A MARKOVIAN ANALYSIS OF HURRICANE TRANSITIONS

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Abstract. The present study considers the random phenomenon that is hurricane tracking in terms of hurricane intensity as measured by hurricane category defined by the newly proposed scale outlined by Wooten and Tsokos, (2007); in conjunction with conditional and unconditional Markov chains. The bi-conditional Markov chains are defined by a previous pressure index and the hurricane stage; that is, before or after the storm reaches its peak intensity or wind speed.

1. INTRODUCTION

Hurricanes are a public and economical concern around the globe. The present study addresses the transitional states of hurricanes from the initial formulations, tropical storms to a level five hurricane, using conditional Markov chains and Monte Carlo simulation to predict hurricane intensity. Markov chains and Monte Carlo simulation used previously to predict annual cyclone counts, (Chu and Zhao, 2004). The proposed modeling has not been used to predict hurricane status nor have conditional Markov chains been used previously in predicting hurricane status by category. To our knowledge this is the first study that uses Markovian modeling for predicting hurricane stage transitions. The results of the present study are encouraging.

Recent studies, (Elsner. and Jagger, 2004; Jagger and Elsner, 2006; Wooten and Tsokos, 2007, Wooten and Tsokos, 2007), among others, have shown that there is a strong correlation between **hurricane force winds** and the **atmospheric temperatures**, however presently temperatures are not available. Consider the *transitional probabilities*; that is, the probability that a storm in a given category transitions into a stronger storm or higher category, transition into a weaker storm or lower category, or remains in the given state or category. Hence, we considered the transitional probabilities based on the *storm stage*; that is, whether the storm has reached maximum intensity: either B for before the storm peak or A for after the storm peaks. Furthermore, under the

assumption that volume is constant, the ideal gas law states that the **atmospheric temperatures** are proportional to **atmospheric pressures**, thus we shall use *pressures index* on which we can generate conditional Markov chains in an effort to accurately predict the probable intensity of a storm.

2. PRELIMINARY REMARKS ON MARKOV CHAINS

When considering a random process regularly over time, the observations of the various outcomes generates a sequence represented by the random variable $x_1, x_2, ..., x_n$, defined on the space X of all possible values that the random variable can assume. The space X is called the *state space* of the sequence and the different values that the random variables can assume are the *states*.

The primary question is given $x_1 = i_1, x_2 = i_2, ..., x_{n-1} = i_{n-1}$, where, for all integers k, $i_k \in X$, what is the probability that $x_n = i_n$; that is, what is the probability that the process is in the state i_n given the previous history of the process.

Readily addressed using the concept of Markov chains; the probabilities of the process being in state i_n, $x_n = i_n$, depends only on the previous state i_{n-1}, $x_{n-1} = i_{n-1}$. More precisely, a Markov chain is such that

$$P(x_n = i_n \mid x_1 = i_1, x_2 = i_2, ..., x_{n-1} = i_{n-1}) = P(x_n = i_n \mid x_{n-1} = i_{n-1}).$$

Let i and j represent any two states. The conditional probabilities that the process moves to state j at time n, given it is in state i at time $n-1$, are called *transitional probabilities*, denoted by p_{ij}:

$$p_{ij} = P(x_n = j \mid x_{n-1} = i),$$

the probability that the process moves from state i to state j in one step or at one trial.

The transition probabilities p_{ij}, satisfies the following two conditions:

1. $p_{ij} \geq 0$ for all i and j, and

2. for every i, $\sum_{j=1}^{k} p_{ij} = 1$, where $k = n(X)$.

The *transition probability matrix* of a process having k states given by

$$P = \begin{array}{c} \\ 1 \\ 2 \\ \vdots \\ k \end{array} \begin{array}{cccc} 1 & 2 & \cdots & k \\ \left[\begin{array}{cccc} p_{11} & p_{12} & \cdots & p_{1k} \\ p_{21} & p_{22} & \cdots & p_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ p_{k1} & p_{k2} & \cdots & p_{kk} \end{array} \right] \end{array}.$$

The basic idea of a finite Markov chain, given as a *directed graph* or *state diagram*, is given in Figure 1. The loops labeled $p_{11}, p_{22}, p_{33}, ..., p_{kk}$ are the probabilities that a process remains in its given state. The arcs, $p_{12}, p_{21}, ...$, are the probabilities that a process changes to a different state; the arrows indicate the direction of the transition.
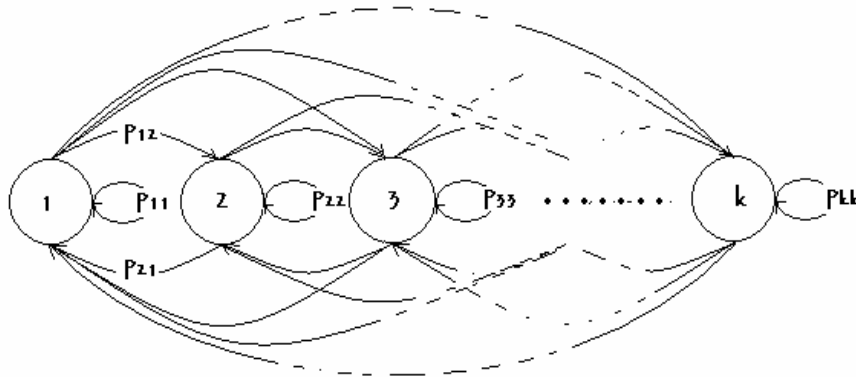

Figure 1:   General state diagram – transitional probaiblities

For example, p_{24} is the probability that a hurricane category two will reorganize and gain strength to a hurricane category four, p_{45} is the probability that a hurricane category four will reorganize and gain strength to a hurricane category five, p_{33} (the loop) is the probability that a hurricane category three will maintain its status, etc.

Markov chains are useful in hurricane analysis in that when we have information from tropical storms, hurricane category 1, hurricane category 2, etc. to develp a Markovian model for such as system. That is, when we are presently at level two hurricane and we wish to determine the probability of becoming a hurricane level three, or remaining level two or return to level one, etc. Knowing such probabilities of

transitions are quite useful for strategic planning, among others. Figure 2 illustrates the state diagram of such a Markovian model.
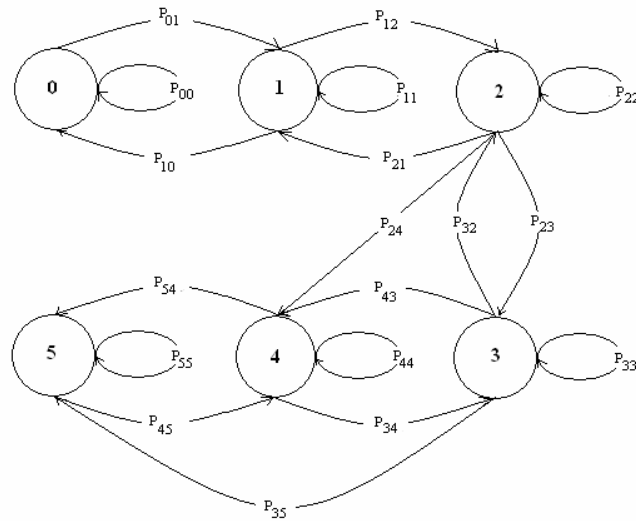

Figure 2:   State diagram for hurricane transformations

3.   CONDITIONAL MARKOVIAN ANALYSIS OF HURRICANE TRANSFORMATIONS

Using real data from twenty-two storms formed from 2004 and 2005 that have reach hurricane status and consider the three random variables: **atmospheric pressure index**, **storm stage**, and **hurricane status** by category as defined by the proposed scale given in Table 1; $P(t)$, $S(t)$, and $C(t)$, respectively. Since the atmosphere pressure, P, is measured on a continue scale and historically ranges between 880 and 1020, a range of 140 units measured in hPa; therefore to reduce this to 14 different levels, definite the atmospheric pressure index to be $\text{int}\left[\frac{P}{10}\right]$; the greatest integer below $\frac{P}{10}$ and ranges from 88 thru 101.

The main variable of interest is the hurricane status as defined by the six categories, $X = \{0,1,2,3,4,5\}$. Then the possible transitions are: p_{00}, p_{01}, p_{10}, p_{11}, p_{12}, p_{21}, p_{22}, p_{23}, p_{24}, p_{32}, p_{33}, p_{34}, p_{35}, p_{43}, p_{44}, p_{45}, p_{54}, and p_{55}; empirically all other transitions are improbable.

Table 1: Proposed Scaling Index for Hurricanes Intensity as Defined by Wind Speed.

Type: Proposed Scale	Category	Pressure (hPa)	Wind (knots)
Tropical Depression/ Tropical Storm	0	995-1010	10-42
Hurricane	1	972-994	43-77
Hurricane	2	951-971	78-102
Hurricane	3	932-950	103-122
Hurricane	4	911-931	123-142
Hurricane	5	<911	>143

Let the number of readings for the transitional between state $i = C(t)$ to state $j = C(t + \Delta t)$ given the pressure index $P = P(t)$ and the hurricane stage $S = S(t) \in \{A, B\}$ be denoted $n(j \text{ given } i, P, S) = n(i \mid j \parallel P, S)$. Then the independent percentages are

$$p(i \mid j \parallel P, S) = \frac{n(i \mid j \parallel P, S)}{n}$$

and the transitional probabilities are characterized by

$$P(i \mid j \parallel P, S) = \frac{p(i \mid j \parallel P, S)}{\sum_j p(i \mid j \parallel P, S)}.$$

Before a storm has reached its peak, B, consider the probability that a storm will transform from a hurricane category 2 to a hurricane category 4 between measurements, that is p_{24}, Table 2 shows that this can occur when the pressure are between 930 and 949, inclusive. However, this will only occur before the storm hits its peak. Once the storm as reached its peak, such a re-intensification is approximately zero; this is illustrated by the lack of p_{24} in Table 3.

Given in Table 2 are the conditional probabilities for each probable transition (given in each column) and for the given pressure index (given in each row) before (B) the storm has obtained peak wind speed, $S = B$. When the pressure is below 900 hPa, the storm will be hurricane category 5, $p_{55} = 1.00$; but when the pressure is between 900 hPa and 910 hPa, there is a two in three chance the storm will increase from hurricane category 4 to hurricane category 5 as defined by the scale outlined by Wooten and Tsokos, (2007); that is, $P(4 \mid 5 \parallel 90, B) = 0.67$.

In addition, the higher the pressure, the lower expected intensity of the storm. In fact, for pressures greater than 1010 hPa, tropical storms will remain a tropical storm,

$P(0\,|\,0\,\|\,101,B)=1.00$; only after the storm drops below 1010 hPa will the storm intensity, $P(0\,|\,1\,\|\,100,B)=0.17$. Furthermore, once a tropical storm transforms into a hurricane category 1, there is a slight chance even at these high pressures that the storm will continue to intensify, $P(1\,|\,2\,\|\,100,B)=0.02$. Moreover, one the pressures drop below 970, a storm will form, $P(0\,|\,1\,\|\,97,B)=1.00$.

Table 2:   Conditional transitional probabilities before the storm hits peak intensity.

(B)	p_{00}	p_{01}	p_{10}	p_{11}	p_{12}	p_{21}	p_{22}	p_{23}	p_{24}	p_{32}	p_{33}	p_{34}	p_{35}	p_{43}	p_{44}	p_{45}	p_{55}
89	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1.00
90	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0.33	0.67	-
91	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1.00	-	1.00
92	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0.93	0.07	-
93	-	-	-	-	-	-	-	-	1.00	-	0.78	0.22	-	0.11	0.89	-	-
94	-	-	-	-	-	-	0.86	-	0.14	-	0.87	0.09	0.04	-	1.00	-	-
95	-	-	-	-	-	-	0.43	0.57	-	0.06	0.92	0.03	-	-	-	-	-
96	-	-	-	-	-	-	0.87	0.13	-	0.25	0.50	0.25	-	-	-	-	-
97	-	1.00	0.04	0.74	0.22	0.03	0.94	0.03	-	-	-	-	-	-	-	-	-
98	-	-	-	0.86	0.14	0.33	0.67	-	-	-	-	-	-	-	-	-	-
99	0.25	0.75	0.01	0.99	-	-	1.00	-	-	-	-	-	-	-	-	-	-
100	0.83	0.17	0.09	0.89	0.02	-	-	-	-	-	-	-	-	-	-	-	-
101	1.00	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

After a storm as reached its peak intensity, A, consider the probability that a storm will transform from a hurricane category 2 to a hurricane category 4 between measurements, that is p_{24}, Table 3 shows that this has never occurred, $P(2\,|\,4,P,A)=0.00$. Before the storm hits its peak, there is enough energy in the atmosphere to quickly intensify, but once a storm has released its energies, it is unlike to intensify, but if so, not as readily.

Table 3, gives the conditional probabilities for each probable transition (given in each column) after the storm has obtained peak wind speed, $S=A$, for the various pressure indices (given in each row). Here, we see that only storms in full force have pressures as low as 882 hPa; it should further be noted that this low pressure occurred during Hurricane Wilma in 2005, which occurred extremely late in the season when temperatures where lower. Furthermore, after the storm has reached its maximum

intensity even with pressures between 1000 hPa and 1010 hPa the storm is more like to weaken, $P(1\,|\,0\,\|\,100,A)=0.67$; or stay as intense, $P(1\,|\,1\,\|\,100,A)=0.33$, but not strengthen, $P(0\,|\,0\,\|\,100,A)=1.00$ and $P(i\,|\,i+1\,\|\,100,A)=0.00$ for $i=0,1,2,3,4$.

Table 3: Conditional transitional probabilities after the storm hits peak intensity

(A)	p_{00}	p_{10}	p_{11}	p_{12}	p_{21}	p_{22}	p_{23}	p_{32}	p_{33}	p_{34}	p_{43}	p_{44}	p_{54}	p_{55}
88	-	-	-	-	-	-	-	-	-	-	-	-	-	1.00
89	-	-	-	-	-	-	-	-	-	-	-	1.00	-	1.00
90	-	-	-	-	-	-	-	-	-	-	-	1.00	0.50	0.50
91	-	-	-	-	-	-	-	-	1.00	-	0.09	0.91	0.25	0.75
92	-	-	-	-	-	-	-	0.11	0.89	-	0.10	0.90	-	-
93	-	-	-	-	-	-	-	-	0.93	0.07	0.17	0.83	-	-
94	-	-	-	-	-	0.67	0.33	0.23	0.77	-	0.50	0.50	-	-
95	-	-	-	-	0.13	0.74	0.13	0.48	0.52	-	1.00	-	-	-
96	-	-	1.00	-	0.08	0.89	0.03	-	-	-	-	-	-	-
97	-	-	1.00	-	0.31	0.69	-	-	-	-	-	-	-	-
98	1.00	0.06	0.92	0.02	0.25	0.75	-	-	-	-	-	-	-	-
99	1.00	0.13	0.87	-	-	-	-	-	-	-	-	-	-	-
100	1.00	0.67	0.33	-	-	-	-	-	-	-	-	-	-	-
101	1.00	-	-	-	-	-	-	-	-	-	-	-	-	-

Once we have established these transitional probabilities, we can use these Markov chains and Monte Carlo simulation to verify the accuracy of these probabilities.

4.   PREDICTING HURRICANE KATRINA

Using these transitional probabilities, we can predict the next state of the hurricane using Monte Carlo simulation. That is, given the pressure index $P(t)$ and the level of intensity indexed by hurricane category $C(t)$, we select the next probable state $C(t+\Delta t)$. To measure the percent correct, let c denote the number of correct prediction and d be the duration of the storm in terms of the number of readings; then, the percent correct $p=\dfrac{c}{d}$. To measure the error in the simulation, we define the absolute error at each step as $\varepsilon(t)=\left|C(t)-\hat{C}(t)\right|$ and the absolute error as $\varepsilon=\sum_{t}\varepsilon(t)$.

Furthermore, define total error squared as $E = \sum_t [\varepsilon(t)]^2$ and the mean error squared as

$$\overline{E} = \frac{E}{d}.$$

Table 4 shows twenty simulations of Hurricane Katrina, the minimum percent correctly predicted is 72%; that is, 43 out of a total of 60 readings taken during Hurricane Katrina. On average, these conditional Markov chains accurately predict the intensity of the storm 78% of the time. The maximum absolute error was 18, which is relative small since at most a hurricane will jump two categories and therefore the maximum possible total error squared is 240. Even if at most there is only a unit difference in the predicted category, the maximum total error squared is 120.

Table 4:   Simulation of Hurricane Katrina

Simulation	Percent Correct	Correct	Absolute Error	Total Error Squared	Mean Error Squared
1	78%	47	15	19	0.32
2	83%	50	12	16	0.27
3	77%	46	16	20	0.33
4	80%	48	12	12	0.20
5	77%	46	17	23	0.38
6	75%	45	17	21	0.35
7	72%	43	18	20	0.33
8	77%	46	16	20	0.33
9	82%	49	12	14	0.23
10	78%	47	14	16	0.27
11	75%	45	18	24	0.40
12	80%	48	14	18	0.30
13	83%	50	10	10	0.17
14	82%	49	11	11	0.18
15	85%	51	9	9	0.15
16	77%	46	15	17	0.28
17	78%	47	14	16	0.27
18	75%	45	18	24	0.40
19	80%	48	12	12	0.20
20	73%	44	17	19	0.32

Similarly, assuming a maximum jump of two categories, the maximum possible mean error squared is four (4); therefore, with a maximum mean error squared of 0.40, these conditional Markov chains accurately predict the next state of a hurricane. That is, the Markov chains accurately predict the next state of a hurricane given the present state and pressure index.

Similar results occurred when simulating the remaining 21 hurricanes used in this study as shown in Table 5. The weaker the storm intensity or the shorter the duration of the storm, the less accurate the simulation; that is, the more intense a hurricane, the better the conditional Markov chains predict the transitional states of the hurricane. However, on average these conditional Markov chains accurately predict 81% of the next hurricane status. The worst prediction was for Hurricane Vince in 2005 which only lasted 42 hours and reach hurricane status category 1 with 63% (5 out of 8) of the next states predicted correctly.

Table 5:   Simulation of remaining hurricanes in the 2004 and 2005 season

Storm	Max Index	Reading	Percent Correct	Correct	Absolute Error	Total Error Squared	Mean Error Squared
Alex	3	32	78%	25	7	7	0.22
Beta	2	32	75%	24	9	11	0.34
Charley	4	48	81%	39	9	9	0.19
Danielle	2	32	91%	29	3	3	0.09
Dennis	4	57	77%	44	14	16	0.28
Emily	4	79	70%	55	24	24	0.30
Epsilon	1	36	97%	35	1	1	0.03
Frances	4	87	85%	74	13	13	0.15
Irene	2	49	86%	42	7	7	0.14
Ivan	5	95	77%	73	22	22	0.23
Jeanne	3	93	83%	77	17	19	0.20
Karl	3	33	76%	25	8	8	0.24
Lisa	1	63	83%	52	11	11	0.17
Maria	2	33	82%	27	6	6	0.18
Nate	2	25	72%	18	7	7	0.28
Ophelia	2	89	87%	77	12	12	0.13
Philippe	1	27	85%	23	4	4	0.15
Rita	5	54	83%	45	10	12	0.22
Stan	1	25	92%	23	2	2	0.08
Vince	1	8	63%	5	3	3	0.38
Wilma	5	79	81%	64	17	21	0.27

5.   TESTING HURRICANE STAGE

To test if the condition of hurricane stage is necessary, consider the Markov chains under the pressure index alone without the hurricane stage: before the storm reaches its peak (B) and after the storm reaches its peak (A). Let the number of readings for the transitional between state $i = C(t)$ to state $j = C(t+\Delta t)$ given the pressure index $P = P(t)$ denoted $n(j \text{ given } i, P) = n(i \mid j \| P)$; then the independent percentages are $p(i \mid j \| P) = \dfrac{n(i \mid j \| P)}{n}$ and the transitional probabilities are given by

$$P(i \mid j \| P) = \frac{p(i \mid j \| P)}{\sum_{j} p(i \mid j \| P)}.$$

The conditional transition probabilities given in Table 6 are the transitional probability p_{ij} given the pressure index P, but not the hurricane stage S.

Table 6: Conditional transitional probabilities by pressure index

	P_{00}	P_{01}	P_{10}	P_{11}	P_{12}	P_{21}	P_{22}	P_{23}	P_{24}	P_{32}	P_{33}	P_{34}	P_{35}	P_{43}	P_{44}	P_{45}	P_{54}	P_{55}
88	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1.00
89	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1.00	-	-	1.00
90	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0.71	0.29	0.50	0.50
91	-	-	-	-	-	-	-	-	-	1.00	-	-	-	0.09	0.91	-	0.20	0.80
92	-	-	-	-	-	-	-	-	-	0.11	0.89	-	-	0.04	0.92	0.04	-	-
93	-	-	-	-	-	-	-	1.00	-	0.89	0.11	-	-	0.15	0.85	-	-	-
94	-	-	-	-	-	0.80	0.10	0.10	0.11	0.82	0.04	0.02	0.43	0.57	-	-	-	-
95	-	-	-	-	-	0.11	0.68	0.21	-	0.21	0.77	0.02	-	1.00	-	-	-	-
96	-	-	-	1.00	-	0.03	0.88	0.09	-	0.25	0.50	0.25	-	-	-	-	-	-
97	-	1.00	0.02	0.86	0.12	0.16	0.83	0.02	-	-	-	-	-	-	-	-	-	-
98	-	-	0.02	0.88	0.09	0.30	0.70	-	-	-	-	-	-	-	-	-	-	-
99	-	1.00	0.03	0.97	-	-	1.00	-	-	-	-	-	-	-	-	-	-	-
100	1.00	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
101	1.00	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-

Shown in Table 7, are twenty simulations using the transitional probabilities given in Table 6 (without this secondary condition) the accuracy of the Markov chains reduces;

on average, only 71% of the next state is predicted accurately compared to 78% when the additional conditions are present. Furthermore, the minimum percent correct drops to a to of 67%; which when compared to a minimum of 72% when the hurricane stage in take into consideration. Moreover, without this additional condition, the maximum percent correct is only 77% compared to 83% with the secondary condition. The mean square error on average is only slightly higher at 1.2 times larger than when both conditions are considered. The mean error squared considering the pressure index range between 0.22 and 0.48.

Table 7: Simulation of Hurricane Katrina: Pressure Index Only

Simulation 2	Percent Correct	Correct	Absolute Error	Total Error Squared	Mean Error Squared
1	68%	41	21	25	0.42
2	72%	43	17	17	0.28
3	75%	45	18	24	0.40
4	72%	43	20	26	0.43
5	72%	43	19	23	0.38
6	78%	47	13	13	0.22
7	75%	45	16	18	0.30
8	65%	39	22	24	0.40
9	73%	44	16	16	0.27
10	72%	43	18	20	0.33
11	72%	43	17	17	0.28
12	67%	40	23	29	0.48
13	68%	41	19	19	0.32
14	67%	40	23	29	0.48
15	67%	40	22	26	0.43
16	77%	46	14	14	0.23
17	73%	44	17	19	0.32
18	70%	42	18	18	0.30
19	75%	45	15	15	0.25
20	70%	42	19	21	0.35

6. TESTING THE PRESSURE INDEX

To test if the condition of pressure index is necessary, consider the Markov chains under the hurricane stage alone without the pressure index. Let the number of readings for the transitional between state $i = C(t)$ to state $j = C(t + \Delta t)$ given the hurricane stage

$S = S(t) \in \{A, B\}$ are denoted $n(j \text{ given } i, S) = n(i \mid j \| S)$. Then the independent percentages are

$$p(i \mid j \| S) = \frac{n(i \mid j \| S)}{n}$$

and the transitional probabilities are

$$P(i \mid j \| S) = \frac{p(i \mid j \| S)}{\sum_j p(i \mid j \| S)}.$$

The conditional transition probabilities given in Table 8 are the transitional probability p_{ij} given the hurricane stage S, but not the pressure index P. Before the storm hit its peak wind speed, there is an 18% chance that a tropical storm with increase in intensity to hurricane category 1, $P(0 \mid 1 \| B) = 0.18$ whereas after a storm as released its energy, the storm is extremely unlikely to reform, $P(0 \mid 0 \| A) = 1.00$.

Table 8: Conditional transitional probabilities by hurricane stage

	P_{00}	P_{01}	P_{10}	P_{11}	P_{12}	P_{21}	P_{22}	P_{23}	P_{24}	P_{32}	P_{33}	P_{34}	P_{35}	P_{43}	P_{44}	P_{45}	P_{54}	P_{55}
A	1.00	-	0.10	0.90	0.01	0.17	0.78	0.06	-	0.20	0.78	0.02	-	0.15	0.85	-	0.25	0.75
B	0.82	0.18	0.02	0.92	0.06	0.03	0.85	0.11	.02	0.04	0.86	0.08	0.01	0.04	0.86	0.11	-	1.00

Table 9, shows twenty simulations, and without this primary condition of pressure, the accuracy of the Markov chains is reduced, but surprisingly not as much as when the secondary condition of hurricane stage is considered. On average, predicting only 77% of the next state accurately compared to 78% when the additional conditions are present and 73% when only the pressure index is considered. The minimum percent correct drops to a to of 67%; which is the same as when only the pressure index is used. However, without the additional pressure index, the maximum percent correct is 82% compared to 85%. Furthermore, the mean square error on average is only slightly higher than when both conditions at 1.05 times larger then the mean square error originally measure. The mean error squared considering the pressure index range between 0.18 and 0.53.

Table 9:   Simulation of Hurricane Katrina: Hurricane Stage Only

Simulation 3	Percent Correct	Correct	Absolute Error	Total Error Squared	Mean Error Squared
1	85%	51	10	12	0.20
2	78%	47	14	16	0.27
3	70%	42	19	21	0.35
4	77%	46	16	20	0.33
5	68%	41	20	22	0.37
6	73%	44	19	25	0.42
7	67%	40	24	32	0.53
8	82%	49	11	11	0.18
9	75%	45	16	18	0.30
10	80%	48	13	15	0.25
11	73%	44	17	19	0.32
12	75%	45	16	18	0.30
13	77%	46	15	17	0.28
14	82%	49	13	17	0.28
15	72%	43	18	20	0.33
16	77%	46	15	17	0.28
17	80%	48	13	15	0.25
18	85%	51	10	12	0.20
19	85%	51	10	12	0.20
20	73%	44	17	19	0.32

Hence, the apparent available heat energy is a better indicator that storm intensity will strengthen. This is consistent with previous studies (Wooten and Tsokos, 2007), that temperature is a better indicator of hurricane force winds.

7.   UNCONDITIONAL MARKOV CHAINS

Consider the transitional probabilities, given in Table 10, without any assumption regarding the hurricane stage or pressure index. Let the number of readings for the transitional between state i to state j be denoted $n(j \text{ given } i) = n(i \mid j)$ and the total number of readings be denoted simply as n. Then the independent percentages are

$$p(i \mid j) = \frac{n(i \mid j)}{n}$$

and the transitional probabilities are

$$p_{ij} = P(i \mid j) = \frac{p(i \mid j)}{\sum_j p(i \mid j)}$$

and the associated state diagram is given in Figure 3, where the various states are the hurricane status of a storm as defined by the index given in Table 1.

As shown in Table 10, in general, the chance that a tropical storm intensifies into a hurricane category 1 is 15%, $p_{01} = 0.15$; however, the majority of the time the tropical storms does not intensify but simply remain a tropical storm and dissipate, $p_{00} = 0.85$. Once a hurricane category 1 is formed, there is a very good chance it will remain a hurricane, $p_{11} + p_{12} = 0.96$ and is very unlikely to weaken and return to tropical storm status, $p_{10} = 0.04$. Additionally, the chance of a hurricane category 1 intensifying, $p_{12} = 0.05$ is only slightly higher than the chance of the storm weakening, $p_{10} = 0.04$.

Table 10:   Transitional Probabilities

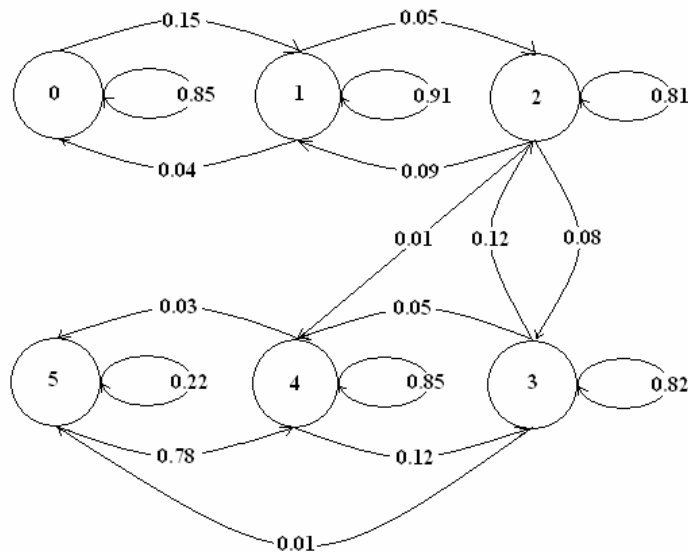|	0	1	2	3	4	5
0	0.85	0.15	-	-	-	-
1	0.04	0.91	0.05	-	-	-
2	-	0.09	0.81	0.08	0.01	-
3	-	-	0.12	0.82	0.05	0.01
4	-	-	-	0.12	0.85	0.03
5	-	-	-	-	0.22	0.78

Figure 3:   Graphic form of unconditional Markov chain

Using Monte Carlo simulation to produce twenty simulations, these unconditional Markov chains correctly predict an average of 75% of the following hurricane status, Table 11; only 3% less than the bi-conditional Markov chains and 2% less than the Markov chains under the condition of hurricane stage. What is surprising is that this is 4% more than the Markov chains under the condition of the pressure index. This indicates that hurricanes for the most part are simple random phenomena that behave similar in nature. As for the mean square error, the mean square error on average is only slightly higher than when both conditions are included with an error on average 1.25 times larger then the mean square error in the originally measured. The mean error squared considering the pressure index range between 0.22 and 0.48; the same as when hurricane pressures are considered.

Table 11:   Simulation of Hurricane Katrina

Simulation 4	Percent Correct	Correct	Absolute Error	Total Error Squared	Mean Error Squared
1	72%	43	20	28	0.47
2	67%	40	23	29	0.48
3	73%	44	18	22	0.37
4	77%	46	16	20	0.33
5	75%	45	16	18	0.30
6	70%	42	23	35	0.58
7	75%	45	16	18	0.30
8	77%	46	16	20	0.33
9	70%	42	19	21	0.35
10	82%	49	12	14	0.23
11	73%	44	17	19	0.32
12	75%	45	17	23	0.38
13	77%	46	17	25	0.42
14	72%	43	19	23	0.38
15	78%	47	15	19	0.32
16	78%	47	14	16	0.27
17	73%	44	18	22	0.37
18	73%	44	18	24	0.40
19	77%	46	16	20	0.33
20	83%	50	11	13	0.22

8. CONCLUSION

The present study shows that hurricanes are a random phenomenon in nature; while there are extraneous conditions such as temperature and pressures that affect the strength of a storm, the developed statistical procedure explains the majority of the transitions probabilistically without additional conditions. Furthermore, the hurricane stage (an indication of the heat released) is more explanatory then the pressures as commonly perceived.

Understanding this is useful when predicting the hurricane tracking at least in terms of hurricane intensity and hurricane force winds. Using recent historical data gathered during the 2004 and 2005 hurricane season including latitude, longitude, day and time as well as atmospheric pressure and wind speeds, then we computed the empirical transitional probabilities and generated simulations using Monte Carlo simulation.

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