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ROBUST STABILITY OF UNCERTAIN MARKOVIAN JUMPING STOCHASTIC COHEN-GROSSBERG TYPE BAM NEURAL NETWORKS WITH TIME-VARYING DELAYS AND REACTION DIFFUSION TERMS

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ABSTRACT. In this paper, the robust exponential stability problem is investigated for a class of uncertain Markovian jumping stochastic Cohen-Grossberg type bidirectional associative memory neural networks (CGBAMNN) with time-varying delays and reaction-diffusion terms. By using the Lyapunov stability theory and linear matrix inequality (LMI) technique, some robust stability conditions guaranteeing the global robust convergence of the equilibrium point are derived. Two numerical examples are given to show the effectiveness of the proposed results.

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1. INTRODUCTION

The Cohen-Grossberg type bidirectional associative memory neural networks (CGBAMNN) model (i.e., the BAM model that possesses Cohen-Grossberg dynamics), initially proposed by Cohen and Grossberg [4], have their promising potential for the tasks of parallel computation, associative memory and have great ability to solve difficult optimization problems. In such applications, it is of prime importance to ensure that the equilibrium points of designed neural networks are stable [7]. Now there have been many results on the stability and convergence of equilibrium point of Cohen-Grossberg-type BAM neural networks with delays, see [1, 3, 5, 18, 19]. For example, by constructing some suitable Lyapunov functionals, authors [5] investigated the asymptotic stability of a class of Cohen-Grossberg-type BAM neural networks with constant delays. In [3], authors further investigated the global exponential stability for Cohen-Grossberg-type BAM neural networks with time-varying delays by using Lyapunov function, M-matrix theory and inequality technique.

Markovian jump system has jumping parameters which are usually governed by a continuous-time discrete-state homogenous Markov process, and each state of the

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parameter represents a mode of the system. Much work has been done for Markovian jumping linear system in the literature, and the stability problems have been extensively investigated, see for example [11, 12, 13] and references therein. Neural networks may also experience such abrupt changes, so Markovian jumping parameters are also introduced into neural networks, see [6, 9, 15, 16]. In [16], the authors studied the exponential stability of delayed neural networks with Markovian jumping parameters. The robust stability problem for a stochastic neural network with both parameter jumping and parameter uncertainties has been considered by [6]. Further the global exponential stability of stochastic BAM neural networks with Markovian jumping parameters has been studied by authors in [8, 10]. In [14], the robust stability of uncertain Markovian jumping Cohen-Grossberg neural networks has been investigated.

Diffusion effect cannot be avoided in neural networks when electrons are moving in asymmetric electromagnetic fields. So it is most important to consider that the activation vary in space as well as in time. Recently several authors have considered the stability of neural networks with reaction-diffusion terms, which are expressed by partial differential equations. The function of actual delayed systems are influenced by unknown disturbances, which may be regarded as stochastic. In order to fix these problems, the system dynamics are suitably approximated by a stochastic linear or nonlinear delayed system. Thus, stochastic delay neural networks have their own characteristic and it is desirable to obtain stability criteria that make full use of these characteristics.

Motivated by the above discussion, in this paper, we are to investigate the robustly exponentially stable of stochastic CGBAMNN with reaction diffusion term and Markovian jumping parameter. However, the LMI based stability criterion for the robust stability stochastic CGBAMNN with reaction diffusion term and Markovian jumping parameter has never been tackled. We have given a new criteria to prove robust exponential stability of stochastic CGBAMNN with reaction-diffusion term and Markovian jump parameters by constructing Lyapunov-Krasovskii functional in terms of LMI, which can be easily calculated by MATLAB toolbox. Numerical examples are given to illustrate the effectiveness and less conservativeness of the proposed system.

2. MODEL DESCRIPTIONS AND PRELIMINARIES

Consider the CGBAMNN with reaction-diffusion terms

$$
(2.1)\quad \begin{cases} \frac{\partial u_\imath(t)}{\partial t} & = \sum_{k=1}^{l} \frac{\partial}{\partial x_k}(D_{\imath k}\frac{\partial u_\imath}{\partial x_k}) - a_\imath(u_\imath(t,x))\Big[b_\imath(u_\imath(t,x)) \\ & \quad - \sum_{\jmath=1}^{m} h_{\imath\jmath} f_\jmath(v_\jmath(t,x)) - \sum_{\jmath=1}^{m} h^*_{\imath\jmath} f_\jmath(v_\jmath(t-h_2(t),x)) - I_\imath\Big] \\ \frac{\partial v_\jmath(t)}{\partial t} & = \sum_{k=1}^{l} \frac{\partial}{\partial x_k}(D^*_{\jmath k}\frac{\partial v_\jmath}{\partial x_k}) - c_\jmath(v_\jmath(t,x))\Big[e_\jmath(v_\jmath(t,x)) \\ & \quad - \sum_{\imath=1}^{n} w_{\jmath\imath} g_\imath(u_\imath(t,x)) - \sum_{\imath=1}^{n} w^*_{\jmath\imath} g_\imath(u_\imath(t-h_1(t),x)) - J_\jmath\Big] \end{cases}
$$

where

- S : A compact set with smooth boundary ∂S and mes $S > 0$ in R^l. $L^2(S)$ be the space of real Lebesgue measurable functions on S and be a Banach space for $L_2 - norm$.

$$\|u(t)\|_2^2 = \int_S u^2(t,x)dx; \quad \|v(t)\|_2^2 = \int_S v^2(t,x)dx$$

- $x = x_1, x_2, \ldots, x_l \in S \subset \mathbb{R}^l$ is a space variable.
- $u(t) = (u_1(t), u_2(t), \ldots, u_n(t))^T \in \mathbb{R}^n$ and $v(t) = (v_1(t), v_2(t), \ldots, v_m(t))^T \in \mathbb{R}^m$
- $u_\imath(t), v_\jmath(t)$ denote the state of the $\imath^{th}$ and $\jmath^{th}$ neuron from neural field F_u and F_v at time t respectively.
- $g_\imath, f_\jmath$ denote the activation functions of the $\imath^{th}, \jmath^{th}$ neuron from F_u and F_v respectively.
- $I_\imath$ and $J_\jmath$ are constants which denote the external inputs on the $\imath^{th}$ and $\jmath^{th}$ neuron from neural field F_u and F_v respectively.
- $h_1(t)$ and $h_2(t)$ correspond to the transmission delays and satisfy $0 \le h_1(t) \le \tau_1 < 1$ and $0 \le h_2(t) \le \tau_2 < 1$. $\dot{h}_1(t)$ and $\dot{h}_2(t)$ are the derivative of $h_1(t)$ and $h_2(t)$, $\max(\dot{h}_1(t)) \le \eta_1 < 1$ and $\max(\dot{h}_2(t)) \le \eta_2 < 1$ respectively.
- $a_\imath(u_\imath(t))$ and $c_\jmath(v_\jmath(t))$ represent amplification functions.
- $b_\imath(u_\imath(t))$ and $e_\jmath(v_\jmath(t))$ are appropriately behaved functions such that the solution of model is remain bounded.
- $h_{\imath\jmath}$, $w_{\jmath\imath}$, $h^*_{\imath\jmath}$ and $w^*_{\jmath\imath}$ denote the connection strengths.
- $D_{\imath k} = D_{\imath k}(t,x,u) \le 0$; $D^*_{\jmath k} = D^*_{\jmath k}(t,x,v) \le 0$ denote smooth functions correspond to the transmission diffusion operators.

The initial conditions are given by

$$
(2.2)\quad \begin{cases} u_\imath(s) = \phi_{u_\imath}(s) & s \in (-\tau_{1\imath}, 0] \\ v_\jmath(s) = \phi_{v_\jmath}(s) & s \in (-\tau_{2\jmath}, 0] \end{cases}
$$

here $\phi_{u_\imath}(s)$ and $\phi_{v_\jmath}(s)$ denote the real-valued continuous functions.

For the purpose of simplicity, we rewrite the eqn (2.1) as the following vector form.

$$
\left\{
\begin{aligned}
\frac{\partial u(t)}{\partial t} &= \nabla\cdot(D(t,x,u)\circ\nabla u)-\mathcal{A}(u(t))\Big[B(u(t))-H_0 f(v(t)) \\
&\quad -H_1 f(v(t-h_2(t)))-I\Big] \\
\frac{\partial v(t)}{\partial t} &= \nabla\cdot(D^*(t,x,v)\circ\nabla v)-\mathcal{B}(v(t))\Big[E(v(t))-W_0 g(u(t)) \\
&\quad -W_1 g(u(t-h_1(t)))-J\Big]
\end{aligned}
\right. \tag{2.3}
$$

where

$$
\begin{aligned}
u(t) &= (u_1(t),u_2(t),\ldots,u_n(t))^T, \quad v(t)=(v_1(t),v_2(t),\ldots,v_m(t))^T, \\
D(t,x,u) &= (D_{ik}(t,x,u))_{n\times l}, \qquad D^*(t,x,v)=(D^*_{jk}(t,x,v))_{m\times l}, \\
\nabla u &= \Big(\nabla u_1,\nabla u_2,\ldots,\nabla u_n\Big), \qquad \nabla v=\Big(\nabla v_1,\nabla v_2,\ldots,\nabla v_m\Big), \\
\nabla u_1 &= \Big(\frac{\partial u_1}{\partial x_1},\frac{\partial u_2}{\partial x_2},\ldots,\frac{\partial u_n}{\partial x_l}\Big), \qquad \nabla v_1=\Big(\frac{\partial v_1}{\partial x_1},\frac{\partial v_2}{\partial x_2},\ldots,\frac{\partial v_m}{\partial x_l}\Big), \\
\mathcal{A}(u(t)) &= diag\Big(a_1(u(t)),a_2(u(t)),\ldots,a_n(u(t))\Big), \\
\mathcal{B}(v(t)) &= diag\Big(c_1(v(t)),c_2(v(t)),\ldots,c_m(v(t))\Big), \\
B &= (b_\imath)_{n\times 1}, \quad E=(e_\jmath)_{m\times 1}, \quad H_0=(h_{\imath\jmath})_{n\times m}, \quad H_1=(h^*_{\imath\jmath})_{n\times m}, \\
W_0 &= (w_{\jmath\imath})_{m\times n}, \quad W_1=(w^*_{\jmath\imath})_{m\times n}, \\
(D\circ\nabla u) &= \left(D_{\imath k}\frac{\partial u_\imath}{\partial x_k}\right), \quad (D^*\circ\nabla v)=\left(D^*_{\jmath k}\frac{\partial v_\jmath}{\partial x_k}\right).
\end{aligned}
$$

here $\circ$ denotes Hadmard product of matrix D and ∇u ; D^* and ∇v. We suppress $u(t,x)$ and $v(t,x)$ as $u(t)$ and $v(t)$ respectively.

3. MAIN RESULT

In this section we state some results and definitions that are needed to prove the main theorem

Lemma 1 ([17]).

$$
\begin{aligned}
&\int_S u^T(t,x)\Big[\nabla\cdot(D(t,x,u)\circ\nabla u(t,x))\Big]dx \\
&\quad = -\int_S\Big(D(t,x,u)\cdot(\nabla u(t,x)\circ\nabla u(t,x))\Big)Edx
\end{aligned}
$$

where

$$
\begin{aligned}
\nabla u\circ\nabla u &= ((\nabla u_1\circ\nabla u_1),\ldots,(\nabla u_n\circ\nabla u_n))^T, \\
\nabla u_i\circ\nabla u_\imath &= ((\frac{\partial u_\imath}{\partial x_1})^2,\ldots,(\frac{\partial u_\imath}{\partial x_l})^2)^T, \\
D\cdot(\nabla u\circ\nabla u) &= ((D_1\cdot(\nabla u_1\circ\nabla u_1)),\ldots,(D_n\cdot(\nabla u_n\circ\nabla u_n))),
\end{aligned}
$$

$$D = (D_1, \ldots, D_n)^T, D_\imath = (D_{\imath 1}, \quad \ldots, D_{\imath l})^T$$

and

$$E = (1, \ldots, 1)^T for \ (\imath = 1, 2, \ldots, n).$$

Further,

$$\int_S v^T(t,x)\Big[\nabla \cdot (D^*(t,x,v) \circ \nabla v(t,x))\Big]dx$$
$$= -\int_S \Big(D^*(t,x,v) \cdot (\nabla v(t,x) \circ \nabla v(t,x))\Big)Edx$$

where

$$\nabla v \circ \nabla v = ((\nabla v_1 \circ \nabla v_1), \ldots, (\nabla v_m \circ \nabla v_m))^T,$$
$$\nabla v_\jmath \circ \nabla v_\jmath = ((\frac{\partial v_\jmath}{\partial x_1})^2, \ldots, (\frac{\partial v_\jmath}{\partial x_l})^2)^T,$$
$$D^* \cdot (\nabla v \circ \nabla v) = ((D_1^* \cdot (\nabla v_1 \circ \nabla v_1)), \ldots, (D_m^* \cdot (\nabla v_m \circ \nabla v_m))),$$
$$D^* = (D_1^*, \ldots, D_m^*)^T, \quad D_\jmath^* = (D_{\jmath 1}^*, \ldots, D_{\jmath l}^*)^T$$

and

$$E = (1, \ldots, 1)^T \ for \ (\jmath = 1, 2, \ldots, m).$$

Lemma 2. *Let x and y be any n dimensional real vectors and ε be a positive scalar. Then the following matrix inequality holds*

$$x^T y + y^T x \le \varepsilon^{-1} x^T x + \varepsilon y^T y.$$

Lemma 3 ([2])**.** *The LMI*

$$\begin{bmatrix} Q(t) & S(t) \\ S^T(t) & R(t) \end{bmatrix} < 0$$

where $Q(t) = Q^T(t)$; $R(t) = R^T(t)$ and $S(t)$ depend on t, is equivalent to any one of the following conditions

$$(S1) \quad R(t) > 0, \quad Q(t) - S(t)R^{-1}(t)S^T(t) > 0$$
$$(S2) \quad Q(t) > 0, \quad R(t) - S(t)Q^{-1}(t)S^T(t) > 0$$

Definition 1. The equilibrium solution of the NNs is said to be robust exponentially stable for all admissible uncertainties in mean square if there exists a pair of constant γ and ρ such that

$$E\|x(t,\phi_1,\phi_2)\|^2 \le \rho e^{-\gamma t} \sup_{\bar{\tau}<s<0} \Big(E\|\phi_1(s)\| + E\|\phi_2(s)\|\Big)$$

Throughout this paper, we make the following assumptions

(H1) The neuron activation functions $f_\imath$ and $g_\jmath$ are bounded Lipschitz continuous that is, there exist constants $L_{1\imath} > 0$ and $L_{2\jmath} > 0$ such that

$$|f_\imath(\xi_1) - f_\imath(\xi_2)| \leq L_{1\imath}|(\xi_1 - \xi_2)|$$

$$|g_\jmath(\xi_1) - g_\jmath(\xi_2)| \leq L_{2\jmath}|(\xi_1 - \xi_2)|$$

where $\xi_1, \xi_2 \in \mathcal{R}, \quad \imath = 1, 2, \dots, n, \quad \jmath = 1, 2, \dots, m$

(H2) $0 < \underline{\mathcal{A}}_i < \mathcal{A}_i(u(t)) \leq \bar{\mathcal{A}}_i < 1$

$0 < \underline{\mathcal{B}}_i < \mathcal{B}_i(v(t)) \leq \bar{\mathcal{B}}_i < 1$

(H3) $z_1(t)\beta_{1i}(z_1(t)) > \mu_i z_1^2(t) \quad \mu_i > 0$

$z_2(t)\beta_{2i}(z_2(t)) > \Delta_i z_2^2(t) \quad \Delta_i > 0$

(H4) $\Delta H_{0i} = M_{0i}\theta(t)N_{0i}; \quad \Delta H_{1i} = M_{1i}\theta(t)N_{1i}; \quad \Delta W_{0i} = M_{2i}\theta(t)N_{2i};$

$\Delta W_{1i} = M_{3i}\theta(t)N_{3i}; \quad \theta^T(t)\theta(t) < I$

(H5)

$$(a) \quad \frac{1}{2}trace\Big[\sigma_1^T(\cdot)\Big(\int_S 2e^{\alpha t}P_i dx\Big)\sigma_1(\cdot)\Big] \leq \int_S \Big\{e^{\alpha t}\Big[F^T(z_2(t))X_{1i}F(z_2(t)) + F^T(z_2(t-h_2(t)))X_{2i}F(z_2(t-h_2(t)))\Big]\Big\}dx$$

$$(b) \quad \frac{1}{2}trace\Big[\sigma_2^T(\cdot)\Big(\int_S 2e^{\alpha t}R_i dx\Big)\sigma_2(\cdot)\Big] \leq \int_S \Big\{e^{\alpha t}\Big[G^T(z_1(t))\bar{X}_{1i}G(z_1(t)) + G^T(z_1(t-h_1(t)))\bar{X}_{2i}G(z_1(t-h_1(t)))\Big]\Big\}dx$$

where M_{ki}, N_{ki} are constant matrices and $\theta(t)$ is an unknown matrix representing the parameter uncertainty.

Let u^*, v^* be the equilibrium point of the equation (2.3). For the purpose of simplicity, we can shift the intended equilibrium u^*, v^* to the origin by letting $z_1 = u - u^*$ and $z_2 = v - v^*$ then the system (2.3) can be rewritten as follows

$$\begin{aligned}
\frac{\partial z_1(t)}{\partial t} &= (\nabla \cdot (D(t,x,z_1) \circ \nabla z_1)) \\
&\quad - \mathcal{A}(z_1(t))\Big[\beta_1(z_1(t)) - H_0F(z_2(t)) - H_1F(z_2(t-h_2(t)))\Big] \\
\frac{\partial z_2(t)}{\partial t} &= (\nabla \cdot (D^*(t,x,z_2) \circ \nabla z_2)) \\
&\quad - \mathcal{B}(z_2(t))\Big[\beta_2(z_2(t)) - W_0G(z_1(t)) - W_1G(z_1(t-h_1(t)))\Big]
\end{aligned}$$

where

$$\begin{aligned}
\nabla \cdot (D(t,x,z_1) \circ \nabla z_1) &= \nabla \cdot (D(t,x,z_1+u^*) \circ \nabla z_1), \\
\nabla \cdot (D^*(t,x,z_2) \circ \nabla z_2) &= \nabla \cdot (D^*(t,x,z_2+v^*) \circ \nabla z_2), \\
F(z_2(t-h_2(t))) &= f(z_2(t-h_2(t)) + v^*) - f(v^*),
\end{aligned}$$

$$
\begin{aligned}
G(z_1(t-h_1(t))) &= g(z_1(t-h_1(t))+u^*)-g(u^*),\\
\mathcal{A}(z_1(t)) &= \mathcal{A}(z_1(t)+u^*), \quad \mathcal{B}(z_2(t))=\mathcal{B}(z_2(t)+u^*),\\
\beta_1(z_1(t)) &= B(z_1(t)+u^*)-B(u^*),\\
\beta_2(z_2(t)) &= E(z_2(t)+v^*)-E(v^*),\\
F(z_2(t)) &= f(z_2(t)+v^*)-f(v^*), \quad G(z_1(t))=g(z_1(t)+u^*)-g(u^*).
\end{aligned}
$$

Now we consider CGBAMNN with uncertain and Markovian jumping parameter

$$
\begin{aligned}
\frac{\partial z_1(t)}{\partial t} &= (\nabla\cdot(D(t,x,z_1)\circ\nabla z_1))-\mathcal{A}(z_1(t),r(t))\Big\{\beta_1(z_1(t),r(t))-\Big[H_0(r(t))\\
&\quad +\Delta H_0(r(t))\Big]F(z_2(t))-\Big[H_1(r(t))+\Delta H_1(r(t))\Big]F(z_2(t-h_2(t)))\Big\}\\
\frac{\partial z_2(t)}{\partial t} &= (\nabla\cdot(D^*(t,x,z_2)\circ\nabla z_2))-\mathcal{B}(z_2(t),r(t))\Big\{\beta_2(z_2(t),r(t))-\Big[W_0(r(t))\\
&\quad +\Delta W_0(r(t))\Big]G(z_1(t))-\Big[W_1(r(t))+\Delta W_1(r(t))\Big]G(z_1(t-h_1(t)))\Big\}
\end{aligned}
$$

where $\{r(t), t>0\}$ is a right-continuous Markov process on the probability space which takes values in the finite space $\mathcal{H}=\{1,2,\ldots,N\}$ with generator $\Gamma=\{\gamma_{ij}\}$ $(i,j\in\mathcal{H})$ (also called jumping transfer matrix) given by

$$
P\{r(t+\Delta)=j|r(t)=i\}=\begin{cases}\gamma_{ij}\Delta+o(\Delta) & \text{if } i\neq j\\ 1+\gamma_{ii}\Delta+o(\Delta) & \text{if } i=j\end{cases}
$$

$\Delta>0$ and $\lim_{\Delta\to 0} o(\Delta)/\Delta=0$, $\gamma_{ij}\geq 0$ is the transition rate from i to j if $i\neq j$ and $\gamma_{ii}=-\sum_{j\neq i}\gamma_{ij}$.

For a fixed network mode, $A(r(t)), W_k(r(t)), H_k(r(t))$ $(k=0,1)$ are known constant matrices with appropriate dimensions. Recall that the Markov process $\{r(t), t>0\}$ takes values in the finite space $\{\mathcal{H}=1,2,\ldots,N\}$. For the sake of simplicity, we write

$$
A(i)=A_i, \quad H_k(i)=H_{ki}, \quad W_k(i)=W_{ki} \quad (k=0,1)
$$

Now we shall work on the network mode $r(t)=i, \forall i\in\mathcal{H}$.

$$
\begin{aligned}
\frac{\partial z_1(t)}{\partial t} &= (\nabla\cdot(D(t,x,z_1)\circ\nabla z_1))-\mathcal{A}_i(z_1(t))\Big\{\beta_{1i}(z_1(t))-H_{0i}(t)F(z_2(t))\\
&\quad -H_{1i}(t)F(z_2(t-h_2(t)))\Big\}\\
\frac{\partial z_2(t)}{\partial t} &= (\nabla\cdot(D^*(t,x,z_2)\circ\nabla z_2))-\mathcal{B}_i(z_2(t))\Big\{\beta_{2i}(z_2(t))-W_{0i}(t)G(z_1(t))\\
&\quad -W_{1i}(t)G(z_1(t-h_1(t)))\Big\}
\end{aligned}
$$

where

$$
\begin{aligned}
H_{0i}(t) &= H_{0i}+\Delta H_{0i}, \quad H_{1i}(t)=H_{1i}+\Delta H_{1i},\\
W_{0i}(t) &= W_{0i}+\Delta W_{0i}, \quad W_{1i}(t)=W_{1i}+\Delta W_{1i}.
\end{aligned}
$$

Now consider the following uncertain Markovian jumping stochastic CGBAMNN with time varying delay and reaction diffusion terms.

$$
\left\{
\begin{aligned}
\partial z_1(t) \quad &= \Big[(\nabla \cdot (D(t,x,z_1) \circ \nabla z_1)) - \mathcal{A}_i(z_1(t))\Big\{\beta_{1i}(z_1(t)) \\
&\quad -H_{0i}(t)F(z_2(t)) - H_{1i}(t)F(z_2(t-h_2(t)))\Big\}\Big]dt \\
&\quad +\sigma_{1i}\Big(t, F(z_2(t)), F(z_2(t-h_2(t)))\Big)d\omega(t) \\
\partial z_2(t) \quad &= \Big[(\nabla \cdot (D^*(t,x,z_2) \circ \nabla z_2)) - \mathcal{B}_i(z_2(t))\Big\{\beta_{2i}(z_2(t)) \\
&\quad -W_{0i}(t)G(z_1(t)) - W_{1i}(t)G(z_1(t-h_1(t)))\Big\}\Big]dt \\
&\quad +\sigma_{2i}\Big(t, G(z_1(t)), G(z_1(t-h_1(t)))\Big)d\omega(t)
\end{aligned}
\right. \tag{3.1}
$$

where $\omega(t)$ is a Brownian motion defined on a complete probability space $(\Omega, \mathcal{F}_t, P)$ with the filtration $\{\mathcal{F}\}_{t\geq 0}$ generated by $\{\omega(s) : 0 \leq s \leq t\}$.

Lemma 4. *The trivial solution of stochastic CGBAMNN for simplified model*

$$dz = \phi(t, z_1(t), z_2(t))dt + \psi(t, z_1(t), z_2(t))d\omega(t) \tag{3.2}$$

where $z = (z_1, z_2)^T$

$$\phi : \mathbb{R}_+ \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n \text{ and } \psi : \mathbb{R}_+ \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^{n\times m}$$

is robust exponentially stable if there exists a function $V(t, z_1, z_2) \in \mathbb{R}_+ \times \mathbb{R}^n \times \mathbb{R}^n$ *which is positive definite in the Lyapunov sense and satisfies*

$$
\begin{aligned}
\mathcal{L}V(t, z_1(t), z_2(t)) = \frac{\partial V(t, z_1(t), z_2(t))}{\partial t} &+ grad(V(t, z_1(t), z_2(t)))\phi \\
&+ \frac{1}{2}tr(\psi^T\psi)Hess(V) < 0
\end{aligned}
$$

The matrix Hess(V) is the Hessian matrix of second-order partial derivatives in which

$$\phi = (\phi_1, \phi_2)^T, \quad \psi = (\psi_1, \psi_2)^T$$

where

$$
\begin{aligned}
\phi_1 &= \Big[(\nabla \cdot (D(t,x,z_1) \circ \nabla z_1)) - \mathcal{A}_i(z_1(t))\Big\{\beta_{1i}(z_1(t)) - H_{0i}(t)F(z_2(t)) \\
&\quad - H_{1i}(t)F(z_2(t-h_2(t)))\Big\}\Big], \\
\phi_2 &= \Big[(\nabla \cdot (D^*(t,x,z_2) \circ \nabla z_2)) - \mathcal{B}_i(z_2(t))\Big\{\beta_{2i}(z_2(t)) - W_{0i}(t)G(z_1(t)) \\
&\quad - W_{1i}(t)G(z_1(t-h_1(t)))\Big\}\Big], \\
\psi_1 &= \sigma_{1i}\Big(t, F(z_2(t)), F(z_2(t-h_2(t)))\Big), \quad \psi_2 = \sigma_{2i}\Big(t, G(z_1(t)), G(z_1(t-h_1(t)))\Big).
\end{aligned}
$$

Theorem 1. *Given positive definite matrices* $W_{0i}, W_{1i}, H_{0i}, H_{1i}$ *and for positive scalar* α, *under the given assumptions, the null solution to the model* (3.1) *is robust exponentially stable in mean square for any time varying delay* $h_1(t)$ *and* $h_2(t)$ *if there* *exist positive symmetric matrices P_i, R_i, Q_1, Q_2 and positive scalars $\varepsilon_0, \varepsilon_1$ such that the following LMI holds*

$$\begin{bmatrix} \Omega_{11} & \Omega_{12} & 0 & \Omega_{14} & P_iM_{0i} & P_iM_{1i} & 0 & 0 & 0 & 0 \\ * & \Omega_{22} & \Omega_{23} & 0 & 0 & 0 & R_iM_{2i} & R_iM_{3i} & 0 & 0 \\ * & * & \Omega_{33} & 0 & 0 & 0 & 0 & 0 & \Omega_{39} & 0 \\ * & * & * & \Omega_{44} & 0 & 0 & 0 & 0 & 0 & \Omega_{410} \\ * & * & * & * & -\frac{\varepsilon_0}{\bar{\mathcal{A}}_i}I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -\frac{\varepsilon_0}{\bar{\mathcal{A}}_i}I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -\frac{\varepsilon_1}{\bar{\mathcal{B}}_i}I & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -\frac{\varepsilon_1}{\bar{\mathcal{B}}_i}I & 0 & 0 \\ * & * & * & * & * & * & * & * & -\varepsilon_1 I & 0 \\ * & * & * & * & * & * & * & * & * & -\varepsilon_0 I \end{bmatrix} < 0 \tag{3.3}$$

$$\Omega_{11} = \alpha P_i + L_1^T Q_1 L_1 - 2\underline{\mathcal{A}}_i P_i \mu_i + \sum_{j=1}^{n} \gamma_{ij} P_j + \bar{\mathcal{B}}_i \varepsilon_1 L_1^T N_{2i}^T N_{2i} L_1 + L_1^T \bar{X}_{1i} L_1,$$

$$\Omega_{12} = \bar{\mathcal{A}}_i P_i H_{0i} L_2 + \bar{\mathcal{B}}_i L_1^T W_{0i}^T R_i, \quad \Omega_{14} = \bar{\mathcal{A}}_i P_i H_{1i} L_2,$$

$$\Omega_{22} = \alpha R_i + L_2^T Q_2 L_2 - 2\underline{\mathcal{B}}_i R_i \Delta_i + \sum_{j=1}^{n} \gamma_{ij} R_j + \bar{\mathcal{A}}_i \varepsilon_0 L_2^T N_{0i}^T N_{0i} L_2 + L_2^T X_{1i} L_2,$$

$$\Omega_{23} = \bar{\mathcal{B}}_i R_i W_{1i} L_1, \quad \Omega_{33} = L_1^T \bar{X}_{2i} L_1 - e^{-\alpha\tau_1} L_1^T Q_1 L_1 (1-\eta_1), \quad \Omega_{39} = \sqrt{\bar{\mathcal{B}}_i} \varepsilon_1 L_1^T N_{3i}^T,$$

$$\Omega_{44} = L_2^T X_{2i} L_2 - e^{-\alpha\tau_2} L_2^T Q_2 L_2 (1-\eta_2), \quad \Omega_{410} = \sqrt{\bar{\mathcal{A}}_i} \varepsilon_0 L_2^T N_{1i}^T.$$

Proof. Consider the following Lyapunov - Krasovskii functional

$$V(t, z_1(t), z_2(t)) = V_{1i}(t, z_1(t), z_2(t)) + V_{2i}(t, z_1(t), z_2(t))$$

where

$$V_{1i}(t, z_1(t), z_2(t)) = \int_S \Big[e^{\alpha t} z_1^T(t) P_i z_1(t) + \int_{t-h_1(t)}^{t} e^{\alpha s} G^T(z_1(s)) Q_1 G^T(z_1(s)) ds \Big] dx,$$

$$V_{2i}(t, z_1(t), z_2(t)) = \int_S \Big[e^{\alpha t} z_2^T(t) R_i z_2(t) + \int_{t-h_2(t)}^{t} e^{\alpha s} F^T(z_2(s)) Q_2 F^T(z_2(s)) ds \Big] dx$$

here $P_i = P_i^T$, $R_i = R_i^T$, $Q_1 = Q_1^T$ and $Q_2 = Q_2^T$. The stability results can be proved using the following steps

By Itô-differential rule, the stochastic derivative of V along trajectories of (3.1) is given by

$$dV(t, z_1(t), z_2(t)) = \mathcal{L}V(t, z_1(t), z_2(t))dt + \frac{\partial V(t, z_1(t), z_2(t))}{\partial z} \psi(t, z_1(t), z_2(t)) d\omega(t).$$

Integrating on both sides of the above equation over $(0, t)$ with respect to t and taking expectation we get

$$E\Big[V(t, z_1(t), z_2(t))\Big] = E\Big[V(0, z_1(0), z_2(0))\Big] + E\Big[\int_0^t \mathcal{L}V(t, z_1(t), z_2(t))dt\Big]. \tag{3.4}$$

The following equalities holds, since $\sum_{j=1}^n \gamma_{ij} = 0$

(3.5)

$$\begin{cases} \sum_{j=1}^n \gamma_{ij} \int_{t-h_1(t)}^t e^{\alpha s} & G^T(z_1(s))Q_1 G^T(z_1(s))ds \\ & = \Big(\sum_{j=1}^n \gamma_{ij}\Big) \int_{t-h_1(t)}^t e^{\alpha s} G^T(z_1(s))Q_1 G^T(z_1(s))ds = 0; \\ \sum_{j=1}^n \gamma_{ij} \int_{t-h_2(t)}^t e^{\alpha s} & F^T(z_2(s))Q_2 F^T(z_2(s))ds \\ & = \Big(\sum_{j=1}^n \gamma_{ij}\Big) \int_{t-h_2(t)}^t e^{\alpha s} F^T(z_2(s))Q_2 F^T(z_2(s))ds = 0. \end{cases}$$

By using Lemma 1, 2, assumptions (H1)–(H5) and equation (3.5) after some manipulation we have

$$\begin{aligned} \mathcal{L}V(t, & z_1(t), z_2(t)) \le e^{\alpha t} \int_S \Big\{ z_1^T(t) \Big[\alpha P_i + L_1^T Q_1 L_1 - 2\underline{\mathcal{A}}_i P_i \mu_i + \varepsilon_1 \bar{\mathcal{B}}_i L_1^T N_{2i}^T N_{2i} L_1 \\ & + L_1^T \bar{X}_{1i} L_1 + \sum_{j=1}^n \gamma_{ij} P_j + \varepsilon_0^{-1} \bar{\mathcal{A}}_i P_i M_{0i} M_{0i}^T P_i + \varepsilon_0^{-1} \bar{\mathcal{A}}_i P_i M_{1i} M_{1i}^T P_i \Big] z_1(t) \\ & + z_2^T(t) \Big[\alpha R_i + L_2^T Q_2 L_2 - 2\underline{\mathcal{B}}_i R_i \Delta_i + \varepsilon_0 \bar{\mathcal{A}}_i L_2^T N_{0i}^T N_{0i} L_2 + L_2^T X_{1i} L_2 \\ & + \sum_{j=1}^n \gamma_{ij} R_j + \varepsilon_1^{-1} \bar{\mathcal{B}}_i R_i M_{2i} M_{2i}^T R_i + \varepsilon_1^{-1} \bar{\mathcal{B}}_i R_i M_{3i} M_{3i}^T R_i \Big] z_2(t) \\ & + z_1^T(t - h_1(t)) \Big[\varepsilon_1 \bar{\mathcal{B}}_i L_1^T N_{3i}^T N_{3i} L_1 + L_1^T \bar{X}_{2i} L_1 - e^{-\alpha\tau_1} L_1^T Q_1 L_1 \\ & \times (1 - \eta_1) \Big] z_1(t - h_1(t)) + z_2^T(t - h_2(t)) \Big[\varepsilon_0 \bar{\mathcal{A}}_i L_2^T N_{1i}^T N_{1i} L_2 \\ & + L_2^T X_{2i} L_2 - e^{-\alpha\tau_2} L_2^T Q_2 L_2 (1 - \eta_2) \Big] z_2(t - h_2(t)) \\ & + z_1^T(t) \Big[\bar{\mathcal{A}}_i P_i H_{0i} L_2 + \bar{\mathcal{B}}_i L_1^T W_{0i}^T R_i \Big] z_2(t) \\ & + z_1^T(t) \bar{\mathcal{A}}_i P_i H_{1i} L_2 z_2(t - h_2(t) + z_2^T(t) \bar{\mathcal{B}}_i R_i W_{1i} L_1 z_1(t - h_1(t)) \Big\} dx \\ & - 2e^{\alpha t} \int_S \Big(D^*(t, x, z_1) \cdot (\nabla z_1 \circ \nabla z_1) \Big) E dx \\ & - 2e^{\alpha t} \int_S \Big(D^*(t, x, z_2) \cdot (\nabla z_2 \circ \nabla z_2) \Big) E dx. \end{aligned}$$

By using schur complement Lemma 3 and (3.3) we get

$$\mathcal{L}V(t, z_1(t), z_2(t)) < 0.$$

which implies the equation (3.4) becomes

$$E\Big[V(t, z_1(t), z_2(t))\Big] \le E\Big[V(t, z_1(0), z_2(0))\Big] \tag{3.6}$$

where

$$E\Big[V(t, z_1(0), z_2(0))\Big] = M_1 E\|\phi_{z_1}\|_2^2 + M_2 E\|\phi_{z_2}\|_2^2$$

in which

$$M_1 = \lambda_M(P_i) + \lambda_M(L_1^T Q_1 L_1)\Big[\frac{1}{\alpha} - \frac{e^{-\alpha h_1(0)}}{\alpha}\Big], \quad E\|\phi_{z_1}\|_2^2 = \sup_{0<s<h_1(0)} E\|z_1(s)\|_2^2,$$

$$M_2 = \lambda_M(R_i) + \lambda_M(L_2^T Q_2 L_2)\Big[\frac{1}{\alpha} - \frac{e^{-\alpha h_2(0)}}{\alpha}\Big], \quad E\|\phi_{z_2}\|_2^2 = \sup_{0<s<h_2(0)} E\|z_2(s)\|_2^2,$$

On the other hand

$$E\Big[V(t, z_1(t), z_2(t))\Big] \geq e^{\alpha t} E\|z_1(t)\|_2^2 + e^{\alpha t} E\|z_2(t)\|_2^2.$$

Therefore the equation (3.6) becomes

$$E\|z_1(t)\|_2^2 + E\|z_2(t)\|_2^2 \leq e^{-\alpha t}\Big[M_1 E\|\phi_{z_1}\|_2^2 + M_2 E\|\phi_{z_2}\|_2^2\Big]$$

This completes the proof.

Remark 2. When Markovian jumping parameter is not present in the system (3.1), then the system can be written as follows

$$\tag{3.7} \begin{cases} \partial z_1(t) &= \Big[(\nabla \cdot (D(t,x,z_1) \circ \nabla z_1)) - \mathcal{A}(z_1(t))\Big\{\beta_1(z_1(t)) \\ & \quad -H_0(t)F(z_2(t)) - H_1(t)F(z_2(t-h_2(t)))\Big\}\Big]dt \\ & \quad +\sigma_1\Big(t, F(z_2(t)), F(z_2(t-h_2(t)))\Big)d\omega(t) \\ \partial z_2(t) &= \Big[(\nabla \cdot (D^*(t,x,z_2) \circ \nabla z_2)) - \mathcal{B}(z_2(t))\Big\{\beta_2(z_2(t)) \\ & \quad -W_0(t)G(z_1(t)) - W_1(t)G(z_1(t-h_1(t)))\Big\}\Big]dt \\ & \quad +\sigma_2\Big(t, G(z_1(t)), G(z_1(t-h_1(t)))\Big)d\omega(t) \end{cases}$$

The stochastic reaction diffusion with uncertainty structure and Markovian jumping parameter is considered in this paper which differs from the papers in the available literature. The LMI has been efficiently solved by utilizing the numerically attractive MATLAB toolbox for the two dimensional systems.

Corollary 1. *Given positive definite matrices W_0, W_1, H_0, H_1 and for positive scalar α, under the given assumptions, the null solution to the model* (3.7) *is robust exponentially stable in mean square for any time-varying delays $h_1(t)$ and $h_2(t)$ if there exist positive symmetric matrices P, R, Q_1, Q_2 and positive scalars $\varepsilon_0, \varepsilon_1$ such that the* *following LMI holds*

$$\begin{bmatrix} \Omega_{11} & \Omega_{12} & 0 & \Omega_{14} & PM_0 & PM_1 & 0 & 0 & 0 & 0 \\ * & \Omega_{22} & \Omega_{23} & 0 & 0 & 0 & RM_2 & RM_3 & 0 & 0 \\ * & * & \Omega_{33} & 0 & 0 & 0 & 0 & 0 & \Omega_{39} & 0 \\ * & * & * & \Omega_{44} & 0 & 0 & 0 & 0 & 0 & \Omega_{410} \\ * & * & * & * & -\frac{\varepsilon_0}{\bar{\mathcal{A}}}I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -\frac{\varepsilon_0}{\bar{\mathcal{A}}}I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -\frac{\varepsilon_1}{\bar{\mathcal{B}}}I & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -\frac{\varepsilon_1}{\bar{\mathcal{B}}}I & 0 & 0 \\ * & * & * & * & * & * & * & * & -\varepsilon_1 I & 0 \\ * & * & * & * & * & * & * & * & * & -\varepsilon_0 I \end{bmatrix} < 0 \tag{3.8}$$

$$\Omega_{11} = \alpha P + L_1^T Q_1 L_1 - 2\underline{\mathcal{A}}P\mu + \bar{\mathcal{B}}\varepsilon_1 L_1^T N_2^T N_2 L_1 + L_1^T \bar{X}_1 L_1,$$

$$\Omega_{12} = \bar{\mathcal{A}}PH_0L_2 + \bar{\mathcal{B}}L_1^T W_0^T R, \quad \Omega_{14} = \bar{\mathcal{A}}PH_1L_2,$$

$$\Omega_{22} = \alpha R + L_2^T Q_2 L_2 - 2\underline{\mathcal{B}}R\Delta + \bar{\mathcal{A}}\varepsilon_0 L_2^T N_0^T N_0 L_2 + L_2^T X_1 L_2,$$

$$\Omega_{23} = \bar{\mathcal{B}}RW_1L_1, \quad \Omega_{33} = L_1^T \bar{X}_2 L_1 - e^{-\alpha\tau_1} L_1^T Q_1 L_1(1-\eta_1), \quad \Omega_{39} = \sqrt{\bar{\mathcal{B}}}\varepsilon_1 L_1^T N_3^T,$$

$$\Omega_{44} = L_2^T X_2 L_2 - e^{-\alpha\tau_2} L_2^T Q_2 L_2(1-\eta_2), \quad \Omega_{410} = \sqrt{\bar{\mathcal{A}}}\varepsilon_0 L_2^T N_1^T.$$

4. EXAMPLE

In this section, the main result is demonstrated with the following example. Our aim is to examine the robust exponential stability of a given stochastic uncertain neural networks.

Example 3. For the sake of simplicity, we consider equation (3.1) with the given parameters

$$p(\xi) = f_1(\xi) = f_2(\xi) = g_1(\xi) = g_2(\xi) = 0.5(|\xi+1| - |\xi-1|).$$

Since for all $\xi_1, \xi_2 \in R$, $|p(\xi_1) - p(\xi_2)| \leqslant |\xi_1 - \xi_2|$, let

$$\tau_1, \tau_2 = 0.5, \quad \eta_1, \eta_2 = 0.7$$

$$\Delta_1 = diag(10 \quad 11),\ \Delta_2 = diag(5 \quad 7), \quad \mu_1 = diag(8 \quad 9),\ \mu_2 = diag(6 \quad 7)$$

$$\underline{\mathcal{A}}_1 = 0.7,\ \underline{\mathcal{A}}_2 = 0.6, \quad \underline{\mathcal{B}}_1 = 0.2,\ \underline{\mathcal{B}}_2 = 0.1,$$

$$\bar{\mathcal{A}}_1 = 0.9,\ \bar{\mathcal{A}}_2 = 0.7, \quad \bar{\mathcal{B}}_1 = 0.4,\ \bar{\mathcal{B}}_2 = 0.8.$$

$$W_{01} = \begin{bmatrix} 0.5 & 0.4 \\ 0.5 & 0.8 \end{bmatrix}, \qquad W_{02} = \begin{bmatrix} 0.6 & 0.1 \\ 0.6 & 0.9 \end{bmatrix}, \qquad W_{11} = \begin{bmatrix} 0.4 & 0.3 \\ 0.1 & 0.8 \end{bmatrix},$$

$$W_{12} = \begin{bmatrix} 0.2 & 0.8 \\ 0.7 & 0.6 \end{bmatrix}, \qquad H_{01} = \begin{bmatrix} 0.2 & 0.7 \\ 0.5 & 0.3 \end{bmatrix}, \qquad H_{02} = \begin{bmatrix} 0.5 & 0.1 \\ 0.2 & 0.7 \end{bmatrix},$$

$$H_{11} = \begin{bmatrix} 0.2 & 0.4 \\ 0.2 & 0.2 \end{bmatrix}, \qquad H_{12} = \begin{bmatrix} 1 & 0.2 \\ 0.5 & 1.5 \end{bmatrix}, \qquad M_{01} = \begin{bmatrix} 0.2 & 0.5 \\ 0.2 & 0.12 \end{bmatrix},$$

$$M_{02} = \begin{bmatrix} 0.5 & 0.1 \\ 0.11 & 10 \end{bmatrix}, \qquad M_{11} = \begin{bmatrix} 0.5 & 0.5 \\ 1 & 0.01 \end{bmatrix}, \qquad M_{12} = \begin{bmatrix} 0.5 & 0.3 \\ 1.7 & 5.2 \end{bmatrix},$$

$$M_{21} = \begin{bmatrix} 0.55 & 0.1010 \\ 0.2 & 1.01 \end{bmatrix}, \qquad M_{22} = \begin{bmatrix} 0.25 & 0.9 \\ 0.2 & 0 \end{bmatrix}, \qquad M_{31} = \begin{bmatrix} 0.4 & 0.15 \\ 0.5 & 1.321 \end{bmatrix},$$

$$M_{32} = \begin{bmatrix} 0.5 & 0.6 \\ 0.01 & 0.3 \end{bmatrix}, \qquad L_0 = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.01 \end{bmatrix}, \qquad L_2 = \begin{bmatrix} 0.7 & 0 \\ 0 & 1.02 \end{bmatrix},$$

$$N_{01} = \begin{bmatrix} 0.02 & 0.07 \\ 7.2157 & 13.057 \end{bmatrix}, \qquad N_{02} = \begin{bmatrix} 3.027 & 10.17 \\ 7.01 & 10.07 \end{bmatrix}, \qquad N_{11} = \begin{bmatrix} 0.015 & 10.15 \\ 1.015 & 1.115 \end{bmatrix},$$

$$N_{12} = \begin{bmatrix} 1.25 & 1.157 \\ 1.0057 & 1.57 \end{bmatrix}, \qquad N_{21} = \begin{bmatrix} 0.015 & 1.05 \\ 0.005 & 0.012 \end{bmatrix}, \qquad N_{22} = \begin{bmatrix} 0.1 & 1.1 \\ 0.01 & 2 \end{bmatrix},$$

$$N_{31} = \begin{bmatrix} 1 & 0 \\ 0.5 & 0.2 \end{bmatrix}, \qquad N_{32} = \begin{bmatrix} 1 & 1 \\ 0.8 & 5 \end{bmatrix}, \qquad \Gamma = \begin{bmatrix} -6 & 6 \\ 7 & -7 \end{bmatrix},$$

$$X_{11} = \begin{bmatrix} 0 & 0.1 \\ 0.1 & 0.002 \end{bmatrix}, \qquad X_{12} = \begin{bmatrix} 0.001 & 0 \\ 0 & 0.01 \end{bmatrix}, \qquad X_{21} = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.06 \end{bmatrix},$$

$$X_{22} = \begin{bmatrix} 0.001 & 2 \\ 2 & 0 \end{bmatrix}, \qquad Y_{11} = \begin{bmatrix} 0 & 0 \\ 0 & 0.001 \end{bmatrix}, \qquad Y_{12} = \begin{bmatrix} 0.002 & 0 \\ 0 & 0.02 \end{bmatrix},$$

$$Y_{21} = \begin{bmatrix} 0.02 & 0 \\ 0 & 0.05 \end{bmatrix}, \qquad Y_{22} = \begin{bmatrix} 0 & 0.1 \\ 0.1 & 0.001 \end{bmatrix}.$$

The feasible solutions of the LMI in the Theorem 1 by MATLAB are given by

$$P_1 = \begin{bmatrix} 2.3934 & -0.0471 \\ -0.0471 & 0.0291 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 1.3292 & -0.0856 \\ -0.0856 & 0.0135 \end{bmatrix},$$

$$R_1 = \begin{bmatrix} 7.1554 & 1.7805 \\ 1.7805 & 19.7273 \end{bmatrix}, \quad R_2 = \begin{bmatrix} 2.5805 & -1.2488 \\ -1.2488 & 1.7059 \end{bmatrix},$$

$$Q_1 = \begin{bmatrix} 49.4714 & 84.5734 \\ 84.5734 & 361.9220 \end{bmatrix}, \quad Q_2 = \begin{bmatrix} 27.6965 & -4.4613 \\ -4.4613 & 61.9711 \end{bmatrix}.$$

Thus all conditions stated in Theorem 1 have been satisfied and hence the uncertain Markovian jumping stochastic Cohen - Grossberg type BAM neural networks with time varying delays and reaction diffusion terms is robust exponentially stable. This completes the proof.

Example 4. In this example, we consider equation (3.7), and parameters W_{01}, W_{11}, H_{01}, H_{11}, M_{01}, M_{11}, M_{21}, M_{31}, L_0, L_1, N_{01}, N_{11}, N_{21}, N_{31}, X_{11}, X_{21}, Y_{11}, Y_{21}, τ_1, τ_2, η_1, η_2, Δ_{11}, $\underline{\mathcal{A}}_1$, $\underline{\mathcal{A}}_2$ $\underline{\mathcal{B}}_1$, $\underline{\mathcal{B}}_2$, $\bar{\mathcal{A}}_1$, $\bar{\mathcal{A}}_2$, $\bar{\mathcal{B}}_1$, $\bar{\mathcal{B}}_2$ as in Example 1.

The feasible solutions of the LMI in the Corollary 1 by MATLAB are given by

$$P_1 = \begin{bmatrix} 2.5004 & -1.4300 \\ -1.4300 & 2.1566 \end{bmatrix}, \quad R_1 = \begin{bmatrix} 17.8912 & 1.7573 \\ 1.7573 & 42.6313 \end{bmatrix},$$

$$Q_1 = \begin{bmatrix} 43.5408 & 13.5689 \\ 13.5689 & 71.5839 \end{bmatrix}, \quad Q_2 = \begin{bmatrix} 51.0761 & -12.6309 \\ -12.6309 & 53.6561 \end{bmatrix}.$$

We conclude that all conditions stated in corollary 1 have been satisfied and hence the uncertain stochastic Cohen - Grossberg type BAM neural networks with time varying delays and reaction diffusion terms is robust exponentially stable. This completes the proof. $\square$

5. CONCLUSION

In this paper, using suitable Lyapunov-Krasovskii functional, inequality techniques and LMI approach, the uncertain Markovian jumping stochastic Cohen - Grossberg type BAM neural networks with time varying delays and reaction diffusion terms have been derived for robust exponentially stability. The derived conditions are expressed in terms of LMI, which have been checked numerically very efficiently for less conservative fact. We have proved the results expand and improve those existing results in the literature with less restrictive conditions.

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