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NONLINEAR FIRST-ORDER SEMIPOSITONE PROBLEMS OF IMPULSIVE DYNAMIC EQUATIONS ON TIME SCALES

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ABSTRACT. By using the well-known Guo-Krasnoselskii fixed point theorem, in this paper, some results of one positive solution to a class of nonlinear first-order semipositone problems of impulsive dynamic equations on time scales are obtained. One example is given to illustrate the main results in this paper.

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1. INTRODUCTION

Let $\mathbf{T}$ be a time scale, i.e., $\mathbf{T}$ is a nonempty closed subset of R. Let $0, T$ be points in $\mathbf{T}$, an interval $(0,T)_{\mathbf{T}}$ denoting time scales interval, that is, $(0,T)_{\mathbf{T}} := (0,T) \cap \mathbf{T}$. Other types of intervals are defined similarly.

The theory of impulsive differential equations is emerging as an important area of investigation, since it is a lot richer than the corresponding theory of differential equations without impulse effects. Moreover, such equations may exhibit several real world phenomena in physics, biology, engineering, etc. (see [3, 4, 20]). At the same time, the boundary value problems for impulsive differential equations and impulsive difference equations have received much attention [2, 10, 17, 18, 21–23, 25–28, 30, 32]. On the other hand, recently, the theory of dynamic equations on time scales has become a new important branch (see, for example, [1, 6, 7, 16, 19]). Naturally, some authors have focused their attention on the boundary value problems of impulsive dynamic equations on time scales [5, 8, 9, 12–15, 24, 33]. However, to the best of our knowledge, few papers concerning PBVPs of impulsive dynamic equations on time scales with semi-position condition.

In this paper, we are concerned with the existence of positive solutions for the following PBVPs of impulsive dynamic equations on time scales with semi-position

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condition

$$(1.1)\qquad \begin{cases} x^{\triangle}(t)+f(t,x(\sigma(t)))=0,\ t\in J:=[0,T]_{\mathbf{T}},\ t\neq t_k,\ k=1,2,\ldots,m,\\ x(t_k^+)-x(t_k^-)=I_k(x(t_k^-)),\ k=1,2,\ldots,m,\\ x(0)=x(\sigma(T)),\end{cases}$$

where $\mathbf{T}$ is an arbitrary time scale, $T>0$ is fixed, $0,T\in\mathbf{T}$, $f\in C(J\times[0,\infty),(-\infty,\infty))$, $I_k\in C([0,\infty),[0,\infty))$, $t_k\in(0,T)_{\mathbf{T}}$, $0<t_1<\ldots<t_m<T$, and for each $k=1,2,\ldots,m$, $x(t_k^+)=\lim_{h\to 0^+}x(t_k+h)$ and $x(t_k^-)=\lim_{h\to 0^-}x(t_k+h)$ represent the right and left limits of $x(t)$ at $t=t_k$. We always assume that following hypothesis holds (semi-position condition):

(H) There exists a positive number $M>0$ such that

$$Mx-f(t,x)\geq 0 \text{ for } x\in[0,\infty),\ t\in[0,T]_{\mathbf{T}}.$$

By using the well-known Guo-Krasnoselskii fixed point theorem [11], some existence criteria of positive solution to problem (1.1) are established. We note that for the case $\mathbf{T}=R$ and $I_k(x)\equiv 0, k=1,2,\ldots,m$, problem (1.1) reduces to the problem studied by [29] and for the case $I_k(x)\equiv 0, k=1,2,\ldots,m$, problem (1.1) reduces to the problem (in the one-dimension case) studied by [31].

In the remainder of this section, we state the Guo-Krasnoselskii fixed point theorem [11].

Theorem 1.1 (Guo-Krasnoselskii). Let X be a Banach space and $K\subset X$ be a cone in X. Assume Ω_1, Ω_2 are bounded open subsets of X with $0\in\Omega_1\subset\overline{\Omega}_1\subset\Omega_2$ and $\Phi: K\cap(\overline{\Omega}_2\backslash\Omega_1)\to K$ is a completely continuous operator such that, either:

(i) $\|\Phi x\|\leq\|x\|$, $x\in K\cap\partial\Omega_1$, and $\|\Phi x\|\geq\|x\|$, $x\in K\cap\partial\Omega_2$; or

(ii) $\|\Phi x\|\geq\|x\|$, $x\in K\cap\partial\Omega_1$, and $\|\Phi x\|\leq\|x\|$, $x\in K\cap\partial\Omega_2$.

Then Φ has at least one fixed point in $K\cap(\overline{\Omega}_2\backslash\Omega_1)$.

2. SOME RESULTS ON TIME SCALES

In this section, we state some fundamental definitions and results concerned time scales, so that the paper is self-contained. For more details, one can refer to [1, 6, 7, 16, 19].

Definition 2.1. Assume that $x:\mathbf{T}\to R$ and fix $t\in\mathbf{T}$ (if $t=\sup\mathbf{T}$, we assume t is not left-scattered). Then x is called differential at $t\in\mathbf{T}$ if there exists a $\theta\in R$ such that for any given $\varepsilon>0$, there is an open neighborhood U of t such that

$$|x(\sigma(t))-x(s)-\theta\,|\sigma(t)-s||\leq\varepsilon\,|\sigma(t)-s|,\ s\in U.$$

In this case, θ is called the delta derivative of x at $t\in\mathbf{T}$ and denote it by $\theta=x^{\triangle}(t)$.

If $F^{\triangle}(t) = f(t)$, then we define the delta integral by

$$\int_a^t f(s)\triangle s = F(t) - F(a).$$

Lemma 2.1. If $f \in C_{rd}$ and $t \in \mathbf{T}^k$, then

$$\int_t^{\sigma(t)} f(s)\triangle s = \mu(t)f(t),$$

where $\mu(t) = \sigma(t) - t$ is the graininess function.

Lemma 2.2. If $f^{\triangle} \geq 0$, then f is increasing.

Lemma 2.3. Assume that f, $g : \mathbf{T} \to R$ are delta derivative at t, then

$$(fg)^{\triangle}(t) = f^{\triangle}(t)g(t) + f(\sigma(t))g^{\triangle}(t) = f(t)g^{\triangle}(t) + f^{\triangle}(t)g(\sigma(t)).$$

Definition 2.2. A function $p : \mathbf{T} \to R$ is regressive provided

$$1 + \mu(t)p(t) \neq 0 \text{ for all } t \in \mathbf{T}^k.$$

The set of all regressive and rd-continuous functions will be denoted by $\mathcal{R}$.

Definition 2.3. We define the set $\mathcal{R}^+$ of all positively regressive elements of $\mathcal{R}$ by

$$\mathcal{R}^+ = \{p \in \mathcal{R} : 1 + \mu(t)p(t) > 0 \text{ for all } t \in \mathbf{T}\}.$$

Definition 2.4. If $p \in \mathcal{R}$, then the delta exponential function is given by

$$e_p(t,s) = \begin{cases} \exp\left(\int_s^t p(\tau)\triangle\tau\right) & \text{if } \mu(\tau) = 0, \\ \exp\left(\int_s^t \frac{1}{\mu(\tau)} Log(1 + p(\tau)\mu(\tau))\triangle\tau\right) & \text{if } \mu(\tau) \neq 0, \end{cases}$$

where Log is the principal logarithm.

Lemma 2.4. If $p \in \mathcal{R}$, then

(1) $e_p(t,t) \equiv 1$;

(2) $e_p(t,s) = \frac{1}{e_p(s,t)}$;

(3) $e_p(t,u)e_p(u,s) = e_p(t,s)$;

(4) $e_p^{\triangle}(t,t_0) = p(t)e_p(t,t_0)$, for $t \in \mathbf{T}^k$ and $t_0 \in \mathbf{T}$.

Lemma 2.5. If $p \in \mathcal{R}^+$ and $t_0 \in \mathbf{T}$, then

$$e_p(t,t_0) > 0 \text{ for all } t \in \mathbf{T}.$$

3. PRELIMINARIES

Throughout the rest of this paper, we always assume that the points of impulse t_k are right-dense for each $k = 1, 2, \ldots, m$.

We define

$$PC = \{x \in [0, \sigma(T)]_{\mathbf{T}} \to R : x_k \in C(J_k, R),\ k = 1, 2, \ldots, m \text{ and there exist } x(t_k^+) \text{ and } x(t_k^-) \text{ with } x(t_k^-) = x(t_k),\ k = 1, 2, \ldots, m\},$$

where x_k is the restriction of x to $J_k = (t_k, t_{k+1}]_{\mathbf{T}} \subset (0, \sigma(T)]_{\mathbf{T}}$, $k = 1, 2, \ldots, m$ and $J_0 = [0, t_1]_{\mathbf{T}}$, $J_{m+1} = \{\sigma(T)\}$.

Let

$$X = \{x : x \in PC,\ x(0) = x(\sigma(T))\}$$

with the norm $\|x\| = \sup_{t\in[0,\sigma(T)]_{\mathbf{T}}} |x(t)|$. Then X is a Banach space.

Lemma 3.1. Suppose $M > 0$ and $h : [0, T]_{\mathbf{T}} \to R$ is rd-continuous, then x is a solution of

$$x(t) = \int_0^{\sigma(T)} G(t, s)h(s)\triangle s + \sum_{k=1}^{m} G(t, t_k)I_k(x(t_k)),\ t \in [0, \sigma(T)]_{\mathbf{T}},$$

where $G(t, s) = \begin{cases} \frac{e_M(s,t)e_M(\sigma(T),0)}{e_M(\sigma(T),0)-1}, & 0 \le s \le t \le \sigma(T), \\ \frac{e_M(s,t)}{e_M(\sigma(T),0)-1}, & 0 \le t < s \le \sigma(T), \end{cases}$ if and only if x is a solution of the boundary value problem

$$\begin{cases} x^{\triangle}(t) + Mx(\sigma(t)) = h(t),\ t \in J := [0, T]_{\mathbf{T}},\ t \ne t_k,\ k = 1, 2, \ldots, m, \\ x(t_k^+) - x(t_k^-) = I_k(x(t_k^-)),\ k = 1, 2, \ldots, m, \\ x(0) = x(\sigma(T)). \end{cases}$$

Proof. Since the proof similar to that of [Lemma 3.1, 33], we omit it here.

Lemma 3.2. Let $G(t, s)$ be defined as Lemma 3.1, then

$$\frac{1}{e_M(\sigma(T), 0) - 1} \le G(t, s) \le \frac{e_M(\sigma(T), 0)}{e_M(\sigma(T), 0) - 1} \text{ for all } t, s \in [0, \sigma(T)]_{\mathbf{T}}.$$

Proof. It is obviously, so we omit it here.

Remark 3.1. Let $G(t, s)$ be defined as Lemma 3.1, then $\int_0^{\sigma(T)} G(t, s)\triangle s = \frac{1}{M}$.

For $u \in X$, we consider the following prblem:

(3.1)

$$\begin{cases} x^{\triangle}(t) + Mx(\sigma(t)) = Mu(\sigma(t)) - f(t, u(\sigma(t))),\ t \in [0, T]_{\mathbf{T}},\ t \ne t_k,\ k = 1, 2, \ldots, m, \\ x(t_k^+) - x(t_k^-) = I_k(x(t_k^-)),\ k = 1, 2, \ldots, m, \\ x(0) = x(\sigma(T)). \end{cases}$$

It follows from Lemma 3.1 that problem (3.1) has a unique solution:

$$x(t) = \int_0^{\sigma(T)} G(t, s)h_u(s)\triangle s + \sum_{k=1}^{m} G(t, t_k)I_k(x(t_k)),\ t \in [0, \sigma(T)]_{\mathbf{T}},$$

where $h_u(s) = Mu(\sigma(s)) - f(s, u(\sigma(s)))$.

We define an operator $\Phi : X \to X$ by

$$\Phi(u)(t) = \int_0^{\sigma(T)} G(t,s)h_u(s)\triangle s + \sum_{k=1}^{m} G(t,t_k)I_k(u(t_k)),\ t \in [0,\sigma(T)]_{\mathbf{T}}.$$

Lemma 3.3. $\Phi : X \to X$ is completely continuous.

Proof. The proof is divided into three steps.

Step 1: To show that $\Phi : X \to X$ is continuous.

Let $\{u_n\}_{n=1}^{\infty}$ be a sequence such that $u_n \to u$ $(n \to \infty)$ in X. Since $f(t,u)$ and $I_k(u)$ are continuous in x, we have

$$|h_{u_n}(t) - h_u(t)| = |M(u_n - u) - (f(t,u_n) - f(t,u))| \to 0\ (n \to \infty),$$

$$|I_k(u_n(t_k)) - I_k(u(t_k))| \to 0\ (n \to \infty).$$

So

$$\begin{aligned}
&|\Phi(u_n)(t) - \Phi(u)(t)| \\
&= \left| \int_0^{\sigma(T)} G(t,s)\left[h_{u_n}(s) - h_u(s)\right]\triangle s + \sum_{k=1}^{m} G(t,t_k)\left[I_k(u_n(t_k)) - I_k(u(t_k))\right] \right| \\
&\le \frac{e_M(\sigma(T),0)}{e_M(\sigma(T),0)-1}\left[\int_0^{\sigma(T)} |h_{u_n}(t) - h_u(t)|\triangle s + \sum_{k=1}^{m} |I_k(u_n(t_k)) - I_k(u(t_k))|\right] \\
&\to 0\ (n \to \infty),
\end{aligned}$$

which lead to $\|\Phi u_n - \Phi u\| \to 0\ (n \to \infty)$. That is, $\Phi : X \to X$ is continuous.

Step 2: To show that Φ maps bounded sets into bounded sets in X.

Let $B \subset X$ be a bounded set, that is, $\exists\ r > 0$ such that $\forall\ u \in B$ we have $\|u\| \le r$. Then, for any $u \in B$, in virtue of the continuity of $f(t,u)$ and $I_k(u)$, there exist $c > 0, c_k > 0$ such that

$$|f(t,u)| \le c,\ \ |I_k(u)| \le c_k,\ k = 1,2,\ldots,m.$$

We get

$$\begin{aligned}
|\Phi(u)(t)| &= \left| \int_0^{\sigma(T)} G(t,s)h_u(s)\triangle s + \sum_{k=1}^{m} G(t,t_k)I_k(u(t_k)) \right| \\
&\le \int_0^{\sigma(T)} G(t,s)\,|h_u(s)|\triangle s + \sum_{k=1}^{m} G(t,t_k)\,|I_k(u(t_k))| \\
&\le \frac{e_M(\sigma(T),0)}{e_M(\sigma(T),0)-1}\left[\sigma(T)\,(Mr + c) + \sum_{k=1}^{m} c_k\right].
\end{aligned}$$

Then we can conclude that Φu is bounded uniformly, and so $\Phi(B)$ is a bounded set.

Step 3: To show that Φ maps bounded sets into equicontinuous sets of X.

Let $t_1,\, t_2 \in [0,\sigma(T)]_{\mathbf{T}}$, $u \in B$, then

$$\begin{aligned} & |\Phi(u)(t_1) - \Phi(u)(t_2)| \\ \leq & \int_0^{\sigma(T)} |G(t_1,s) - G(t_2,s)|\,|h_u(s)| \triangle s + \sum_{k=1}^{m} |G(t_1,t_k) - G(t_2,t_k)|\,|I_k(u(t_k))|\,. \end{aligned}$$

The right-hand side tends to uniformly zero as $|t_1 - t_2| \to 0$.

Consequently, Step 1-3 together with the Arzela-Ascoli Theorem show that $\Phi : X \to X$ is completely continuous.

Let

$$K = \{u \in X : u(t) \geq \delta \|u\|,\ t \in [0,\sigma(T)]_{\mathbf{T}}\},$$

where $\delta = \frac{1}{e_M(\sigma(T),\,0)} \in (0,1)$. It is not difficult to verify that K is a cone in X.

From Lemma 3.2, it is easy to obtain following result:

Lemma 3.4. Φ maps K into K.

4. MAIN RESULTS

Let

$$f^0 = \lim_{u\to 0^+} \sup \max_{t\in[0,T]_{\mathbf{T}}} \frac{f(t,u)}{u},\ f^\infty = \lim_{u\to\infty} \sup \max_{t\in[0,T]_{\mathbf{T}}} \frac{f(t,u)}{u},$$

$$f_0 = \lim_{u\to 0^+} \inf \min_{t\in[0,T]_{\mathbf{T}}} \frac{f(t,u)}{u},\ \ f_\infty = \lim_{u\to\infty} \inf \min_{t\in[0,T]_{\mathbf{T}}} \frac{f(t,u)}{u},$$

and

$$I_0 = \lim_{u\to 0^+} \frac{I_k(u)}{u},\ \ I_\infty = \lim_{u\to\infty} \frac{I_k(u)}{u}.$$

Now we state our main results.

Theorem 4.1. Suppose that

$$f_0 > 0,\ \ f^\infty < \frac{\delta - 1}{\delta} M;\ \ I_0 = 0,\ \text{for any } k.$$

Then the problem (1.1) has at least one positive solutions.

Proof. From the hypotheses we know there exist $\varepsilon > 0$ and $L_1 > r_1 > 0$ such that

$$f(t,u) \geq \varepsilon u,\ \ I_k(u) \leq \frac{(e_M(\sigma(T),0) - 1)\varepsilon}{Mme_M(\sigma(T),0)} u,\ \text{for any } k,\ 0 < u \leq r_1;$$

$$f(t,u) \leq \left(\frac{\delta - 1}{\delta} M - \varepsilon\right) u,\ \ u \geq L_1.$$

Let $\Omega_1 = \{u \in X : \|u\| < r_1\}$. It follows that for $u \in K$ with $\|u\| = r_1$, we have

$$\begin{aligned} \Phi(u)(t) &= \int_0^{\sigma(T)} G(t,s) h_u(s) \triangle s + \sum_{k=1}^{m} G(t,t_k) I_k(u(t_k)) \\ &\leq \int_0^{\sigma(T)} G(t,s)\,(M - \varepsilon)\, u(\sigma(s)) \triangle s + \sum_{k=1}^{m} G(t,t_k) \frac{(e_M(\sigma(T),0) - 1)\varepsilon}{Mme_M(\sigma(T),0)} u(t_k) \end{aligned}$$

$$
\begin{aligned}
&\leq \frac{(M-\varepsilon)}{M}\|u\|+\frac{e_M(\sigma(T),0)}{e_M(\sigma(T),0)-1}\sum_{k=1}^{m}\frac{(e_M(\sigma(T),0)-1)\varepsilon}{Mme_M(\sigma(T),0)}\|u\| \\
&= \|u\|,
\end{aligned}
$$

which yields

$$\|\Phi u\| \leq \|u\|,\ u \in K \cap \partial\Omega_1. \tag{4.1}$$

Set $\Omega_2 = \left\{u \in X : \|u\| < \frac{L_1}{\delta}\right\}$. Since $u \in K \cap \partial\Omega_2$, we have $u(t) \geq \delta\|u\| = L_1$. Hence for $u \in K \cap \partial\Omega_2$, we have

$$
\begin{aligned}
\Phi(u)(t) &= \int_0^{\sigma(T)} G(t,s)h_u(s)\triangle s + \sum_{k=1}^{m} G(t,t_k)I_k(u(t_k)) \\
&\geq \int_0^{\sigma(T)} G(t,s)h_u(s)\triangle s \\
&\geq \int_0^{\sigma(T)} G(t,s)\left(M+\frac{1-\delta}{\delta}M+\varepsilon\right)u(\sigma(s))\triangle s \\
&\geq \frac{1}{M}\left(\frac{1}{\delta}M+\varepsilon\right)\delta\|u\| \\
&\geq \|u\|,
\end{aligned}
$$

which implies

$$\|\Phi u\| \geq \|u\|,\ x \in K \cap \partial\Omega_2. \tag{4.2}$$

Therefore, from (4.1), (4.2) and Theorem 1.1, it follows that Φ has a fixed point in $K \cap \left(\overline{\Omega}_2 \backslash \Omega_1\right)$, that is, the problem (1.1) has at least one positive solution.

Theorem 4.2. Suppose that

$$f_\infty > 0,\ f^0 < \frac{\delta-1}{\delta}M;\ I_\infty = 0,\ \text{for any } k.$$

Then the problem (1.1) has at least one positive solutions.

Proof. From the hypotheses we know there exist $\varepsilon' > 0$ and $L_2 > r_2 > 0$ such that

$$f(t,u) \geq \varepsilon' u,\ I_k(u) \leq \frac{(e_M(\sigma(T),0)-1)\varepsilon'}{Mme_M(\sigma(T),0)}u,\ \text{for any } k,\ u \geq L_2;$$

$$f(t,u) \leq \left(\frac{\delta-1}{\delta}M-\varepsilon'\right)u,\ 0 < u \leq r_2.$$

Let $\Omega_1 = \left\{u \in X : \|u\| < \frac{L_2}{\delta}\right\}$. Since $u \in K \cap \partial\Omega_1$, we have $u(t) \geq \delta\|u\| = L_2$. Hence for $u \in K \cap \partial\Omega_1$, we have

$$
\begin{aligned}
\Phi(u)(t) &= \int_0^{\sigma(T)} G(t,s)h_u(s)\triangle s + \sum_{k=1}^{m} G(t,t_k)I_k(u(t_k)) \\
&\leq \int_0^{\sigma(T)} G(t,s)\left(M-\varepsilon'\right)u(\sigma(s))\triangle s + \sum_{k=1}^{m} G(t,t_k)\frac{(e_M(\sigma(T),0)-1)\varepsilon'}{Mme_M(\sigma(T),0)}u(t_k)
\end{aligned}
$$

$$\begin{aligned} &\leq \frac{(M-\varepsilon')}{M}\|u\| + \frac{e_M(\sigma(T),0)}{e_M(\sigma(T),0)-1}\sum_{k=1}^{m}\frac{(e_M(\sigma(T),0)-1)\varepsilon'}{Mme_M(\sigma(T),0)}\|u\| \\ &= \|u\|, \end{aligned}$$

which yields

$$\|\Phi u\| \leq \|u\|,\ u \in K \cap \partial\Omega_1. \tag{4.3}$$

Set $\Omega_2 = \{x \in X : \|u\| < r_2\}$. It follows that for $u \in K$ with $\|u\| = r_2$, we have

$$\begin{aligned} \Phi(u)(t) &= \int_0^{\sigma(T)} G(t,s)h_u(s)\triangle s + \sum_{k=1}^{m} G(t,t_k)I_k(u(t_k)) \\ &\geq \int_0^{\sigma(T)} G(t,s)h_u(s)\triangle s \\ &\geq \int_0^{\sigma(T)} G(t,s)\left(M + \frac{1-\delta}{\delta}M + \varepsilon'\right)u(\sigma(s))\triangle s \\ &\geq \frac{1}{M}\left(\frac{1}{\delta}M + \varepsilon'\right)\delta\|u\| \\ &\geq \|u\|, \end{aligned}$$

which implies

$$\|\Phi u\| \geq \|u\|,\ u \in K \cap \partial\Omega_2. \tag{4.4}$$

Hence, from (4.3), (4.4) and Theorem 1.1, it follows that Φ has a fixed point in $K \cap \left(\overline{\Omega}_1 \backslash \Omega_2\right)$, that is, the problem (1.1) has at least one positive solution.

5. EXAMPLE

Example 5.1. Let $\mathbf{T} = [0,1] \cup [2,3]$. We consider the following problem on $\mathbf{T}$

$$\begin{cases} x^{\triangle}(t) + f(t, x(\sigma(t))) = 0,\ t \in [0,3]_{\mathbf{T}},\ t \neq \frac{1}{2}, \\ x\left(\frac{1}{2}^{+}\right) - x\left(\frac{1}{2}^{-}\right) = I(x(\frac{1}{2})), \\ x(0) = x(3), \end{cases} \tag{5.1}$$

where $T = 3$, $f(t,x) = x - (t+1)x^2$, and $I(x) = x^2$.

Let $M = 1$, then $\delta = \frac{1}{2e^2}$, it is easy to see that

$$Mx - f(t,x) = (t+1)x^2 \geq 0 \text{ for } x \in [0,\infty),\ t \in [0,3]_{\mathbf{T}},$$

and

$$f_0 \geq 1,\ f^{\infty} = -\infty,\ \text{and } I_0 = 0.$$

Therefore, by Theorem 4.1, it follows that the problem (5.1) has at least one positive solution.

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