Neural, Parallel & Scientific Computations 15 (2007) 393-410

Robust Adaptive Neural Network Control for a Class of Nonlinear Systems

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Abstract
In this paper, a stable robust adaptive control approach is presented for a class of unknown nonlinear systems in the strict-feedback form with disturbances. The key assumption is that neural network approximation errors and external disturbances satisfy certain bounding conditions. By combining neural network technique with backstepping method and introducing a special type of Lyapunov functions, the controller singularity problem is avoided perfectly. As the estimates of unknown neural network approximation error bound and external disturbance bound are adjusted adaptively, the robustness of the closed-loop system is improved and the application scope of nonlinear systems is extended. The overall neural network control systems can guarantee that all the signals of the closed-loop system are uniformly ultimately bounded and the tracking error converges to a small neighborhood of zero by suitably choosing the design parameters. The feasibility of the control approach is demonstrated through simulation results.

Keywords - neural network, adaptive control, nonlinear strict-feedback system, backstepping

1. INTRODUCTION

Since the late 1980s, great deals of approaches have been developed on the adaptive control of nonlinear systems with linearly parameterized uncertainty. However, the research results during that period are required to satisfy some assumptions, such as matching conditions, or extended matching conditions (Isidori, 1989; Nam & Arapostations, 1988). Recently, many new results emerge and they do not rely on these assumptions (Krstic, Kanellakopoulos & Kokotovic, 1995; Seto, Annaswamy & Baillieul, 1994). In these papers, adaptive control laws are obtained by adopting backstepping method for strict-feedback or lower triangular systems. For systems with high uncertainty, for example, the uncertainty that cannot be linearly parameterized or is completely unknown, the adaptive neural network control approach based on backstepping is developed further.

Although significant progress has been made by combining backstepping method with neural network technique, there are still lots of problems that need to be solved in practice. In (Polycarpou, 1996; Polycarpou & Mears, 1998), an adaptive NN theorem with boundedness was

Received February 15, 2007

proposed and estimated values of unknown bounds of the neural network approximation error were on-line adaptively adjusted. In (Kwan & Lewis, 2000; Zhou, Feng & Feng, 2005) by using fuzzy and neural network approaches and based on backstepping, a robust adaptive control scheme was developed for a class of MIMO nonlinear systems with modeling uncertainties. An adaptive fuzzy controller with H∞ tracking performance was designed in (Wang, Chan, Lee & Liu, 2000). H∞ tracking performance was applied to substantially attenuate the effect of the modeling errors and disturbances. However, in order to avoid the controller singularity problem, the gain functions $g_i(\bar{x}_i), i=1,2,\cdots,n$ (see system (1) in section II) are assumed to be constants (Polycarpou, 1996; Polycarpou & Mears, 1998) or known functions (Kwan & Lewis, 2000; Zhou, Feng & Feng, 2005). This assumption cannot be satisfied in many practical systems. The situation that the gain functions are unknown has been studied in (Zhang, Ge & Hang, 2000; Ge & Wang, 2002). By introducing the integral-type Lyapunov functions, an adaptive backstepping neural network scheme is presented in (Zhang, Ge & Hang, 2000). However, because of the introduction of the integral operation, this scheme is very complex and difficult to apply in practice. Shuzhi S. Ge (Ge & Wang, 2002) proposed an adaptive neural network control approach without the requirement for the integral-type Lyapunov functions and the stability of the closed-loop systems is guaranteed. But the derivatives of the virtual controllers are included in the neural networks so that computational burden increases. In addition, authors in (Zhang, Ge & Hang, 2000; Ge & Wang, 2002) assumed that unknown bounds of the neural network approximation error are less than bounded constants. If unknown bounds are larger than the assumed bounds, the performance of systems cannot be guaranteed. In most research results, the systems do not have external disturbances. Even though external disturbances exist, they are required to satisfy square integrable conditions (Chen, Lee & Chang, 1996; Wang, Chan, Lee & Liu, 2000). These requirements are also difficult to realize in practice.

Taking above disadvantages into account, a stable robust adaptive control approach is presented for a class of unknown nonlinear systems in the strict-feedback form with disturbances in this paper. This approach does not require that the gain functions $g_i(\bar{x}_i), i=1,2,\cdots,n$ are known. The key assumption is that neural network approximation errors and external disturbances satisfy certain bounding conditions. In addition, the derivatives of the virtual controllers are not included in the neural networks so that the computational burden is reduced. By combining neural network technique with backstepping method and introducing a special type of Lyapunov functions, the controller singularity problem is avoided perfectly. As the estimates of unknown neural network approximation error bound and external disturbance bound are adjusted adaptively, the robustness of the closed-loop system is improved and the application scope of nonlinear systems is extended. Simulation results demonstrate the effectiveness of the control approach.

2. PROBLEM FORMULATION

The model of many practical nonlinear systems can be expressed as a special state-space form

$$\begin{aligned} \dot{x}_i &= f_i(\bar{x}_i) + g_i(\bar{x}_i)x_{i+1} + d_i \quad 1 \le i \le n-1 \\ \dot{x}_n &= f_n(\bar{x}_n) + g_n(\bar{x}_n)u + d_n \quad n \ge 2 \\ y &= x_1 \end{aligned} \tag{1}$$

where $\bar{x}_i = [x_1, x_2, ..., x_i]^T \in R^i, i = 1,2,...,n$, $u \in R$, $y \in R$ are state variables, system input and output, respectively, and d_i is external disturbance. The control objective is to design an adaptive neural network controller for system (1) such that 1) all the signals in the closed-loop remain semi-globally uniformly ultimately bounded and 2) the output y follows a desired trajectory y_d.

Note that in the following derivation of the adaptive neural network controller, neural network approximation is only guaranteed within some compact sets. Accordingly, the stability results obtained in this work are semi-global in the sense that, as long as desired, there exist controllers with sufficiently large number of neural network nodes such that all the signals in the closed-loop remain bounded.

Since $g_i(\cdot), i = 1,2,...,n$ are smooth functions, they are therefore bounded within some compact sets. Accordingly, we can make the following two assumptions as commonly being done in the literature.

Assumption 1: The signs of $g_i(\cdot)$ are bounded, i.e., there exist constants $g_{i1} > g_{i0} > 0$, such that $g_{i1} \ge |g_i(\cdot)| \ge g_{i0}$, $\forall \bar{x}_n \in \Omega \in R^n$.

The above assumption implies that the smooth functions $g_i(\cdot)$ are strictly either positive or negative. Without losing generality, we shall assume $g_{i1} \ge g_i(\cdot) \ge g_{i0}, \forall \bar{x}_n \in \Omega \in R^n$.

Assumption 2: There exist constants $g_{id} > 0$ such that $|\dot{g}_i(\cdot)| \le g_{id}, \forall \bar{x}_n \in \Omega \in R^n$.

The controller design presented in this paper employs RBF neural networks to approximate the nonlinear functions. Now let us review the approximation property of RBF neural networks briefly. The general form of RBF neural networks is $\theta^T \xi(x)$, where $\theta \in R^N$ is a vector of adjustable weights and $\xi(x) = [\zeta_1(x), \cdots, \zeta_N(x)]^T \in R^N$ is a vector-valued function with basis functions $\zeta_j(x), j = 1, \cdots, N$ being chosen as commonly used Gaussian functions, which have the form

$$\zeta_j(x) = \exp\left(-\frac{\|x-\mu_j\|^2}{2\sigma^2}\right), \sigma \ge 0, j = 1, \cdots, N \tag{2}$$

where $\mu_j(x) \in R^n, j = 1, \cdots, N$ is the centre of basis functions and σ is the width of basis functions.

According to the approximation property of RBF neural networks, for a continuous function $f(x): \Omega \in R^n \to R$, given a large enough integer N, by choosing $\mu_j(x) \in R^n$, $j = 1, \cdots, N$ and

σ suitably, there exists an optimal weight vector $\theta^* \in R^N$ such that

$$f(x) = \theta^{*T}\xi(x) + \delta(x) \tag{3}$$

where $\delta(x)$ denotes the approximation error of the neural networks. Since θ^* is unknown, we will use θ to denote the estimate of θ^* and design adaptive laws to adjust θ.

3. CONTROLLER DESIGN

The detailed design procedure is described in the following steps. For clarity and conciseness, Step1 are described with detailed explanations, while Step i are simplified, with the relevant equations and the explanations being omitted.

Step1: Let $x_{1d} = y_d$ and define $e_1 = x_1 - x_{1d}$. Its derivative is

$$\dot{e}_1 = \dot{x}_1 - \dot{x}_{1d} = f_1(x_1) + g_1(x_1)x_2 + d_1 - \dot{x}_{1d} = g_1(x_1)\left[g_1^{-1}(x_1)f_1(x_1) + x_2 + g_1^{-1}(x_1)d_1 - g_1^{-1}(x_1)\dot{x}_{1d}\right] \tag{4}$$

If we view x_2 as a virtual control input, there exists a desired control input

$$x_{2d}^* = x_2 = -g_1^{-1}(x_1)f_1(x_1) - g_1^{-1}(x_1)d_1 + g_1^{-1}(x_1)\dot{x}_{1d} - k_1 e_1 \tag{5}$$

where k_1 is a positive constant and a Lyapunov function $V_1 = \frac{1}{2}e_1^2$ such that $\dot{V}_1 = -g_1(x_1)k_1 e_1^2 \le -g_{10}k_1 e_1^2 \le 0$. Therefore, e_1 is asymptotically stable.

However, since the functions $f_1(x_1)$ and $g_1(x_1)$ are unknown, the desired controller cannot be implemented. Instead, RBF neural networks can be used as follows to approximate $g_1^{-1}(x_1)f_1(x_1)$ and $g_1^{-1}(x_1)$

$$g_1^{-1}(x_1)f_1(x_1) = \theta_1^{*T}\xi_1(x_1) + p_1(x_1) \tag{6}$$

$$g_1^{-1}(x_1) = \delta_1^{*T}\eta_1(x_1) + q_1(x_1) \tag{7}$$

where θ_1^* and δ_1^* are the optimal weight vectors of $g_1^{-1}(x_1)f_1(x_1)$ and $g_1^{-1}(x_1)$, respectively. $p_1(x_1)$ and $q_1(x_1)$ are approximation errors.

Throughout this paper, we introduce $\theta_i^T\xi_i(\bar{x}_i)$ and $\delta_i^T\eta_i(\bar{x}_i)$ as RBF neural networks to approximate $g_i^{-1}(\bar{x}_i)f_i(\bar{x}_i)$ and $g_i^{-1}(\bar{x}_i)$, respectively and make the following assumptions for $p_i(\bar{x}_i)$ and $q_i(\bar{x}_i)$.

Assumption 3: On the compact region Ω_i

$$|p_i(\bar{x}_i)| \le \psi_{pi}^* s_{pi}(\bar{x}_i) \quad i = 1,2,\ldots,n \tag{8}$$

$$|q_i(\bar{x}_i)| \le \psi_{qi}^* s_{qi}(\bar{x}_i) \quad i = 1,2,\ldots,n \tag{9}$$

where $\psi_{pi}^* \ge 0$ and $\psi_{qi}^* \ge 0$ are unknown bounding parameters and $s_{pi}(\bar{x}_i): U_c \to R^+$ and $s_{qi}(\bar{x}_i): U_c \to R^+$ are known smooth bounding functions.

We make the following assumptions for the external disturbance d_i of system (1).

Assumption 4: On the compact region Ω_i

$$|d_i(\bar{x}_i)| \le \psi_{di}^* s_{di}(\bar{x}_i) \quad i = 1,2,\dots,n \tag{10}$$

where $\psi_{di}^* \ge 0$ is an unknown bounding parameter and $s_{di}(\bar{x}_i): U_c \to R^+$ is a known smooth bounding function.

Let the virtual control input be chosen as

$$x_{2d} = -\theta_1^T \xi_1(x_1) + \delta_1^T \eta_1(x_1)\dot{x}_{1d} - k_1 e_1 + H_1 \tag{11}$$

where θ_1 and δ_1 are estimates of θ_1^* and δ_1^*, respectively and H_1 is a bounding control function which will be defined later on.

Defining $e_2 = x_2 - x_{2d}$, $\dot{e}_1$ can be obtained as

$$\dot{e}_1 = f_1(x_1) + g_1(x_1)x_2 + d_1 - \dot{x}_{1d} = g_1(x_1)\left[g_1^{-1}(x_1)f_1(x_1) + e_2 + x_{2d} + g_1^{-1}(x_1)d_1 - g_1^{-1}(x_1)\dot{x}_{1d}\right] \tag{12}$$

Substituting (6), (7) and (11) into (12), we can obtain

$$\begin{aligned}\dot{e}_1 &= g_1(x_1)\left[\left(\theta_1^{*T}\xi_1(x_1) + p_1(x_1)\right) + \left(-\theta_1^T\xi_1(x_1) + \delta_1^T\eta_1(x_1)\dot{x}_{1d} + e_2 + g_1^{-1}(x_1)d_1 - \left(\delta_1^{*T}\eta_1(x_1) + q_1(x_1)\right)\dot{x}_{1d}\right]\\ &= g_1(x_1)\left[\tilde{\theta}_1^T\xi_1(x_1) - \tilde{\delta}_1^T\eta_1(x_1)\dot{x}_{1d} - k_1e_1 + e_2 + p_1(x_1) - q_1(x_1)\dot{x}_{1d} + g_1^{-1}(x_1)d_1 + H_1\right]\end{aligned} \tag{13}$$

where $\tilde{\theta}_1 = \theta_1^* - \theta_1$ and $\tilde{\delta}_1 = \delta_1^* - \delta_1$. Throughout this paper, we shall define $\tilde{(\cdot)} = (\cdot)^* - (\cdot)$.

Consider the following Lyapunov candidate

$$V_1 = \frac{1}{2g_1(x_1)}e_1^2 + \frac{1}{2}\tilde{\theta}_1^T\Gamma_{11}^{-1}\tilde{\theta}_1 + \frac{1}{2}\tilde{\delta}_1^T\Gamma_{12}^{-1}\tilde{\delta}_1 + \frac{1}{2\tau_1}\tilde{\psi}_1^T\tilde{\psi}_1 \tag{14}$$

where $\Gamma_{11} = \Gamma_{11}^T > 0$ and $\Gamma_{12} = \Gamma_{12}^T > 0$ are adaptive gain matrices, $\tau_1 > 0$ is an adaptive gain scalar and $\tilde{\psi}_1 = \psi^*_1 - \psi_1$ with $\tilde{\psi}_1 = \left[\tilde{\psi}_{p1}, \tilde{\psi}_{q1}, \tilde{\psi}_{d1}\right]^T$, $\psi_1^* = \left[\psi_{p1}^*, \psi_{q1}^*, \psi_{d1}^*\right]^T$, $\psi_1 = \left[\psi_{p1}, \psi_{q1}, \psi_{d1}\right]^T$. ψ_1 denotes the estimate of ψ^*_1.

The derivative of V_1 is

$$\begin{aligned}\dot{V}_1 &= \frac{e_1\dot{e}_1}{g_1(x_1)} - \frac{\dot{g}_1(x_1)}{2g_1^2(x_1)}e_1^2 - \tilde{\theta}_1^T\Gamma_{11}^{-1}\dot{\theta}_1 - \tilde{\delta}_1^T\Gamma_{12}^{-1}\dot{\delta}_1 - \frac{1}{\tau_1}\tilde{\psi}_1^T\dot{\psi}_1\\ &= e_1e_2 - \left(k_1 + \frac{\dot{g}_1(x_1)}{2g_1^2(x_1)}\right)e_1^2 + \tilde{\theta}_1^T\left[e_1\xi_1(x_1) - \Gamma_{11}^{-1}\dot{\theta}_1\right] - \tilde{\delta}_1^T\left[e_1\eta_1(x_1)\dot{x}_{1d} + \Gamma_{12}^{-1}\dot{\delta}_1\right] + \Lambda_1\end{aligned} \tag{15}$$

where $\Lambda_1 = e_1p_1(x_1) - e_1q_1(x_1)\dot{x}_{1d} + e_1g_1^{-1}(x_1)d_1 + e_1H_1 - \frac{1}{\tau_1}\tilde{\psi}_1^T\dot{\psi}_1$.

Choose the following adaptation laws

$$\begin{aligned}\dot{\theta}_1 &= \Gamma_{11}\left[e_1\xi_1(x_1) - \sigma_1\theta_1\right]\\ \dot{\delta}_1 &= \Gamma_{12}\left[-e_1\eta_1(x_1)\dot{x}_{1d} - \gamma_1\delta_1\right]\end{aligned} \tag{16}$$

where $\sigma_1 > 0$ and $\gamma_1 > 0$ are given scalars.

Substituting (16) into (15), we can obtain

$$\dot{V}_1 = e_1 e_2 - \left(k_1 + \frac{\dot{g}_1(x_1)}{2g_1^2(x_1)}\right)e_1^2 + \sigma_1 \tilde{\theta}_1^T \theta_1 + \gamma_1 \tilde{\delta}_1^T \delta_1 + \Lambda_1 \tag{17}$$

By completion of squares, we have

$$\sigma_1 \tilde{\theta}_1^T \theta_1 = \sigma_1 \tilde{\theta}_1^T (\theta_1^* - \tilde{\theta}_1) \le \sigma_1 \|\tilde{\theta}_1\| \|\theta_1^*\| - \sigma_1 \|\tilde{\theta}_1\|^2 \le -\frac{\sigma_1 \|\tilde{\theta}_1\|^2}{2} + \frac{\sigma_1 \|\theta_1^*\|^2}{2} \tag{18}$$

$$\gamma_1 \tilde{\delta}_1^T \delta_1 = \gamma_1 \tilde{\delta}_1^T (\delta_1^* - \tilde{\delta}_1) \le \gamma_1 \|\tilde{\delta}\| \|\delta_1^*\| - \gamma_1 \|\tilde{\delta}\|^2 \le -\frac{\gamma_1 \|\tilde{\delta}\|^2}{2} + \frac{\gamma_1 \|\delta_1^*\|^2}{2} \tag{19}$$

According to assumption 1 and 2, the following inequality holds

$$-\left(k_1 + \frac{\dot{g}_1(x_1)}{2g_1^2(x_1)}\right)e_1^2 \le -\left(k_1 - \frac{g_{1d}}{2g_1^2(x_1)}\right)e_1^2 \le -\left(k_1 - \frac{g_{1d}}{2g_{10}^2}\right)e_1^2 \le -k_1^* e_1^2 \tag{20}$$

where k_1 is chosen such that $k_1^* = k_1 - \frac{g_{1d}}{2g_{10}^2} > 0$.

Choose the bounding control function H_1 as follows

$$H_1 = -\psi_1^T w_1 \tag{21}$$

$$w_1 = \begin{bmatrix} w_{p1} \\ w_{q1} \\ w_{d1} \end{bmatrix} = \begin{bmatrix} s_{p1} \tanh(e_1 s_{p1} / \varepsilon) \\ \dot{x}_{1d} s_{q1} \tanh(e_1 \dot{x}_{1d} s_{q1} / \varepsilon) \\ (s_{d1} / g_{10}) \tanh(e_1 s_{d1} / g_{10} \varepsilon) \end{bmatrix} \tag{22}$$

$$\begin{aligned} \dot{\psi}_1 &= \tau_1 \left[e_1 w_1 - \mu_1 (\psi_1 - \psi_1^0) \right] \\ &= \tau_1 \begin{bmatrix} e_1 w_{p1} - \mu_1 (\psi_{p1} - \psi_{p1}^0) \\ e_1 w_{q1} - \mu_1 (\psi_{q1} - \psi_{q1}^0) \\ e_1 w_{d1} - \mu_1 (\psi_{d1} - \psi_{d1}^0) \end{bmatrix} \end{aligned} \tag{23}$$

where $\mu_1 > 0$ is a given scalar, $\psi_1^0 = [\psi_{p1}^0, \psi_{q1}^0, \psi_{d1}^0]^T$ is the initial estimate of ψ_1, the scalar ε is a small design constant and $\tanh(\cdot)$ denotes the hyperbolic tangent function.

For any $\varepsilon > 0$ and for $\Pi \in R$, the following inequality holds

$$0 \le |\Pi| - \Pi \tanh(\Pi / \varepsilon) \le \kappa\varepsilon \tag{24}$$

where κ is a constant that satisfies $\kappa = \exp(-(\kappa + 1))$, i.e., $\kappa = 0.2785$.

Using (21), (22), (23), assumption 1, assumption 3, assumption 4 and the inequality (24), we have

$$\begin{aligned} \Lambda_1 &= e_1 p_1(x_1) - e_1 q_1(x_1)\dot{x}_{1d} + e_1 g_1^{-1}(x_1) d_1 + e_1 H_1 - \frac{1}{\tau_1}\tilde{\psi}_1^T \dot{\psi}_1 \\ &\le |e_1| \psi_{p1}^* s_{p1} + |e_1 \dot{x}_{1d}| \psi_{q1}^* s_{q1} + |e_1| \psi_{d1}^* s_{d1} / g_{10} - e_1 \psi_1^T w_1 - \frac{1}{\tau_1} \tau_1 \tilde{\psi}_1^T \left[e_1 w_1 - \mu_1 (\psi_1 - \psi_1^0) \right] \end{aligned}$$

$$=\left[\psi_{p1}^{*},\psi_{q1}^{*},\psi_{d1}^{*}\right]\begin{bmatrix}|e_1|s_{p1}\\|e_1\dot{x}_{1d}|s_{q1}\\|e_1|s_{d1}/g_{10}\end{bmatrix}-e_1\psi_1^{*T}w_1+\mu_1\tilde{\psi}_1^T\left(\psi_1-\psi_1^0\right)$$

$$=\left[\psi_{p1}^{*},\psi_{q1}^{*},\psi_{d1}^{*}\right]\begin{bmatrix}|e_1|s_{p1}-e_1s_{p1}\tanh\left(e_1s_{p1}/\varepsilon\right)\\|e_1\dot{x}_{1d}|s_{q1}-e_1\dot{x}_{1d}s_{q1}\tanh\left(e_1\dot{x}_{1d}s_{q1}/\varepsilon\right)\\|e_1|s_{d1}/g_{10}-\left(e_1s_{d1}/g_{10}\right)\tanh\left(e_1s_{d1}/g_{10}\varepsilon\right)\end{bmatrix}+\mu_1\tilde{\psi}_1^T\left(\psi_1^*-\tilde{\psi}_1-\psi_1^0\right)$$

$$\le\psi_{p1}^{*}\kappa\varepsilon+\psi_{q1}^{*}\kappa\varepsilon+\psi_{d1}^{*}\kappa\varepsilon+\frac{\mu_1\left\|\psi_1^*-\psi_1^0\right\|^2}{2}-\frac{\mu_1\left\|\tilde{\psi}_1\right\|^2}{2}$$

$$=\kappa\varepsilon\left\|\psi_1^*\right\|_1+\frac{\mu_1\left\|\psi_1^*-\psi_1^0\right\|^2}{2}-\frac{\mu_1\left\|\tilde{\psi}_1\right\|^2}{2}\tag{25}$$

Substituting (18), (19), (20) and (25) into (17), we can obtain the following inequality

$$\dot{V}_1\le e_1e_2-k_1^*e_1^2-\left(\frac{\sigma_1\left\|\tilde{\theta}_1\right\|^2}{2}+\frac{\gamma_1\left\|\tilde{\delta}_1\right\|^2}{2}+\frac{\mu_1\left\|\tilde{\psi}_1\right\|^2}{2}\right)+\left(\frac{\sigma_1\left\|\theta_1^*\right\|^2}{2}+\frac{\gamma_1\left\|\delta_1^*\right\|^2}{2}+\frac{\mu_1\left\|\psi_1^*-\psi_1^0\right\|^2}{2}\right)+\kappa\varepsilon\left\|\psi_1^*\right\|_1\tag{26}$$

where the coupling term e_1e_2 will be cancelled in Step 2.

Step2: Similar to the procedure in Step 1, the virtual controller x_{3d} will be designed to make the error $e_2=x_2-x_{2d}$ as small as possible. The derivative of e_2 is

$$\begin{aligned}\dot{e}_2&=\dot{x}_2-\dot{x}_{2d}\\&=f_2(\bar{x}_2)+g_2(\bar{x}_2)x_3+d_2-\dot{x}_{2d}\\&=g_2(\bar{x}_2)\left[g_2^{-1}(\bar{x}_2)f_2(\bar{x}_2)+e_3+x_{3d}+g_2^{-1}(\bar{x}_2)d_2-g_2^{-1}(\bar{x}_2)\dot{x}_{2d}\right]\\&=g_2(\bar{x}_2)\left[\left(\theta_2^{*T}\xi_2(\bar{x}_2)+p_2(\bar{x}_2)\right)+e_3+x_{3d}+g_2^{-1}(\bar{x}_2)d_2-\left(\delta_2^{*T}\eta_2(\bar{x}_2)+q_2(\bar{x}_2)\right)\dot{x}_{2d}\right]\end{aligned}\tag{27}$$

Introduce the error variable $e_3=x_3-x_{3d}$ and let the virtual controller be chosen as

$$x_{3d}=-e_1-\theta_2^T\xi_2(\bar{x}_2)+\delta_2^T\eta_2(\bar{x}_2)\dot{x}_{2d}-k_2e_2+H_2\tag{28}$$

Substituting (28) into (27), we can obtain

$$\dot{e}_2=g_2(\bar{x}_2)\left[\tilde{\theta}_2^T\xi_2(\bar{x}_2)-\tilde{\delta}_2^T\eta_2(\bar{x}_2)\dot{x}_{2d}-e_1-k_2e_2+e_3+p_2(\bar{x}_2)-q_2(\bar{x}_2)\dot{x}_{2d}+g_2^{-1}(\bar{x}_2)d_2+H_2\right]\tag{29}$$

Consider the following Lyapunov candidate

$$V_2=V_1+\frac{1}{2g_2(\bar{x}_2)}e_2^2+\frac{1}{2}\tilde{\theta}_2^T\Gamma_{21}^{-1}\tilde{\theta}_2+\frac{1}{2}\tilde{\delta}_2^T\Gamma_{22}^{-1}\tilde{\delta}_2+\frac{1}{2\tau_2}\tilde{\psi}_2^T\tilde{\psi}_2\tag{30}$$

where $\Gamma_{21}=\Gamma_{21}^T>0$ and $\Gamma_{22}=\Gamma_{22}^T>0$ are adaptive gain matrices and $\tau_2>0$ is an adaptive gain scalar.

The derivative of V_2 is

$$\begin{aligned}\dot{V}_2&=\dot{V}_1+\frac{e_2\dot{e}_2}{g_2(\bar{x}_2)}-\frac{\dot{g}_2(\bar{x}_2)}{2g_2^2(\bar{x}_2)}e_2^2-\tilde{\theta}_2^T\Gamma_{21}^{-1}\dot{\theta}_2-\tilde{\delta}_2^T\Gamma_{22}^{-1}\dot{\delta}_2-\frac{1}{\tau_2}\tilde{\psi}_2^T\dot{\psi}_2\\&=\dot{V}_1-e_1e_2+e_2e_3-\left(k_2+\frac{\dot{g}_2(\bar{x}_2)}{2g_2^2(\bar{x}_2)}\right)e_2^2+\tilde{\theta}_2^T\left[e_2\xi_2(\bar{x}_2)-\Gamma_{21}^{-1}\dot{\theta}_2\right]-\tilde{\delta}_2^T\left[e_2\eta_2(\bar{x}_2)\dot{x}_{2d}+\Gamma_{22}^{-1}\dot{\delta}_2\right]+\Lambda_2\end{aligned}\tag{31}$$

where $\Lambda_2 = e_2 p_2(\bar{x}_2) - e_2 q_2(\bar{x}_2)\dot{x}_{2d} + g_2^{-1}(\bar{x}_2)d_2 + e_2 H_2 - \frac{1}{\tau_2}\tilde{\psi}_2^T \dot{\psi}_2$.

Choose the following adaptation laws

$$\dot{\theta}_2 = \Gamma_{21}\left[e_2 \xi_2(\bar{x}_2) - \sigma_2 \theta_2\right]$$
$$\dot{\delta}_2 = \Gamma_{22}\left[-e_2 \eta_2(\bar{x}_2)\dot{x}_{2d} - \gamma_2 \delta_2\right] \tag{32}$$

where $\sigma_2 > 0$ and $\gamma_2 > 0$ are given scalars.

Choose the bounding control function H_2 as follows

$$H_2 = -\psi_2^T w_2 \tag{33}$$

$$w_2 = \begin{bmatrix} w_{p2} \\ w_{q2} \\ w_{d2} \end{bmatrix} = \begin{bmatrix} s_{p2}\tanh(e_2 s_{p2}/\varepsilon) \\ \dot{x}_{2d} s_{q2}\tanh(e_2 \dot{x}_{2d} s_{q2}/\varepsilon) \\ (s_{d2}/g_{20})\tanh(e_2 s_{d2}/g_{20}\varepsilon) \end{bmatrix} \tag{34}$$

$$\dot{\psi}_2 = \tau_2\left[e_2 w_2 - \mu_2(\psi_2 - \psi_2^0)\right] = \tau_2 \begin{bmatrix} e_2 w_{p2} - \mu_2\left[\psi_{p2} - \psi_{p2}^0\right] \\ e_2 w_{q2} - \mu_2\left[\psi_{q2} - \psi_{q2}^0\right] \\ e_2 w_{d2} - \mu_2\left[\psi_{d2} - \psi_{d2}^0\right] \end{bmatrix} \tag{35}$$

where $\mu_2 > 0$ is a given scalar, $\psi_2^0 = \left[\psi_{p2}^0, \psi_{q2}^0, \psi_{d2}^0\right]^T$ is the initial estimate of ψ_2, the scalar ε is a small design constant and $\tanh(\cdot)$ denotes the hyperbolic tangent function.

By using (32)-(35), assumptions 1, assumptions 2 and with some completion of squares and straightforward derivation similar to those employed in step 1, the derivative of V_2 is obtained as

$$\dot{V}_2 \le e_2 e_3 - \sum_{j=1}^{2} k_j^* e_j^2 - \sum_{j=1}^{2}\left(\frac{\sigma_j\|\tilde{\theta}_j\|^2}{2} + \frac{\gamma_j\|\tilde{\delta}_j\|^2}{2} + \frac{\mu_j\|\tilde{\psi}_j\|^2}{2}\right) + \sum_{j=1}^{2}\left(\frac{\sigma_j\|\theta_j^*\|^2}{2} + \frac{\gamma_j\|\delta_j^*\|^2}{2} + \frac{\mu_j\|\psi_j^* - \psi_j^0\|^2}{2}\right) + \kappa\varepsilon\sum_{j=1}^{2}\|\psi_j^*\|_1 \tag{36}$$

where k_2 is chosen such that $k_2^* = k_2 - \frac{g_{2d}}{2g_{20}^2} > 0$ and the coupling term $e_2 e_3$ will be cancelled in Step 3.

Step i ($3 \le i \le n-1$): Similar to the procedure in Step 2, the virtual controller x_{id} will be designed to make the error $e_i = x_i - x_{id}$ as small as possible. The derivative of e_i is

$$\begin{aligned} \dot{e}_i &= \dot{x}_i - \dot{x}_{id} \\ &= f_i(\bar{x}_i) + g_i(\bar{x}_i)x_{i+1} + d_i - \dot{x}_{id} \\ &= g_i(\bar{x}_i)\left[g_i^{-1}(\bar{x}_i)f_i(\bar{x}_i) + e_{i+1} + x_{(i+1)d} + g_i^{-1}(\bar{x}_i)d_i - g_i^{-1}(\bar{x}_i)\dot{x}_{id}\right] \\ &= g_i(\bar{x}_i)\left[\left(\theta_i^{*T}\xi_i(\bar{x}_i) + p_i(\bar{x}_i)\right) + e_{i+1} + x_{(i+1)d} + g_i^{-1}(\bar{x}_i)d_i - \left(\delta_i^{*T}\eta_i(\bar{x}_i) + q_i(\bar{x}_i)\right)\dot{x}_{id}\right] \end{aligned} \tag{37}$$

Introduce the error variable $e_{i+1} = x_{i+1} - x_{(i+1)d}$ and let the virtual controller be chosen as

$$x_{(i+1)d} = -e_{i-1} - \theta_i^T \xi_i(\bar{x}_i) + \delta_i^T \eta_i(\bar{x}_i)\dot{x}_{id} - k_i e_i + H_i \tag{38}$$

Substituting (38) into (37), we can obtain

$$\dot{e}_i = g_i(\bar{x}_i)\left[\tilde{\theta}_i^T \xi_i(\bar{x}_i) - \tilde{\delta}_i^T \eta_i(\bar{x}_i)\dot{x}_{id} - e_{i-1} - k_i e_i + e_{i+1} + p_i(\bar{x}_i) - q_i(\bar{x}_i)\dot{x}_{id} + g_i^{-1}(\bar{x}_i)d_i + H_i\right] \tag{39}$$

Consider the following Lyapunov candidate

$$V_i = V_{i-1} + \frac{1}{2g_i(\bar{x}_i)}e_i^2 + \frac{1}{2}\tilde{\theta}_i^T \Gamma_{i1}^{-1}\tilde{\theta}_i + \frac{1}{2}\tilde{\delta}_i^T \Gamma_{i2}^{-1}\tilde{\delta}_i + \frac{1}{2\tau_i}\tilde{\psi}_i^T \tilde{\psi}_i \tag{40}$$

where $\Gamma_{i1} = \Gamma_{i1}^T > 0$ and $\Gamma_{i2} = \Gamma_{i2}^T > 0$ are adaptive gain matrices and $\tau_i > 0$ is an adaptive gain scalar.

Choose the following adaptation laws

$$\dot{\theta}_i = \Gamma_{i1}\left[e_i \xi_i(\bar{x}_i) - \sigma_i \theta_i\right]$$
$$\dot{\delta}_i = \Gamma_{i2}\left[-e_i \eta_i(\bar{x}_i)\dot{x}_{id} - \gamma_i \delta_i\right] \tag{41}$$

Where $\sigma_i > 0$ and $\gamma_i > 0$ are given scalars.

Choose the bounding control function H_i as follows

$$H_i = -\psi_i^T w_i \tag{42}$$

$$w_i = \begin{bmatrix} w_{pi} \\ w_{qi} \\ w_{di} \end{bmatrix} = \begin{bmatrix} s_{pi}\tanh(e_i s_{pi}/\varepsilon) \\ \dot{x}_{id}s_{qi}\tanh(e_i \dot{x}_{id}s_{qi}/\varepsilon) \\ (s_{di}/g_{i0})\tanh(e_i s_{di}/g_{i0}\varepsilon) \end{bmatrix} \tag{43}$$

$$\begin{aligned}\dot{\psi}_i &= \tau_i\left[e_i w_i - \mu_i(\psi_i - \psi_i^0)\right] \\ &= \tau_i \begin{bmatrix} e_i w_{pi} - \mu_i(\psi_{pi} - \psi_{pi}^0) \\ e_i w_{qi} - \mu_i(\psi_{qi} - \psi_{qi}^0) \\ e_i w_{di} - \mu_i(\psi_{di} - \psi_{di}^0) \end{bmatrix}\end{aligned} \tag{44}$$

where $\mu_i > 0$ is a given scalar, $\psi_i^0 = \left[\psi_{pi}^0, \psi_{qi}^0, \psi_{di}^0\right]^T$ is the initial estimate of ψ_1, the scalar ε is a small design constant and $\tanh(\cdot)$ denotes the hyperbolic tangent function.

By using (41)-(44), assumptions 1, assumptions 2 and with some completion of squares and straightforward derivation similar to those employed in step 1, the derivative of V_i is obtained as

$$\dot{V}_i \le e_i e_{i+1} - \sum_{j=1}^{i} k_j^* e_j^2 - \sum_{j=1}^{i}\left(\frac{\sigma_j\|\tilde{\theta}_j\|^2}{2} + \frac{\gamma_j\|\tilde{\delta}_j\|^2}{2} + \frac{\mu_j\|\tilde{\psi}_j\|^2}{2}\right) + \sum_{j=1}^{i}\left(\frac{\sigma_j\|\theta_j^*\|^2}{2} + \frac{\gamma_j\|\delta_j^*\|^2}{2} + \frac{\mu_j\|\psi_j^* - \psi_j^0\|^2}{2}\right) + \kappa\varepsilon\sum_{j=1}^{i}\|\psi_j^*\|_1 \tag{45}$$

where k_i is chosen such that $k_i^* = k_i - \frac{g_{id}}{2g_{i0}^2} > 0$ and the coupling term $e_i e_{i+1}$ will be cancelled in Step $i+1$, $i = 3, \ldots, n-1$.

Step n: This is the final step. Define $e_n = x_n - x_{nd}$. Its derivative is

$$\begin{aligned}\dot{e}_n &= \dot{x}_n - \dot{x}_{nd} \\ &= f_n(\bar{x}_n) + g_n(\bar{x}_n)u + d_n - \dot{x}_{nd} \\ &= g_n(\bar{x}_n)\left[g_n^{-1}(\bar{x}_n)f_n(\bar{x}_n) + u + g_n^{-1}(\bar{x}_n)d_n - g_n^{-1}(\bar{x}_n)\dot{x}_{nd}\right] \\ &= g_n(\bar{x}_n)\left[(\theta_n^{*T}\xi_n(\bar{x}_n) + p_n(\bar{x}_n)) + u + g_n^{-1}(\bar{x}_n)d_n - (\delta_n^{*T}\eta_n(\bar{x}_n) + q_n(\bar{x}_n))\dot{x}_{nd}\right]\end{aligned} \tag{46}$$

Let the practical control input be chosen as

$$u = -e_{n-1} - \theta_n^T \xi_n(\bar{x}_n) + \delta_n^T \eta_n(\bar{x}_n)\dot{x}_{nd} - k_n e_n + H_n \tag{47}$$

Substituting (47) into (46), we can obtain

$$\dot{e}_n = g_n(\bar{x}_n)\left[\tilde{\theta}_n^T \xi_n(\bar{x}_n) - \tilde{\delta}_n^T \eta_n(\bar{x}_n)\dot{x}_{nd} - e_{n-1} - k_n e_n + p_n(\bar{x}_n) - q(\bar{x}_n)\dot{x}_{nd} + g_n^{-1}(\bar{x}_n)d_n + H_n\right] \tag{48}$$

Consider the following Lyapunov candidate

$$V_n = V_{n-1} + \frac{1}{2g_n(\bar{x}_n)}e_n^2 + \frac{1}{2}\tilde{\theta}_n^T \Gamma_{n1}^{-1}\tilde{\theta}_n + \frac{1}{2}\tilde{\delta}_n^T \Gamma_{n2}^{-1}\tilde{\delta}_n + \frac{1}{2\tau_n}\tilde{\psi}_n^T \tilde{\psi}_n \tag{49}$$

where $\Gamma_{n1} = \Gamma_{n1}^T > 0$ and $\Gamma_{n2} = \Gamma_{n2}^T > 0$ are adaptive gain matrices and $\tau_n > 0$ is an adaptive gain scalar.

Choose the following adaptation laws

$$\begin{aligned} \dot{\theta}_n &= \Gamma_{n1}\left[e_n \xi_n(\bar{x}_n) - \sigma_n \theta_n\right] \\ \dot{\delta}_n &= \Gamma_{n2}\left[-e_n \eta_n(\bar{x}_n)\dot{x}_{nd} - \gamma_n \delta_n\right] \end{aligned} \tag{50}$$

where $\sigma_n > 0$ and $\gamma_n > 0$ are given scalars.

Choose the bounding control function H_n as follows

$$H_n = -\psi_n^T w_n \tag{51}$$

$$w_n = \begin{bmatrix} w_{pn} \\ w_{qn} \\ w_{dn} \end{bmatrix} = \begin{bmatrix} s_{pn}\tanh(e_n s_{pn}/\varepsilon) \\ \dot{x}_{nd}s_{qn}\tanh(e_n \dot{x}_{nd}s_{qn}/\varepsilon) \\ (s_{dn}/g_{n0})\tanh(e_n s_{dn}/g_{n0}\varepsilon) \end{bmatrix} \tag{52}$$

$$\dot{\psi}_n = \tau_n\left[e_n w_n - \mu_n(\psi_n - \psi_n^0)\right] = \tau_n \begin{bmatrix} e_n w_{pn} - \mu_n(\psi_{pn} - \psi_{pn}^0) \\ e_n w_{qn} - \mu_n(\psi_{qn} - \psi_{qn}^0) \\ e_n w_{dn} - \mu_n(\psi_{dn} - \psi_{dn}^0) \end{bmatrix} \tag{53}$$

where $\mu_n > 0$ is a given scalar, $\psi_n^0 = \left[\psi_{pn}^0, \psi_{qn}^0, \psi_{dn}^0\right]^T$ is the initial estimate of ψ_n, the scalar ε is a small design constant and $\tanh(\cdot)$ denotes the hyperbolic tangent function.

By using (50)-(53), assumptions 1, assumptions 2 and with some completion of squares and straightforward derivation similar to those employed in step 1, the derivative of V_n is obtained as

$$\dot{V}_n \le -\sum_{j=1}^{n} k_j^* e_j^2 - \sum_{j=1}^{n}\left(\frac{\sigma_j\|\tilde{\theta}_j\|^2}{2} + \frac{\gamma_j\|\tilde{\delta}_j\|^2}{2} + \frac{\mu_j\|\tilde{\psi}_j\|^2}{2}\right) + \sum_{j=1}^{n}\left(\frac{\sigma_j\|\theta_j^*\|^2}{2} + \frac{\gamma_j\|\delta_j^*\|^2}{2} + \frac{\mu_j\|\psi_j^* - \psi_j^0\|^2}{2}\right) + \kappa\varepsilon\sum_{j=1}^{n}\|\psi_j^*\|_1 \tag{54}$$

where k_n is chosen such that $k_n^* = k_n - \frac{g_{nd}}{2g_{n0}^2} > 0$

Let $\phi = \sum_{j=1}^{n}\left(\frac{\sigma_j\|\theta_j^*\|^2}{2} + \frac{\gamma_j\|\delta_j^*\|^2}{2} + \frac{\mu_j\|\psi_j^* - \psi_j^0\|^2}{2}\right) + \kappa\varepsilon\sum_{j=1}^{n}\|\psi_j^*\|_1$ .Choose k_j^* such that $k_j^* \ge \beta/2g_{j0}$, i.e., choose k_j such that $k_j \ge (\beta/2g_{j0})+(g_{jd}/2g_{j0}^2)$, where β is a positive constant. Choose $\sigma_j, \gamma_j, \mu_j, \Gamma_{j1}, \Gamma_{j2}$ and τ_j, such that $\sigma_j \ge \beta\lambda_{\max}\{\Gamma_{j1}^{-1}\}$, $\gamma_j \ge \beta\lambda_{\max}\{\Gamma_{j2}^{-1}\}$ and $\mu_j \ge \beta/\tau_{j\min}$, where $\lambda_{\max}\{\cdot\}$ is the largest eigenvalue of matrices. Then, from (54), we have the following inequality

$$\begin{aligned}\dot{V}_n &\le -\sum_{j=1}^{n} k_j^{*} e_j^2 - \sum_{j=1}^{n}\left(\frac{\sigma_j\|\tilde{\theta}_j\|^2}{2}+\frac{\gamma_j\|\tilde{\delta}_j\|^2}{2}+\frac{\mu_j\|\tilde{\psi}_j\|^2}{2}\right)+\phi \\ &\le -\sum_{j=1}^{n}\frac{\beta}{2g_{j0}}e_j^2 - \beta\sum_{j=1}^{n}\left(\frac{\tilde{\theta}_j^T\Gamma_{j1}^{-1}\tilde{\theta}_j}{2}+\frac{\tilde{\delta}_j^T\Gamma_{j2}^{-1}\tilde{\delta}_j}{2}+\frac{\tilde{\psi}_j^T\tilde{\psi}_j}{2\tau_j}\right)+\phi \\ &\le -\beta\left[\sum_{j=1}^{n}\frac{1}{2g_{j0}}e_j^2+\sum_{j=1}^{n}\left(\frac{\tilde{\theta}_j^T\Gamma_{j1}^{-1}\tilde{\theta}_j}{2}+\frac{\tilde{\delta}_j^T\Gamma_{j2}^{-1}\tilde{\delta}_j}{2}+\frac{\tilde{\psi}_j^T\tilde{\psi}_j}{2\tau_j}\right)\right]+\phi \\ &\le -\beta V_n+\phi \end{aligned} \tag{55}$$

The following theorem shows the stability and control performance of the closed-loop adaptive system.

Theorem1: Under the assumptions 1-4, consider the closed-loop system consisting of (1) and the known bounded reference signal $y_d(t), t \ge 0$, and choose the virtual control inputs (11), (28) and (38), the practical control input (47), the neural network weight adaptation laws(16), (32), (41) and (50). Assume that there exist sufficiently large compact sets Ω_{θ_i}, Ω_{δ_i}, Ω_{ψ_i} and Ω_i $i=1,\cdots,n$ with proper dimensions, such that $\theta_i \in \Omega_{\theta_i}$, $\delta_i \in \Omega_{\delta_i}$, $\psi_i \in \Omega_{\psi_i}$ and $e_i \in \Omega_i$ for all $t \ge 0$. Then for bounded initial conditions, all signals in the closed-loop system remain bounded and the output tracking error converges to a small neighbourhood around zero by an appropriate choice of the design parameters.

Proof:

From (55), we have

$$\dot{V}_n \le -k_{\min}^{*}\|e\|^2 - \frac{\sigma_{\min}\|\tilde{\theta}\|^2+\gamma_{\min}\|\tilde{\delta}\|^2+\mu_{\min}\|\tilde{\psi}\|^2}{2}+\phi \tag{56}$$

where $e=[e_1,e_2,\ldots,e_n]^T$, $\tilde{\theta}=[\tilde{\theta}_1^T,\tilde{\theta}_2^T,\ldots,\tilde{\theta}_n^T]^T$, $\tilde{\delta}=[\tilde{\delta}_1^T,\tilde{\delta}_2^T,\ldots,\tilde{\delta}_n^T]^T$, $\tilde{\psi}=[\tilde{\psi}_1^T,\tilde{\psi}_2^T,\ldots,\tilde{\psi}_n^T]^T$ and $(\cdot)_{\min}$ is the minimum of $(\cdot)_i, i=1,2,\ldots,n$. Therefore, the derivative of global Lyapunov function is negative as long as

$$e \notin \Omega = \left\{ e \,\middle|\, \|e\| \le \sqrt{\frac{\phi}{k_{\min}^{*}}} \right\} \tag{57}$$

or

$$\tilde{\theta} \notin \Omega_\theta = \left\{ \tilde{\theta} \,\middle|\, \|\tilde{\theta}\| \le \sqrt{\frac{2\phi}{\sigma_{\min}}} \right\} \tag{58}$$

or

$$\tilde{\delta} \notin \Omega_\delta = \left\{ \tilde{\delta} \,\middle|\, \|\tilde{\delta}\| \le \sqrt{\frac{2\phi}{\gamma_{\min}}} \right\} \tag{59}$$

or

$$\tilde{\psi} \notin \Omega_\psi = \left\{ \tilde{\psi} \,\middle|\, \|\tilde{\psi}\| \le \sqrt{\frac{2\phi}{\mu_{\min}}} \right\} \tag{60}$$

According to a standard Lyapunov theorem extension, these demonstrate the uniformly ultimately boundedness (UUB) of e, $\tilde{\theta}$, $\tilde{\delta}$ and $\tilde{\psi}$. Since $e_1 = x_1 - x_{1d}$ and x_{1d} is bounded, we have that x_1 is bounded. From $e_i = x_i - x_{id}, i = 2,3,\cdots n$ and the definitions of virtual controls x_{id}, we have that x_{id} remains bounded. Using (47), we conclude that the control input u is also bounded. According to the assumption of the system that the functions $f_i(x)$ and $g_i(x)$ are continuous, the functions $f_i(x)$ and $g_i(x)$ are bounded in any certainty compact. Thus, the optimal weights θ_i^* and δ_i^* are bounded, and the weights θ_i and δ_i of the neural networks are bounded. So, all the signals in the closed-loop system remain bounded.

Let $\rho = \phi / \beta$, then (55) satisfies

$$0 \le V_n(t) \le \rho + \left(V(0) - \rho\right)\exp(-\beta t) \tag{61}$$

From (61), we have

$$\sum_{i=1}^{n} \frac{1}{2g_i} e_i^2 < \rho + \left(V_n(0) - \rho\right)\exp(-\beta t) < \rho + V_n(0)\exp(-\beta t) \tag{62}$$

Let $g_{\max} = \max_{1 \le i \le n}\{g_{i1}\}$. Then we have

$$\frac{1}{2g_{\max}} \sum_{i=1}^{n} e_i^2 \le \sum_{i=1}^{n} \frac{1}{2g_i} e_i^2 < \rho + V_n(0)\exp(-\beta t) \tag{63}$$

that is

$$\sum_{i=1}^{n} e_i^2 < 2g_{\max}\rho + 2g_{\max}V_n(0)\exp(-\beta t) \tag{64}$$

which implies that given $\Delta > \sqrt{2g_{\max}\rho}$, there exists $T(\Delta)$, such that for all $t \ge T(\Delta)$, the tracking error satisfies $|e_1| = |x_1 - x_{1d}| = |y - y_d| < \Delta$. This concludes the proof.

Remark1: In (Ge & Wang, 2002), a neural network is adopted to approximate the nonlinear function $(f_i(x_i) - \dot{x}_{id}) / g_i(x_i)$. However, the derivatives of the virtual control x_{id} are included in the neural networks, so the dimensions of the input vectors of the neural networks become twice as much as those of the corresponding state vectors and the computational burden increases. Therefore, the approach in (Ge & Wang, 2002) is difficult to implement and apply in practice. In this paper, although two neural networks are adopted to approximate nonlinear functions $g_i^{-1}(x) f_i^{-1}(x)$ and $g_i^{-1}(x)$ respectively in every step, there are no dimensional increments and no additional parameters must be calculated. Compared with the approach in (Ge & Wang, 2002), the method presented in this paper is much simpler to understand and apply in practice.

4. SIMULATION

In this paper, two examples are used to verify the performance of the proposed controller.

Example 1

In this example, we discuss a real nonlinear system with nonlinear functions $f(x)$, $g(x)$ and disturbances $d(x)$. Let x_1 be the angle of the pendulum with respect to the vertical line. The dynamic equations of the inverted pendulum system are

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \frac{g_a \sin(x_1) - \dfrac{mlx_2^2 \cos(x_1)\sin(x_1)}{m_c+m}}{l\left(\dfrac{4}{3} - \dfrac{m\cos^2(x_1)}{m_c+m}\right)} + \frac{\dfrac{\cos(x_1)}{m_c+m}}{l\left(\dfrac{4}{3} - \dfrac{m\cos^2(x_1)}{m_c+m}\right)} u + d \\ &= f(\bar{x}_2) + g(\bar{x}_2)u + d_2 \\ y &= x_1 \end{aligned} \tag{65}$$

where

$d = 0.01\sin(20t)$	external disturbance;
$g_a = 9.8\, m/s^2$	acceleration due to gravity;
$m_c = 1\, kg$	mass of the cart;
$m = 0.1\, kg$	mass of the pole;
$l = 0.5\, m$	half-length of the pole;
u	applied force (control signal).

It is noticed that this system is non-triangular. The initial condition is $x_0 = [x_1(0), x_2(0)]^T = [0.1, 0]^T$ and the desired reference signal of the system is $y_d = 0.1\sin(t)$. In this example, we only need neural networks $\theta_2^T \xi_2(\bar{x}_2)$ and $\delta_2^T \eta_2(\bar{x}_2)$ to approximate $g_2^{-1}(\bar{x}_2) f_2(\bar{x}_2)$ and $g_2^{-1}(\bar{x}_2)$. In this paper, all the basis functions of the neural networks have the following form

$$G(\bar{x}_i) = \exp\left[-\frac{(\bar{x}_i - u_i)^T(\bar{x}_i - u_i)}{v_i^2}\right] \tag{66}$$

where $u_i = [u_{i1}, u_{i2}, \dots, u_{ij}]^T$ is the centre of the receptive field and v_i is the width of the Gaussian function. The neural networks $\theta_2^T \xi_2(\bar{x}_2)$ and $\delta_2^T \eta_2(\bar{x}_2)$ all contain 169 nodes, with centers u_j evenly spaced in $[-6,6]\times[-6,6]$ and widths $v_j = 1$ $(j = 1, 2, \dots, 169)$.

Step 1: Let $x_{1d} = y_d$ and define $e_1 = x_1 - x_{1d}$. The virtual controller is chosen as

$$x_{2d} = \dot{x}_{1d} - k_1 e_1 \tag{67}$$

Step 2: Define $e_2 = x_2 - x_{2d}$. We obtain the control law

$$u = -e_1 - \theta_2^T \xi_2(\bar{x}_2) + \delta_2^T \eta_2(\bar{x}_2)\dot{x}_{2d} - k_2 e_2 + H_2 \tag{68}$$

and adaptation laws

$$\dot{\theta}_2 = \Gamma_{21}\left[e_2\xi_2(\bar{x}_2) - \sigma_2\theta_2\right]$$

$$\dot{\delta}_2 = \Gamma_{22}\left[-e_2\eta_2(\bar{x}_2)\dot{x}_{2d} - \gamma_2\delta_2\right] \tag{69}$$

The bounding control function H_2 is chosen as

$$H_2 = -\psi_2^T w_2 \tag{70}$$

$$w_2 = \begin{bmatrix} w_{p2} \\ w_{q2} \\ w_{d2} \end{bmatrix} = \begin{bmatrix} s_{p2}\tanh\left(e_2 s_{p2}/\varepsilon\right) \\ \dot{x}_{2d}s_{q2}\tanh\left(e_2\dot{x}_{2d}s_{q2}/\varepsilon\right) \\ \left(s_{d2}/g_{20}\right)\tanh\left(e_2 s_{d2}/g_{20}\varepsilon\right) \end{bmatrix} \tag{71}$$

$$\begin{aligned}\dot{\psi}_2 &= \tau_2\left[e_2 w_2 - \mu_2\left(\psi_2 - \psi_2^0\right)\right] \\ &= \tau_2\begin{bmatrix} e_2 w_{p2} - \mu_2\left[\psi_{p2} - \psi_{p2}^0\right] \\ e_2 w_{q2} - \mu_2\left[\psi_{q2} - \psi_{q2}^0\right] \\ e_2 w_{d2} - \mu_2\left[\psi_{d2} - \psi_{d2}^0\right] \end{bmatrix}\end{aligned} \tag{72}$$

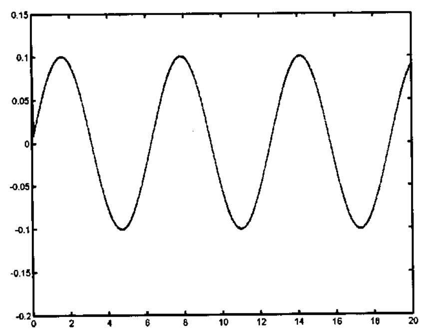

Fig. 1 The output y and the reference signal y_d (y-solid line and y_d-dash line)

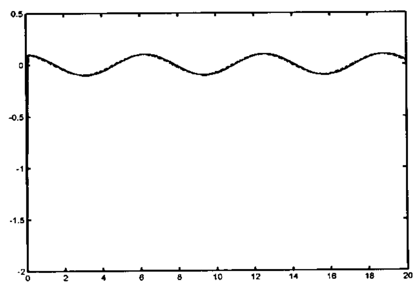

Fig. 2 The state x_2 and the reference signal $\dot{y}_d$ (x_2-solid line and $\dot{y}_d$-dash line)

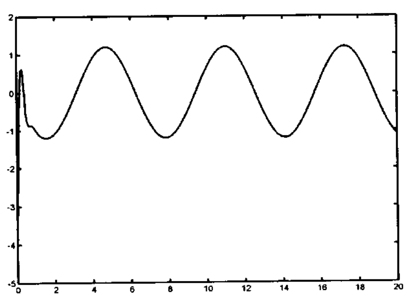

Fig.3 The control input u of the system

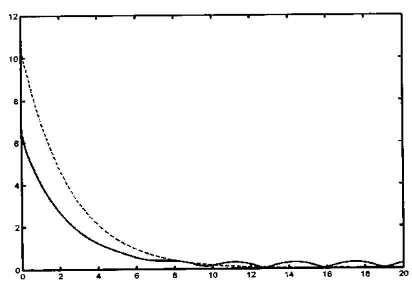

Fig. 4 L_2 norms of the neural network weights (θ_2-solid line and δ_2- dash line)

The design parameters of the controller are $k_1 = k_2 = 5$, $\Gamma_2 = diag\{5\}$, $\sigma_2 = \gamma_2 = 0.2$, $\tau_2 = 10$, $\mu_2 = 0.1$ and $\varepsilon = 0.05$. The initial weights θ_2 are all given arbitrarily in $[-1,1]$, and δ_2 in $[0,1]$. The initial values of updating parameters are $\psi_2(0) = [1,1,1]^T$. For simplicity, let $s_{p2} = s_{q2} = s_{d2} = 1$. The simulation results are shown in Fig.1-4.

From Fig.1 and 2, it can be inferred that the system output and state track the desired signals very well by the proposed controller. The boundedness of the control input u and neural network weights are shown in Fig.3 and Fig.4. The inverted pendulum simulation results demonstrate the effectiveness of the proposed approach. It also indicates that this method can be used for non- triangular systems.

Example 2

The model of the strict-feedback systems is described as follows

$$\begin{aligned} \dot{x}_1 &= 0.5x_1 + \left(1 + 0.1x_1^2\right)x_2 + d_1 \\ \dot{x}_2 &= x_1 x_2 + \left[2 + \cos(x_1)\right]u + d_2 \\ y &= x_1 \end{aligned} \tag{73}$$

where x_1 and x_2 are states, y is the output of the system, and $d_1 = -\sin x_1$ and $d_1 = -0.5(e^{x_1} + e^{-x_1})$ are external disturbances. The initial condition is $x_0 = \left[x_1(0), x_2(0)\right]^T = [1,0]^T$ and the desired reference signal of the system is $y_d = \sin(t)$.

Step 1: Let $x_{1d} = y_d$ and define $e_1 = x_1 - x_{1d}$. The virtual controller is chosen as

$$x_{2d} = -\theta_1^T \xi_1(x_1) + \delta_1^T \eta_1(x_1)\dot{x}_{1d} - k_1 e_1 + H_1 \tag{74}$$

Choose the following adaptation laws

$$\begin{aligned} \dot{\theta}_1 &= \Gamma_{11}\left[e_1\xi_1(x_1) - \sigma_1\theta_1\right] \\ \dot{\delta}_1 &= \Gamma_{12}\left[-e_1\eta_1(x_1)\dot{x}_{1d} - \gamma_1\delta_1\right] \end{aligned} \tag{75}$$

and the bounding control function H_1

$$H_1 = -\psi_1^T w_1 \tag{76}$$

$$w_1 = \begin{bmatrix} w_{p1} \\ w_{q1} \\ w_{d1} \end{bmatrix} = \begin{bmatrix} s_{p1}\tanh\left(e_1 s_{p1}/\varepsilon\right) \\ \dot{x}_{1d} s_{q1}\tanh\left(e_1\dot{x}_{1d}s_{q1}/\varepsilon\right) \\ \left(s_{d1}/g_{10}\right)\tanh\left(e_1 s_{d1}/g_{10}\varepsilon\right) \end{bmatrix} \tag{77}$$

$$\begin{aligned} \dot{\psi}_1 &= \tau_1\left[e_1 w_1 - \mu_1\left(\psi_1 - \psi_1^0\right)\right] \\ &= \tau_1\begin{bmatrix} e_1 w_{p1} - \mu_1\left(\psi_{p1} - \psi_{p1}^0\right) \\ e_1 w_{q1} - \mu_1\left(\psi_{q1} - \psi_{q1}^0\right) \\ e_1 w_{d1} - \mu_1\left(\psi_{d1} - \psi_{d1}^0\right) \end{bmatrix} \end{aligned} \tag{78}$$

Step 2 is the same way as that of example 1.

The neural networks $\theta_1^T\xi_1(x_1)$ and $\delta_1^T\eta_1(x_1)$ all contain 13 nodes, with centers u_j evenly spaced in $[-6,6]$ and widths $v_j=1\,(j=1,2,\ldots,13)$. The neural networks $\theta_2^T\xi_2(\bar{x}_2)$ and $\delta_2^T\eta_2(\bar{x}_2)$ all contain 169 nodes, with centers u_j evenly spaced in $[-6,6]\times[-6,6]$ and widths $v_j=1\,(j=1,2,\ldots,169)$. The design parameters of the controller are $k_1=k_2=2$, $\Gamma_1=\Gamma_2=diag\{2\}$, $\sigma_1=\sigma_2=\gamma_1=\gamma_2=0.2$, $\tau_1=\tau_2=10$, $\mu_1=\mu_2=0.1$ and $\varepsilon=0.05$. The initial weights θ_1 and θ_2 are all given arbitrarily in $[-1,1]$, and δ_1 and δ_2 in $[0,1]$. The initial values of updating parameters are $\psi_1(0)=\psi_2(0)=[1,1,1]^T$. For simplicity, let $s_{p1}=s_{p2}=s_{q1}=s_{q2}=s_{d1}=s_{d2}=1$. Fig.5-8 show the simulation results.

From Fig.5 and 6, we can see that tracking convergence is very fast and good tracking performance is obtained by applying the design controller. Fig.7 and 8 show the boundedness of the control input u and neural network weights, respectively. The simulation results demonstrate the feasibility of the proposed approach.

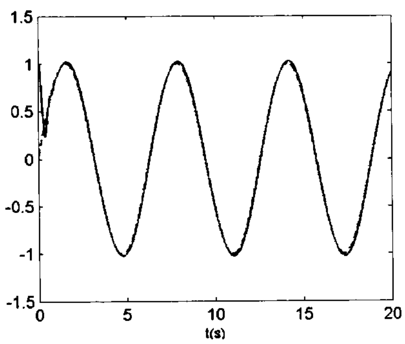

Fig. 5 The output y and the reference signal y_d (y-solid line and y_d-dash line)

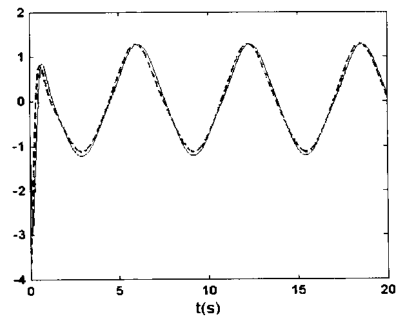

Fig. 6 The state x_2 and the reference signal $\dot{y}_d$ (x_2-solid line and $\dot{y}_d$-dash line)

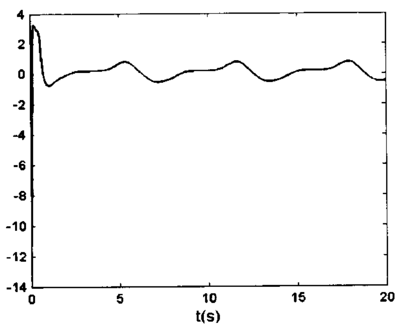

Fig. 7 The control input u of the system

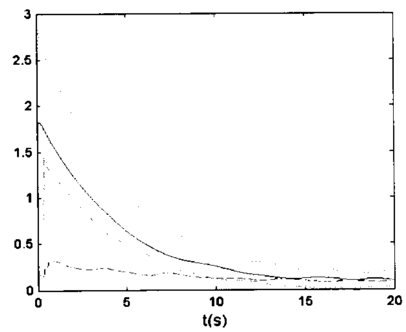

Fig.8 L_2 norms of the neural network weights (θ_1- solid line, θ_2- dash line, δ_1- dot line and δ_2- dash-dotted line)

5. CONCLUSION

In this paper, a stable adaptive neural network control approach is proposed for a class of unknown nonlinear systems in the strict-feedback form with disturbances based on backstepping. The developed approach can avoid controller singularity problem perfectly. As the estimates of unknown neural network approximation error bound and external disturbances bound are adjusted adaptively, the robustness of the closed-loop system is improved and the application scope of nonlinear systems is extended. All the signals of the closed-loop system are guaranteed to be uniformly ultimately bounded, and the output of system is proven to converge to a small neighbourhood of zero. The simulation results demonstrate the feasibility of the control approach.

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