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DISCRETE-TIME COUNTERPARTS OF IMPULSIVE COHEN-GROSSBERG NEURAL NETWORKS OF NEUTRAL TYPE

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ABSTRACT. The discrete-time counterpart of an impulsive Cohen-Grossberg neural network of neutral type is introduced. Sufficient conditions for the existence and global asymptotic stability of a unique equilibrium point of the discrete-time system considered are obtained.

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1. INTRODUCTION

An artificial neural network is an information processing paradigm that is inspired by the way biological nervous systems, such as the brain, process information. The key element of this paradigm is the novel structure of the information processing system. It is composed of a large number of highly interconnected processing elements (neurons) working in unison to solve specific problems. Although the initial intent of artificial neural networks was to explore and reproduce human information processing tasks such as speech, vision, and knowledge processing, artificial neural networks also demonstrated their superior capability for classification and function approximation problems. This has great potential for solving complex problems such as systems control, data compression, optimization problems, pattern recognition, and system identification.

Cohen-Grossberg neural network [9] and its various generalizations with or without transmission delays and impulsive state displacements have been the subject of intense investigation recently [3, 5, 6, 13, 16, 17]. In a Cohen-Grossberg neural network model, the feedback terms consist of amplification and stabilizing functions

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which are generally nonlinear. These terms provide the model with a special kind of generalization wherein many neural network models that are capable for content addressable memory such as additive neural networks, cellular neural networks and bidirectional associative memory networks and also biological models such as Lotka-Volterra models of population dynamics are included as special cases.

In contrast to retarded systems, in neutral systems time delays appear explicitly in the state velocity vector. Neutral systems can be applied to describe more complicated nonlinear engineering and bioscience models, including those describing chemical reactors, transmission lines, partial element equivalent circuits in very large-scale integrated systems, and Lotka-Volterra systems [18, 14, 4, 1, 2, 8, 15]. Neural networks can be implemented using very large-scale integrated circuits. Therefore, both retarded-type delays and neutral type delays are inherent in the dynamics of neural networks.

In the present paper we consider a Cohen-Grossberg neural network of neutral type as in [7] provided with impulse conditions. A modification of the semi-discretization method given in [12] is used for obtaining a discrete-time analogue. Sufficient conditions for global asymptotic stability of the unique equilibrium point of the discrete-time system are obtained by exploiting an appropriate Lyapunov sequence.

2. PRELIMINARIES

We consider an impulsive Cohen-Grossberg neural network of neutral type consisting of $m \geq 2$ elementary processing units (or neurons) whose state variables x_i ($i = \overline{1,m}$ which henceforth will stand for $i = 1, 2, \ldots, m$) are governed by the system

$$\begin{aligned} \dot{x}_i(t) + \sum_{j=1}^{m} e_{ij}\dot{x}_j(x-\tau_j) = a_i(x_i(t)) \Bigg[-b_i(x_i(t)) \\ + \sum_{j=1}^{m} c_{ij} f_j(x_j(t)) + \sum_{j=1}^{m} d_{ij} g_j(x_j(t-\tau_j)) + I_i \Bigg], \\ t > t_0 = 0, \quad t \neq t_k, \end{aligned} \tag{2.1}$$

$$\begin{aligned} \Delta x_i(t_k) = \gamma_{ik} x_i(t_k) + \sum_{j=1}^{m} \delta_{ijk} x_j(t_k - \tau_j) + \zeta_{ik}, \\ i = \overline{1,m}, \quad k \in \mathbb{N} = \{1, 2, 3, \ldots\}, \end{aligned} \tag{2.2}$$

with initial values prescribed by piecewise-continuous functions $x_i(s) = \phi_i(s)$ with discontinuities of the first kind for $s \in [-\tau, 0]$, $\tau = \max_{j=\overline{1,m}}\{\tau_j\}$. In (2.1), $a_i(x_i)$ denotes an amplification function; $b_i(x_i)$ denotes an appropriate function which supports the stabilizing (or negative) feedback term $-a_i(x_i)b_i(x_i)$ of the unit i; $f_j(x_j)$, $g_j(x_j)$ denote activation functions; the parameters c_{ij}, d_{ij} are real numbers that represent the weights (or strengths) of the synaptic connections between the j-th unit and the i-th unit, respectively without and with time delays τ_j; the real numbers e_{ij} show how the state velocities of the neurons are delay feed-forward connected in the network; the real constant I_i represents an input signal introduced from outside the network to the i-th unit; in (2.2) $\Delta x_i(t_k) = x_i(t_k+0) - x_i(t_k-0)$ denote impulsive state displacements at fixed moments of time t_k, $k \in \mathbb{N}$, involving time delays τ_j. Here it is assumed that $x_i(t_k+0) = \lim\limits_{t\to t_k+0} x_i(t)$ and $x_i(t_k-0) = \lim\limits_{t\to t_k-0} x_i(t)$, and the sequence of times $\{t_k\}_{k=1}^{\infty}$ satisfies $0 = t_0 < t_1 < t_2 < \cdots < t_k \to \infty$ as $k \to \infty$.

As usual in the theory of impulsive differential equations, at the points of discontinuity t_k of the solution $t \mapsto x(t) = (x_1(t), x_2(t), \ldots, x_m(t))^T$ we assume that $x_i(t_k) \equiv x_i(t_k-0)$. It is clear that, in general, the derivatives $\dot{x}_i(t_k)$ do not exist. On the other hand, according to (2.1) there exist the limits $\dot{x}_i(t_k \mp 0)$. According to the above convention, we assume $\dot{x}_i(t_k) \equiv \dot{x}_i(t_k-0)$.

The assumptions that accompany the impulsive network (2.1), (2.2) are given as follows:

(H1) The amplification functions $a_i : \mathbb{R} \to \mathbb{R}^+$ are continuous and bounded in the sense that

$$0 < \underline{a}_i \le a_i(x) \le \overline{a}_i \quad \text{for } \ x \in \mathbb{R},\ i = \overline{1,m}.$$

(H2) The stabilizing functions $b_i : \mathbb{R} \to \mathbb{R}$ are Lipschitz continuous and monotone increasing, namely,

$$0 < \underline{b}_i \le \frac{b_i(x) - b_i(y)}{x - y} \le \overline{b}_i \quad \text{for } \ x \ne y,\ x, y \in \mathbb{R},\ i = \overline{1,m}.$$

(H3) The activation functions $f_j, g_j : \mathbb{R} \to \mathbb{R}$ are Lipschitz continuous, that is, there exist positive constants F_j, G_j such that

$$F_j = \sup_{x\ne y} \left| \frac{f_j(x) - f_j(y)}{x - y} \right|, \ G_j = \sup_{x\ne y} \left| \frac{g_j(x) - g_j(y)}{x - y} \right|$$

for $x, y \in \mathbb{R},\ j = \overline{1,m}$.

Under these assumptions and the given initial conditions, there is a unique solution of the impulsive network (2.1), (2.2). The solution is a vector $x(t) = (x_1(t), x_2(t), \ldots, x_m(t))^T$ in which $x_i(t)$ are piecewise continuous for $t \in (0, \beta)$, where β is some positive number, possibly ∞, such that the limits $x_i(t_k+0)$ and $x_i(t_k-0)$ exist and $x_i(t)$ are differentiable for $t \in (t_{k-1}, t_k) \subset (0, \beta)$. An equilibrium point of the impulsive network (2.1), (2.2) is denoted by $x^* = (x_1^*, x_2^*, \ldots, x_m^*)^T$ where the components

x_i^* are governed by the algebraic system

$$b_i(x_i^*) = \sum_{j=1}^{m} c_{ij} f_j(x_j^*) + \sum_{j=1}^{m} d_{ij} g_j(x_j^*) + I_i, \quad i = \overline{1,m}, \tag{2.3}$$

and satisfy the linear equations

$$\gamma_{ik} x_i^* + \sum_{j=1}^{m} \delta_{ijk} x_j^* + \zeta_{ik} = 0, \quad k \in \mathbb{N},\ i = \overline{1,m}. \tag{2.4}$$

3. FORMULATION OF AN IMPULSIVE DISCRETE-TIME ANALOGUE

Till recently, the semi-discretization model had not been exploited for obtaining a discrete-time analogue of Cohen-Grossberg neural network mainly due to the non-linearity of the feedback terms $-a_i(x_i)b_i(x_i)$. An appropriate extension of the method was presented in [12]. Exploiting the same idea, we start by rewriting the differential system (2.1) as

$$\begin{aligned} &\dot{x}_i(t) + \beta_i x_i(t) + \sum_{j=1}^{m} e_{ij} \left(\dot{x}_j(t-\tau_j) + \beta_i x_j(t-\tau_j)\right) \\ &= \beta_i x_i(t) + \sum_{j=1}^{m} e_{ij} \beta_i x_j(t-\tau_j) + a_i(x_i(t)) \Bigg[-b_i(x_i(t)) \\ &+ \sum_{j=1}^{m} c_{ij} f_j(x_j(t)) + \sum_{j=1}^{m} g_j(x_j(t-\tau_{ij})) + I_i \Bigg], \\ &i = \overline{1,m}, \quad t > 0, \quad t \neq t_k, \end{aligned} \tag{3.1}$$

where $\beta_i = \underline{a}_i \underline{b}_i > 0$. Let the value $h > 0$ of the discretization step be fixed, and $n = [t/h]$, $\sigma_j = [\tau_j/h]$, where $[r]$ denotes the greatest integer contained in the real number r. On any interval $[nh, (n+1)h)$ not containing a moment of impulse effect t_k we multiply equation (3.1) by $e^{\beta_i t}$ and approximate it by an equation with constant arguments of the form

$$\begin{aligned} &\frac{d}{dt}\left(x_i(t) e^{\beta_i t} + \sum_{j=1}^{m} e_{ij} x_j(t - \sigma_j h) e^{\beta_i t} \right) \\ &= \beta_i e^{\beta_i t} \left(x_i(nh) + \sum_{j=1}^{m} e_{ij} x_j((n-\sigma_j)h) \right) \\ &+ e^{\beta_i t} a_i(x_i(nh)) \Bigg[-b_i(x_i(nh)) + \sum_{j=1}^{m} c_{ij} f_j(x_j(nh)) \\ &+ \sum_{j=1}^{m} d_{ij} g_j(x_j((n-\sigma_j)h) + I_i \Bigg], \quad i = \overline{1,m}, \end{aligned} \tag{3.2}$$

with $[t/h]h = nh \to t$, $[\tau_j/h]h = \sigma_j h \to \tau_j$ for a fixed time t as $h \to 0$. Upon integrating (3.2) over the interval $[nh, (n+1)h)$, one obtains a discrete analogue of the differential system (2.1) given by

$$\begin{aligned}
&x_i(n+1)e^{\beta_i(n+1)h} - x_i(n)e^{\beta_i nh} \\
&\quad + \sum_{j=1}^{m} e_{ij}\left(x_j(n+1-\sigma_j)e^{\beta_i(n+1)h} - x_j(n-\sigma_j)e^{\beta_i nh}\right) \\
&= \left(e^{\beta_i(n+1)h} - e^{\beta_i nh}\right)\left(x_i(n) + \sum_{j=1}^{m} e_{ij}x_j(n-\sigma_j)\right) \\
&\quad + \frac{e^{\beta_i(n+1)h} - e^{\beta_i nh}}{\beta_i} a_i(x_i(n))\Bigg[-b_i(x_i(n)) \\
&\quad + \sum_{j=1}^{m} c_{ij}f_j(x_j(n)) + \sum_{j=1}^{m} d_{ij}g_j(x_j(n-\sigma_j)) + I_i\Bigg]
\end{aligned}$$

for $i = \overline{1,m}$, $n \in \{0\} \cup \mathbb{N}$, wherein the notation $w(n) \equiv w(nh)$ has been adopted for simplicity. We multiply the i-th equation of this system by $e^{-\beta_i(n+1)h}$ and obtain the difference system

$$\begin{aligned}
x_i(n+1) &= x_i(n) + \sum_{j=1}^{m} e_{ij}(x_j(n-\sigma_j) - x_j(n+1-\sigma_j)) \\
&\quad + \psi_i(h)a_i(x_i(n))\Bigg[-b_i(x_i(n)) + \sum_{j=1}^{m} c_{ij}f_j(x_j(n)) \\
&\quad + \sum_{j=1}^{m} d_{ij}g_j(x_j(n-\sigma_j)) + I_i\Bigg], \qquad i = \overline{1,m}, \quad n \in \{0\} \cup \mathbb{N},
\end{aligned} \tag{3.3}$$

where we have denoted $\psi_i(h) = \frac{1-e^{-\beta_i h}}{\beta_i}$. Observe that $0 < \psi_i(h) < \frac{1}{\beta_i}$ for $h > 0$ and $\psi_i(h) = h + O(h^2)$ for small $h > 0$.

The analogue (3.3) is supplemented with an initial vector sequence $\phi(\ell) = (\phi_1(\ell), \phi_2(\ell), \ldots, \phi_m(\ell))^T$ for $\ell = \overline{-\sigma, 0}$, $\sigma = \max_{j=\overline{1,m}} \sigma_j$. Next we discretize the impulse conditions (2.2). If we denote $n_k = \left[\frac{t_k}{h}\right]$, we obtain a sequence of positive integers $\{n_k\}$ satisfying $0 < n_1 < n_2 < \cdots < n_k \to \infty$ as $k \to \infty$. With each such integer n_k we associate two values of the solution $x(n)$, namely, $x(n_k)$ which can be regarded as the value of the solution before the impulse effect and whose components are evaluated by equations (3.3), and $x^+(n_k)$ which can be regarded as the value of the solution after the impulse effect and whose components are evaluated by the equations

$$x_i^+(n_k) = (1+\gamma_{ik})x_i(n_k) + \sum_{j=1}^{m} \delta_{ijk}x_j(n_k - \sigma_j) + \zeta_{ik},\ i = \overline{1,m},\ k \in \mathbb{N}. \tag{3.4}$$

If a value of $x(n)$ in the right-hand side of (3.3) or (3.4) must be evaluated at a member of the sequence $\{n_k\}_{k\in\mathbb{N}}$, we take $x^+(n_k)$ evaluated from (3.4). The existence of a unique solution $x(n) = (x_1(n), x_2(n), \ldots, x_m(n))^T$ of the impulsive analogue (3.3), (3.4) for $n \in \{0\} \cup \mathbb{N}$ is therefore justified.

If we want to give a formal description of the discrete-time analogue of the impulsive system (2.1), (2.2), we should write

$$\begin{aligned} x_i^-(n+1) = x_i^+(n) &+ \sum_{j=1}^m e_{ij}(x_j^+(n-\sigma_j) - x_j^+(n+1-\sigma_j)) \\ &+ \psi_i(h)a_i(x_i^+(n)) \Bigg[-b_i(x_i^+(n)) + \sum_{j=1}^m c_{ij} f_j(x_j^+(n)) \\ &+ \sum_{j=1}^m d_{ij} g_j(x_j^+(n-\sigma_j)) + I_i \Bigg], \qquad n \in \{0\} \cup \mathbb{N}, \end{aligned}$$

$$x_i^+(n) = \begin{cases} x_i^-(n) & \text{for } n \neq n_k, \\ (1+\gamma_{ik})x_i^-(n_k) + \sum_{j=1}^m \delta_{ijk} x_j^-(n_k - \sigma_j) + \zeta_{ik} & \text{for } n = n_k, \end{cases}$$

$i = \overline{1,m}$. Systems (2.1), (2.2) and (3.3), (3.4) have the same equilibrium points if any. Their components must satisfy (2.3), (2.4).

Definition 3.1. The equilibrium point $x^* = (x_1^*, x_2^*, \ldots, x_m^*)^T$ of system (3.3), (3.4) is said to be *globally asymptotically stable* if any other solution $x(n) = (x_1(n), x_2(n), \ldots,$ $x_m(n))^T$ of system (3.3), (3.4) is defined for all $n \in \mathbb{N}$ and satisfies

$$\lim_{n\to\infty} x(n) = x^*.$$

4. EXISTENCE AND GLOBAL ASYMPTOTIC STABILITY OF AN EQUILIBRIUM POINT

Our first task is to prove the existence and uniqueness of the solution x^* of the algebraic system (2.3). To this end we will need the following lemma.

Lemma 4.1 ([10])**.** *A locally invertible C^0 map $\Phi : \mathbb{R}^m \to \mathbb{R}^m$ is a homeomorphism of $\mathbb{R}^m$ onto itself if and only if it is proper.*

In fact, this assertion is due to Hadamard [11]. A mapping is proper if the preimage of every compact is compact. In the finite-dimensional case it suffices to show that $\|\Phi(x)\| \to \infty$ as $\|x\| \to \infty$.

Theorem 4.2. *Let the assumptions* (H2), (H3) *hold. Suppose, further, that the following inequalities are valid:*

$$\text{(4.1)} \qquad \underline{b}_i - \frac{1}{2}\sum_{j=1}^m (|c_{ij}|F_j + |c_{ji}|F_i) - \frac{1}{2}\sum_{j=1}^m (|d_{ij}|G_j + |d_{ji}|G_i) > 0, \ i = \overline{1,m}.$$

Then the system without impulses (2.1) *has a unique equilibrium point* $x^* = (x_1^*, x_2^*, \ldots, x_m^*)^T$.

Proof. Let us define a mapping $\Phi : \mathbb{R}^m \to \mathbb{R}^m$ by $\Phi(x) = (\Phi_1(x), \Phi_2(x), \ldots, \Phi_m(x))^T$ for $x \in \mathbb{R}^m$, where

$$\Phi_i(x) = b_i(x_i) - \sum_{j=1}^{m} c_{ij} f_j(x_j) - \sum_{j=1}^{m} d_{ij} g_j(x_j) - I_i, \quad i = \overline{1, m}.$$

The space $\mathbb{R}^m$ is endowed with the Euclidean norm $\|x\| = \left(\sum_{i=1}^{m} x_i^2\right)^{1/2}$. We denote by $\langle \cdot, \cdot \rangle$ the respective inner product. Under the assumptions (H2), (H3), $\Phi(x) \in C^0$. It is known that if $\Phi(x) \in C^0$ is a homeomorphism of $\mathbb{R}^m$, then there is a unique point $x^* = (x_1^*, x_2^*, \ldots, x_m^*)^T \in \mathbb{R}^m$ such that $\Phi(x^*) = 0$, that is, $\Phi_i(x^*) = 0,\ i = \overline{1, m}$. The last equalities are, in fact, (2.3), so $x^* = (x_1^*, x_2^*, \ldots, x_m^*)^T$ is the equilibrium point we are looking for.

To demonstrate the one-to-one property of $\Phi(x)$, we take arbitrary vectors $x, y \in \mathbb{R}^m$ and assume that $\Phi(x) = \Phi(y)$. We multiply the equalities

$$b_i(x_i) - b_i(y_i) = \sum_{j=1}^{m} c_{ij} \left(f_j(x_j) - f_j(y_j)\right) + \sum_{j=1}^{m} d_{ij} \left(g_j(x_j) - g_j(y_j)\right), \quad i = \overline{1, m},$$

respectively by $x_i - y_i$ and add them together to obtain

$$\begin{aligned} \sum_{i=1}^{m} \left(b_i(x_i) - b_i(y_i)\right)(x_i - y_i) &= \sum_{i=1}^{m} \sum_{j=1}^{m} c_{ij} \left(f_j(x_j) - f_j(y_j)\right)(x_i - y_i) \\ &\quad + \sum_{i=1}^{m} \sum_{j=1}^{m} d_{ij} \left(g_j(x_j) - g_j(y_j)\right)(x_i - y_i). \end{aligned}$$

According to the assumptions (H2), (H3) we derive

$$\begin{aligned} \sum_{i=1}^{m} \underline{b}_i (x_i - y_i)^2 &\leq \sum_{i=1}^{m} \sum_{j=1}^{m} |c_{ij}|\, F_j\, |x_j - y_j|\, |x_i - y_i| \\ &\quad + \sum_{i=1}^{m} \sum_{j=1}^{m} |d_{ij}|\, G_j\, |x_j - y_j|\, |x_i - y_i| \\ &\leq \frac{1}{2} \sum_{i=1}^{m} \sum_{j=1}^{m} |c_{ij}|\, F_j\, \left[(x_j - y_j)^2 + (x_i - y_i)^2\right] \\ &\quad + \frac{1}{2} \sum_{i=1}^{m} \sum_{j=1}^{m} |d_{ij}|\, G_j\, \left[(x_j - y_j)^2 + (x_i - y_i)^2\right] \end{aligned}$$

$$= \sum_{i=1}^{m} \left\{ \frac{1}{2} \sum_{j=1}^{m} \left(|c_{ij}| F_j + |c_{ji}| F_i\right) + \frac{1}{2} \sum_{j=1}^{m} \left(|d_{ij}| G_j + |d_{ji}| G_i\right) \right\} (x_i - y_i)^2,$$

that is,

$$\sum_{i=1}^{m}\left\{\underline{b}_i-\frac{1}{2}\sum_{j=1}^{m}(|c_{ij}|F_j+|c_{ji}|F_i)-\frac{1}{2}\sum_{j=1}^{m}(|d_{ij}|G_j+|d_{ji}|G_i)\right\}(x_i-y_i)^2\le 0.$$

Now the assertion $x_i = y_i$, $i=\overline{1,m}$, follows by virtue of inequalities (4.1). Thus, $\Phi(x)=\Phi(y)$ implies $x=y$.

Next we show that $\|\Phi(x)\|\to\infty$ as $\|x\|\to\infty$. It suffices to show that $\|\tilde{\Phi}(x)\|\to\infty$, where $\tilde{\Phi}(x)=\Phi(x)-\Phi(0)$. We have $\tilde{\Phi}(x)=(\tilde{\Phi}_1(x),\tilde{\Phi}_2(x),\ldots,\tilde{\Phi}_m(x))^T$, where

$$\tilde{\Phi}_i(x)=(b_i(x_i)-b_i(0))-\sum_{j=1}^{m}c_{ij}(f_j(x_j)-f_j(0))-\sum_{j=1}^{m}d_{ij}(g_j(x_j)-g_j(0)).$$

Then

$$\begin{aligned}\langle\tilde{\Phi}(x),x\rangle=\sum_{i=1}^{m}\tilde{\Phi}_i(x)x_i=\sum_{i=1}^{m}\Bigg\{&(b_i(x_i)-b_i(0))x_i\\ &-\sum_{j=1}^{m}c_{ij}(f_j(x_j)-f_j(0))x_i-\sum_{j=1}^{m}d_{ij}(g_j(x_j)-g_j(0))x_i\Bigg\},\end{aligned}$$

from which by virtue of the assumptions (H2), (H3) we derive

$$\begin{aligned}\left|\langle\tilde{\Phi}(x),x\rangle\right|&\ge\sum_{i=1}^{m}\left\{\underline{b}_ix_i^2-\sum_{j=1}^{m}|c_{ij}|\,F_j\,|x_j|\,|x_i|-\sum_{j=1}^{m}|d_{ij}|\,G_j\,|x_j|\,|x_i|\right\}\\&\ge\sum_{i=1}^{m}\left\{\underline{b}_ix_i^2-\frac{1}{2}\sum_{j=1}^{m}|c_{ij}|\,F_j\,(x_j^2+x_i^2)-\frac{1}{2}\sum_{j=1}^{m}|d_{ij}|\,G_j\,(x_j^2+x_i^2)\right\}\\&=\sum_{i=1}^{m}\left\{\underline{b}_i-\frac{1}{2}\sum_{j=1}^{m}(|c_{ij}|F_j+|c_{ji}|F_i)-\frac{1}{2}\sum_{j=1}^{m}(|d_{ij}|G_j+|d_{ji}|G_i)\right\}x_i^2.\end{aligned}$$

By virtue of inequalities (4.1) there exists a number $\mu>0$ such that

$$\underline{b}_i-\frac{1}{2}\sum_{j=1}^{m}(|c_{ij}|F_j+|c_{ji}|F_i)-\frac{1}{2}\sum_{j=1}^{m}(|d_{ij}|G_j+|d_{ji}|G_i)\ge\mu,\qquad i=\overline{1,m}.$$

Then $\|\tilde{\Phi}(x)\|\cdot\|x\|\ge|\langle\tilde{\Phi}(x),x\rangle|\ge\mu\|x\|^2$ and $\|\tilde{\Phi}(x)\|\ge\mu\|x\|\to\infty$ as $\|x\|\to\infty$.

According to Lemma 4.1, $\Phi(x)\in C^0$ is a homeomorphism of $\mathbb{R}^m$. Thus, there is a unique point $x^*\in\mathbb{R}^m$ such that $\Phi(x^*)=0$. The point represents a unique solution of the algebraic system (2.3). □

Theorem 4.3. *Let the assumptions* (H1)–(H3) *hold. Suppose, further, that the inequalities*

$$\underline{a}_i\underline{b}_i-\frac{1}{2}\sum_{j=1}^{m}(\overline{a}_i|c_{ij}|F_j+\overline{a}_j|c_{ji}|F_i) \tag{4.2}$$

$$- \frac{1}{2} \sum_{j=1}^{m} (\overline{a}_i |d_{ij}| G_j + \overline{a}_j |d_{ji}| G_i) - \frac{1}{2} \sum_{j=1}^{m} \left(\overline{a}_i \overline{b}_i |e_{ij}| + \overline{a}_j \overline{b}_j |e_{ji}| \right)$$

$$- \frac{1}{2} \sum_{j=1}^{m} \overline{a}_j \sum_{k=1}^{m} (|c_{ji}|\, |e_{jk}|\, F_i + |c_{jk}|\, |e_{ji}|\, F_k)$$

$$- \frac{1}{2} \sum_{j=1}^{m} \overline{a}_j \sum_{k=1}^{m} (|d_{ji}|\, |e_{jk}|\, G_i + |d_{jk}|\, |e_{ji}|\, G_k) > 0, \quad i = \overline{1, m},$$

and the conditions

$$\delta_{ijk} = \gamma_{ik} e_{ij}, \quad -2 \le \gamma_{ij} \le 0, \qquad i, j = \overline{1, m}, \quad k \in \mathbb{N}, \tag{4.3}$$

are valid and the system (3.3), (3.4) *has an equilibrium point* $x^* = (x_1^*, x_2^*, \ldots, x_m^*)^T$ *whose components satisfy* (2.3), (2.4). *Then the equilibrium point* x^* *is globally asymptotically stable for all sufficiently small values of* $h > 0$.

Remark 4.4. Inequalities (4.1) can be deduced from (4.2) for $\underline{a}_i = \overline{a}_i = 1$, $e_{ij} = 0$ for $i, j = \overline{1, m}$. However, in general inequalities (4.2) do not imply (4.1).

Proof. Upon introducing the translations

$$u_i(n) = x_i(n) - x_i^*, \qquad \varphi_i(\ell) = \phi_i(\ell) - x_i^*$$

we derive the system

$$u_i(n+1) + \sum_{j=1}^{m} e_{ij} u_j(n+1-\sigma_j) = u_i(n) + \sum_{j=1}^{m} e_{ij} u_j(n-\sigma_j) \tag{4.4}$$

$$+\psi_i(h) \tilde{a}_i(u_i(n)) \left[-\tilde{b}_i(u_i(n)) + \sum_{j=1}^{m} c_{ij} \tilde{f}_j(u_j(n)) + \sum_{j=1}^{m} d_{ij} \tilde{g}_j(u_j(n-\sigma_j)) \right],$$

$$i = \overline{1, m}, \quad n \in \mathbb{N},$$

$$u_i^+(n_k) = (1+\gamma_{ik}) u_i(n_k) + \sum_{j=1}^{m} \delta_{ijk} u_j(n_k - \sigma_j), \tag{4.5}$$

$$i = \overline{1, m}, \quad k \in \mathbb{N},$$

$$u_i(\ell) = \varphi_i(\ell), \qquad i = \overline{1, m}, \quad \ell = \overline{-\sigma, 0},$$

where

$$\tilde{a}_i(u_i) = a_i(u_i + x_i^*), \qquad \tilde{b}_i(u_i) = b_i(u_i + x_i^*) - b_i(x_i^*),$$
$$\tilde{f}_j(u_j) = f_j(u_j + x_j^*) - f_j(x_j^*), \qquad \tilde{g}_j(u_j) = g_j(u_j + x_j^*) - g_j(x_j^*).$$

This system inherits the assumptions (H1)–(H3) given before. It suffices to examine the stability characteristics of the trivial equilibrium point $u^* = 0$ of system (4.4), (4.5).

We define a Lyapunov sequence $\{V(n)\}_{n=0}^{\infty}$ by

$$V(n) = \frac{1}{2}\sum_{i=1}^{m}\left[u_i(n) + \sum_{j=1}^{m} e_{ij}u_j(n-\sigma_j)\right]^2 + h\sum_{i=1}^{m}\omega_i \sum_{\ell=n-\sigma_i}^{n-1} u_i^2(\ell),$$

where ω_i, $i = \overline{1,m}$, will be determined later. First we notice that the value $V(0)$ is completely determined from the initial values of the system. Then we successively find

$$\begin{aligned} V(n+1) &= \frac{1}{2}\sum_{i=1}^{m}\left[u_i(n+1) + \sum_{j=1}^{m} e_{ij}u_j(n+1-\sigma_j)\right]^2 \\ &\quad + h\sum_{i=1}^{m}\omega_i \sum_{\ell=n+1-\sigma_i}^{n} u_i^2(\ell) \\ = &\ \frac{1}{2}\sum_{i=1}^{m}\left\{\left[u_i(n) + \sum_{j=1}^{m} e_{ij}u_j(n-\sigma_j)\right] + \psi_i(h)\tilde{a}_i(u_i(n))\left[-\tilde{b}_i(u_i(n))\right.\right. \\ + &\left.\left. \sum_{j=1}^{m} c_{ij}\tilde{f}_j(u_j(n)) + \sum_{j=1}^{m} d_{ij}\tilde{g}_j(u_j(n-\sigma_j))\right]\right\}^2 + h\sum_{i=1}^{m}\omega_i \sum_{\ell=n+1-\sigma_i}^{n} u_i^2(\ell), \end{aligned}$$

$$\begin{aligned} V(n+1) - V(n) &= \sum_{i=1}^{m}\psi_i(h)\tilde{a}_i(u_i(n))\left[u_i(n) + \sum_{j=1}^{m} e_{ij}u_j(n-\sigma_j)\right] \\ &\times\left[-\tilde{b}_i(u_i(n)) + \sum_{j=1}^{m} c_{ij}\tilde{f}_j(u_j(n)) + \sum_{j=1}^{m} d_{ij}\tilde{g}_j(u_j(n-\sigma_j))\right] \\ &+\frac{1}{2}\sum_{i=1}^{m}\psi_i^2(h)\tilde{a}_i^2(u_i(n))\left[-\tilde{b}_i(u_i(n)) + \sum_{j=1}^{m} c_{ij}\tilde{f}_j(u_j(n))\right. \\ &\left. + \sum_{j=1}^{m} d_{ij}\tilde{g}_j(u_j(n-\sigma_j))\right]^2 + h\sum_{i=1}^{m}\omega_i\left(u_i^2(n) - u_i^2(n-\sigma_i)\right) \\ = &\ h\sum_{i=1}^{m}\left\{\tilde{a}_i(u_i(n))\left[u_i(n) + \sum_{j=1}^{m} e_{ij}u_j(n-\sigma_j)\right]\right. \\ &\times\left[-\tilde{b}_i(u_i(n)) + \sum_{j=1}^{m} c_{ij}\tilde{f}_j(u_j(n)) + \sum_{j=1}^{m} d_{ij}\tilde{g}_j(u_j(n-\sigma_j))\right] \\ &\left. + \left(C_i(h) + \omega_i\right)u_i^2(n) + \left(D_i(h) - \omega_i\right)u_i^2(n-\sigma_i)\right\}, \end{aligned}$$

where $C_i(h)$, $D_i(h) = O(h)$ for $i = \overline{1,m}$. For the sake of brevity we do not write in details the terms of order $O(h^2)$. Then

$$V(n+1) - V(n) = h\sum_{j=1}^{m}\left\{-\tilde{a}_i(u_i(n))\tilde{b}_i(u_i(n))u_i(n)\right.$$

$$
\begin{aligned}
&+ \quad \tilde{a}_i(u_i(n))u_i(n)\left[\sum_{j=1}^{m} c_{ij}\tilde{f}_j(u_j(n)) + \sum_{j=1}^{m} d_{ij}\tilde{g}_j(u_j(n-\sigma_j))\right] \\
&+ \quad \tilde{a}_i(u_i(n))\sum_{j=1}^{m} e_{ij}u_j(n-\sigma_j)\left[-\tilde{b}_i(u_i(n)) + \sum_{j=1}^{m} c_{ij}\tilde{f}_j(u_j(n))\right. \\
&+ \quad \left.\sum_{j=1}^{m} d_{ij}\tilde{g}_j(u_j(n-\sigma_j))\right] + (C_i(h)+\omega_i)\, u_i^2(n) + (D_i(h)-\omega_i)\, u_i^2(n-\sigma_i)\Bigg\} \\
&\le \quad h\sum_{i=1}^{m}\Bigg\{-\underline{a}_i\underline{b}_i u_i^2(n) + \overline{a}_i|u_i(n)|\left[\sum_{j=1}^{m}|c_{ij}|\,F_j\,|u_j(n)|\right. \\
&\qquad \left.+\sum_{j=1}^{m}|d_{ij}|\,G_j\,|u_j(n-\sigma_j)|\right] + \overline{a}_i\sum_{j=1}^{m}|e_{ij}|\,|u_j(n-\sigma_j)|\left[\overline{b}_i\,|u_i(n)|\right. \\
&\qquad \left.+\sum_{j=1}^{m}|c_{ij}|\,F_j\,|u_j(n)| + \sum_{j=1}^{m}|d_{ij}|\,G_j\,|u_j(n-\sigma_j)|\right] \\
&\qquad + (C_i(h)+\omega_i)\, u_i^2(n) + (D_i(h)-\omega_i)\, u_i^2(n-\sigma_i)\Bigg\} \\
&\le \quad h\sum_{i=1}^{m}\Bigg\{-\underline{a}_i\underline{b}_i u_i^2(n) + \overline{a}_i\frac{1}{2}\sum_{j=1}^{m}|c_{ij}|\,F_j\left(u_i^2(n)+u_j^2(n)\right) \\
&\qquad +\overline{a}_i\frac{1}{2}\sum_{j=1}^{m}|d_{ij}|\,G_j\left(u_i^2(n)+u_j^2(n-\sigma_j)\right) \\
&\qquad +\overline{a}_i\overline{b}_i\frac{1}{2}\sum_{j=1}^{m}|e_{ij}|\left(u_i^2(n)+u_j^2(n-\sigma_j)\right) \\
&\qquad +\overline{a}_i\frac{1}{2}\sum_{j=1}^{m}\sum_{k=1}^{m}|e_{ij}|\,|c_{ik}|\,F_k\left(u_k^2(n)+u_j^2(n-\sigma_j)\right) \\
&\qquad +\overline{a}_i\frac{1}{2}\sum_{j=1}^{m}\sum_{k=1}^{m}|e_{ij}|\,|d_{ik}|\,G_k\left(u_j^2(n-\sigma_j)+u_k^2(n-\sigma_k)\right) \\
&\qquad + (C_i(h)+\omega_i)\, u_i^2(n) + (D_i(h)-\omega_i)\, u_i^2(n-\sigma_i)\Bigg\} \\
&= \quad h\sum_{i=1}^{m}\Bigg\{-\Bigg[\underline{a}_i\underline{b}_i - \frac{1}{2}\left(\overline{a}_i\sum_{j=1}^{m}|c_{ij}|F_j + F_i\sum_{j=1}^{m}|c_{ji}|\overline{a}_j\right) - \frac{a_i}{2}\sum_{j=1}^{m}|d_{ij}|G_j \\
&- \quad \frac{\overline{a}_i\overline{b}_i}{2}\sum_{j=1}^{m}|e_{ij}| - \frac{F_i}{2}\sum_{j=1}^{m}\sum_{k=1}^{m}|c_{ki}|\,|e_{kj}|\overline{a}_k - C_i(h) - \omega_i\Bigg]\, u_i^2(n) \\
&+ \quad \Bigg[\frac{G_i}{2}\sum_{j=1}^{m}|d_{ji}|\overline{a}_j + \frac{1}{2}\sum_{j=1}^{m}|e_{ji}|\overline{a}_j\overline{b}_j + \frac{1}{2}\sum_{j=1}^{m}\sum_{k=1}^{m}|e_{ji}|\,|c_{jk}|\overline{a}_jF_k
\end{aligned}
$$

$$+\quad \frac{1}{2}\sum_{j=1}^{m}\sum_{k=1}^{m}\left(|e_{ji}|\,|d_{jk}|\overline{a}_jG_k+|e_{kj}|\,|d_{ki}|\overline{a}_kG_i\right)+D_i(h)-\omega_i\Bigg]\,u_i^2(n-\sigma_i)\Bigg\}.$$

Choose

$$\begin{aligned}\omega_i &= \frac{G_i}{2}\sum_{j=1}^{m}|d_{ji}|\overline{a}_j+\frac{1}{2}\sum_{j=1}^{m}|e_{ji}|\overline{a}_j\overline{b}_j+\frac{1}{2}\sum_{j=1}^{m}\sum_{k=1}^{m}|e_{ji}|\,|c_{jk}|\overline{a}_jF_k\\ &+\frac{1}{2}\sum_{j=1}^{m}\sum_{k=1}^{m}\left(|e_{ji}|\,|d_{jk}|\overline{a}_jG_k+|e_{kj}|\,|d_{ki}|\overline{a}_kG_i\right)+D_i(h),\end{aligned}$$

then after some simplifications we obtain

$$\begin{aligned}&V(n+1)-V(n)\le -h\sum_{i=1}^{m}\Bigg\{\underline{a}_i\underline{b}_i-\frac{1}{2}\sum_{j=1}^{m}\left(\overline{a}_i|c_{ij}|F_j+\overline{a}_j|c_{ji}|F_i\right)\\ &-\frac{1}{2}\sum_{j=1}^{m}\left(\overline{a}_i|d_{ij}|G_j+\overline{a}_j|d_{ji}|G_i\right)-\frac{1}{2}\sum_{j=1}^{m}\left(\overline{a}_i\overline{b}_i|e_{ij}|+\overline{a}_j\overline{b}_j|e_{ji}|\right)\\ &-\frac{1}{2}\sum_{j=1}^{m}\overline{a}_j\sum_{k=1}^{m}\left(|c_{ji}|\,|e_{jk}|\,F_i+|c_{jk}|\,|e_{ji}|\,F_k\right)\\ &-\frac{1}{2}\sum_{j=1}^{m}\overline{a}_j\sum_{k=1}^{m}\left(|d_{ji}|\,|e_{jk}|\,G_i+|d_{jk}|\,|e_{ji}|\,G_k\right)-C_i(h)-D_i(h)\Bigg\}\,u_i^2(n).\end{aligned}$$

According to conditions (4.2) there exists $\delta>0$ such that

$$\begin{aligned}\delta &= \min_{i=\overline{1,m}}\Bigg\{\underline{a}_i\underline{b}_i-\frac{1}{2}\sum_{j=1}^{m}\left(\overline{a}_i|c_{ij}|F_j+\overline{a}_j|c_{ji}|F_i\right)-\frac{1}{2}\sum_{j=1}^{m}\left(\overline{a}_i|d_{ij}|G_j+\overline{a}_j|d_{ji}|G_i\right)\\ &-\frac{1}{2}\sum_{j=1}^{m}\left(\overline{a}_i\overline{b}_i|e_{ij}|+\overline{a}_j\overline{b}_j|e_{ji}|\right)-\frac{1}{2}\sum_{j=1}^{m}\overline{a}_j\sum_{k=1}^{m}\left(|c_{ji}|\,|e_{jk}|\,F_i+|c_{jk}|\,|e_{ji}|\,F_k\right)\\ &-\frac{1}{2}\sum_{j=1}^{m}\overline{a}_j\sum_{k=1}^{m}\left(|d_{ji}|\,|e_{jk}|\,G_i+|d_{jk}|\,|e_{ji}|\,G_k\right)\Bigg\}.\end{aligned}$$

Further on, we can choose the discretization step h so small that $|C_i(h)+D_i(h)|<\delta/2$, then

$$V(n+1)-V(n)\le -\frac{h\delta}{2}\|u(n)\|^2,\quad n\in\{0\}\cup\mathbb{N}. \tag{4.6}$$

In case $n=n_k$, in the above inequality instead of $V(n)$ we should take the value $V^+(n)$ evaluated for $u_i^+(n_k)$ given by (4.5). Thus

$$\begin{aligned}V^+(n_k)-V(n_k) &= \frac{1}{2}\sum_{i=1}^{m}\Bigg\{\Bigg[(1+\gamma_{ik})u_i(n_k)+\sum_{j=1}^{m}(e_{ij}+\delta_{ijk})u_j(n_k-\sigma_j)\Bigg]^2\\ &\qquad -\Bigg[u_i(n_k)+\sum_{j=1}^{m}e_{ij}u_j(n_k-\sigma_j)\Bigg]^2\Bigg\}\end{aligned}$$

$$= \frac{1}{2}\sum_{i=1}^{m}\left[\gamma_{ik}u_i(n_k)+\sum_{j=1}^{m}\delta_{ijk}u_j(n_k-\sigma_j)\right]$$
$$\times \left[(2+\gamma_{ik})u_i(n_k)+\sum_{j=1}^{m}(2e_{ij}+\delta_{ijk})u_j(n_k-\sigma_j)\right].$$

According to conditions (4.3) we have

$$V^+(n_k)-V(n_k)=\frac{1}{2}\sum_{i=1}^{m}\gamma_{ik}(2+\gamma_{ik})\left[u_i(n_k)+\sum_{j=1}^{m}e_{ij}u_j(n_k-\sigma_j)\right]^2\le 0,$$

which implies the validity of (4.6) also for $n=n_k$ and $V(n_k)$ evaluated for $u_i(n_k)$.

The inequalities (4.6) show that for any solution $u(n)$ of system (4.4), (4.5) the sequence $\{V(n)\}_{n=0}^{\infty}$ is monotone decreasing and it is bounded below by 0. Thus there exists the limit $\lim_{n\to\infty} V(n)\ge 0$. Passing to the limit as $n\to\infty$ in (4.6), we find that $\lim_{n\to\infty}\|u(n)\|=0$, that is, $\lim_{n\to\infty}\|x(n)-x^*\|=0$. This means that the equilibrium point x^* of system (3.3), (3.4) is globally asymptotically stable. □

Example 4.5. Consider the system

(4.7)
$$\begin{aligned} &\dot{x}_1(t)+0.1\dot{x}_1(t-\tau_1)+0.15\dot{x}_2(t-\tau_2)\\ =\;&(2+0.01\sin x_1(t))\left[-2x_1(t)+0.1\arctan x_1(t)+0.15\arctan x_2(t)\right.\\ &\left.+\,0.1\arctan x_1(t-\tau_1)+0.15\arctan x_2(t-\tau_2)+1\right],\\ &\dot{x}_2(t)-0.2\dot{x}_1(t-\tau_1)+0.1\dot{x}_2(t-\tau_2)\\ =\;&(3-0.02\sin x_2(t))\left[-3x_2(t)+0.15\arctan x_1(t)-0.2\arctan x_2(t)\right.\\ &\left.+\,0.1\arctan x_1(t-\tau_1)-0.2\arctan x_2(t-\tau_2)+1\right],\ t>0,\ t\ne t_k,\end{aligned}$$

(4.8)
$$\begin{aligned} \Delta x_1(t_k)&=-1.1x_1(t_k)-0.11x_1(t_k-\tau_1)\\ &\quad-0.165x_2(t_k-\tau_2)+0.7847118585,\\ \Delta x_2(t_k)&=-0.9x_2(t_k)+0.18x_1(t_k-\tau_1)\\ &\quad-0.09x_2(t_k-\tau_2)+0.2235363329,\quad k\in\mathbb{N},\end{aligned}$$

with arbitarary initial conditions $x_i(s)=\phi(s)$, $i=1,2$, $s\in[-\max\{\tau_1,\tau_2\},0]$.

System (4.7), (4.8) has the form (2.1), (2.2). It satisfies assumptions (H1)–(H3) with $\underline{a}_1=1.99$, $\overline{a}_1=2.01$, $\underline{a}_2=2.98$, $\overline{a}_2=3.02$, $\underline{b}_1=\overline{b}_1=2$, $\underline{b}_2=\overline{b}_2=3$, $F_1=F_2=G_1=G_2=1$.

The discrete-time counterpart of system (4.7), (4.8) is

(4.9)
$$\begin{aligned} &x_1(n+1)=x_1(n)+0.1\left[x_1(n-\sigma_1)-x_1(n+1-\sigma_1)\right]\\ +\;&0.15\left[x_2(n-\sigma_2)-x_2(n+1-\sigma_2)\right]+\frac{1-e^{-3.98h}}{3.98}(2+0.01\sin x_1(n))\\ \times\;&\left[-2x_1(n)+0.1\arctan x_1(n)+0.15\arctan x_2(n)\right.\end{aligned}$$

$$
\begin{aligned}
& \quad + 0.1 \arctan x_1(n-\sigma_1) + 0.15 \arctan x_2(n-\sigma_2) + 1 \right], \\
& \quad x_2(n+1) = x_2(n) - 0.2 \left[x_1(n-\sigma_1) - x_1(n+1-\sigma_1) \right] \\
+ & \quad 0.1 \left[x_2(n-\sigma_2) - x_2(n+1-\sigma_2) \right] + \frac{1-e^{-8.94h}}{8.94} \left(3 - 0.02 \sin x_2(n) \right) \\
\times & \quad \left[-3x_2(n) + 0.15 \arctan x_1(n) - 0.2 \arctan x_2(n) \right. \\
& \quad \left. + 0.1 \arctan x_1(n-\sigma_1) - 0.2 \arctan x_2(n-\sigma_2) + 1 \right] \quad n \in \{0\} \cup \mathbb{N},
\end{aligned}
$$

$$
\begin{aligned}
x_1^+(n_k) &= -0.1 x_1(n_k) - 0.11 x_1(n_k-\sigma_1) \\
&\qquad - 0.165 x_2(n_k-\sigma_2) + 0.7847118585, \\
x_2^+(n_k) &= 0.1 x_2(t_k) + 0.18 x_1(n_k-\sigma_1) \\
&\qquad - 0.09 x_2(n_k-\sigma_2) + 0.2235363329, \quad k \in \mathbb{N},
\end{aligned}
\tag{4.10}
$$

with initial conditions $x_i(\ell) = \phi(\ell)$, $\ell = \overline{-\sigma, 0}$, $\sigma = \max\{\sigma_1, \sigma_2\}$ and $\sigma_i = [\tau_i/h]$, $i = 1, 2$.

It is easy to see that system (4.7) satisfies inequalities (4.1). In fact, the left-hand sides of these inequalities are equal respectively to 1.525 and 2.325 for $i = 1$ and 2. Thus system (4.7) or (4.9) has a unique equilibrium point x^*. We can find that $x^* = (0.6027869379, 0.3353919007)^T$. Its components satisfy the linear equations (2.4), thus x^* is the unique equilibrium of the continuous-time impulsive system (4.7), (4.8) and its discrete-time analogue (4.9), (4.10).

Further on, system (4.9), (4.10) satisfies the assumptions of Theorem 4.3. In fact, the left-hand sides of inequalities (4.2) are equal respectively to 0.87945 and 4.5471375 for $i = 1$ and 2. Thus the equilibrium point x^* of system (4.9), (4.10) is globally asymptotically stable for sufficiently small values of the discretization step $h > 0$.

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