed the following software reliability model incorporating the uncertainty of the system fault detection rate per unit of time subject to the operating environment:

$$M(t) = N \left(1 - \left(\frac{\beta}{\beta + (at)^b} \right)^\alpha \right). \quad (1)$$

Definition 2. Song, Chang and Pham [2] developed the following new software reliability model with consideration of a three-parameter fault-detection rate in the software development process:

$$M_1(t) = N \left(1 - \frac{\beta}{\beta - \frac{a}{b} \ln \left(\frac{(1+c)e^{-bt}}{1+ce^{-bt}} \right)} \right). \quad (2)$$

Definition 3. The shifted Heaviside function is defined as:

$$h_{t_0}(t) = \begin{cases} 0, & \text{if } t < t_0, \\ [0, 1], & \text{if } t = t_0 \\ 1, & \text{if } t > t_0 \end{cases} \quad (3)$$

The confidence intervals of the models $M(t)$ and $M_1(t)$ are visualized on Fig.1 (see, [2]). We will note that the determination of compulsory in area of the Software Reliability Theory components, such as confidence intervals and confidence bounds, should also be accompanied by a serious analysis of the value of the best Hausdorff approximation of the function $h_{t_0}(t)$ by cumulative functions of type (1)–(2) - the subject of study in the present paper.

Definition 4. [3] The Hausdorff distance (the H-distance) $\rho(f, g)$ between two interval functions f, g on $\Omega \subseteq \mathbb{R}$, is the distance between their completed graphs $F(f)$ and $F(g)$ considered as closed subsets of $\Omega \times \mathbb{R}$. More precisely,

$$\rho(f, g) = \max \left\{ \sup_{A \in F(f)} \inf_{B \in F(g)} \|A - B\|, \sup_{B \in F(g)} \inf_{A \in F(f)} \|A - B\| \right\},$$

wherein $\|\cdot\|$ is any norm in \mathbb{R}^2 , e. g. the maximum norm $\|(t, x)\| = \max\{|t|, |x|\}$; hence the distance between the points $A = (t_A, x_A)$, $B = (t_B, x_B)$ in \mathbb{R}^2 is $\|A - B\| = \max(|t_A - t_B|, |x_A - x_B|)$.

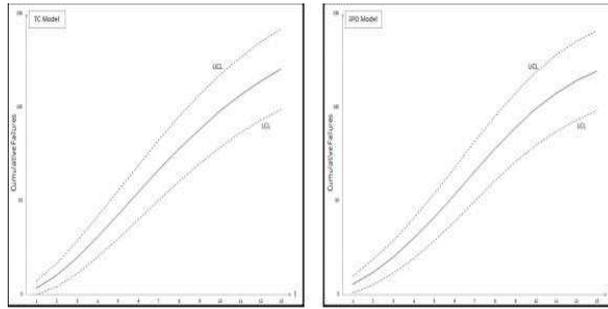


Figure 1: The confidence intervals of the models (1) and (2) [2].

2. MAIN RESULTS

2.1. A NOTE ON THE "TESTING COVERAGE MODEL" (1) [1]

Without losing of generality we will look at the following "cumulative sigmoid":

$$M^*(t) = 1 - \left(\frac{\beta}{\beta + (at)^b} \right)^\alpha, \tag{4}$$

with $N = 1, b = \beta$ (see (1)), and

$$t_0 = \frac{1}{a} \left(\beta \left(\frac{1}{\left(\frac{1}{2}\right)^{\frac{1}{\alpha}}} - 1 \right) \right)^{\frac{1}{\beta}}; \quad M^*(t_0) = \frac{1}{2}. \tag{5}$$

The one-sided Hausdorff distance d between the function $h_{t_0}(t)$ and the sigmoid ((4)–(5)) satisfies the relation

$$M^*(t_0 + d) = 1 - d. \tag{6}$$

The following theorem gives upper and lower bounds for d .

Theorem 1. *Let*

$$p = -\frac{1}{2}, \quad q = 1 + a\alpha \left(\beta \left(\frac{1}{\left(\frac{1}{2}\right)^{\frac{1}{\alpha}}} - 1 \right) \right)^{\frac{\beta-1}{\beta}} \left(\frac{1}{2} \right)^{\frac{\alpha+1}{\alpha}}.$$

For the one-sided Hausdorff distance d between $h_{t_0}(t)$ and the sigmoid ((4)–(5)) the following inequalities hold for: $2.1q > e^{1.05}$

$$d_l = \frac{1}{2.1q} < d < \frac{\ln(2.1q)}{2.1q} = d_r. \tag{7}$$

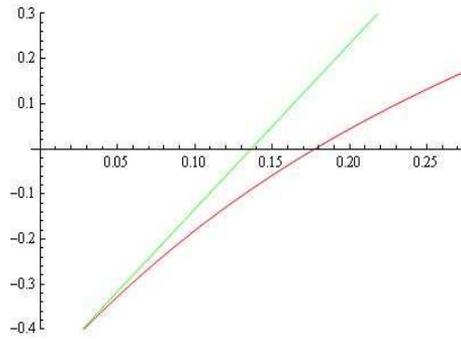


Figure 2: The functions $F(d)$ and $G(d)$.

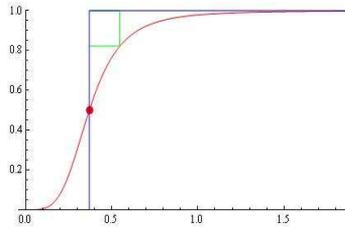


Figure 3: The model ((4)–(5)) for $\beta = 4$, $\alpha = 0.95$, $a = 3.9$, $t_0 = 0.369175$; H-distance $d = 0.177566$, $d_l = 0.129912$, $d_r = 0.265137$.

Proof. Let us examine the function:

$$F(d) = M^*(t_0 + d) - 1 + d. \tag{8}$$

From $F'(d) > 0$ we conclude that function F is increasing. Consider the function

$$G(d) = p + qd. \tag{9}$$

From Taylor expansion we obtain $G(d) - F(d) = O(d^2)$. Hence $G(d)$ approximates $F(d)$ with $d \rightarrow 0$ as $O(d^2)$ (see Fig. 2). In addition $G'(d) > 0$. Further, for $2.1q > e^{1.05}$ we have $G(d_l) < 0$ and $G(d_r) > 0$.

This completes the proof of the theorem.

The model ((4)–(5)) for $\beta = 4$, $\alpha = 0.95$, $a = 3.9$, $t_0 = 0.369175$ is visualized on Fig. 3. From the nonlinear equation (6) and inequalities (7) we have: $d = 0.177566$, $d_l = 0.129912$ and $d_r = 0.265137$. The model ((4)–(5)) for $\beta = 6$, $\alpha = 0.99$, $a = 9$, $t_0 = 0.150127$, $d = 0.0773387$, $d_l = 0.0434456$, $d_r = 0.136256$ is visualized on Fig. 4.

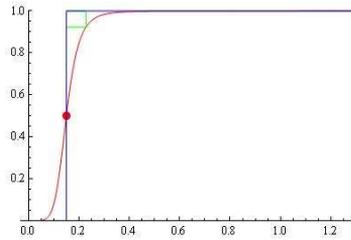


Figure 4: The model ((4)–(5)) for $\beta = 6$, $\alpha = 0.99$, $a = 9$, $t_0 = 0.150127$; H-distance $d = 0.0773387$, $d_l = 0.0434456$, $d_r = 0.136256$.

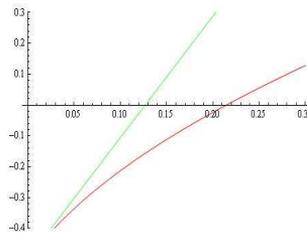


Figure 5: The functions $F_1(d_1)$ and $G_1(d_1)$.

2.2. A NOTE ON THE "DETERMINISTIC MODEL" (2) [2]

We consider the following "cumulative sigmoid":

$$M_1^*(t) = 1 - \frac{\beta}{\beta - \frac{a}{b} \ln \left(\frac{(1+c)e^{-bt}}{1+ce^{-bt}} \right)}, \tag{10}$$

with $N = 1$ (see (2)), and

$$t_0 = \frac{\beta}{a} + \frac{1}{b} \ln \left(1 + c \left(1 - e^{-\frac{\beta b}{a}} \right) \right); \quad M_1^*(t_0) = \frac{1}{2}. \tag{11}$$

The one-sided Hausdorff distance d_1 between the function $h_{t_0}(t)$ and the sigmoid ((10)–(11)) satisfies the relation

$$M_1^*(t_0 + d_1) = 1 - d_1. \tag{12}$$

The following theorem gives upper and lower bounds for d_1

Theorem 2. *Let*

$$p_1 = -\frac{1}{2}, \quad q_1 = \frac{\left(1 + c \left(1 - e^{-\frac{\beta b}{a}} \right) \right) (a + 4\beta + 4c\beta)}{4\beta(1 + c)}.$$

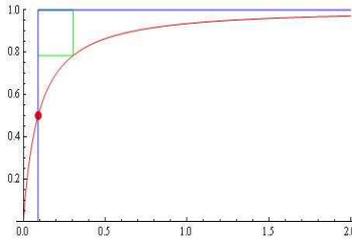


Figure 6: The model ((10)–(11)) for $\beta = 4$, $b = 1.55$, $a = 9$, $c = 1.1$, $t_0 = 0.0901371$; H-distance $d_1 = 0.214728$, $d_{l_1} = 0.120875$, $d_{r_1} = 0.25548$.

For the one-sided Hausdorff distance d_1 between h_{t_0} and the sigmoid $M_1^*(t)$ the following inequalities hold for: $2.1q_1 > e^{1.05}$

$$d_{l_1} = \frac{1}{2.1q_1} < d_1 < \frac{\ln(2.1q_1)}{2.1q_1} = d_{r_1}. \tag{13}$$

Proof. Let us examine the functions:

$$F_1(d_1) = M_1^*(t_0 + d_1) - 1 + d_1. \tag{14}$$

$$G_1(d_1) = p_1 + q_1 d_1. \tag{15}$$

From Taylor expansion we obtain $G_1(d_1) - F_1(d_1) = O(d_1^2)$. Hence $G_1(d_1)$ approximates $F_1(d_1)$ with $d_1 \rightarrow 0$ as $O(d_1^2)$ (see Fig. 5). In addition $G'(d_1) > 0$. Further, for $2.1q_1 > e^{1.05}$ we have $G_1(d_{l_1}) < 0$ and $G(d_{r_1}) > 0$.

This completes the proof of the theorem.

The model ((10)–(11)) for $\beta = 4$, $b = 1.55$, $a = 9$, $c = 1.1$, $t_0 = 0.0901371$ is visualized on Fig. 6. From the nonlinear equation (12) and inequalities (13) we have: $d_1 = 0.214728$, $d_{l_1} = 0.120875$ and $d_{r_1} = 0.25548$.

3. NUMERICAL EXAMPLE. CONCLUDING REMARKS.

The research of each new model in the field of debugging and test theory compulsory passes through the experimental phase with imposed in practice databases. One of them is the data provided in [4]. The operating time of the software is 167,900 days. 115 failures are detected for these days which contain 71 unique failures. Table 1 shows the failures data which are united for each of the 13 months. Dataset included [5] Year 2000 compatibility modifications, operating system upgrade, and signaling message processing. Below, we will illustrate the fitting of this data, for example,

with the $M_1(t)$ model, and will show the connection to discussed in this article - approximate task.

Month In- dex	System Days (Days)	System Days (Cu- mulative)	Failures	Cumulative Failures
1	961	961	7	7
2	4170	5131	3	10
3	8789	13,920	14	24
4	11,858	25,778	8	32
5	13,110	38,888	11	43
6	14,198	53,086	8	51
7	14,265	67,351	7	58
8	15,175	82,526	19	77
9	15,376	97,902	17	94
10	15,704	113,606	6	100
11	18,182	131,788	11	111
12	17,760	149,548	4	115
13	18,352	167,900	0	115

Table 1. Field failure data [4].

The fitted model

$$M_1(t) = N \left(1 - \frac{\beta}{\beta - \frac{a}{b} \ln \left(\frac{(1+c)e^{-bt}}{1+ce^{-bt}} \right)} \right)$$

based on the data of Table 1 for the estimated parameters: $N = 115$; $a = 2.07563$; $b = 0.464771$; $c = 12181.7$; $\beta = 0.00620473$ is plotted on Fig. 7.

With such found parameters N, a, b, c, β for the value t_0 we have $t_0 = 6.21147$. The value of the best Hausdorff distance between the function $h_{t_0}(t)$ and sigmoid $M_1(t)$ satisfies the following nonlinear equation: $M_1(t_0 + d) - N + d = 0$. In this case, d is the length of the side of the square shown on Fig. 7. Obviously, studying of phenomenon super saturation is mandatory element along with other important components - confidence bounds and confidence intervals (see, Fig. 1) when dealing with questions from Software Reliability Models domain.

Remark. Song, Chang and Pham [6] developed the following new software reliability model with consideration of a five-parameter fault-detection rate in the software development process:

$$M_2(t) = N \left(1 - \frac{\beta}{\beta - \frac{a}{b} \ln \left(\frac{(1+c)e^{-bpt}}{1+ce^{-bpt}} \right)} \right)^\alpha \tag{16}$$

where $a, b, c, \beta, \alpha > 0$ and $0 \leq p \leq 1$. The one-sided Hausdorff distance d_2 between

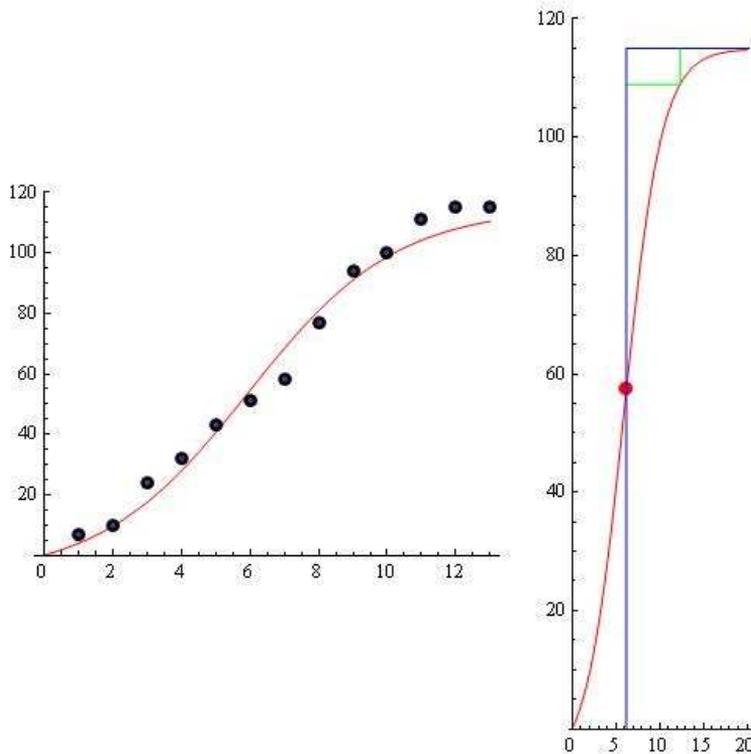


Figure 7: a.) The model $M_1(t)$ with $N = 115$; $a = 2.07563$; $b = 0.464771$; $c = 12181.7$; $\beta = 0.00620473$; b.) The Hausdorff approximation of the shifted Heaviside function $h_{t_0}(t)$ by sigmoid $M_1(t)$.

the function $h_{t_0}(t)$ and the sigmoid (16) satisfies the relation

$$M_2(t_0 + d_2) = 1 - d_2. \quad (17)$$

The model (16) for $N = 1$, $\beta = 0.1$, $b = 1.8$, $a = 9$, $c = 1.5$, $p = 0.95$, $\alpha = 0.9$, $t_0 = 0.0248855$, $d_2 = 0.132733$ is visualized on Fig. 8.

The fitted model based on the data of Table 1 for the estimated parameters: $N = 111.5$; $a = 1.57493$; $b = 0.453278$; $c = 46.7291$; $\beta = 0.310662$; $\alpha = 1.7115$; $p = 0.902566$ is plotted on Fig. 9.

In particular, the discipline "Approximation and Modelling Aspects in Debugging and Test Theory" will be made, the materials for which will be in Distributed Platform for e-Learning DisPeL [69]-[75].

Based on the methodology proposed in the present note, the reader may formulate the corresponding approximation problems on his/her own. We hope that the results will be useful for specialists in this scientific area.

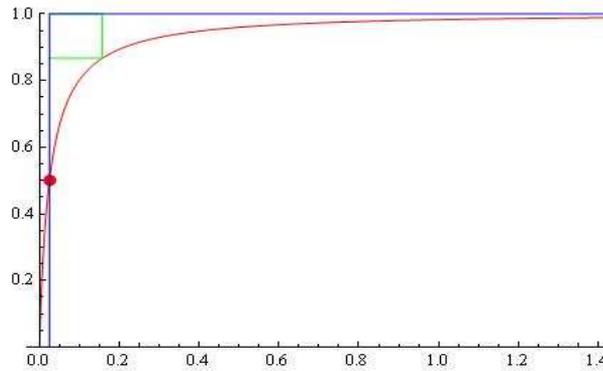


Figure 8: The model (16) for $N = 1$, $\beta = 0.1$, $b = 1.8$, $a = 9$, $c = 1.5$, $p = 0.95$, $\alpha = 0.9$, $t_0 = 0.0248855$; H-distance $d_2 = 0.132733$.

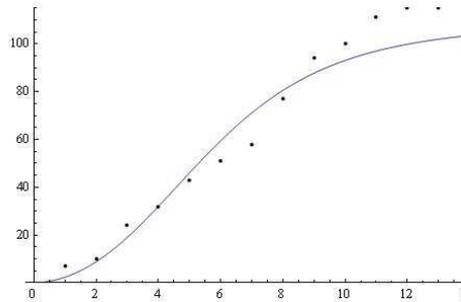


Figure 9: The fitted model $M_2(t)$ with $N = 111.5$; $a = 1.57493$; $b = 0.453278$; $c = 46.7291$; $\beta = 0.310662$; $\alpha = 1.7115$; $p = 0.902566$.

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