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GENERATOR ALGORITHMS FOR PRIME k-TUPLES USING BINOMIAL EXPRESSIONS

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ABSTRACT. In this paper generator algorithms for prime k-tuples based on the the divisibility properties of binomial coefficients are introduced. The mathematical foundation lies in the connection that exists between binomial coefficients and the number of carries that result in the sum in different bases of the variables that form the binomial coefficent and, characterizations of k-tuple primes in terms of binomial coefficients.

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1. INTRODUCTION

In this paper generator algorithms of prime k-tuples based on the divisibility properties of binomial coefficients are presented. Their mathematical justification results from the work done by Kummer in 1852 [1] in relation to the connection that exists between binomial coefficients and the number of carries that result in the sum in different bases of the variables that form the binomial coefficient. The necessary and sufficient conditions provided for k-tuple primes verification in terms of binomial coefficients were inspired in the work presented in [2], however its proof is based on generating functions which is distinct to the argument provided here to prove it, for another characterization of this type see [3]. The mathematical approach applied to prove the presented results is novice, and the algorithms are new. The paper is organized as follows. Section 1, gives the mathematical preliminaries needed to understand the rest of the paper. Section 2, deals with the prime generator algorithm which is used in the next section. Section 3, with the prime k-tuple generator algorithms. Finally, some concluding remarks are presented.

2. PRELIMINARIES

Definition 1. Let n and p be integers, the p-adic expansion of n (which is the representation of n in base p) is given by,

$$n = a_0 + a_1 p + a_2 p^2 + \cdots + a_m p^m \tag{2.1}$$

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where the digits $a_i \in \{o, \dots, p-1\}$ and m is an integer. Alternatively n is said to have the p-adic expansion,

$$n = (a_m a_{m-1} \cdots a_1 a_0)_p \tag{2.2}$$

Definition 2. Let n and k be integers, the p-adic addition of n and k consists in, the addition of its respective p-adic representations in base p. The number of carries in the p-adic addition of n and k will be denoted by $\tau = c_p(n,k)$.

The next stated result, is due to Kummer 1852 [1], (for completeness purposes the proof is supplied).

Theorem 3. *Let $\tau = c_p(n,k)$ be the number of carries in the p-adic addition of n and k then, $\binom{n+k}{k}$ is divisible by the prime power p^τ but not by $p^{\tau+1}$.*

In order to derive this beautiful theorem the following result, called Legendre's formula (1808), is used.

Lemma 4. *Let $\mu(n)$ be the largest exponent of the prime power $p^{\mu(n)}$ which divides $n!$ then,*

$$\mu(n) = \frac{n-\sigma}{p-1} \tag{2.3}$$

where σ is the sum of the p-adic coefficients of $a_i \in \{0, \dots, p-1\}$ of n.

Proof. From the identity $\mu(n) = \sum_{i=1}^{\infty} \left\lfloor \frac{n}{pi} \right\rfloor$ Legendre's formula is equivalent to $\sum_{i=1}^{\infty} \left\lfloor \frac{n}{pi} \right\rfloor (p-1) = n - \sigma$ which is next established. Using the p-adic representation of n and the definition of the floor function $\left\lfloor \frac{n}{pi} \right\rfloor = a_i + a_{i+1}p + a_2 p^2 + \cdots + a_m p^{m-i}$, $i \le m$. Next, computing the two sums one gets

$$\sum_{i=1}^{\infty} \left\lfloor \frac{n}{pi} \right\rfloor (p) = \sum_{i=1}^{\infty} (a_i p + a_{i+1}p^2 + \cdots + a_m p^{(m-i+1)})$$

$$\begin{aligned} = a_1 p \;+\; & a_2 p^2 + a_3 p^3 + \cdots + a_m p^m \\ +\; & a_2 p + a_3 p^2 + \cdots + a_m p^{m-1} \\ & \qquad + a_3 p + \dots + a_m p^{m-2} \\ & \qquad\qquad \vdots \\ & \qquad\qquad + a_m p \end{aligned}$$

$$\sum_{i=1}^{\infty} \left\lfloor \frac{n}{p^i} \right\rfloor = \sum_{i=1}^{\infty} (a_i + a_{i+1}p + \cdots + a_m p^{(m-i)})$$

$$\begin{array}{llllllll} = a_1 & + \; a_2 p & + \; a_3 p^2 & + & \cdots & + & a_m p^{m-1} \\ & + \; a_2 & + \; a_3 p & + & \cdots & + & a_m p^{m-2} \\ & & + \; a_3 & + & \ldots & + & a_m p^{m-3} \\ & & & & & & \vdots \\ & & & & & + & a_m . \end{array}$$

Finally performing the difference between the two sums

$$\sum_{i=1}^{\infty} \left\lfloor \frac{n}{p^i} \right\rfloor (p-1) = (a_1 p + a_2 p^2 + \cdots + a_m p^m) - (a_1 + a_2 + \cdots + a_m)$$
$$= (n - a_0) - (\sigma - a_0) = n - \sigma,$$

which proves the formula. □

Next, the theorem is proved.

Proof. Let the p-adic expansion of n and k be $n = a_0 + a_1 p + a_2 p^2 + \cdots + a_m p^m$, and $k = b_0 + b_1 p + b_2 p^2 + \cdots + b_m p^m$ where $a_i, b_i \in \{o, \ldots, p-1\}$. Now if p^τ is the largest prime power which divides $\begin{pmatrix} n+k \\ k \end{pmatrix}$ then $\nu = \mu(n+k) - \mu(n) - \mu(k)$. Therefore, it remains to prove that the following identity holds

$$c_p(n,k) = \mu(n+k) - \mu(n) - \mu(k). \tag{2.4}$$

Carrying out the p-adic addition of n and k produces carries $\epsilon_0, \epsilon_1, \ldots$ (obtained from $\epsilon_0 = \left\lfloor \frac{a_0+b_0}{p} \right\rfloor$ and $\epsilon_i = \left\lfloor \frac{a_i+b_i+\epsilon_{i-1}}{p} \right\rfloor, i = 1, 2, \ldots$), therefore, the sum of carries takes the form $c_p(n,k) = \sum_{i=0}^{\infty} \epsilon_i$. On the other hand, the p-adic representation of the sum $n+k$ can be expressed as $n+k = \sum_{i=0}^{\infty} c_i p^i$ where $c_i \in \{0, \ldots, p-1\}$. Moreover, the c_i digits of this addition in terms of those of n and k and the carries ϵ_i is given by the formula $c_i = a_i + b_i + \epsilon_{i-1} - \epsilon_i p, \epsilon{-1} = 0$ for $i = 0, 1, \ldots$ Finally, employing this last formula and Legendre‘s identity, 2.4 is shown to be true as can be seen in the next computation,

$$\begin{aligned} \nu &= \mu(n+k) - \mu(n) - \mu(k) \\ &= \frac{n+k-\sum_{i=0}^{\infty} c_i}{p-1} - \frac{n-\sum_{i=0}^{\infty} a_i}{p-1} - \frac{k-\sum_{i=0}^{\infty} b_i}{p-1} \\ &= \frac{1}{p-1}\left(\sum_{i=0}^{\infty} a_i + \sum_{i=0}^{\infty} b_i - \sum (a_i + b_i + \epsilon_{i-1} - \epsilon_i p)\right) = \frac{1}{p-1}\left(\sum_{i=0}^{\infty} \epsilon_i p - \epsilon_{i-1}\right) \\ &= \frac{1}{p-1}\left(\sum_{i=0}^{\infty} \epsilon_i (p-1)\right) \\ &= \sum_{i=0}^{\infty} \epsilon_i = c_p(n,k). \end{aligned}$$

Therefore, Theorem 3 is established. □

Corollary 5. Let n be any integer and p a prime number $(\leq n)$. Set $\tau = c_p(n-1,1)$ then, n is divided by the prime power p^{τ} but not by $p^{\tau+1}$.

In number theory, a prime k-tuple is an ordered set of values (i.e. a vector) representing a repeatable pattern of prime numbers, being some of the most common:

- Twin primes, a set of two prime numbers that differ by two except for the pair $(2,3)$.
- Cousin primes, a set of two prime numbers that differ by four.
- Sexy primes, a set of pairs of prime numbers that differ by six
- Prime triplets, a set of three prime numbers of the form $(p, p+2, p+6)$ or $(p, p+4, p+6)$ with the exceptions of (2, 3, 5) and (3, 5, 7).
- Prime quadruplets, a set of four primes of the form $(p, p+2, p+6, p+8)$.

Notice that a prime triplet contains a pair of twin primes (p and $p+2$, or $p+4$ and $p+6$), a pair of cousin primes (p and $p+4$, or $p+2$ and $p+6$), and a pair of sexy primes (p and $p+6$) and also that a prime quadruplet contains two pairs of twin primes and two overlapping prime triplets.

Theorem 6 ([4]). *Let $2n+1$ be any number with $n \geq 2$ and consider the set $\Pi_T = \{p : p$ is a prime such that $3 \leq p < 2n+1\}$. Then the number $2n+1$ is a prime number if and only if $\forall p \in \Pi_T,\ p \mid \binom{n+\frac{p-1}{2}}{p-1}$.*

Theorem 7. *Let $2n-1$ and $2n+1$ be any two numbers with $n \geq 3$ and consider the set $\Pi_T = \{p : p$ is a prime such that $3 \leq p < 2n-1\}$. Then the pair $(2n-1, 2n+1)$ is a twin prime pair if and only if $\forall p \in \Pi_T,\ p \mid \binom{n+\frac{p-3}{2}}{p-2}$.*

Proof. First, assume that the pair $(2n-1, 2n+1)$ is a twin prime pair we must show that $\forall p \in \Pi,\ p \mid \binom{n+\frac{p-3}{2}}{p-2}$. Since each one of them is a prime number, from Theorem 6: $p \mid \binom{n+\frac{p-3}{2}}{p-1}$ and $p \mid -\binom{n+\frac{p-1}{2}}{p-1}$ but

$$\binom{n+\frac{p-3}{2}}{p-2} = \binom{n+\frac{p-1}{2}}{p-1} - \binom{n+\frac{p-3}{2}}{p-1} > 0 \tag{2.5}$$

Therefore $p \mid \binom{n+\frac{p-3}{2}}{p-2}$ as desired.

Now let us prove the converse. Assume that the pair $(2n-1, 2n+1)$ is not a twin prime. If $p \mid (2n-1) \Rightarrow 2n-1 = p(2m+1)$ with $m \geq 1$. Then $n + \frac{p-3}{2} = p-1+mp$ and the binomial coefficient becomes

$$\frac{(p-1+mp)(p-2+mp)\cdots(2+mp)}{(p-2)!}$$

In case $p \mid (2n+1)$ we get that

$$\frac{(p-2+mp)(p-1+mp)\cdots(1+mp)}{(p-2)!}$$

In either case p does not divide $\binom{n+\frac{p-3}{2}}{p-2}$. $\square$

Remark 8. The characterization given in theorem 7 for twin primes is expressed in terms of one binomial coefficient thanks to the recursive formula for binomial coefficients (2.5) applied to the binomial coefficient conditions for each one of the primes $2n-1$ and $2n+1$ (this can be interpreted as a linear combination with coefficients equal to one). Unfortunately, it is not possible to obtain a similar recursive formula when the difference between the pair of primes is higher than two (as can be proved by direct computation). This will be seen in the next results whose proofs are obtained straightforwardly by applying Theorems 6 and 7.

Theorem 9. *Let $2n+1$ and $2n+5$ be any two numbers with $n \geq 2$ and consider the sets $\Pi_{T_1} = \{p : p$ is a prime such that $3 \leq p < 2n+1\}$ and $\Pi_{T_2} = \{p : p$ is a prime such that $3 \leq p < 2n+5\}$. Then the pair $(2n+1, 2n+5)$ is a cousin prime pair if and only if $\forall p \in \Pi_{T_1}$, $p \mid \binom{n+\frac{p-1}{2}}{p-1}$ and $\forall p \in \Pi_{T_2}$, $p \mid \binom{n+\frac{p+3}{2}}{p-1}$.*

Theorem 10. *Let $2n+1$ and $2n+7$ be any two numbers with $n \geq 2$ and consider the sets $\Pi_{T_1} = \{p : p$ is a prime such that $3 \leq p < 2n+1\}$ and $\Pi_{T_2} = \{p : p$ is a prime such that $3 \leq p < 2n+7\}$. Then the pair $(2n+1, 2n+7)$ is a sexy prime pair if and only if $\forall p \in \Pi_{T_1}$, $p \mid \binom{n+\frac{p-1}{2}}{p-1}$ and $\forall p \in \Pi_{T_2}$, $p \mid \binom{n+\frac{p+5}{2}}{p-1}$.*

Theorem 11. *Let $2n-1$, $2n+1$, $2n+3$ and $2n+5$ be any four numbers with $n \geq 3$ and consider the sets $\Pi_{T_1} = \{p : p$ is a prime such that $3 \leq p < 2n-1\}$, $\Pi_{T_2} = \{p : p$ is a prime such that $3 \leq p < 2n+3\}$. Then $(2n-1, 2n+1, 2n+5)$ or $(2n-1, 2n+3, 2n+5)$ are a prime triplet if and only if $\forall p \in \Pi_{T_1}$, $p \mid \binom{n+\frac{p-3}{2}}{p-2}$ and $\forall p \in \Pi_{T_2}$, $p \mid \binom{n+\frac{p+3}{2}}{p-1}$ or $\forall p \in \Pi_{T_1}$, $p \mid \binom{n+\frac{p-3}{2}}{p-1}$ and $\forall p \in \Pi_{T_2}$, $p \mid \binom{n+\frac{p+1}{2}}{p-2}$.*

Theorem 12. *Let $2n-1$, $2n+1$, $2n+5$ and $2n+7$ be any four numbers with $n \geq 3$ and consider the sets $\Pi_{T_1} = \{p : p$ is a prime such that $3 \leq p < 2n-1\}$ and $\Pi_{T_2} = \{p : p$ is a prime such that $3 \leq p < 2n+5\}$. Then $(2n-1, 2n+1, 2n+5, 2n+7)$ is a prime quadruplet if and only if $\forall p \in \Pi_{T_1}$, $p \mid \binom{n+\frac{p-3}{2}}{p-2}$ and $\forall p \in \Pi_{T_2}$, $p \mid \binom{n+\frac{p+3}{2}}{p-2}$.*

Remark 13. Notice that the conditions given in Theorem 12 are not unique since a prime quadruplet can be considered as two pairs of twin primes (as was done here), or as two overlapping prime triplets. This means, that Theorem 12 could have been stated in terms of prime triplets however, the characterization provided here results to be more economical.

3. GENERATOR ALGORITHMS FOR PRIMES

This section provides an algorithm for generation of primes whose proof follows directly from Theorems 3 and 6. The algorithm proposed works in such a way that at the same time that generates the primes gives a factorization of the input n in terms of prime numbers in case of existing.

Algorithm

Step 1. Enter $n = 2$ and the set of primes $\Pi = \{3\}$.
Step 2. Do $\forall p \in \Pi_F = \{p : p \text{ is a prime} < 2n+1\}$ (where a numeration of the elements of Π_F will be denoted by $\{q_{k_i}\}_{i=1}^{\#(\Pi_F)}$ and Π_F denotes the restriction of the set of primes Π to the condition specified in Π_F) the q_{k_i}adic addition of $n + \frac{p-1}{2} - (p-1)$ and $p-1$, compute the number of carries $c_{q_{k_i}}$. Set $k = q_{k_1}^{c_{q_{k_1}}} \cdot q_{k_2}^{c_{q_{k_2}}} \cdot \cdots \cdot q_{k_{\sqrt{n}}}^{c_{q_{k_{\#(\Pi_F)}}}}$.
Step 3. If $k = 1$ then n is prime, set $\Pi = \{n\} \bigcup \Pi$ and $n = n + 2$ otherwise, n is composite and its prime factorization is given by $n = kq$ with $q = \frac{n}{k}$, set $n = n + 1$.
Step 4. Go to step 2.

4. GENERATOR ALGORITHMS FOR PRIME k-TUPLES

In this section the primes k-tuple generator algorithms, based on Theorems 3, 7, 9, 10, 11 and 12 for prime k-tuples are presented.

Twin primes generator algorithm

Step 1. Enter $n = 3$, Π and the set of twin primes $\Pi_{TP} = \{(2, 3), (3, 5)\}$.
Step 2. Do $\forall p \in \Pi_T = \{p : p \text{ is a prime such that } 3 \leq p < 2n - 1\}$ (where a numeration of the elements of Π_T will be denoted by $\{q_{k_i}\}_{i=1}^{\#(\Pi_T)}$ and Π_T denotes the restriction of the set of primes Π to the condition specified in Π_T) the q_{k_i}adic addition of $n + \frac{p-3}{2} - (p-2)$ and $p-2$, compute the number of carries $c_{q_{k_i}}$.
Step 3. If the number of carries is such that $c_{q_{k_i}} \geq 1 \, \forall p \in \Pi_T$ then, the pair of integers $(2n - 1, 2n + 1)$ is a twin prime pair, include the twin prime pair in the set Π_{TP} otherwise is not and p divides either $2n - 1$ or $2n + 1$.
Step 4. Set $n = n + 1$ and go to step 2.

Cousin primes generator algorithm

Step 1. Enter $n = 2$, Π and the set of cousin primes $\Pi_{CP} = \{(3, 7)\}$.
Step 2. Do $\forall p \in \Pi_{T_1} = \{p : p \text{ is a prime such that } 3 \leq p < 2n + 1\}$ and $\forall p \in \Pi_{T_2} = \{p : p \text{ is a prime such that } 3 \leq p < 2n + 5\}$ (where a numeration of the elements of

Π_{T_1} will be denoted by $\{q^1_{k_i}\}_{i=1}^{\#(\Pi_{T_1})}$ and a numeration of the elements of Π_{T_2} will be denoted by $\{q^2_{k_i}\}_{i=1}^{\#(\Pi_{T_2})}$ and Π_{T_1} and Π_{T_2} denote the restriction of the set of primes Π to the conditions specified in Π_{T_1} and Π_{T_2} respectively) the $q^1_{k_i}$ adic addition of $n+\frac{p-1}{2}-(p-1)$ and $p-1$, and the $q^2_{k_i}$adic addition of $n+\frac{p+3}{2}-(p-1)$ and $p-1$, compute the number of carries $c^1_{q_{k_i}}$ and $c^2_{q_{k_i}}$.
Step 3. If the number of carries is such that $c^1_{q_{k_i}} \geq 1$ $\forall p \in \Pi_{T_1}$ and $c^2_{q_{k_i}} \geq 1$ $\forall p \in \Pi_{T_2}$ then, the pair of integers $(2n+1, 2n+5)$ is a cousin prime, include the cousin prime in the set Π_{CP} otherwise is not and p divides either $2n+1$ or $2n+5$.
Step 4. Set $n = n+1$ and go to step 2.

Sexy primes generator algorithm
Step 1. Enter $n = 2$, Π and the set of sexy primes $\Pi_{SP} = \{\emptyset\}$.
Step 2. Do $\forall p \in \Pi_{T_1} = \{p : p$ is a prime such that $3 \leq p < 2n+1\}$ and $\forall p \in \Pi_{T_2} = \{p : p$ is a prime such that $3 \leq p < 2n+7\}$ (where a numeration of the elements of Π_{T_1} will be denoted by $\{q^1_{k_i}\}_{i=1}^{\#(\Pi_{T_1})}$ and a numeration of the elements of Π_{T_2} will be denoted by $\{q^2_{k_i}\}_{i=1}^{\#(\Pi_{T_2})}$ and Π_{T_1} and Π_{T_2} denote the restriction of the set of primes Π to the conditions specified in Π_{T_1} and Π_{T_2} respectively) the $q^1_{k_i}$ adic addition of $n+\frac{p-1}{2}-(p-1)$ and $p-1$, and the $q^2_{k_i}$adic addition of $n+\frac{p+5}{2}-(p-1)$ and $p-1$, compute the number of carries $c^1_{q_{k_i}}$ and $c^2_{q_{k_i}}$.
Step 3. If the number of carries is such that $c^1_{q_{k_i}} \geq 1$ $\forall p \in \Pi_{T_1}$ and $c^2_{q_{k_i}} \geq 1$ $\forall p \in \Pi_{T_2}$ then, the pair of integers $(2n+1, 2n+7)$ is a sexy prime, include the sexy prime in the set Π_{SP} otherwise is not and p divides either $2n+1$ or $2n+7$.
Step 4. Set $n = n+1$ and go to step 2.

Prime triplets generator algorithm
Case a) Prime triplets of the form $(2n-1, 2n+1, 2n+5)$.
Step 1. Enter $n = 3$, Π and the set of prime triplets $\Pi_{TP} = \{(2,3,5), (3,5,7)\}$.
Step 2. Do $\forall p \in \Pi_{T_1} = \{p : p$ is a prime such that $3 \leq p < 2n-1\}$ and $\forall p \in \Pi_{T_2} = \{p : p$ is a prime such that $3 \leq p < 2n+5\}$ (where a numeration of the elements of Π_{T_1} will be denoted by $\{q^1_{k_i}\}_{i=1}^{\#(\Pi_{T_1})}$ and a numeration of the elements of Π_{T_2} will be denoted by $\{q^2_{k_i}\}_{i=1}^{\#(\Pi_{T_2})}$ and Π_{T_1} and Π_{T_2} denote the restriction of the set of primes Π to the conditions specified in Π_{T_1} and Π_{T_2} respectively) the $q^1_{k_i}$ adic addition of $n+\frac{p-3}{2}-(p-2)$ and $p-2$, and the $q^2_{k_i}$adic addition of $n+\frac{p+3}{2}-(p-1)$ and $p-1$, compute the number of carries $c^1_{q_{k_i}}$ and $c^2_{q_{k_i}}$.
Step 3. If the number of carries is such that $c^1_{q_{k_i}} \geq 1$ $\forall p \in \Pi_{T_1}$ and $c^2_{q_{k_i}} \geq 1$ $\forall p \in \Pi_{T_2}$ then, the triplet of integers $(2n-1, 2n+1, 2n+5)$ is a prime triplet, include the prime triplet in the set Π_{TP} otherwise is not and p divides $2n-1$ or $2n+1$ or $2n+5$.
Step 4. Set $n = n+1$ and go to step 2.

Case b) Prime triplets of the form $(2n-1, 2n+3, 2n+5)$.
Step 1. Enter $n = 3$, Π and the set of prime triplets $\Pi_{TP} = \{(2,3,5), (3,5,7)\}$.

Step 2. Do $\forall p \in \Pi_{T_1} = \{p : p \text{ is a prime such that } 3 \leq p < 2n-1\}$ and $\forall p \in \Pi_{T_2} = \{p : p \text{ is a prime such that } 3 \leq p < 2n+3\}$ (where a numeration of the elements of Π_{T_1} will be denoted by $\{q^1_{k_i}\}_{i=1}^{\#(\Pi_{T_1})}$ and a numeration of the elements of Π_{T_2} will be denoted by $\{q^2_{k_i}\}_{i=1}^{\#(\Pi_{T_2})}$ and Π_{T_1} and Π_{T_2} denote the restriction of the set of primes Π to the conditions specified in Π_{T_1} and Π_{T_2} respectively) the $q^1_{k_i}$ adic addition of $n + \frac{p-3}{2} - (p-1)$ and $p-1$, and the $q^2_{k_i}$adic addition of $n + \frac{p+1}{2} - (p-2)$ and $p-2$, compute the number of carries $c^1_{q_{k_i}}$ and $c^2_{q_{k_i}}$.
Step 3. If the number of carries is such that $c^1_{q_{k_i}} \geq 1 \; \forall p \in \Pi_{T_1}$ and $c^2_{q_{k_i}} \geq 1 \; \forall p \in \Pi_{T_2}$ then, the triplet of integers $(2n-1, 2n+3, 2n+5)$ is a prime triplet, include the prime triplet in the set Π_{TP} otherwise is not and p divides $2n-1$ or $2n+3$ or $2n+5$.
Step 4. Set $n = n+1$ and go to step 2.

Prime quadruplets generator algorithm

Step 1. Enter $n = 3$, Π and the set of prime quadruplet $\Pi_{QP} = \{\emptyset\}$.
Step 2. Do $\forall p \in \Pi_{T_1} = \{p : p \text{ is a prime such that } 3 \leq p < 2n-1\}$ and $\forall p \in \Pi_{T_2} = \{p : p \text{ is a prime such that } 3 \leq p < 2n+5\}$ (where a numeration of the elements of Π_{T_1} will be denoted by $\{q^1_{k_i}\}_{i=1}^{\#(\Pi_{T_1})}$ and a numeration of the elements of Π_{T_2} will be denoted by $\{q^2_{k_i}\}_{i=1}^{\#(\Pi_{T_2})}$ and Π_{T_1} and Π_{T_2} denote the restriction of the set of primes Π to the conditions specified in Π_{T_1} and Π_{T_2} respectively) the $q^1_{k_i}$ adic addition of $n + \frac{p-3}{2} - (p-2)$ and $p-2$, and the $q^2_{k_i}$adic addition of $n + \frac{p+3}{2} - (p-2)$ and $p-2$, compute the number of carries $c^1_{q_{k_i}}$ and $c^2_{q_{k_i}}$.
Step 3. If the number of carries is such that $c^1_{q_{k_i}} \geq 1 \; \forall p \in \Pi_{T_1}$ and $c^2_{q_{k_i}} \geq 1 \; \forall p \in \; \Pi_{T_2}$ then, $(2n-1, 2n+1, 2n+5, 2n+7)$ is a prime quadruplet, include the quadruplet prime in the set Π_{QP} otherwise is not and p divides $2n-1$ or $2n+1$ or $2n+5$ or $2n+7$.
Step 4. Set $n = n+1$ and go to step 2.

5. CONCLUSIONS

In this paper algorithms for generation of prime k-tuples were presented. The methodology proposed, based on the divisibility properties of binomial expressions, results in a new computing approach where divisions are substituted by p-adic additions. Theorems 7, 9, 10, 11 and 12 give new characterizations for k-tuple primes which permit to check if a given k-tuple number is a k-tuple prime or not.

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