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A NEW CLASS OF ACTIVATION FUNCTIONS BASED ON THE CORRECTING AMENDMENTS OF GOMPERTZ–MAKEHAM TYPE

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ABSTRACT: We will explore the interesting methodological task for constructing new activation functions using ”correcting amendments” of ”Gompertz–Makeham–type” (GMAF). We also define the new family of recurrence generated activation functions based on ”Gompertz–Makeham correction” - (RGGMAF).

We prove upper and lower estimates for the Hausdorff approximation of the sign function by means of this new class of parametric activation functions - (RGGMAF). Numerical examples, illustrating our results are given.

AMS Subject Classification: 41A46

Key Words: parametric activation function based on ”amendments” of ”Gompertz Makeham type” (GMAF), recurrence generated activation functions based on ”Gompertz–Makeham correction” - (RGGMAF), sign function, Hausdorff distance, upper and lower bounds

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1. INTRODUCTION

Sigmoidal functions (also known as ”activation functions”) find multiple applications to neural networks [1]–[11].

In a series of papers, we have explored the interesting task of approximating the functions – Heaviside function $h(t)$ and $sign(t)$ with all-knowing functions such as Hyperbolic tangent, Logistic, Gompertz and others (see, for instance [12]–[14]).

The task is important in the treatment of questions related to the study of the "super saturation" - the object of the research in various fields - neural networks, nucleation theory, machine learning and others.

A survey of neural transfer activation functions can be found in [15].

The function $sign(t)$ plays an important role in the theory of impulse technic.

One consequence from Haar's theorem [18] is the assertion that for any natural number n and any number $0 < \lambda < 1$ there exists an unique polynomial

$$Q_{2n+1}(t) = \sum_{k=0}^{n} q_k t^{2k+1}$$

on the best uniform approximation of the constant 1 in the interval $[\lambda, 1]$.

The polynomials Q_{2n+1} take part in some technical problems like antenna synthesis and electrical schemes [19].

Therefore their explicit finding excites a certain interest.

It follows from the generalized Chebyshev theorem in [18] that: if $\sigma_1, \sigma_2, \dots, \sigma_n$ are internal points of maximum deviation of Q_{2n+1} in the interval $[\lambda, 1]$, then $Q'_{2n+1}(\pm\sigma_i) = 0$ for $i = 1, \dots, n$ which leads to

$$Q'_{2n+1}(t) = \frac{dQ_{2n+1}(t)}{dt} = \prod_{i=1}^{n} (t^2 - \sigma_i^2).$$

The solution is

$$Q_{2n+1}(t) = C(n) I_n(t),$$

where

$$I_n(t) = \int_0^t \prod_{i=1}^{n} (z^2 - \sigma_i^2)\, dz$$

and $C(n)$ is a constant.

In [20] a numerical method is proposed for determination of alternation points $\sigma_1, \sigma_2, \dots, \sigma_n$ whereupon the polynomial Q_{2n+1}, can be explicitly represented.

For the best Hausdorff approximation $E_{n,\alpha}$ with parameter $\alpha > 0$ of the function $sign(t)$ by algebraic polynomial pf degree $\leq n$ is valid: $E_{n,\alpha} = c(\alpha)\frac{\ln n}{n}$.

The numerical experiments [20] show that the solution is a hard problem and the question for computation of the polynomial Q_{2n+1} for large n is open (see, also [24]).

The polynomial Q_{2n+1} for $n = 8$ is visualized on Fig. 1.

For other results, see [21]–[24].

We will explore the interesting methodological task for constructing new activation functions using "correcting amendments" of "Gompertz–Makeham–type" and prove

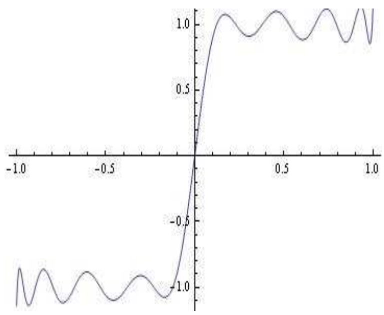

Figure 1: The polynomial Q_{2n+1} for $n = 8$.

upper and lower estimates for the Hausdorff approximation of the sign function by means of this new family of parametric activation functions.

2. PRELIMINARIES

Definition 1. The sign function of a real number t is defined as follows:

$$sgn(t) = \begin{cases} -1, & \text{if} \quad t < 0, \\ 0, & \text{if} \quad t = 0, \\ 1, & \text{if} \quad t > 0. \end{cases} \tag{1}$$

Definition 2. [16], [17] The Hausdorff distance (the H–distance) $\rho(f, g)$ between two interval functions f, g on $\Omega \subseteq \mathbb{R}$, is the distance between their completed graphs $F(f)$ and $F(g)$ considered as closed subsets of $\Omega \times \mathbb{R}$. More precisely,

$$\rho(f,g) = \max\{ \sup_{A \in F(f)} \inf_{B \in F(g)} ||A - B||, \sup_{B \in F(g)} \inf_{A \in F(f)} ||A - B||\}, \tag{2}$$

wherein $||.||$ is any norm in $\mathbb{R}^2$, e. g. the maximum norm $||(t,x)|| = \max\{|t|,|x|\}$; hence the distance between the points $A = (t_A, x_A)$, $B = (t_B, x_B)$ in $\mathbb{R}^2$ is $||A - B|| = \max(|t_A - t_B|, |x_A - x_B|)$.

Definition 3. The new parametric activation function based on "amendments" of "Gompertz–Makeham - type" - (GMAF) is defined as follows

$$\varphi_0(t) = \frac{e^{-t-c(e^{-at}-1)} - e^{-t-c(e^{at}-1)}}{e^{-t-c(e^{-at}-1)} + e^{-t-c(e^{at}-1)}}. \tag{3}$$

It is natural to define the following special class of recurrence generated activation functions

Definition 4. The new family of recurrence generated activation functions based on "Gompertz–Makeham correction" - (RGGMAF) is defined as follows

$$\varphi_i(t) = \varphi_0(t + \varphi_{i-1}(t)); \quad i = 1, 2, \dots, p, \tag{4}$$

where $\varphi_0(t)$ is defined by (3).

3. MAIN RESULTS

In this Section we prove upper and lower estimates for the Hausdorff approximation of the sign function by means of families $\varphi(t)$ and $\varphi_i(t) = \varphi_0(t+\varphi_{i-1}(t))$; $i = 1, 2, \dots, p$, where p is the number of recursions.

The H-distance $d_0(sgn(t), \varphi_0(t))$ between the sgn function and the function φ_0 satisfies the relation:

$$\varphi_0(d_0) = \frac{e^{-d_0 - c(e^{-ad_0}-1)} - e^{-d_0 - c(e^{ad_0}-1)}}{e^{-d_0 - c(e^{-ad_0}-1)} + e^{-d_0 - c(e^{ad_0}-1)}} = 1 - d_0. \tag{5}$$

The nonlinear equation (5) has unique positive root d_0.

The following Theorem gives upper and lower bounds for d_0

Theorem 3.1. *Let*

$$W_0 = 1 + ac; \; r_0 = 1.1W_0.$$

For the Hausdorff distance d_0 between the sgn function and the function φ_0 the following inequalities hold for

$$r_0 > e^{1.1} \approx 3.00417$$

$$d_{l_0} = \frac{1}{r_0} < d_0 < \frac{\ln r_0}{r_0} = d_{r_0}. \tag{6}$$

Proof. We define the functions

$$F_0(d) = \frac{e^{-d-c(e^{-ad}-1)} - e^{-d-c(e^{ad}-1)}}{e^{-d-c(e^{-ad}-1)} + e^{-d-c(e^{ad}-1)}} - 1 + d \tag{7}$$

and

$$G_0(d) = -1 + (1 + ac)d = -1 + W_0 d. \tag{8}$$

From Taylor expansion we find $F_0(d) - G_0(d) = O(d_0^3)$ (see, Fig. 2)

In addition $G_0'(d) > 0$. We look for two reals d_{l_0} and d_{r_0} such that $G_0(d_{l_0}) < 0$ and $G_0(d_{r_0}) > 0$ (leading to $G_0(d_{l_0}) < G_0(d_0) < G(d_{r_0})$ and thus $d_{l_0} < d_0 < d_{r_0}$).

Trying $d_{l_0} = \frac{1}{r_0}$ and $d_{r_0} = \frac{\ln r_0}{r_0}$ we obtain for $r_0 > e^{1.1} \approx 3.00417$

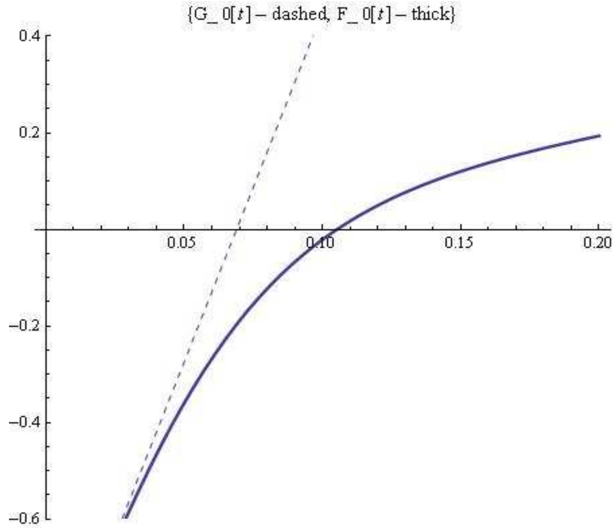

Figure 2: The functions $F_0(d)$ and $G_0(d)$ for $a = 3$; $c = 4.5$.

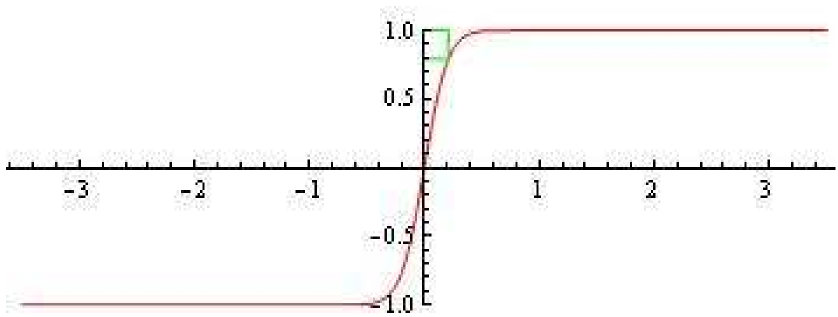

Figure 3: Approximation of the $sgn(t)$ by (GMAF) for $a = 2$; $c = 2.5$; Hausdorff distance: $d = 0.208803$.

$$G_0(d_{l_0}) < 0; \quad G_0(d_{r_0}) > 0.$$

This completes the proof of the inequalities (6).

Approximations of the $sgn(t)$ by (GMAF)–functions for various a and c are visualized on Fig. 3–Fig. 4.

From the graphics it can be seen that the "saturation" is faster.

Some computational examples using relations (6) are presented in Table 1. The last column of Table 1 contains the values of d_0 computed by solving the nonlinear

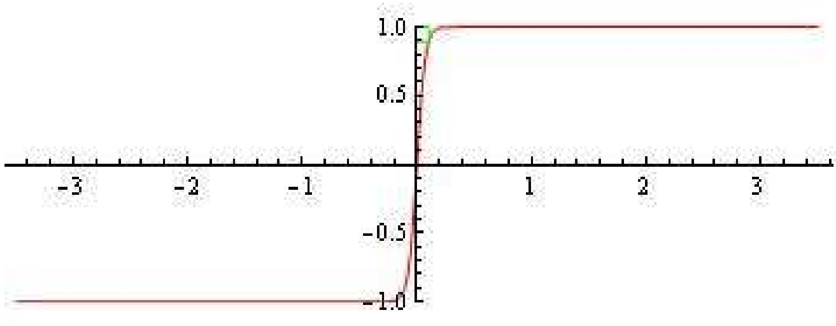

Figure 4: Approximation of the $sgn(t)$ by (GMAF) for $a = 3$; $c = 4.5$; Hausdorff distance: $d = 0.105284$.

equation (5).

a	c	d_{l_0}	d_{r_0}	d_0 from (5)
2	2.5	0.151515	0.28592	0.208803
3	4.5	0.0626959	0.173634	0.105284
4	6.5	0.03367	0.11418	0.0646451
5	10	0.0178253	0.071785	0.0389444
6	20	0.00751315	0.0367476	0.0192617

Table 1: Bounds for d_0 computed by (6) for various a and c.

From the above table, it can be seen that the right estimates for the value of the best Hausdorff distance are precise.

The General Case.

The H-distance $d_p(sgn(t), \varphi_p(t))$ between the sgn function and the function φ_p satisfies the relation:

$$\varphi_p(d_p) = \varphi_0(t + \varphi_{p-1}(t)) = 1 - d_p, \quad p = 1, 2, 3, ... \tag{9}$$

The following Theorem gives upper and lower bounds for d_p

Theorem 3.2. *Let*

$$W_p = 1 + \sum_{i=1}^{p+1} (ac)^i; \;\; p = 1, 2, 3, ...; \;\;\; r_p = 1.1 W_p.$$

For the Hausdorff distance d_p between the sgn function and the function φ_p the following inequalities hold for

$$r_p > e^{1.1}$$

$$d_{l_p} = \frac{1}{r_p} < d_p < \frac{\ln r_p}{r_p} = d_{r_p}. \tag{10}$$

Proof. We define the functions

$$F_p(d_p) = \varphi_p(d_p) - 1 + d_p \tag{11}$$

and

$$G_p(d_p) = -1 + W_p d_p. \tag{12}$$

From Taylor expansion we find $F_p(d_p) - G_p(d_p) = O(d_p^3)$.

The proof follows the ideas given in Theorem 3.1 and will be omitted.

We note that

$$G_p(d_{l_p}) < 0; \quad G_p(d_{r_p}) > 0.$$

This completes the proof of the inequalities (10).

4. REMARKS

Remark 1. We also formulate the following mew Half Gompertz–Makeham Activation Function (HGMAF) by:

$$M_0(t) = \frac{1 - e^{-t-c(e^{at}-1)}}{1 + e^{-t-c(e^{at}-1)}}. \tag{13}$$

Theorem 3.3. *Let*

$$q = \frac{1}{2}(3 + ac); \; r_0 = 1.1q.$$

For the Hausdorff distance d_0 between the sgn function and the function $M_0(t)$ the following inequalities hold for

$$r_0 > e^{1.1}$$

$$d_{l_0} = \frac{1}{r_0} < d_0 < \frac{\ln r_0}{r_0} = d_{r_0}. \tag{14}$$

The proof follows the ideas given in this note and will be omitted.

Approximation of the $sgn(t)$ by $M(t)$ for $a = 4$ and $c = 5$ is visualized on Fig. 6.

Remark 2. It is natural to define the following special class of recurrence generated activation functions

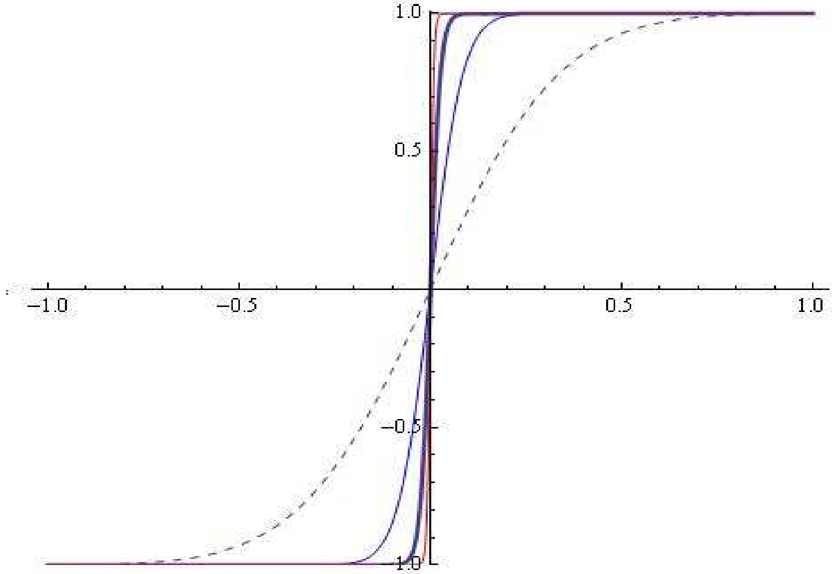

Figure 5: The activation functions for $a = 1.5$; $c = 2$: φ_0 (dashed); Hausdorff distance $d_0 = 0.288019$; φ_1 (blue); Hausdorff distance $d_1 = 0.112234$; φ_2 (thick); Hausdorff distance $d_2 = 0.0464705$; φ_3 (red); Hausdorff distance $d_3 = 0.0192846$.

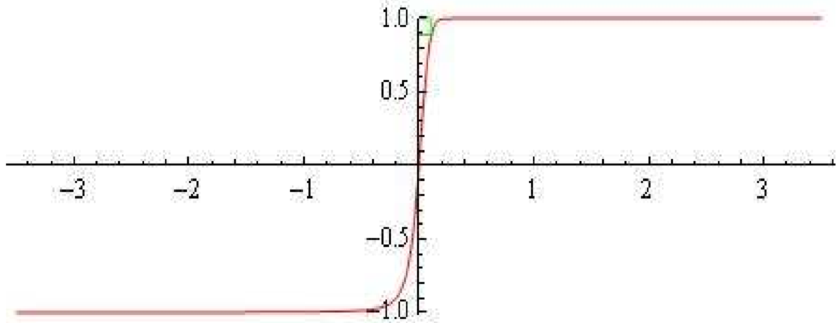

Figure 6: Approximation of the $sgn(t)$ by $M_0(t)$ for $a = 4$ and $c = 5$; Hausdorff distance: $d = 0.109287$.

Definition 5. The new family of recurrence generated activation functions based on the "Half Gompertz–Makeham correction" - (RHGMAF) is defined as follows

$$M_i(t) = M_0(t + M_{i-1}(t)); \quad i = 1, 2, \ldots, p, \tag{15}$$

where $M_0(t)$ is defined by (13) and p is the number of recursions.

Let

$$W_p = \frac{1}{2^{p+1}} \left(D_p^1 + \sum_{i=1}^{p} D_p^{i+1}(ac)^i + D_p^{p+2}(ac)^{p+1} \right); \; p = 1, 2, 3, ...$$

where

$$D_1^1 = 7; \; D_p^1 = 2D_{p-1} + 1; \; p \geq 2,$$

$$D_1^2 = 4; \; D_p^{i+1} = D_{p-1}^i + D_{p-1}^{i+1}; \; p \geq 2; \; 1 \leq i \leq p,$$

$$D_k^{k+2} = 1; \; k \geq 1.$$

For example

$$\begin{aligned} W_1 &= \tfrac{1}{4}\left(7 + 4ac + a^2c^2\right) \\ W_2 &= \tfrac{1}{8}\left(15 + 11ac + 5a^2c^2 + a^3c^3\right) \\ W_3 &= \tfrac{1}{16}\left(31 + 26ac + 16a^2c^2 + 6a^3c^3 + a^4c^4\right) \end{aligned}$$

The following theorem is valid

Theorem 3.4. *Let $r_p = 1.1W_p$. For the one-sided Hausdorff distance d_p between the sgn function and the function $M_p(t)$ the following inequalities hold for $r_p > e^{1.1}$*

$$d_{l_p} = \frac{1}{r_p} < d_p < \frac{\ln r_p}{r_p} = d_{r_p}. \tag{16}$$

Proof. We will briefly sketch the proof. Let

$$F_p(d_p) = M_p(d_p) - 1 + d_p; \; G_p(d_p) = -1 + W_p d_p.$$

From Taylor expansion we find $F_p(d_p) - G_p(d_p) = O(d_p^2)$ and

$$G_p(d_{l_p}) < 0; \quad G_p(d_{r_p}) > 0.$$

Remark 3.

After the substitution $t = kl\cos\theta + a$, where

– $k = \frac{2\pi}{\lambda}$, λ is the wave length;

– b is the phase difference;

– θ is the azimuthal angle;

– l is the distance between the emitters ($l = \frac{\lambda}{2}$ is fixed),

the activation function $\varphi_0(\theta)$ has a form of emitting chart of antenna factor.

```
Manipulate[PolarPlot[(Exp[-(Pi*Cos[θ]+b)-c*(Exp[-a*(Pi*Cos[θ]+b)]-1)]
    -Exp[-(Pi*Cos[θ]+b)-c*(Exp[a*(Pi*Cos[θ]+b)]-1)])/
  (Exp[-(Pi*Cos[θ]+b)-c*(Exp[-a*(Pi*Cos[θ]+b)]-1)]
    +Exp[-(Pi*Cos[θ]+b)-c*(Exp[a*(Pi*Cos[θ]+b)]-1)]),
 {θ, -2 Pi, 2 Pi}], {a, 0.05, 10., Appearance → "Open"}, {c, 0.0005, 10,
 Appearance → "Open"}, {b, -2*Pi, 2*Pi, Appearance → "Open"}]
```

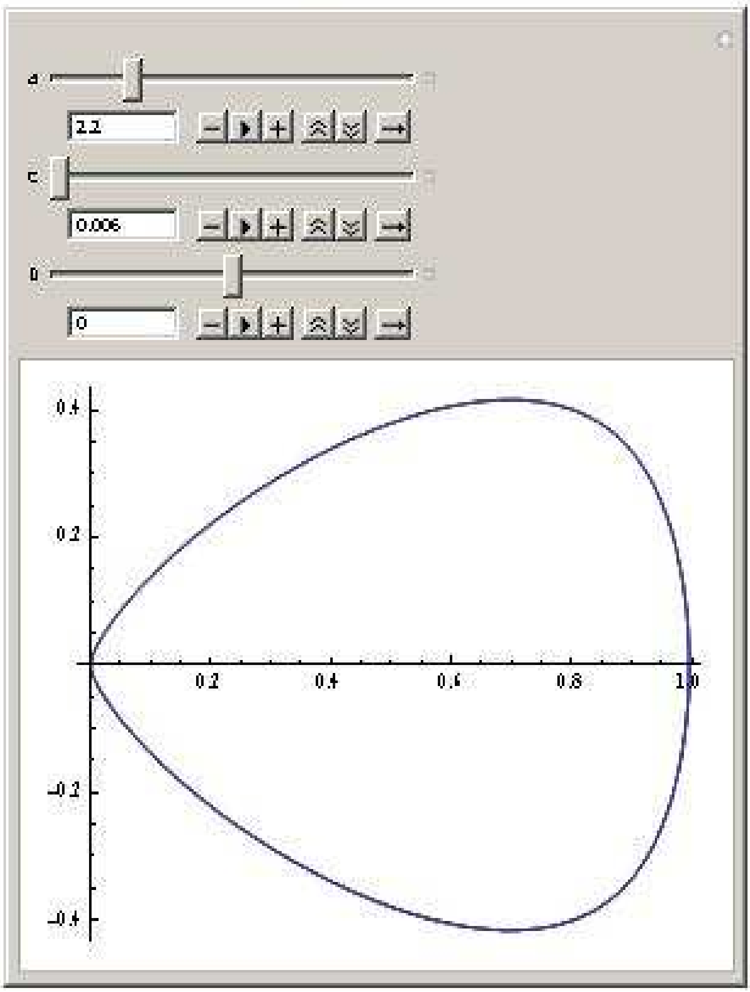

Figure 7: Typical emitting chart of antenna factor for $a = 2.2$; $c = 0.006$.

Typical emitting chart is visualized on Fig. 7.

Of course, the question of the practical realization of the activation functions which are generated as emitting charts remains open.

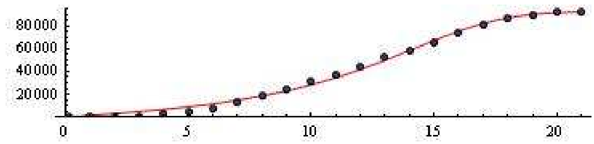

Figure 8: The model (17) for $\omega = 92500$; $a = 0.211503$ and $c = 0.0775344$.

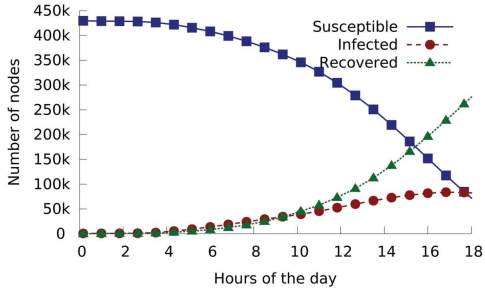

Figure 9: Numbers of susceptible nodes $S(t)$, infected nodes $I(t)$ and recovered nodes $R(t)$ as a function of time t, as inferred from CAIDA's dataset on 21/Nov/ 2008, the day of Conficker's outbreak [37].

Remark 4. Here we will present a new analysis of Conficker propagation in 2008 and we explore the Network Telescope project's daily dataset [37], [38] collected on November 21, 2008. We will approximate the number of infected nodes $I(t)$, see Fig. 9.

Consider the function

$$H(t) = \omega \frac{e^{-t-c(e^{-at}-1)} - e^{-t-c(e^{at}-1)}}{e^{-t-c(e^{-at}-1)} + e^{-t-c(e^{at}-1)}}. \tag{17}$$

for $t \in [0, \infty]$ and $c > 0$, $a > 0$.

The model (17) for $\omega = 92500$; $a = 0.211503$ and $c = 0.0775344$ is visualized on Fig. 8.

For a new analysis and more precise modelling of the data of computer virus epidemics see [39], [40].

5. CONCLUSION

A family of parametric activation functions (PGHAF) based on ”correcting amendments” of ”Gompertz–Makeham - type” is introduced finding application in neural network theory and practice.

Theoretical and numerical results on the approximation in Hausdorff sense of the sgn function by means of functions belonging to the family are reported in the paper.

We propose a software module within the programming environment *CAS Mathematica* for the analysis of the considered family of (RGGMAF) functions.

The module offers the following possibilities:

- generation of the activation functions under user defined values of the parameters a, c and number of recursions p;

- calculation of the H-distance d_p, $p = 1, 2, \ldots$, between the sgn function and the activation functions $\varphi_p(t)$;

- simulation of the emitting chart of antenna factor;

- software tools for animation and visualization.

For other results, see [25]–[36].

In conclusion, we will note that the newly constructed recurrently general families of sigmoidal and activation functions can be used with success in creating a new higher order recurrent neural networks.

6. ACKNOWLEDGMENTS

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REFERENCES

[1] N. Guliyev, V. Ismailov, A single hidden layer feedforward network with only one neuron in the hidden layer san approximate any univariate function, *Neural Computation*, **28** (2016), 1289–1304.

[2] D. Costarelli, R. Spigler, Approximation results for neural network operators activated by sigmoidal functions, *Neural Networks*, **44** (2013), 101–106.

[3] D. Costarelli, G. Vinti, Pointwise and uniform approximation by multivariate neural network operators of the max-product type, *Neural Networks*, **81** (2016), 81–90.

[4] D. Costarelli, R. Spigler, Solving numerically nonlinear systems of balance laws by multivariate sigmoidal functions approximation, *Computational and Applied Mathematics*, **37**, No. 1 (2018), 99–133.

[5] D. Costarelli, G. Vinti, Convergence for a family of neural network operators in Orlicz spaces, *Mathematische Nachrichten*, **290**, No. 2-3 (2017), 226–235.

[6] J. Dombi, Z. Gera, The Approximation of Piecewise Linear Membership Functions and Lukasiewicz Operators, *Fuzzy Sets and Systems*, **154**, No. 2 (2005), 275–286.

[7] I. A. Basheer, M. Hajmeer, Artificial Neural Networks: Fundamentals, Computing, Design, and Application, *Journal of Microbiological Methods*, **43** (2000), 3–31.

[8] Z. Chen, F. Cao, The Approximation Operators with Sigmoidal Functions, *Computers & Mathematics with Applications*, **58** (2009), 758–765.

[9] Z. Chen, F. Cao, The Construction and Approximation of a Class of Neural Networks Operators with Ramp Functions, *Journal of Computational Analysis and Applications*, **14** (2012), 101–112.

[10] Z. Chen, F. Cao, J. Hu, Approximation by Network Operators with Logistic Activation Functions, *Applied Mathematics and Computation*, **256** (2015), 565–571.

[11] D. Costarelli, R. Spigler, Constructive Approximation by Superposition of Sigmoidal Functions, *Anal. Theory Appl.*, **29** (2013), 169–196.

[12] N. Kyurkchiev, A new class of activation function based on the correcting amendments, *Int. J. for Sci., Res. and Developments*, **6**, No. 2 (2018), 565–568.

[13] N. Kyurkchiev, A. Iliev, S. Markov, *Some techniques for recurrence generating of activation functions*, LAP LAMBERT Academic Publishing (2017), ISBN: 978-3-330-33143-3.

[14] A. Iliev, N. Kyurkchiev, S. Markov, A Note on the New Activation Function of Gompertz Type, *Biomath Communications*, **4**, No. 2 (2017).

[15] W. Duch, N. Jankowski, Survey of neural transfer functions, *Neural Computing Surveys*, **2** (1999), 163–212.

[16] F. Hausdorff, *Set Theory*, 2nd ed., Chelsea Publ., New York (1962).

[17] B. Sendov, *Hausdorff Approximations*, Kluwer, Boston (1990).

[18] N. I. Akhiezer, *Theory of approximation*, Nauka, Moscow (1965), 2nd ed., (in Russian), English translation (of the 1st edition): N. I. Achiezer, *Theory of approximation*, Translated by Charles J. Hyman: Frederick Ungar Publishing, New York (1956).

[19] E. Voronovskaya, Odd polynomials of least deviation, *Dokl. AN SSSR*, **159**, No. 4 (1964), 715–718. (in Russian)

[20] S. Markov, Bl. Sendov, On the numerical evaluation of a class of polynomials of best approximation, *Ann. Univ. Sofia, Fac. Math.*, **61** (1966/67), 17–27. (in Bulgarian)

[21] A. Andreev, N. Kyurkchiev, Approximation of some impulse functions - implementation in programming environment MATHEMATICA, *Proceedings of the 43 Spring Conference of the Union of Bulgarian Mathematicians*, Borovetz, April 2-6, 2014, 111–117.

[22] N. Kyurkchiev, A. Andreev, Hausdorff approximation of functions different from zero at one point - implementation in programming environment MATHEMATICA, *Serdica J. of Computing*, **7**, No. 2 (2013), 135–142.

[23] N. Kyurkchiev, A. Andreev, Synthesis of slot aerial grids with Hausdorff–type directive patterns – implementation in programming envoronment Mathematica, *C. R. Acad. Bulg. Sci.*, **66**, No. 11 (2013), 1521–1528.

[24] N. Kyurkchiev, A. Andreev, *Approximation and Antenna and Filters synthesis: Some Moduli in Programming Environment MATHEMATICA*, LAP LAMBERT Academic Publishing, Saarbrucken (2014), ISBN: 978-3-659-53322-8.

[25] N. Kyurkchiev, A. Iliev, On the Hausdorff distance between the Heaviside function and some transmuted activation functions, *Mathematical Modelling in Engineering Applications*, **2**, No. 1 (2016), 1–5.

[26] N. Kyurkchiev, S. Markov, Hausdorff approximation of the sign function by a class of parametric activation functions, *Biomath Communications*, **3**, No. 2 (2016).

[27] A. Iliev, N. Kyurkchiev, S. Markov, A Family of Recurrence Generated Parametric Activation Functions with Applications to Neural Networks, *International Journal on Research Innovations in Engineering Science and Technology*, **2**, No. 1 (2017), 60–68.

[28] N. Kyurkchiev, A. Iliev, S. Markov, Families of Recurrence Generated Three and Four Parametric Activation Functions, *International Journal for Scientific Research & Development*, **5**, No. 1 (2017), 746–750.

[29] V. Kyurkchiev, A. Iliev, N. Kyurkchiev, On Some Families of Recurrence Generated Activation Functions, *International Journal of Scientific Engineering and Applied Science*, **3**, No. 3 (2017), 243–248.

[30] N. Kyurkchiev, A. Iliev, A Note On The New Fibonacci Hyperbolic Tangent Activation Function, *International Journal of Innovative Science Engineering and Technology*, **4**, No. 5 (2017), 364–368.

[31] A. Golev, A. Iliev, N. Kyurkchiev, A Note on the Soboleva' Modified Hyperbolic Tangent Activation Function, *Int. J. of Sci., Eng. and Techn.*, **4**, No. 6 (2017), 177–182.

[32] A. Malinova, A. Golev, A. Iliev, N. Kyurkchiev, A Family of Recurrence Generating Activation Functions Based on Gudermann Function, *International Journal*

of Engineering Researches and Management Studies, **4**, No. 8 (2017), 38–48.

[33] N. Kyurkchiev, A Note on the Volmer's Activation (VA) Function, C. R. Acad. Bulg. Sci., **70**, No. 6 (2017), 769–776.

[34] V. Kyurkchiev, N. Kyurkchiev, A Family of Recurrence Generated Functions Based on "Half-Hyperbolic Tangent Activation Function", *Biomedical Statistics and Informatics*, **2**, No. 3 (2017), 87–94.

[35] N. Kyurkchiev, A. Iliev, A new class of 2-parametric deterministic activation functions, *Int. J. of Advanced Research in Computer and Communication Engineering*, **7**, No. 3 (2018), 481–483.

[36] N. Kyurkchiev, A new class activation functions with application in the theory of impulse technics, *Journal of Mathematical Sciences and Modelling*, **1**, No. 1 (2018), 15–20.

[37] C. Zhang, S. Zhou, B. Chain, Hybrid Epidemics–A Case Study on Computer Worm Conficker, PLoS ONE, **10**, No. 5 (2015), e0127478.

[38] P. Porras, H. Saidi, V. Yegneswaran, An Analysis of Conficker'S Logic and Rendezvous Points, SRI international technical report, March 19, (2009).

[39] N. Kyurkchiev, A. Iliev, A. Rahnev, T. Terzieva, A new analysis of Code Red and Witty worms behavior, Communications in Applied Analysis, 23 (2), 2019, 267–285.

[40] N. Kyurkchiev, A. Iliev, A. Rahnev, T. Terzieva, Some New Approaches for Modelling Large–Scale Worm Spreading on the Internet. II, Neural, Parallel, and Scientific Computations, **27** (2019). (to appear)