

## DYNAMICAL ANALYSIS OF A VIRAL INFECTION MODEL FOR PEST MANAGEMENT

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**ABSTRACT.** In this paper, a mathematical model for insect viruses attacking pests is constructed and two models of continuous and impulsive pest control strategies are analyzed. In case of a continuous control, it is shown that the model admits a globally asymptotically stable positive equilibrium under certain condition. As a result, the global asymptotic stability of the unique positive equilibrium point is used to establish a procedure to maintain the pests at an acceptably low level in the long term. In case of an impulsive control, it is observed that there exists a globally asymptotically stable pest-eradication periodic solution on condition that the amount of viruses released periodically is larger than some critical value. When the amount of viruses released is less than some critical value, the system is shown to be permanent, which implies that the trivial pest-eradication solution loses its stability. Furthermore, the mathematical results are also confirmed by means of numerical simulation. Finally, the efficiency of continuous and impulsive control policies is compared.

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### 1. INTRODUCTION

Currently, the application of chemical pesticides to combat pests is still one of the main measures to improve crop yields. However, the heavy and unreasonable use of chemical pesticides results in pest and disease resistance, a large quantity of pesticide residues, serious environmental pollution and people and livestock poisoned from time to time, which is seriously harmful to people's lives. It has become an urgent issue to reduce the use of high toxicity of high pesticide residues and to promote the technology of pollution-free plant. Therefore it is necessary to apply the biological control of plant diseases and insect pests.

Biological control is, generally, human use of a suitably chosen living organism, referred as the biocontrol agent, to control another. Biocontrol agents can be predators, pathogens or parasites of the organism to be controlled that either kill the harmful organism or interfere with its biological processes [1]. In a large number

of bio-pesticides, the insect virus pesticide plays an important role in pest biological control for its high pathogenicity, specificity, and simple production. The insect viruses for the biological control of pests are mainly baculoviruses. Baculoviruses comprise a family of double-stranded DNA viruses which are pathogenic for arthropods, mainly insects. The polyhedral occlusion body (OB) is the characteristic phenotypic appearance of baculoviruses and, in case of a nucleopolyhedrovirus (NPV) typically comprised of a proteinaceous matrix with a large number of embedded virus particles. Baculoviruses have a long history as effective and environmentally benign insect control agents in field crops, vegetables, forests, and pastures [2].

The attempts to use baculoviruses for the protection of European forests dated back to 19<sup>th</sup> century but the first introduction of baculovirus into the environment which resulted in successful regulation of the pest in a large area occurred accidentally in 1930 s [2]. A parasitoid was imported from Scandinavia to Canada to control spruce sawfly *Diprion hercyniae*. Along with a parasitoid, an NPV specific for spruce sawfly was introduced which established itself in Canada. Since then no control measures have been required against *Diprion hercyniae*. This example of “introduction-establishment” approach-baculovirus became a permanent part of an ecosystem in which it was not previously presented. One notable example was the *A. gemmatalis* nucleopolyhedrovirus (AgMNPV) used to control the velvet bean caterpillar in soybean [2, 3, 4]. The use of AgMNPV in Brazil brought about many economic, ecological and social benefits. At the soybean grower level, the financial savings from the use of the virus may reach ca. U \$7/ha/season, including product cost and application cost. The current annual savings at the grower level, in the total area sprayed with the virus is over U\$11,000,000. Since the beginning of the program more than 17 million liters of chemical insecticides have not been sprayed in the environment, resulting in considerable environmental benefits [3, 4].

Transmission is also key to the persistence of baculoviruses in the environment. Transmission occurs primarily when a NPV-infected larva dies and lyses, releasing a massive number of OBs onto foliage and soil. Susceptible hosts become infected when they ingest OBs while feeding. Defecation and regurgitation by infected larvae have been reported as additional routes of contamination of host plants with viruses [5, 6, 7, 8]. Moreover, some studies suggest that cannibalism and predation may also be routes of virus transmission. Environmental factors such as rainfall, wind transport, and contaminated ovipositors of parasitic hymenopterans could contribute to NPV transmission as well [9, 10, 11].

In view of the epidemiological dynamics, it is necessary to predict optimal timing, frequency, and dosage of virus application and to assess the shorter and longer term persistence of NPV in insect populations and the environment. Modelling studies can help to obtain preliminary assessments of expected ecological dynamics in the shorter

and longer term. In this paper, we will build a mathematical model to predict the behavior of viruses attacking pests under reasonable assumptions.

According to the above practical background, we introduce a model to investigate the dynamics of insect viruses attacking pests, which is described as the following system:

$$(1.1) \quad \begin{cases} S'(t) = rS(t)(1 - \frac{S(t)}{K}) - \beta S(t)bV(t)S(t) - bV(t)S(t), \\ V'(t) = \omega(\beta S(t)bV(t)S(t) + bV(t)S(t)) - ubV(t)S(t) - dV(t). \end{cases}$$

Where  $S(t)$  and  $V(t)$  denote the density of pests and viruses at time  $t$ , respectively. The assumptions in the model are:

(i) The pests  $S$  have a logistic growth rate with intrinsic birth rate  $r$  and carrying capacity  $K(> 0)$  [12].

(ii) The infection rate is the form  $\beta S b V S$ , in which  $\beta, b$  are positive constants and  $b V S$  denotes the quantity of disease pests invaded by virus.

(iii)  $\omega(\beta S b V S + b V S)$  denotes the number of the viruses released by the pests died because of disease.

(iv) The disease pest population does not recover and disease pests cannot attack crops. For biological reasons, we restrict our discussion to the feasible region  $\{(S, V) | S \geq 0, V \geq 0\}$ .

Set  $a = \beta b$ ,  $e = \beta b \omega$ ,  $\mu = -ub + b\omega > 0$ , then the system (1.1) becomes:

$$(1.2) \quad \begin{cases} S'(t) = rS(t)(1 - \frac{S(t)}{K}) - aS^2(t)V(t) - bS(t)V(t), \\ V'(t) = S(t)V(t)(eS(t) + \mu) - dV(t). \end{cases}$$

It is easy to obtain that system (1.2) has two trival equilibrium points  $E_1(0, 0)$ ,  $E_2(K, 0)$  and if  $K > \frac{-\mu + \sqrt{\mu^2 + 4ed}}{2e}$ , it also has a positive equilibrium point  $E(S^*, V^*)$ , where  $S^* = \frac{-\mu + \sqrt{\mu^2 + 4ed}}{2e}$ ,  $V^* = \frac{(r - \frac{r}{K})S^*}{aS^* + b}$ . By calculating, we have  $E_1(0, 0)$  is saddle point, and if the positive equilibrium point  $E(S^*, V^*)$  exists,  $E_2(K, 0)$  is also a saddle point. Moreover, system (1.2) has no pest-eradication equilibrium point. Therefore, the approach of this kind in pest control is not effective.

System with impulsive effects describing evolution processes is characterized by the fact that at certain moments of time they abruptly experience a change of state. Processes of such type are studied in almost every domain of applied science. Impulsive differential equations have been recently used in population dynamics in relation to impulsive vaccination, population ecology, the chemotherapeutic treatment of disease, the theory of the chemostat [13, 14, 15, 16, 17, 18, 19, 20, 21].

The main purpose of this paper is to construct two realistic models of viruses attacking pests for pest management, investigate their dynamics and compare the results obtained for the ordinary differential model, corresponding to the continuous

control, with those obtained for the impulsive differential model, corresponding to the impulsive control. The rest of this paper is organized as follows: in Section 2, by using the qualitative theory of ordinary differential equations, we investigate the behavior of the ordinary system which models the process of continuous release of viruses. In Section 3, we construct an impulsive system which models the process of periodic release of viruses at fixed moments. By using Floquets theory for impulsive differential equations, small-amplitude perturbation methods and comparison techniques, we investigate the global asymptotic stability of the pest-eradication periodic solution and the conditions for the permanence of the system. A brief discussion of some optimal control issues arising in pest management is also provided in the last section.

## 2. CONTINUOUS RELEASE VIRUSES FOR PEST MANAGEMENT

In this section, we introduce the continuous release of viruses into the system (1.2), then we have the following system:

$$(2.1) \quad \begin{cases} S'(t) = rS(t)\left(1 - \frac{S(t)}{K}\right) - aS^2(t)V(t) - bS(t)V(t), \\ V'(t) = S(t)V(t)(eS(t) + \mu) - dV(t) + p, \end{cases}$$

where  $p > 0$  is the amount of release viruses. Other parameters are the same as system (1.2).

In the following, we investigate the dynamics of the system (2.1) by means of stability analysis and apply the subsequently obtained stability results to the study of our control problem. We define

$$P(S, V) = rS(t)\left(1 - \frac{S(t)}{K}\right) - aS^2(t)V(t) - bS(t)V(t),$$

$$Q(S, V) = S(t)V(t)(eS(t) + \mu) - dV(t) + p.$$

Obviously, system (2.1) has a pest-eradication equilibrium point  $E_1(0, \frac{p}{d})$ , and if  $\frac{r}{b} > \frac{p}{d}$ , the equilibrium point  $E_1$  is a saddle point, the positive equilibrium point  $E_2(S, V)$  exists. We shall prove the existence of the positive equilibrium point  $E_2$  by means of geometric methods.

For existence of E, if  $\frac{r}{b} > \frac{p}{d}$ , the two isoclines

$$(2.2) \quad l1 : P(S, V) = 0,$$

$$(2.3) \quad l2 : Q(S, V) = 0,$$

must intersect in the region  $R_+^2 = \{(S, V) | S > 0, V > 0\}$ . The isocline  $l1$  includes the line  $l3 : S = 0$  and the curve

$$(2.4) \quad l4 : r\left(1 - \frac{S}{K}\right) - aSV - bV = 0.$$

It follows from Eq.(2.3) that

$$\frac{dV}{dS} = \frac{p(2eS + \mu)}{(eS^2 + \mu S - d)^2}.$$

Obviously, when  $S \in (-\infty, -\frac{\mu}{2e})$ , the isocline  $l2$  is strictly decreasing as  $S$  increases and when  $S \in (-\frac{\mu}{2e}, \infty)$  the isocline  $l2$  is strictly increasing as  $S$  increases. Therefore, when  $S > 0$  the isocline  $l2$  is strictly increasing as  $S$  increases. From Eq. (2.4), we derive that  $\frac{dV}{dS} = \frac{-\frac{rb}{K}-ra}{(aS+b)^2} < 0$ , which implies that the curve  $l4$  is strictly decreasing as  $S$  increases.

We define  $P_1 = (0, \frac{p}{d})$ , which is the point of intersection of curve  $l2$  and line  $l3$ ,  $P_2 = (0, \frac{r}{b})$ , which is the point of intersection of line  $l3$  and curve  $l4$ ,  $P_3 = (K, 0)$  and  $P_4 = (-\frac{\mu}{2e}, \frac{\mu^2+4ed}{4e})$ , which is the point on the curve  $l2$  with the lowest S-coordinate. In the following we shall investigate the system (2.1) with  $0 < S < K$ . Clearly, if  $\frac{r}{b} > \frac{p}{d}$ , the monotonicity property of the curves  $l2$  and  $l4$  guarantees that the system (2.1) has a unique positive equilibrium point  $E_2 = E_2(S^*, V^*)$  (see FIGURE 1). Also,  $0 < S^* < K, \frac{p}{d} < V^* < \frac{r}{b}$ .

A traditional approach to gain preliminary insight into the stability of the equilibrium of a dynamic system is to carry out a slope field analysis of the system. The slope field diagram provides information about whether or not the equilibrium  $E_2$  is locally stable (see FIGURE 2 for details). Having the information provided by FIGURE 2 in mind, we are now ready to analyze the local stability of the previously found positive equilibrium by means of Jacobian matrix analysis. With regard to this, it is seen the Jacobian matrix of system (2.1) at  $E_2$  in the form of

$$J_{E_2} = \begin{pmatrix} -aS^*V^* - \frac{r}{K}S^* & -a(S^*)^2 - bS^* \\ 2eS^*V^* + \mu V^* & e(S^*)^2 + \mu S^* - d \end{pmatrix}$$

The eigenvalue problem for the  $J_{E_2}$  provides the characteristic equation

$$\lambda^2 + Q_1\lambda + Q_2 = 0,$$

where the coefficients  $Q_1 = aS^*V^* + \frac{r}{K}S^* + \frac{p}{V^*} > 0, Q_2 = (a(S^*)^2 + bS^*)(2eS^*V^* + \mu V^*) + (aS^*V^* + \frac{r}{K}S^*)\frac{p}{V^*} > 0$ , then both of the eigenvalues have negative real part, we have that  $E_2$  is locally asymptotically stable. Therefore, we have the following result.

**Theorem 2.1.** *If  $\frac{r}{b} > \frac{p}{d}$ , then system (2.1) has a positive equilibrium point  $E_2$  which is locally asymptotically stable.*

Forward we shall prove the positive equilibrium point  $E_2$  is globally asymptotically stable. For this purpose, we shall prove that each positive solution of system (2.1) is uniformly ultimately bounded.

**Lemma 2.2.** *The system (2.1) is uniformly ultimately bounded.*

*Proof.* Define a function  $L(t) = eS(t) + aV(t)$ , then we have

$$\begin{aligned} \frac{dL(t)}{dt} + dL(t) &= -\frac{er}{K}S^2(t) + (de + er)S(t) + (a\mu - eb)S(t)V(t) + ap \\ &\leq (d + r)eS(t) - \frac{erS^2(t)}{K} + (a\mu - eb)S(t)V(t) + ap, \end{aligned}$$

for  $a\mu - eb = -b^2\beta u < 0$ , then  $\frac{dL(t)}{dt} + dL(t) \leq (d + r)eS(t) - \frac{erS^2(t)}{K} + ap$ . Obviously, the right hand of the above equality is bounded, thus, there exists  $M_0 > 0$  such that

$$\frac{dL}{dt} \leq -dL + M_0.$$

It follows that

$$\liminf_{t \rightarrow \infty} L(t) \leq \limsup_{t \rightarrow \infty} L(t) \leq \frac{M_0}{d}.$$

Therefore, by the definition of  $L(t)$  and the positivity of  $S(t)$  and  $V(t)$ , we obtain that the system (2.1) is uniformly ultimately bounded. The proof is completed.  $\square$

**Lemma 2.3.** *Suppose  $\Gamma(T) = (S(t), V(t))$  is a periodic orbit with  $T$  of system (2.1),  $\mathfrak{R}$  is the set which consists of all the points in phase plane  $\Gamma$ . Denote*

$$N = \int_0^T \left( \frac{\partial f_1}{\partial S}(S(t), V(t)) + \frac{\partial f_2}{\partial V}(S(t), V(t)) \right) dt,$$

where  $S'(t) = f_1(S(t), V(t))$ ,  $V'(t) = f_2(S(t), V(t))$ . Then we can obtain  $N < 0$ .

*Proof.*

$$\begin{aligned} N &= \int_0^T \left[ r \left( 1 - \frac{S(t)}{K} \right) - 2aS(t)V(t) - bV(t) - \frac{rS(t)}{K} + S(t)(eS(t) + \mu) - d \right] dt \\ &= \int_0^T \left[ r \left( 1 - \frac{S(t)}{K} \right) - aS(t)V(t) - bV(t) \right] dt + \int_0^T \left[ -aS(t)V(t) - \frac{rS(t)}{K} - \frac{p}{V} \right] dt \\ &+ \int_0^T \frac{V'(t)}{V} dt, \end{aligned}$$

for  $S(t)$  and  $V(t)$  are period function with  $T$ , so

$$\int_0^T \left[ r \left( 1 - \frac{S(t)}{K} \right) - aS(t)V(t) - bV(t) \right] dt = \int_0^T d \ln S(t) = 0,$$

$$\int_0^T \frac{V'}{V} dt = \int_0^T d \ln V(t) = 0,$$

hence

$$N = \int_0^T \left[ -aS(t)V(t) - \frac{rS(t)}{K} - \frac{p}{V} \right] dt,$$

It is evident that  $N < 0$ , the proof is complete.  $\square$

**Theorem 2.4.** *If  $\frac{r}{b} > \frac{p}{d}$ , then the positive equilibrium point  $E_2$  is globally asymptotically stable.*

*Proof.* From Theorem 2.1, we know that  $E_2$  is locally stable. According to Lemma 2.3, we can obtain if there exists periodic solution  $(S(t), V(t))$  around  $E_2(S^*, V^*)$ , then it is stable for any periodic solution. This is impossible. According to Poincare-Bendixson Theorem, limit set  $\omega$  of all orbits must be equilibrium point  $E_2$ . This implies that  $E(S^*, V^*)$  is globally asymptotically stable in the region  $R_+^2 = \{(S, V) | S > 0, V > 0\}$ . This completes the proof.  $\square$

**Example 2.5.** As an application of our main result, we consider the following system:

$$(2.5) \quad \begin{cases} S'(t) = 1.8S(t)(1 - \frac{S(t)}{2}) - 0.6S^2(t)V(t) - 0.7S(t)V(t), \\ V'(t) = V(t)(0.9S^2(t) + 0.3S(t)) - 0.7V(t) + 1, \end{cases}$$

Obviously, the system (2.5) has a unique positive globally asymptotically stable equilibrium  $E_2(0.26984, 1.80663)$  (see FIGURE 3).

To make our biological control strategy successful, we should regulate target pests to densities below the economic injury level (EIL), which indicates the pest densities (numbers of pests per unit area) at which artificial control measures are economically justified. In other words, at this level the cost of implementing the control measures is less than the loss of the farmer, or other resource producer would suffer if control action were not taken. We now let  $L$  be the number of the pest population reaching the economic injury level and discuss the strategy to control target pests. From Eqs. (2.2) and (2.3), we see that  $S$  can be thought as a function of the independent variable  $p$ , that is  $S = f(p)$ . Further, we know from Eq.(2.3) that  $S$  is strictly decreasing as a function of  $p$ . Obviously,  $f$  is invertible. We denote the inverse of function  $f$  by  $p = f^{-1}(S)$ . Thus, for any positive  $\varepsilon$  small enough, we may choose the control variable  $p \geq f^{-1}(L - \varepsilon)$  to control the target pest population below  $L$ .

### 3. IMPULSIVE RELEASE VIRUSES FOR PEST MANAGEMENT

In this section, we introduce the impulsive release viruses into the system (1.2), then we have the following system:

$$(3.1) \quad \left\{ \begin{array}{l} S'(t) = rS(t)(1 - \frac{S(t)}{K}) - aS^2(t)V(t) - bS(t)V(t), \\ V'(t) = V(t)(eS^2(t) + \mu S(t) - d), \\ \Delta S(t) = 0, \\ \Delta V(t) = p \end{array} \right\} \begin{array}{l} t \neq nT, \\ t = nT, n = 1, 2, \dots \end{array}$$

where  $T$  is the impulsive period,  $n = \{1, 2, \dots\}$ ,  $p$  is the release amount of viruses,  $\Delta S(t) = S(t^+) - S(t)$ ,  $\Delta V(t) = V(t^+) - V(t)$ . Other parameters are the same as system (2.1).

First, we give some definitions, notations and lemmas which will be useful for stating and proving our main results. Let  $R_+ = [0, \infty)$ ,  $R_+^2 = \{(x_1, x_2) \mid x_i > 0, i = 1, 2\}$ . Denote  $f = (f_1, f_2)^T$  the map defined by the right hand of the first two equations in system (3.1). Let  $V : R_+ \times R_+^2 \rightarrow R_+$ , then  $V \in V_0$  if

(i)  $V$  is continuous in  $(nT, (n + 1)T] \times R_+^2$  and for each  $z \in R_+^2, n \in N$

$$\lim_{(t,z) \rightarrow (nT^+, z)} V(t, z) = V(nT^+, z)$$

exists.

(ii)  $V$  is locally Lipschitzian in  $z$ .

**Definition 3.1.**  $V \in V_0$ , then for  $(t, z) \in (nT, (n + 1)T] \times R^2$ , the upper right derivative of  $V(t, z)$  with respect to system (3.1) is defined as

$$D^+V(t, z) = \limsup_{h \rightarrow 0^+} \frac{1}{h} [V(t + h, z + hf(t, z)) - V(t, z)].$$

The solution of (3.1), denoted by  $z(t) = (S(t), V(t))$ , is a piecewise continuous function  $z(t): R_+ \rightarrow R_+^2$ ,  $z(t)$  is continuous on  $(nT, (n + 1)T]$ ,  $n \in N$  and  $z(nT^+) = \lim_{t \rightarrow nT^+} z(t)$  exists. Obviously, the existence and uniqueness of the solution of (3.1) is guaranteed by the smoothness properties of  $f$  (for more details see [13, 14]).

**Lemma 3.2.** *Suppose  $z(t)$  is a solution of (3.1) with  $z(0^+) \geq 0$ , then  $z(t) \geq 0$  for all  $t \geq 0$ . Moreover, if  $z(0^+) > 0$ , then  $z(t) > 0$  for all  $t \geq 0$ .*

**Lemma 3.3.** *Let  $V : R_+ \times R_+^n \rightarrow R_+$  and  $V \in V_0$ . Assume that*

$$(3.2) \quad \begin{cases} D^+V(t, z(t)) \leq (\geq)g(t, V(t, z)), t \neq \tau_k, \\ V(t, z(t^+)) \leq (\geq)\Psi_n(V(t, z(t))), t = \tau_k, k \in N, \\ z(0^+) = z_0, \end{cases}$$

where  $g : R_+ \times R_+^n \rightarrow R^n$  is continuous in  $(\tau_k, \tau_{k+1}] \times R_+^n$  and for each  $\nu \in R_+^n, n \in N$

$$\lim_{(t, \iota) \rightarrow (\tau_k^+, \nu)} g(t, \iota) = g(\tau_k^+, \nu)$$

exists,  $g(t, U)$  is quasimonotone nondecreasing in  $U$  and  $\Psi_n : R_+^n \rightarrow R_+^n$  is non-decreasing. Let  $\mathfrak{R}(t) = \mathfrak{R}(t, 0, U_0)$  be the maximal (minimal) solution of the scalar impulsive differential equation

$$(3.3) \quad \begin{cases} U'(t) = g(t, U), t \neq \tau_k, \\ U(t^+) = \Psi_n(U(t)), t = \tau_k, k \in N, \\ U(0^+) = U_0, \end{cases}$$

existing on  $[0, \infty]$ . Then  $V(0^+, z_0) \leq (\geq)U_0$  implies that

$$V(t, z(t)) \leq (\geq)\mathfrak{R}(t), t \geq 0,$$

where  $z(t)$  is any solution of (3.1) existing on  $[0, \infty]$ . Note that if we have some smoothness conditions of  $g$  to guarantee the existence and uniqueness of solutions for (3.3), then  $\mathfrak{R}(t)$  is exactly the unique solution of (3.3).

**Lemma 3.4.** *There exists a constant  $M > 0$  such that  $S(t) \leq M, V(t) \leq M$  for each positive solution  $(S(t), V(t))$  of (3.1) with  $t$  large enough.*

*Proof.* Define a function  $L$  such that  $L(t) = eS(t) + aV(t)$ . Then we have

$$\begin{aligned} D^+L(t) |_{(3.1)} + dL(t) &= -\frac{er}{K}S^2(t) + (de + er)S(t) + (a\mu - eb)S(t)V(t) \\ &\leq (d + r)eS(t) - \frac{erS^2(t)}{K} + (a\mu - eb)S(t)V(t), \end{aligned}$$



for  $a\mu - eb = -b^2\beta u < 0$ , then  $D^+L(t) \Big|_{(3.1)} + dL(t) \leq (d+r)eS(t) - \frac{erS^2(t)}{K}$  for  $t \in (nT, (n+1)T]$ . Obviously, the right hand of the above equality is bounded, thus, there exists  $M_0 > 0$  such that

$$(3.4) \quad \begin{cases} D^+L(t) \leq -dL(t) + M_0, & t \neq nT, \\ L(nT^+) = L(nT) + \eta, & t = nT, \end{cases}$$

where  $\eta = ap$ . According to Lemma 2.2 in [13], we derive

$$\begin{aligned} L(t) &\leq L(0)e^{-dt} + \int_0^t M_0 e^{-d(t-s)} ds + \sum_{0 < kT < t} \eta e^{-d(t-kT)} \\ &\rightarrow \frac{M_0}{d} + \frac{\eta e^{dT}}{e^{dT}-1} \text{ as } t \rightarrow \infty \end{aligned}$$

Therefore, by the definition of  $L(t)$  we obtain that each positive solution of system (3.1) is uniformly ultimately bounded. The proof is complete.  $\square$

**Remark 3.5.** From Lemma 3.4, it is clear that  $\limsup_{t \rightarrow \infty} V(t) < \frac{M_0}{d} + \frac{\eta e^{dT}}{e^{dT}-1}$ . For convenience, let  $M = \frac{1}{\kappa}(\frac{M_0}{d} + \frac{\eta e^{dT}}{e^{dT}-1})$ , where  $\kappa = \min\{e, a\}$ .

Next, we give some basic property of the following subsystem:

$$(3.5) \quad \begin{cases} y'(t) = -dy(t), & t \neq nT \\ \Delta y(t) = p, & t = nT \end{cases}$$

**Lemma 3.6.** *System (3.5) has a positive periodic solution  $y^*(t)$  and for every positive solution  $y(t)$  of system (3.5),  $|y(t) - y^*(t)| \rightarrow 0$  as  $t \rightarrow \infty$ , where*

$$y^*(t) = \frac{pe^{-d(t-nT)}}{1 - e^{-dT}}$$

and

$$y^*(0^+) = \frac{p}{1 - e^{-dT}}$$

When  $S(t) = 0$ , for all  $t \geq 0$ . we get the subsystem of system (3.1)

$$(3.6) \quad \begin{cases} V'(t) = -dV(t), & t \neq nT \\ \Delta V(t) = p, & t = nT \end{cases}$$

By Lemma 3.6, we can obtain the unique positive periodic solution of system (3.6):  $V^*(t) = \frac{pe^{-d(t-nT)}}{1 - e^{-dT}}, nT < t \leq (n+1)T$ , with initial value  $V^*(0^+) = \frac{p}{1 - e^{-dT}}$ . Thus the pest-eradication solution is explicitly shown. That is, system (3.1) has a so called pest-eradication periodic solution  $(0, V^*(t))$ . We shall now give a condition which assures its global asymptotic stability.

**Theorem 3.7.** *Let  $(S(t), V(t))$  be any solution of system (3.1) with positive initial values. Then the pest-eradication periodic solution  $(0, V^*(t))$  is globally asymptotically stable provided*

$$(3.7) \quad rT < b \int_0^T V^*(t) dt.$$

*Proof.* The local stability of periodic solution  $(0, V^*(t))$  may be determined by considering the behavior of small amplitude perturbation of the solution. Let  $S(t) = u(t)$ ,  $V(t) = v(t) + V^*(t)$ . The corresponding linearized system of (3.1) at  $(0, V^*)$  is

$$(3.8) \quad \begin{cases} \left. \begin{aligned} u'(t) &= (r - bV^*(t))u(t), \\ v'(t) &= \mu V^*(t)u(t) - dv(t), \end{aligned} \right\} & t \neq nT, \\ u(t^+) &= u(t), \quad v(t^+) = v(t), & t = nT, n = 1, 2, \dots \end{cases}$$

Let  $\Phi(t)$  be the fundamental matrix of (3.8), then  $\Phi(t)$  satisfies

$$\frac{d\Phi(t)}{dt} = \begin{pmatrix} r - bV^*(t) & 0 \\ \mu V^*(t) & -d \end{pmatrix} \Phi(t)$$

and  $\Phi(0) = E_2$  (unit  $2 \times 2$  matrix). Hence, the fundamental solution matrix is

$$\Phi(t) = \begin{pmatrix} e^{\int_0^t (r - bV^*(t))dt} & 0 \\ * & e^{-dt} \end{pmatrix}$$

The resetting impulsive condition of (3.8) becomes

$$\begin{pmatrix} u(nT^+) \\ v(nT^+) \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} u(nT) \\ v(nT) \end{pmatrix}.$$

Hence, if all the eigenvalues of

$$M = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \Phi(T)$$

have absolute values less than one, then the periodic solution  $(0, V^*(t))$  is locally stable. Since the eigenvalues of  $M$  are

$$\lambda_1 = e^{-dT} < 1, \lambda_2 = e^{\int_0^T (r - bV^*(t))dt}$$

and  $|\lambda_2| < 1$  if and only if (3.7) holds. According to Floquet's theory of impulsive differential equation, the pest-eradication periodic solution  $(0, V^*(t))$  is locally stable. In the following, we shall prove its global attractivity.

Choose  $\varepsilon_1 > 0$  small enough such that

$$\int_0^T [r - b(V^*(t) - \varepsilon_1)]dt \doteq \eta < 0$$

Note that  $V'(t) \geq -dV(t)$ , by Lemma 3.3 and 3.6, there exists a  $n_1$  such that for all  $t \geq n_1T$

$$(3.9) \quad V(t) \geq V^*(t) - \varepsilon_1,$$

thus we have

$$\begin{aligned} S'(t) &= rS(t)\left(1 - \frac{S(t)}{K}\right) - aS^2(t)V(t) - bS(t)V(t) \\ &\leq S(t)[r - bV(t)] \\ &\leq S(t)[r - b(V^*(t) - \varepsilon_1)] \end{aligned}$$

Integrating the above inequality on  $((n_1 + k)T, (n_1 + k + 1)T]$ ,  $k \in N$ , yields

$$S(t) \leq S(n_1T)e^{\int_{n_1T}^{(n_1+1)T} [r-b(V^*(t)-\varepsilon_1)]dt} \leq S(n_1T)e^{k\eta}$$

Since  $\eta < 0$ , we can easily get  $S(t) \rightarrow 0$  as  $t \rightarrow \infty$ . In the following, we prove  $V(t) \rightarrow V^*(t)$ , as  $t \rightarrow +\infty$ . For  $\varepsilon_2 > 0$  small enough ( $e\varepsilon_2^2 + \mu\varepsilon_2 < d$ ), there must exist a  $n_2(n_2 > n_1)$  such that  $0 < S(t) < \varepsilon_2$ , for  $t \geq n_2T$ . Then from system (3.1) we have

$$V'(t) \leq (-d + e\varepsilon_2^2 + \mu\varepsilon_2)V(t),$$

by Lemma 3.3 and 3.6, there exists a  $n_3(n_3 > n_2)$  such that

$$(3.10) \quad V(t) \leq V_2^*(t) + \varepsilon_1 \quad \text{for all } t \geq nT, \quad n > n_3,$$

where  $V_2^*(t) = \frac{pe^{-(d-e\varepsilon_2^2-\mu\varepsilon_2)(t-nT)}}{1-e^{-(d-e\varepsilon_2^2-\mu\varepsilon_2)T}}$ . Let  $\varepsilon_2 \rightarrow 0$ , we have  $V_2^*(t) \rightarrow V^*(t)$ . Together with (3.9), (3.10), we get  $V(t) \rightarrow V^*(t)$  as  $t \rightarrow +\infty$ . Therefore,  $(0, V^*(t))$  is globally attractive. This completes the proof.  $\square$

In fact, for condition (3.7),  $rT$  represents the normalized gain of the pest in a period, while  $b \int_0^T y^*(t)dt$  represents the normalized loss of the pest in a period due to disease. That is, this condition is a balance condition for the pest near the pest-eradication periodic solution, which asserts the fact that in a vicinity of this solution  $(0, V^*(t))$  the pest are depleted faster than they can recover and consequently the pest is condemned to extinction.

**Corollary 3.8.** *If  $p > p_1^* = \frac{rdT}{b}$  or  $T < T_1^* = \frac{bp}{rd}$ , then the pest-eradication periodic solution  $(0, V^*(t))$  is globally asymptotically stable.*

We have proved that, if  $p > p_1^* = \frac{rdT}{b}$  or  $T < T_1^* = \frac{bp}{rd}$ , the pest-eradication periodic solution  $(0, V^*(t))$  is globally asymptotically stable, that is, the pest population is eradicated totally. But in practice, from the view point of keeping ecosystem balance and preserving biological resources, it is not necessary to eradicate the pest population. Next we focus our attention on the permanence of system (3.1). Before starting our result, we give the definition of permanence.

**Definition 3.9.** System (3.1) is said to be permanent if there are constants  $m, M > 0$  (independent of initial value) and a finite time  $T_0$  such that all solutions  $z(t) = (x(t), y(t))$  with initial values  $z(0^+) > 0$ ,  $m \leq z(t) \leq M$  holds for all  $t \geq T_0$ . Here  $T_0$  may depend on the initial values  $z(0^+) > 0$ .

**Theorem 3.10.** *Let  $(S(t), V(t))$  be any positive solution of (3.1) with positive values  $z(0^+) > 0$ . Then system (3.1) is permanent provided*

$$(3.11) \quad rT > (a + b) \int_0^T V^*(t)dt.$$

*Proof.* Suppose  $z(t) = (S(t), V(t))$  is a solution of system (3.1) with initial values  $z(0^+) > 0$ . By Lemma 3.4, there exists a positive constant  $M$  such that  $S(t) \leq M$ , and  $V(t) \leq M$  for  $t$  large enough. We may assume  $S(t) \leq M, V(t) \leq M$  for all  $t \geq 0$ . From (3.9), we know

$$V(t) \geq V^*(t) - \varepsilon_1 \geq \frac{pe^{-dT}}{1 - e^{-dT}} - \varepsilon_1 \doteq m_2 > 0,$$

for all  $t$  large enough. Thus we only need to find  $m_1 > 0$  such that  $S(t) \geq m_1$  for  $t$  large enough. We shall do it in two steps.

**Step 1:** Since  $rT > (a+b) \int_0^T V^*(t)dt$ , that is  $rT > \frac{(a+b)p}{d}$ , we can select  $m_3 > 0$ ,  $\varepsilon > 0$  small enough such that

$$em_3^2 + \mu m_3 < d,$$

and

$$\delta \doteq rT - \left( \frac{rm_3}{K}T + am_3\varepsilon T + b\varepsilon T + \frac{am_3p}{d - em_3^2 - \mu m_3} + \frac{bp}{d - em_3^2 - \mu m_3} \right) > 0.$$

We shall prove  $S(t) < m_3$  cannot hold for all  $t > 0$ . Otherwise

$$V'(t) = V(t)(eS^2(t) + \mu S(t) - d) \leq (-d + em_3^2 + \mu m_3)V(t),$$

Then we obtain  $V(t) \leq u(t)$  and  $u(t) \rightarrow u^*(t)$  as  $t \rightarrow \infty$ , where  $u^*(t)$  is the solution of

$$(3.12) \quad \begin{cases} u'(t) = (-d + em_3^2 + \mu m_3)u(t), & t \neq nT, \\ \Delta u(t) = p, & t = nT, \\ u(0^+) = V(0^+) > 0, \end{cases}$$

and

$$u^*(t) = \frac{pe^{(-d+em_3^2+\mu m_3)(t-(n-1)T)}}{1 - e^{(-d+em_3^2+\mu m_3)T}}, \quad t \in ((n-1)T, nT].$$

Therefore, there exists a  $\tilde{T} > 0$  such that

$$V(t) \leq u(t) \leq u^*(t) + \varepsilon$$

for  $t > \tilde{T}$ . Therefore, there exists  $T_1 > \tilde{T}$  such that

$$\begin{aligned} S'(t) &= rS(t)\left(1 - \frac{S(t)}{K}\right) - aS^2(t)V(t) - bS(t)V(t) \\ &\geq S(t)\left[r - \frac{rm_3}{K} - am_3V(t) - bV(t)\right] \\ &\geq S(t)\left[r - \frac{rm_3}{K} - am_3(u^*(t) + \varepsilon) - b(u^*(t) + \varepsilon)\right] \end{aligned}$$

for all  $t > T_1$ . Let  $N_0 \in \mathbb{N}$  such that  $(N_0 - 1)T \geq T_1$ . Integrating the above inequality on  $((n-1)T, nT], n \geq N_0$ , we have

$$\begin{aligned} S(nT) &\geq S((n-1)T)e^{\int_{(n-1)T}^{nT} (r - \frac{rm_3}{K} - am_3(u^*(t)+\varepsilon) - b(u^*(t)+\varepsilon))dt} \\ &= S((n-1)T)e^{rT - (\frac{rm_3}{K}T + am_3\varepsilon T + b\varepsilon T + \frac{am_3p}{d - em_3^2 - \mu m_3} + \frac{bp}{d - em_3^2 - \mu m_3})} \\ &= S((n-1)T)e^\delta. \end{aligned}$$

Then  $S((n+k)T) \geq S(nT)e^{k\delta} \rightarrow \infty$  as  $k \rightarrow \infty$ , which is a contradiction to the boundedness of  $S(t)$ . Thus, there exists a  $t_1 > 0$  such that  $S(t_1) \geq m_3$ .

**Step 2:** If  $S(t) \geq m_3$  for all  $t \geq t_1$ , then our aim is obtained. Hence we need only to consider the situation that  $S(t) \geq m_3$  is not always true for  $t \geq t_1$ , we denote  $t^* = \inf_{t \geq t_1} \{S(t) < m_3\}$ . Then  $S(t) \geq m_3$  for  $t \in [t_1, t^*)$  and  $S(t^*) = m_3$ , since  $S(t)$  is continuous. Suppose  $t^* \in (n_1T, (n_1+1)T]$ ,  $n_1 \in N$ . Select  $n_2, n_3 \in N$  such that

$$n_2T > T_2 = \frac{\ln \frac{\varepsilon}{M+u_0^*}}{-d + em_3^2 + \mu m_3}, \quad e^{\delta_1(n_2+1)T} e^{\delta n_3} > 1,$$

where  $u_0^* = \frac{p}{1 - e^{-(d+em_3^2+\mu m_3)T}}$ ,  $\delta_1 = r - \frac{rm_3}{K} - am_3M - bM < 0$ . Let  $\widehat{T} = (n_2+n_3)T$ . We claim there must be a  $t_2 \in [(n_1+1)T, (n_1+1)T + \widehat{T}]$  such that  $S(t_2) \geq m_3$ . Otherwise  $S(t) < m_3, t_2 \in [(n_1+1)T, (n_1+1)T + \widehat{T}]$ . Consider (3.12) with  $u((n_1+1)T^+) = V((n_1+1)T^+)$ . We have

$$u(t) = (u(n_1+1)T^+ - u_0^*)e^{(-d+em_3^2+\mu m_3)(t-(n_1+1)T)} + u^*(t),$$

$$t \in (nT, (n+1)T], n_1+1 \leq n \leq n_1+1+n_2+n_3.$$

Thus

$$|u(t) - u^*(t)| \leq (M + u_0^*)e^{(-d+em_3^2+\mu m_3)n_2T} < \varepsilon,$$

and

$$V(t) \leq u(t) \leq u^*(t) + \varepsilon, \quad (n_1+1+n_2)T \leq t \leq (n_1+1)T + \widehat{T}.$$

Thus, we have

$$\begin{aligned} S'(t) &= rS(t)\left(1 - \frac{S(t)}{K}\right) - aS^2(t)V(t) - bS(t)V(t) \\ &\geq S(t)\left[r - \frac{rm_3}{K} - am_3V(t) - bV(t)\right] \\ &\geq S(t)\left[r - \frac{rm_3}{K} - am_3(u^*(t) + \varepsilon) - b(u^*(t) + \varepsilon)\right] \end{aligned}$$

for  $(n_1+1+n_2)T \leq t \leq (n_1+1)T + \widehat{T}$ . As in step 1, we have

$$S((n_1+1+n_2+n_3)T) \geq S((n_1+1+n_2)T)e^{\delta n_3}.$$

On the interval  $t \in [t^*, (n_1+1+n_2)T]$ , we have

$$\begin{aligned} S'(t) &= rS(t)\left(1 - \frac{S(t)}{K}\right) - aS^2(t)V(t) - bS(t)V(t) \\ &\geq S(t)\left[r - \frac{rm_3}{K} - aS(t)V(t) - bV(t)\right] \\ &\geq S(t)\left(r - \frac{rm_3}{K} - am_3M - bM\right), \end{aligned}$$

and

$$\begin{aligned} S((n_1+1+n_2)T) &\geq S(t^*)e^{\int_{t^*}^{(n_1+1+n_2)T} (r - \frac{rm_3}{K} - am_3M - bM)dt} \\ &\geq m_3e^{(r - \frac{rm_3}{K} - am_3M - bM)(n_2+1)T} \\ &= m_3e^{\delta_1(n_2+1)T}. \end{aligned}$$

Thus  $S((n_1+1+n_2+n_3)T) \geq m_3e^{\delta_1(n_2+1)T} e^{\delta n_3} > m_3$ , which is a contradiction. Let  $\bar{t} = \inf_{t \geq t^*} \{S(t) \geq m_3\}$ , then  $S(\bar{t}) \geq m_3$ , for  $t \in [t^*, \bar{t})$ , we have  $S(t) \geq S(t^*)e^{(t-t^*)\delta_1} \geq m_3e^{(1+n_2+n_3)T\delta_1} = m_1$ . For  $t > \bar{t}$ , the same arguments can be continued, since  $S(\bar{t}) \geq$

$m_3$ , and  $m_1, m_3$  are  $t_1$ -independent. Hence  $S(t) \geq m_1$  for all  $t \geq t_1$ . The proof is complete.  $\square$

**Corollary 3.11.** *If  $p < p_2^* = \frac{rdT}{a+b}$  or  $T > T_2^* = \frac{(a+b)p}{rd}$ , then system (3.1) is permanent.*

**Example 3.12.** Let us consider the following system

$$(3.13) \quad \left\{ \begin{array}{l} S'(t) = 1.8S(t)\left(1 - \frac{S(t)}{2}\right) - 0.2S^2(t)V(t) - 0.8S(t)V(t), \\ V'(t) = V(t)(0.9S^2(t) + 0.5S(t) - 0.7), \\ \Delta S(t) = 0, \\ \Delta V(t) = p \end{array} \right\} \begin{array}{l} t \neq nT, \\ t = nT, n = 1, 2, \dots \end{array}$$

According to Theorem 3.7, Theorem 3.10 and Corollary 3.8, Corollary 3.11, we know that if  $p > 2.3625$ , then  $(0, V^*(t))$  is globally asymptotically stable (see FIGURE 4), if  $p < 1.8900$ , then the system is permanent (see FIGURE 5).

#### 4. NUMERICAL SIMULATIONS AND DISCUSSION

In this paper, a mathematical model for insect viruses attacking pest is constructed and two models of continuous and impulsive release viruses for pest management are analyzed. We first examine the situation in which a continuous control is used and obtain that if  $\frac{r}{b} > \frac{p}{d}$ , the system (2.1) has a unique positive equilibrium point  $E_2(S^*, V^*)$  which is globally asymptotically stable. We then show that in case that an impulsive control is employed, by using Floquet's theorem, small-amplitude perturbation skills and comparison theorem, we establish the sufficient conditions for the global asymptotical stability of the pest-eradication periodic solution as well as the permanence of the impulsive system (3.1). It is clear that the conditions for the global stability and permanence of the system depend on the parameters  $p, T$ , which implies that the parameters  $p, T$  play a very important role in the model.

From Corollary 3.8, we know that the pest-eradication periodic solution  $(0, V^*(t))$  is globally asymptotically stable when  $p > p_1^*$  or  $T < T_1^*$ . In order to drive the pests to extinction, we can determine the impulsive release amount  $p$  such that  $p > p_1^*$  or the impulsive period  $T$  such that  $T < T_1^*$ . If we choose parameters as  $r = 1.8$ ,  $K = 2$ ,  $a = 0.2$ ,  $e = 0.9$ ,  $b = 0.8$ ,  $\mu = 0.5$ ,  $d = 0.7$ , then we have  $T_1^* = 1.524$ ,  $p_1^* = 2.3625$ , so we can make the impulsive release amount  $p$  larger than 2.3625 or the impulsive period  $T$  smaller than 1.524 in order to eradicate the pests (see FIGURE 4). At the same time,  $T_2^* = 1.905$ ,  $p_2^* = 1.8900$ , so we can make the impulsive release amount  $p$  less than 1.8900 or the impulsive period  $T$  larger than 1.905 in order to maintain the system permanent (see FIGURE 5). However, from a pest control point of view, our aim is to keep pests at acceptably low levels; not to eradicate them, only to control their population size. To compare the efficiency of both continuous and impulsive control strategies, we study Eqs. (2.1) and (3.1) with the same parameter under the

	EIL	Time	The amount of release viruses	Initial value	Numerical study
Continuous	0.28	2.3	$1 * 2.3(p = 1)$	(0.9, 0)	Figure 6
Impulsive	0.28	2.4	$1.7 * 1.6(T = 1.5, p = 1.7)$	(0.9, 0)	Figure 7
Impulsive	0.28		Useless control( $T = 2, p = 1.7$ )	(0.9, 0)	Figure 8

TABLE 1. A comparison between the efficiency of continuous and impulsive controls.

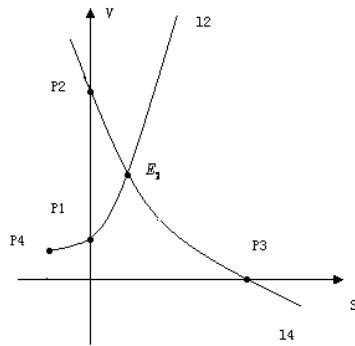


FIGURE 1. The unique positive equilibrium of the system (2.1).

assumption that (2.1) has a unique positive equilibrium. We choose  $r = 1.8$ ,  $K = 2$ ,  $a = 0.2$ ,  $e = 0.9$ ,  $b = 0.8$ ,  $\mu = 0.5$ ,  $d = 0.7$ . It is noted that  $p$  represents the rate of release of viruses in the system (2.1), while in the system (3.1)  $p$  is the release amount of viruses(see TABLE 1). With regard to this, the optimal control strategy in the management of a pest population is to drive the pest population below a given level and to do so in a manner which minimizes the cost of using the control and the time it takes to drive the system to the target. From Table 1, it is preferable to control the target pest population by using a continuous control, but in practice, it is impossible. Therefore, according to the growth characteristics of pests, we should choose a reasonable impulsive time and an appropriate impulsive period  $T$  such that the density of the pest is below the given level. We hope our results provide an insight to practical pest management. However, in the real world, for the seasonal damages of pests, should we consider impulsive releasing viruses on a finite interval? Such work will be beneficial to pest management, and it is reasonable. We leave it as a future work.

### 5. ACKNOWLEDGMENTS

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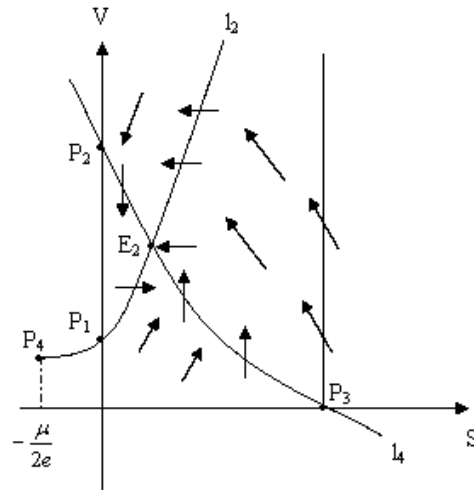


FIGURE 2. The slope field of the system (2.5).

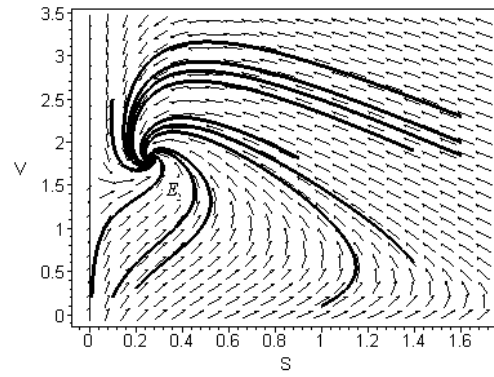


FIGURE 3. Phase portraits of the system (2.5).



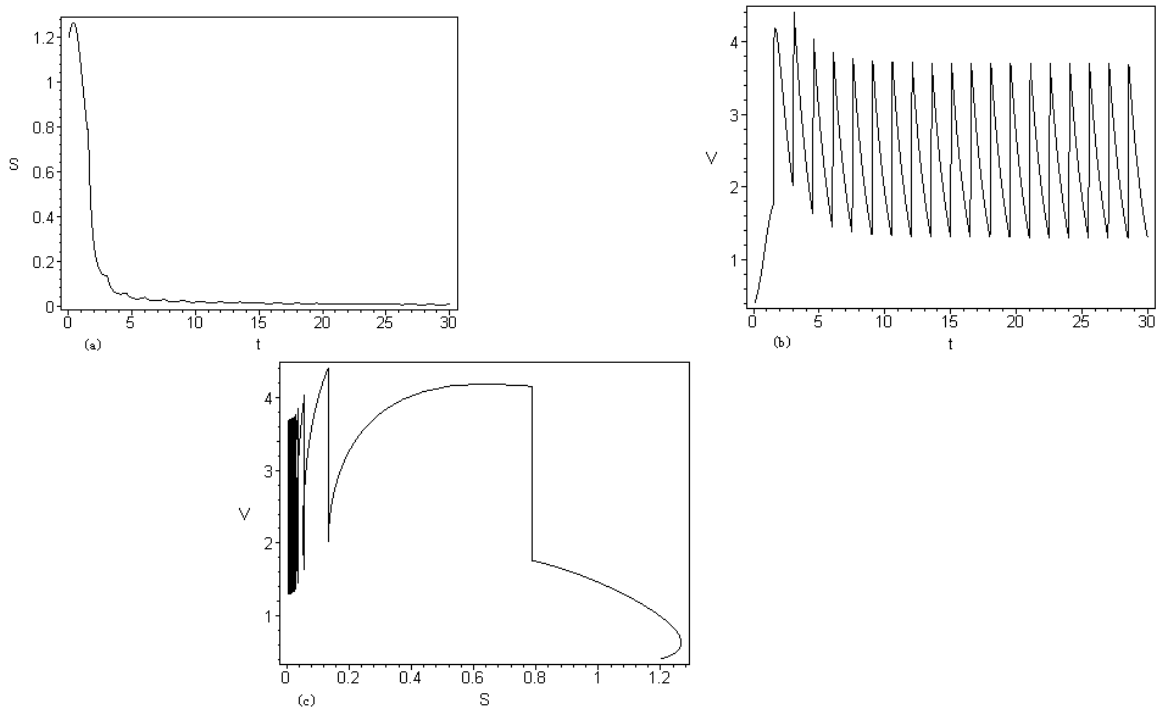


FIGURE 4. Dynamical behavior of the system (3.1) with impulsive control  $p = 2.4$  (a) time-series of the pest population. (b) time-series of the virus population. (c) Phase portraits of the system.

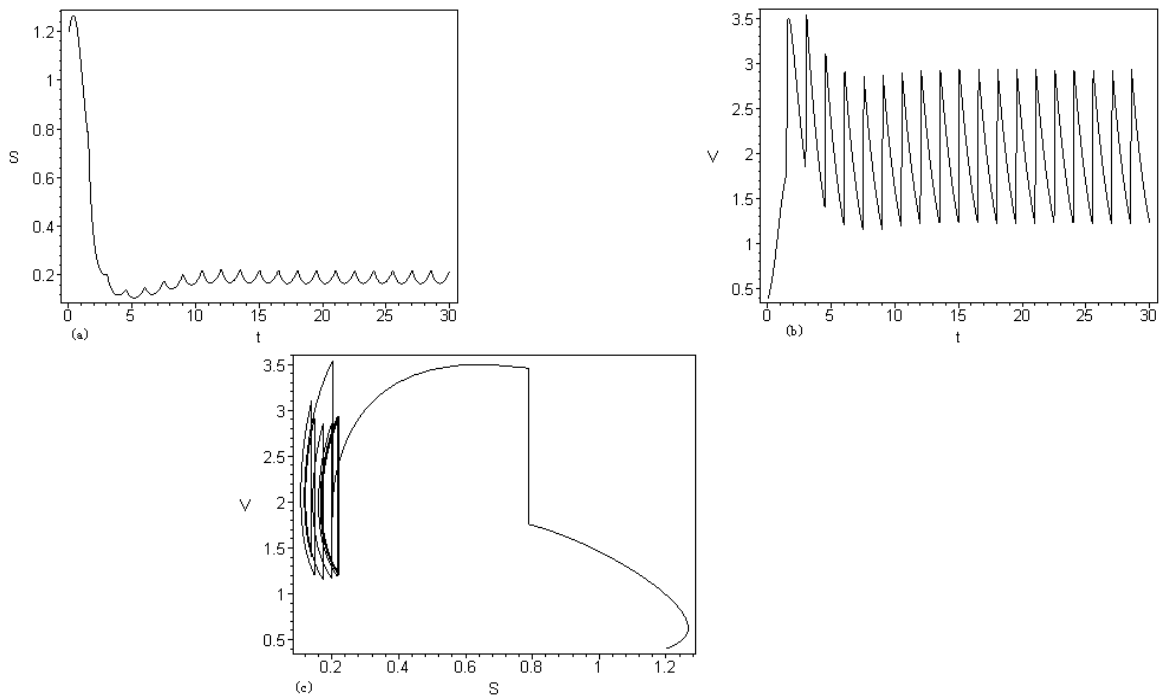


FIGURE 5. Dynamical behavior of the system (3.1) with impulsive control  $p = 1.7$  (a) time-series of the pest population. (b) time-series of the virus population. (c) Phase portraits of the system.

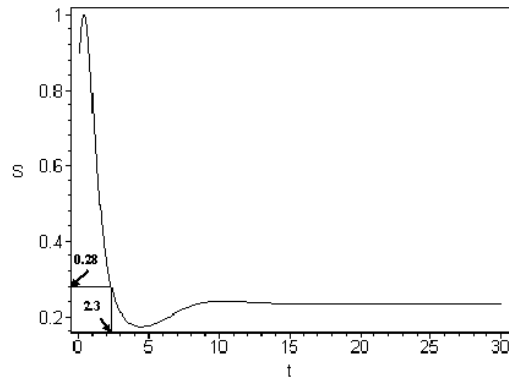


FIGURE 6. Dynamical behavior of the system (2.1) with continuous control ( $p=1$ ).

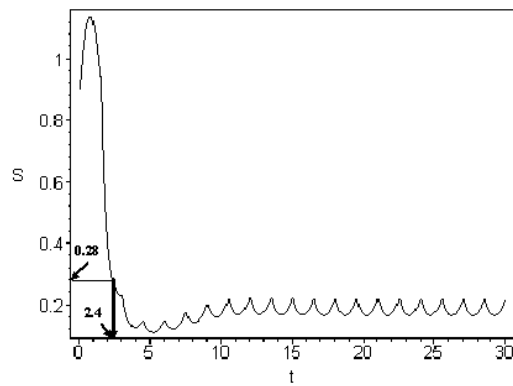


FIGURE 7. Dynamical behavior of the system (3.1) with impulsive control ( $p=1.7$  and  $T=1.5$ ).

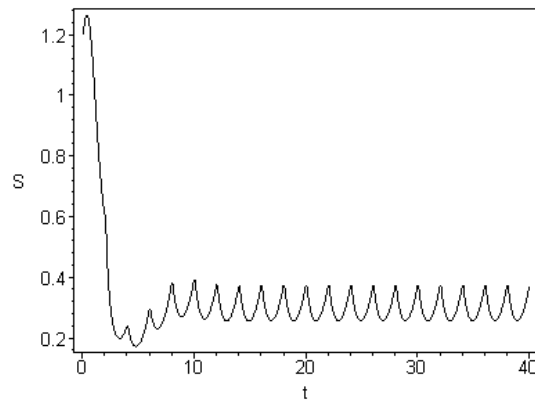


FIGURE 8. Dynamical behavior of the system (3.1) with impulsive control ( $p=1.7$  and  $T=2$ ).

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