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ROBUST STABILITY AND STABILIZATION OF LINEAR POLYTOPIC DELAY-DIFFERENCE EQUATIONS WITH INTERVAL TIME-VARYING DELAYS

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ABSTRACT. This paper addresses the robust stability for a class of linear delay-difference equations with interval time-varying delays. Based on the parameter-dependent Lyapunov-Krasovskii functional, new delay-dependent conditions for the robust stability are established in terms of linear matrix inequalities. An application to robust stabilization of linear discrete-time control systems is given. Numerical examples are included to illustrate the effectiveness of our results.

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1. INTRODUCTION

During the last decades, the problem of stability and stabilization of dynamical systems with time delays has received considerable attention, and lots of interesting results have reported in the literature, see, e.g; [1, 3, 5, 12, 15–18] and the references therein. Some delay-dependent stability criteria for discrete-time systems with time-varying delay are investigated in [3, 9, 13, 16, 19], where the discrete Lyapunov functional method are employed to prove stability conditions in terms of linear matrix inequalities (LMIs). A number research works for dealing with asymptotic stability problem for discrete systems with interval time-varying delays have been presented in [2, 6, 10, 20, 21]. Theoretically, stability analysis of the systems with time-varying delays is more complicated, especially for the case where the system matrices belong to some convex polytope. In this case, the parameter-dependent Lyapunov-Krasovskii functionals are constructed as the convex combination of a set of functions assures the robust stability of the nominal systems and the stability conditions must be solved upon a grid on the parameter space, which results in testing a finite number of linear matrix inequalities (LMI) [7, 8, 19]. To the best of the authors' knowledge, the stability for linear discrete-time systems with both time-varying delays and polytopic uncertainties has not been fully investigated. The papers [4, 11] propose sufficient conditions for robust stability of discrete and continuous polytopic systems without time delays. More recently, combining the ideas in [7, 8], improved conditions for

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$\mathcal{D}$-stability and $\mathcal{D}$-stabilization of linear polytopic delay-difference equations with constant delays have been proposed in [14].

In this paper, we consider linear polytopic discrete equations with interval time-varying delays. By using the parameter-dependent Lyapunov-Krasovskii functional combined with LMI techniques, we propose new criteria for the robust stability of the system. The delay-dependent stability conditions are formulated in terms of LMIs, being thus solvable by the numeric technology available in the literature to date. The result is applied to robust stabilization of linear discrete control systems. Compared to other results, our result has its own advantages. First, it deals with the delay-difference system, where the state-space data belong to the convex polytope of uncertainties and the rate of change of the state depends not only on the current state of the systems, but also its state at some times in the past. Second, the time-delay is assumed to be a time-varying function belonging to a given interval, which means that the lower and upper bounds for the time-varying delay are available. Third, our approach allows us to apply in robust stabilization of the linear system subjected to polytopic uncertainties and external controls. Therefore, our results are more general than the related previous results.

The paper is organized as follows. Section 2 introduces the main notations, definitions and some lemmas needed for the development of the main results. In Section 3, sufficient conditions are derived for robust stability, stabilization of linear discrete-time systems with interval time-varying delays and polytopic uncertainties. They are followed by some remarks. Illustrative examples are given in Section 4.

2. PRELIMINARIES

The following notations will be used throughout this paper.

- R^+ denotes the set of all real non-negative numbers; R^n denotes the n-dimensional space with the scalar product $\langle \cdot, \cdot \rangle$ and the vector norm $\| \cdot \|$;
- $R^{n\times r}$ denotes the space of all matrices of $(n \times r)$-dimension. A^T denotes the transpose of A; a matrix A is symmetric if $A = A^T$.
- Matrix A is semi-positive definite $(A \geq 0)$ if $\langle Ax, x\rangle \geq 0$, for all $x \in R^n$; A is positive definite $(A > 0)$ if $\langle Ax, x\rangle > 0$ for all $x \neq 0$; $A \geq B$ means $A - B \geq 0$.

Consider a delay-difference systems with polytopic uncertainties of the form

$$
(\Sigma_\xi) \qquad \begin{aligned} x(k+1) &= A(\xi)x(k) + D(\xi)x(k-h(k)), \quad k = 0, 1, 2, \ldots \\ x(k) &= v_k, \quad k = -h_2, -h_2+1, \ldots, 0, \end{aligned}
$$

where $x(k) \in R^n$ is the state, the system matrices are subjected to uncertainties and belong to the polytope Ω given by

$$\Omega = \left\{ [A, D](\xi) := \sum_{i=1}^{p} \xi_i [A_i, D_i], \quad \sum_{i=1}^{p} \xi_i = 1, \xi_i \geq 0 \right\},$$

where $A_i, D_i, i = 1, 2, \ldots, p$, are given constant matrices with appropriate dimensions. The time-varying function $h(k)$ satisfies the condition:

$$0 < h_1 \leq h(k) \leq h_2, \quad \forall k = 0, 1, 2, \ldots.$$

Remark 2.1. It is worth noting that the time delay is a time-varying function belonging to a given interval, which allows the time-delay to be a fast time-varying function and the lower bound is not restricted to being zero as considered in [3, 9, 13, 16, 19].

Definition 2.1. The system Σ_ξ is robustly stable if the zero solution of the system is asymptotically stable for all uncertainties in Ω.

Proposition 2.1. *For real numbers $\xi_i \geq 0, i = 1, 2, \ldots, p$, $\sum_{i=1}^{p} \xi_i = 1$, the following inequality hold*

$$(p-1) \sum_{i=1}^{p} \xi_i^2 - 2 \sum_{i=1}^{p-1} \sum_{j=i+1}^{p} \xi_i \xi_j \geq 0.$$

Proof. The proof is followed from the completing the square:

$$(p-1) \sum_{i=1}^{p} \xi_i^2 - 2 \sum_{i=1}^{p-1} \sum_{j=i+1}^{p} \xi_i \xi_j = \sum_{i=1}^{p-1} \sum_{j=i+1}^{p} (\xi_i - \xi_j)^2 \geq 0.$$

3. MAIN RESULTS

A. Robust stability. In this section, we present sufficient delay-dependent conditions for the robust stability of system (Σ_ξ). Let us set

$$\|x_k\| = \sup_{s \in [-h_2, 0]} \|x(k+s)\|,$$

$$\mathcal{M}_{ij}(P, Q, R, S) = \begin{pmatrix} (h_2 - h_1 + 1)Q_i - P_i - A_j^T R_i - R_i^T A_j & R_i^T - A_j^T S_i & -R_i^T D_j \\ R_i - S_i^T A_j & P_i + S_i + S_i^T & -S_i^T D_j \\ -D_j^T R_i & D_j^T S_i & -Q_i \end{pmatrix},$$

$$\mathcal{S} = \begin{pmatrix} S & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad P(\xi) = \sum_{i=1}^{p} \xi_i P_i, \quad Q(\xi) = \sum_{i=1}^{p} \xi_i Q_i,$$

$$R(\xi) = \sum_{i=1}^{p} \xi_i R_i, \quad S(\xi) = \sum_{i=1}^{p} \xi_i S_i.$$

Theorem 3.1. *The system Σ_ξ is robustly stable if there exist symmetric matrices $P_i > 0$, $Q_i > 0$, $S \geq 0$, R_i, S_i, $i = 1, 2 \ldots, p$ satisfying the following LMIs*

(i) $\mathcal{M}_{ii}(P,Q,R,S)+\mathcal{S}<0$, $i=1,2,\dots,p$.
(ii) $\mathcal{M}_{ij}(P,Q,R,S)+\mathcal{M}_{ji}(P,Q,R,S)-\frac{2}{p-1}\mathcal{S}<0$, $i=1,2,\dots,p-1$; $j=i+1,\dots,p$.

Proof. Consider the following parameter-dependent Lyapunov-Krasovskii functional for system (Σ_ξ)

$$V(k)=V_1(k)+V_2(k)+V_3(k),$$

where

$$V_1(k)=x^T(k)P(\xi)x(k),\quad V_2(k)=\sum_{i=k-h(k)}^{k-1}x^T(i)Q(\xi)x(i),$$

$$V_3(k)=\sum_{j=-h_2+2}^{-h_1+1}\sum_{l=k+j+1}^{k-1}x^T(l)Q(\xi)x(l),$$

We can verify that

$$\lambda_1\|x(k)\|^2\le V(k)\le\lambda_2\|x_k\|^2. \tag{3.1}$$

Let us set $z(k)=[x(k)\,x(k+1)\,x(k-h(k))]^T$, and

$$E(\xi)=\begin{pmatrix}0&0&0\\0&P(\xi)&0\\0&0&0\end{pmatrix},\quad F(\xi)=\begin{pmatrix}P(\xi)&0&0\\R(\xi)&S(\xi)&0\\0&0&I\end{pmatrix}.$$

Then, the difference of $V_1(k)$ along the solution of the system is given by

$$\begin{aligned}\Delta V_1(k)&=x^T(k+1)P(\xi)x(k+1)-x^T(k)P(\xi)x(k)\\&=z^T(k)E(\xi)z(k)-2z^T(k)F^T(\xi)\begin{pmatrix}0.5x(k)\\0\\0\end{pmatrix}.\end{aligned} \tag{3.2}$$

Using the expression of system (Σ_ξ)

$$0=-x(k+1)+A(\xi)x(k)+D(\xi)x(k-h(k)),$$

we have

$$-2z^T(k)F^T(\xi)\begin{pmatrix}0.5x(k)\\-x(k+1)+A(\xi)x(k)+D(\xi)x(k-h(k))\\0\end{pmatrix}z(k)$$

$$=-z^T(k)F^T(\xi)\begin{pmatrix}0.5I&0&0\\A(\xi)&-I&D(\xi)\\0&0&0\end{pmatrix}z(k)-z^T(k)\begin{pmatrix}0.5I&A^T(\xi)&0\\0&-I&0\\0&D^T(\xi)&0\end{pmatrix}F(\xi)z(k).$$

Therefore, from (3.2) it follows that

$$\Delta V_1(k)=z^T(k)M(\xi)z(k), \tag{3.3}$$

where

$$M(\xi) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & P(\xi) & 0 \\ 0 & 0 & 0 \end{pmatrix} - F^T(\xi) \begin{pmatrix} 0.5I & 0 & 0 \\ A(\xi) & -I & D(\xi) \\ 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} 0.5I & A^T(\xi) & 0 \\ 0 & -I & 0 \\ 0 & D^T(\xi) & 0 \end{pmatrix} F(\xi).$$

The difference of $V_2(k)$ is given by

(3.4)

$$\begin{aligned} \Delta V_2(k) = & \sum_{i=k+1-h(k+1)}^{k} x^T(i)Q(\xi)x(i) - \sum_{i=k-h(k)}^{k-1} x^T(i)Q(\xi)x(i) \\ = & \sum_{i=k+1-h(k+1)}^{k-h_1} x^T(i)Q(\xi)x(i) + x^T(k)Q(\xi)x(k) - x^T(k-h(k))Q(\xi)x(k-h(k)) \\ & + \sum_{i=k+1-h_1}^{k-1} x^T(i)Q(\xi)x(i) - \sum_{i=k+1-h(k)}^{k-1} x^T(i)Q(\xi)x(i). \end{aligned}$$

Since $h(k) \geq h_1$ we have

$$\sum_{i=k+1-h_1}^{k-1} x^T(i)Q(\xi)x(i) - \sum_{i=k+1-h(k)}^{k-1} x^T(i)Q(\xi)x(i) \leq 0,$$

and hence from (3.4) we have

(3.5)

$$\Delta V_2(k) \leq \sum_{i=k+1-h(k+1)}^{k-h_1} x^T(i)Q(\xi)x(i) + x^T(k)Q(\xi)x(k) - x^T(k-h(k))Q(\xi)x(k-h(k)).$$

The difference of $V_3(k)$ is given by

$$\begin{aligned} \Delta V_3(k) &= \sum_{j=-h_2+2}^{-h_1+1} [x^T(k)Q(\xi)x(k) - x^T(k+j-1)Q(\xi)x(k+j-1)] \\ &= (h_2-h_1)x^T(k)Q(\xi)x(k) - \sum_{l=k+1-h_2}^{k-h_1} x^T(l)Q(\xi)x(l). \end{aligned} \tag{3.6}$$

Since

$$\sum_{i=k=1-h(k+1)}^{k-h_1} x^T(i)Q(\xi)x(i) - \sum_{i=k+1-h_2}^{k-h_1} x^T(i)Q(\xi)x(i) \leq 0,$$

we obtain from (3.5) and (3.6) that

$$\Delta V_2(k) + \Delta V_3(k) \leq (h_2-h_1+1)x^T(k)Q(\xi)x(k) - x^T(k-h(k))Q(\xi)x(k-h(k)). \tag{3.7}$$

Therefore, combining the inequalities (3.3), (3.7) gives

$$\Delta V(k) \leq z^T(k)T(\xi)z(k), \tag{3.8}$$

where

$$T(\xi) = \begin{pmatrix} W(\xi) & -A^T(\xi)S(\xi) + R^T(\xi) & -R^T(\xi)D(\xi) \\ R(\xi) - S^T(\xi)A(\xi) & P(\xi) + S(\xi) + S^T(\xi) & -S^T(\xi)D(\xi) \\ -D^T(\xi)R(\xi) & -D^T(\xi)S(\xi) & -Q(\xi) \end{pmatrix},$$

and

$$W(\xi) = (h_2 - h_1 + 1)Q(\xi) - P(\xi) - A^T(\xi)R(\xi) - R(\xi)A(\xi).$$

Let us denote

$$\begin{aligned} W_{ij} &:= (h_2 - h_1 + 1)Q_i - P_i - A_j^T R_i - R_i^T A_j, \\ (A^T S)_{ij} &:= A_j^T S_i + A_i^T S_j, \quad Q_{ij} = Q_i + Q_j, \\ (R^T D)_{ij} &= R_i^T D_j + R_j^T D_i, \quad (S^T D)_{ij} = S_i^T D_j S_j D_i \\ (A^T R)_{ij} &= A_i^T R_j + A_j^T R_i, \quad S_{ij} = S_i + S_j, \quad R_{ij} = R_i + R_j. \end{aligned}$$

From the convex combination of the expression of $P(\xi), Q(\xi), R(\xi), S(\xi), A(\xi), D(\xi)$, we have

$$\begin{aligned} T(\xi) &= \sum_{i=1}^{p} \xi_i^2 \begin{pmatrix} W_{ii} & R_i^T - A_i^T S_i & -R_i^T D_i \\ R_i - S_i^T A_i & P_i + S_i + S_i^T & -S_i^T D_i \\ -D_i^T R_i & -D_i^T S_i & -Q_i \end{pmatrix} \\ &\quad + \sum_{i=1}^{p-1} \sum_{j=i+1}^{p} \xi_i \xi_j \begin{pmatrix} W_{ij} + W_{ji} & (A^T S)_{ij} + R_{ij} & -(R^T D)_{ij} \\ R_{ij}^T - (S^T A)_{ij} & P_{ij} + S_{ij} + S_{ij}^T & -(S^T D)_{ij} \\ -(D^T R)_{ij} & -(D^T S)_{ij} & -Q_{ij} \end{pmatrix} \\ &= \sum_{i=1}^{p} \xi_i^2 \mathcal{M}_i(P,Q,R,S) + \sum_{i=1}^{p-1} \sum_{j=i+1}^{p} \xi_i \xi_j [\mathcal{M}_{ij}(P,Q,R,S) + \mathcal{M}_{ji}(P,Q,R,S)]. \end{aligned}$$

Then the conditions (i), (ii) give

$$T(\xi) < -\sum_{i=1}^{p} \xi_i^2 \mathcal{S} + \frac{2}{p-1} \sum_{i=1}^{p-1} \sum_{j=i+1}^{p} \xi_i \xi_j \mathcal{S} \le 0,$$

because of Proposition 2.1:

$$(p-1)\sum_{i=1}^{p} \xi_i^2 - 2\sum_{i=1}^{p-1} \sum_{j=i+1}^{p} \xi_i \xi_j = \sum_{i=1}^{p-1} \sum_{j=i+1}^{p} (\xi_i - \xi_j)^2 \ge 0,$$

and hence, we finally obtain from (3.8) that

$$\Delta V(k) < 0, \quad \forall k = 0, 1, 2, \ldots.$$

which together with (3.1) implies that the system is robustly stable. This completes the proof of the theorem.

Remark 3.1. The stability conditions of Theorem 3.1 are more appropriate for practical systems since practically it is impossible to know exactly the delay but lower and upper bounds are always possible.

B. Robust stabilization. This section deals with a stabilization problem considered in [20] for constructing a delayed feedback controller, which stabilizes the resulting closed-loop system. The robust stability condition obtained in previous section will be applied to design a time-delayed state feedback controller for the discrete-time control system described by

$$x(k+1) = A(\xi)x(k) + B(\xi)u(k), \quad k = 0, 1, 2, \dots \tag{3.9}$$

where $u(k) \in R^n$ is the control input, the system matrices are subjected to uncertainties and belong to the polytope Ω given by

$$\Omega = \{[A, B](\xi) := \sum_{i=1}^{p} \xi_i [A_i, B_i], \quad \sum_{i=1}^{p} \xi_i = 1, \xi_i \geq 0\},$$

where A_i, B_i, $i = 1, 2, \dots, p$, are given constant matrices with appropriate dimensions. As in [18], we consider a parameter-dependent delayed feedback control law

$$u(k) = F(\xi)x(k - h(k)), k = -h_2, \dots, 0, \tag{3.10}$$

where $h(k)$ is the time-varying delay function satisfying $0 < h_1 \leq h(k) \leq h_2$, and $F(\xi)$ is the controller gain to be determined. Applying the feedback controller (3.10) to the system (3.9), the closed-loop time-delay system is

$$x(k+1) = A(\xi)x(k) + B(\xi)F(\xi)x(k - h(k)), \quad k = 0, 1, 2, \dots \tag{3.11}$$

Definition 3.1. The system (3.9) is robustly stablilizable if there is a delayed feedback control (3.10) such that the closed-loop delay system (3.11) is robustly stable. Let us

$$\mathcal{M}_{ij}(P, Q, R) = \begin{pmatrix} (h_2 - h_1 + 1)Q_i - P_i - A_j^T R_i - R_i^T A_j & R_i^T - A_j^T R_i & -P_i \\ R_i - R_i^T A_j & P_i + R_i + R_i^T & -P_i \\ -P_i & -P_i & -Q_i \end{pmatrix},$$

$$\mathcal{S} = \begin{pmatrix} S & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

The following theorem can be derived from Theorem 3.1.

Theorem 3.2. The system (3.9) is robustly stabilizable by the delayed feedback control (3.10), where

$$F(\xi) = B^T(\xi)[B(\xi)B^T(\xi)]^{-1} R(\xi)[R^T(\xi)R(\xi)]^{-1} P(\xi),$$

if there exist symmetric matrices $P_i > 0, Q_i > 0, i = 1, 2 \dots, p$ and constant matrices $R_i, i = 1, 2, \dots, p, S \geq 0$ satisfying the following LMIs (i) $\mathcal{M}_{ii}(P, Q, R) + \mathcal{S} < 0, i = 1, 2, \dots, p$.

(ii) $\mathcal{M}_{ij}(P, Q, R) + \mathcal{M}_{ji}(P, Q, R) - \frac{2}{p-1}\mathcal{S} < 0, \quad i = 1, 2, \dots, p-1; j = i+1, \dots, p.$

Proof. Taking $R_i = S_i$ and using the feedback control (3.10), the closed-loop system becomes system (Σ_ξ), where $D(\xi) = B(\xi)F(\xi) = R(\xi)[R^T(\xi)R(\xi)]^{-1}P(\xi)$. Since $R^T(\xi)D(\xi) = P(\xi)$, the robust stability condition of the closed-loop system (3.11), by Theorem 3.1, is immediately derived.

Remark 3.2. Note that the approach developed in this paper is based on the use of the modified Lyapunov-Krasovskii functional, but our solution is derived for a more generic class of linear systems with polytopic uncertainties and interval time-varying delays. When $p = 1$, taking $S_i = S$, $R_i = R$, $\mathcal{S} = 0$, the conditions of Theorem 3.1 become

$$\begin{pmatrix} (h_2 - h_1 + 1)Q - P - A^T R - R^T A & R^T - A^T R & -P \\ R - R^T A & P + R + R^T & -P \\ -P & -P & -Q \end{pmatrix} < 0,$$

which derive the conditions obtained in [2, 20]. Moreover, the results of [7, 19] and [11] can be derived from ours as special cases when $h(k) = 0$, and $h(k) = h_1 = h_2$, respectively.

4. NUMERICAL EXAMPLES

To illustrate the effectiveness of the previous theoretical results, we consider the following numerical examples.

Example 4.1. (Robust stability) Consider system Σ_ξ for $p = 2$, and

$$A_1 = \begin{pmatrix} -0.8 & 0.01 \\ 0.02 & -0.8 \end{pmatrix}, A_2 = \begin{pmatrix} -0.9 & 0.004 \\ 0.005 & -0.05 \end{pmatrix},$$

$$D_1 = \begin{pmatrix} -0.03 & 0.004 \\ 0.005 & -0.05 \end{pmatrix}, D_2 = \begin{pmatrix} -0.02 & 0.006 \\ 0.007 & -0.03 \end{pmatrix},$$

with any time-varying delay function $h(k)$ with $h_1 = 2, h_2 = 3$. By using the LMI Toolbox in MATLAB, the LMIs (i) and (ii) of Theorem 3.1 are feasible with

$$P_1 = \begin{pmatrix} 0.1500 & 0.0902 \\ 0.0902 & 0.0589 \end{pmatrix}, P_2 = \begin{pmatrix} 0.0549 & 0.0389 \\ 0.0389 & 0.0281 \end{pmatrix},$$

$$Q_1 = \begin{pmatrix} 0.0022 & 0.0015 \\ 0.0015 & 0.0012 \end{pmatrix}, Q_2 = \begin{pmatrix} 0.0006 & 0.0004 \\ 0.0004 & 0.0003 \end{pmatrix},$$

$$S = \begin{pmatrix} 0.0008 & 0.0005 \\ 0.0005 & 0.0004 \end{pmatrix},$$

$$S_1 = \begin{pmatrix} 0.0008 & -0.5507 \\ -0.5507 & -0.3554 \end{pmatrix}, S_2 = \begin{pmatrix} -0.6927 & -0.4614 \\ -0.4614 & -0.3108 \end{pmatrix},$$

$$R_1 = \begin{pmatrix} -0.8803 & -0.5003 \\ -0.5003 & -0.3127 \end{pmatrix}, R_2 = \begin{pmatrix} -0.6602 & -0.4385 \\ -0.4385 & -0.2941 \end{pmatrix},$$

Therefore, the system is robustly stable.

Example 4.2 (Robust stabilization) Consider system Σ_ξ for $p=2$ and

$$A_1 = \begin{pmatrix} -0.52130 & 0.34646 \\ -0.21218 & -0.71280 \end{pmatrix}, \quad A_2 = \begin{pmatrix} -0.63410 & 0.26354 \\ -0.25410 & -0.71280 \end{pmatrix},$$

$$B_1 = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}, \quad B_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

with any time-varying delay function $h(k)$ with $h_1 = 2, h_2 = 4$. By using the LMI Toolbox in MATLAB, the LMIs (i) and (ii) of Theorem 3.2 are feasible with

$$P_1 = \begin{pmatrix} 0.1089 & 0.0283 \\ 0.0283 & 0.0595 \end{pmatrix}, \quad P_2 = \begin{pmatrix} 0.8331 & 0.2523 \\ 0.2523 & 0.7416 \end{pmatrix},$$

$$Q_1 = \begin{pmatrix} 1.3666 & 0.0713 \\ 0.4311 & 1.2885 \end{pmatrix}, \quad Q_2 = \begin{pmatrix} 1.2795 & -0066 \\ -0.0066 & 1.2584 \end{pmatrix},$$

$$R_1 = \begin{pmatrix} -0.0937 & -0.0209 \\ -0.0209 & -0.3326 \end{pmatrix}, \quad R_2 = \begin{pmatrix} -0.1274 & -0.0235 \\ -0.0235 & -0.2903 \end{pmatrix},$$

$$S = \begin{pmatrix} 0.5340 & 0.0241 \\ 0.0241 & 0.6009 \end{pmatrix}.$$

Therefore, the system is robustly stabilizable with the feedback control

$$\begin{aligned}
u(k) &= B^T(\xi)[B(\xi)B^T(\xi)]^{-1}R(\xi)[R^T(\xi)R(\xi)]^{-1}P(\xi)x(k-h(k)) \\
&= (\xi_1 B_1 + \xi_2 B_2)^T[(\xi_1 B_1 + \xi_2 B_2)(\xi_1 B_1 + \xi_2 B_2)^T]^{-1} \\
&\quad \times (\xi_1 R_1 + \xi_2 R_2)^T[(\xi_1 R_1 + \xi_2 R_2)^T(\xi_1 R_1 + \xi_2 R_2)]^{-1} \\
&\quad \times (\xi_1 P_1 + \xi_2 P_2)(\xi)x(k-h(k)) \\
&= \begin{pmatrix} \xi_1 + 2\xi_2 & 0 \\ 0 & \xi_1 + \xi_2 \end{pmatrix} \begin{pmatrix} (\xi_1 + 2\xi_2)^{-2} & 0 \\ 0 & (\xi_1 + \xi_2)^{-2} \end{pmatrix} \\
&\quad \times \begin{pmatrix} -0.0933\xi_1 - 0.1274\xi_2 & -0.0209\xi_1 - 0.0235\xi_2 \\ -0.0209\xi_1 - 0.0235\xi_2 & -0.3326\xi_1 - 0.2903\xi_2 \end{pmatrix} \\
&\quad \times \begin{pmatrix} -0.0933\xi_1 - 0.1274\xi_2 & -0.0209\xi_1 - 0.0235\xi_2 \\ -0.0209\xi_1 - 0.0235\xi_2 & -0.3326\xi_1 - 0.2903\xi_2 \end{pmatrix}^{-2} \\
&\quad \times \begin{pmatrix} 0.1089\xi_1 + 0.8331\xi_2 & 0.0283\xi_1 + 0.2523\xi_2 \\ 0.0283\xi_1 + 0.2523\xi_2 & 0.0595\xi_1 + 0.7416\xi_2 \end{pmatrix} x(k-h(k)) \\
&= \begin{pmatrix} (\xi_1 + 2\xi_2)(0.1089\xi_1 + 0.8331\xi_2) & (\xi_1 + 2\xi_2)(0.0283\xi_1 + 0.2523\xi_2) \\ (\xi_1 + \xi_2)(0.0283\xi_1 + 0.2523\xi_2) & (\xi_1 + \xi_2)(0.0595\xi_1 + 0.7416\xi_2) \end{pmatrix} \\
&\quad \times x(k-h(k))
\end{aligned}$$

Therefore, the feedback delayed controller is

$$\begin{cases} u_1(k) = [0.1089\xi_1^2 + 1.0509\xi_1\xi_2 + 1.6662\xi_2^2]x_1(k-h(k)) \\ \qquad\qquad +[0.0283\xi_1^2 + 0.3089\xi_1\xi_2 + 0.5046\xi_2^2]x_2(k-h(k)), \\ u_2(k) = [0.0283\xi_1^2 + 0.2806\xi_1\xi_2 + 0.2523\xi_2^2]x_1(k-h(k)) \\ \qquad\qquad +[0.0595\xi_1^2 + 0.8011\xi_1\xi_2 + 0.7416\xi_2^2]x_2(k-h(k)). \end{cases}$$

5. CONCLUSION

In this paper, new delay-dependent robust stability conditions for linear polytopic delay-difference equations with interval time-varying delays have been presented in terms of LMIs. An application in robust stabilization of discrete control systems with time-delayed feedback controllers has been studied. Numerical examples have been given to demonstrate the effectiveness of the proposed conditions.

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