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FATIGUE FAILURE ANALYSIS OF NON-STATIONARY GAUSSIAN STRESS PROCESS:RANDOM ALGEBRAIC POLYNOMIALS

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ABSTRACT. Fatigue is considered as a primary model of failure for metallic structures or mechanical devices subjected to oscillatory stress processes. In this paper we study fatigue failure and consider certain random polynomial as the underlying stress process. Let $Q_n(t) = \sum_{k=0}^{n} A_i t^i$ be a random algebraic polynomial in which the coefficients $A_0, A_1, A_2, \ldots, A_n$ form a sequence of i.i.d random variables with standard normal distribution. We obtain the distribution of peaks's magnitude of $Q_n(t)$. We also evaluate the behavior of the distribution, expectation and variance of the peaks magnitude. Finally we provide a method for evaluation of time to failure and the number of cycles to failure for such a situation.

AMS (MOS) Subject Classification. Primary 60H42, Secondary 60G99

1. Introduction

Let $Q_n(t)$ be the dynamic response, (either a deflection, a strain or a stress) at a critical point in a given structure. Damage to the structure accumulated as $Q_n(t)$ fluctuates at small or moderate excursions, and failure occurs when the accumulated damage reaches a fixed total. We call the failures due to this reason fatigue failures. Fatigue is considered as a primary model of failure for metallic structures or mechanical devices subjected to oscillatory stress processes. Customary stationary processes are assumed as the underlying stress process. In this paper we consider certain random polynomials as the underlying stress processes. Let $Q_n(t)$ be a continuous function of t, which is also a continuous valued random process. Miles [8] studied fatigue damage under some proposed random loading. Lin [6] is a rich literature in the theory of random vibration and fatigue failure for stationary processes. The stationary assumption is rather restricted so that it can be met under laboratory conditions. Lutes and Sarkani [7] and Benasciutti and Tovo [1] used spectral method for fatigue life prediction under non stationary random processes. Rezakhah and Soltani [10] studied structural fatigue design, by assuming stress process to be

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random polynomials with wave coefficients. In this work we propose certain random algebraic polynomials with normal coefficients as the underlying stress process. There is a rich literature on the theory of the number of real zeros of random polynomials. This area of research was elaborated by the fundamental work of M. Kac [5] and Rice [15]. The books of Bharucha-Ried and Sambandham [2] and [4], presents fundamental contributions to the subject. Recently there has been much interest in cases where the coefficients form certain random processes, see for e.g. Rezakhah and Shemehsavar [11, 12, 13, 14]. This paper is organized as follows: We present some preliminaries about random algebraic polynomial $Q_n(t)$, expected number of peaks below a prescribed level u and expected total number of peaks of $Q_n(t)$ in section 2. We obtain probability density, mean and variance of peak's magnitude of $Q_n(t)$ in section 3. Finally we study fatigue failure and provide a method for evaluating time to failure and also expected number of cycles to failure, and provide some graphical and numerical evaluation for these in section 4.

2. **Preliminaries**

Let

$$Q_n(t) = \sum_{i=0}^{n} A_i t^i \tag{2.1}$$

be a random algebraic polynomial whose coefficients $A_0, A_1, \ldots, A_n$ form a sequence of $i.i.d$ random variables with standard normal distribution.

In this paper we study the behavior of the distribution of peak's magnitude for such polynomials. A peak (or maxima) of a random algebraic polynomial $Q_n(t)$ occurs whenever $Q'_n(t)$ is zero and $Q''_n(t)$ is negative, then the magnitude of such a peak could be positive as well as negative. The peak's magnitude distribution of $Q_n(t)$ can be obtained from the joint distribution of $Q_n(t)$, $Q'_n(t)$ and $Q''_n(t)$.

Lutes and Sarkani [7] showed, the probability distribution of the peak's magnitude of stress process can be found by deriving the rates of the occurrence of the peaks below every level u. We follow this method of deriving the probability distribution of peak's magnitude of $Q_n(t)$.

Thus we define $\gamma_z[t, Q_n(t) \le u]$ as the expected rate of occurrence of those peaks that their magnitude do not exceed the level u. So in an infinitesimal interval Δt, the expected number of peaks is the same as the probability of one peak in the interval. This is by neglecting the probability of two or more peaks in such infinitesimal interval. Thus

$$\gamma_z[t, Q_n(t) \le u]\Delta t = P(\text{peak} \le u \text{ during } [t, t+\Delta t]) \tag{2.2}$$

just as

$$\gamma_z(t)\Delta t = P(\text{peak during } [t, t+\Delta t]) \tag{2.3}$$

in which $\gamma_z(t)$ is the expected rate of occurrence of peaks of all magnitudes, which is the limit of $\gamma_z[t, Q_n(t) \leq u]$ as u goes to infinity. Furthermore

$$P(\text{peak} \leq u, \text{ peak} \in [t, t+\Delta t]) = \\ P(\text{peak } \in [t, t+\Delta t]) P(\text{peak} \leq u \mid \text{ peak } \in [t, t+\Delta t])$$

The final term in this expression is the cumulative distribution function of peak's magnitude at point t:

$$F_{Z(t)}(u) = P(\text{peak} \leq u \mid \text{peak during } [t, t+\Delta t]).$$

By (2.2) and (2.3) we find that

$$F_{Z(t)}(u) = \frac{\gamma_z[t, Q_n(t) \leq u]}{\gamma_{Z(t)}} \tag{2.4}$$

Thus the probability distribution of the peak's magnitude $F_{Z(t)}(u)$ depends on the rate of occurrence of peaks below level u.

Rice [15] showed that for any function of the random variables $A_0, A_1, \ldots A_n$ like $Q_n(t)$, the expected number of peaks within the interval (a, b) is equal to

$$\int_a^b \int_{-\infty}^{\infty} \int_{-\infty}^{0} |x| p_t(r, 0, x) dx\, dr\, dt \tag{2.5}$$

where $p_t(r, s, x)$ is the joint probability density function of $Q_n(t)$, $Q_n'(t)$ and $Q_n''(t)$. Let $M_u(a, b)$ be the number of peaks of $Q_n(t)$ inside interval (a, b), whose magnitudes are smaller than or equal to u. Using (2.5) we find that

$$E(M_u(a, b)) = \int_a^b \int_{-\infty}^{u} \int_{-\infty}^{0} |x| p_t(r, 0, x) dx\, dr\, dt$$

Now we call

$$E(M_u(t)) := \int_{-\infty}^{u} \int_{-\infty}^{0} |x| p_t(r, 0, x) dx\, dr = -\int_{-\infty}^{u} \int_{-\infty}^{0} x p_t(r, 0, x) dx\, dr, \tag{2.6}$$

the expected rate of the number of peaks of $Q_n(t)$ below level u.

Therefore the expected rate of total number of peaks, regardless of their magnitude is obtained from (2.6) by letting $u \to \infty$, say

$$E(M_T(t)) := -\int_{-\infty}^{\infty} \int_{-\infty}^{0} x p_t(r, 0, x) dx\, dr \tag{2.7}$$

In this way, we obtain the rate of occurrence of peaks not exceeding the level u as

$$\gamma_z[t, Q_n(t) \leq u] = -\int_{-\infty}^{u} \int_{-\infty}^{0} x p_t(r, 0, x) dx\, dr.$$

Thus from (2.4) we have that

$$F_{Z(t)}(u) = \frac{E(M_u(t))}{E(M_T(t))} = \frac{-\int_{-\infty}^{u} \int_{-\infty}^{0} x p_t(r, 0, x) dx\, dr}{-\int_{-\infty}^{\infty} \int_{-\infty}^{0} x p_t(r, 0, x) dx\, dr}$$

3. Distribution of the Peak's Magnitude of $Q_n(t)$

In this section we obtain probability distribution function of the peak's magnitude of $Q_n(t)$ defined in (2.1).

Theorem: Let $Q_n(t)$ be the random algebraic polynomial given by (2.1) in which $A_0, A_1, A_2, \dots, A_n$ are $i.i.d$ random variables with standard normal distribution. Then the probability density of the peak's magnitude of $Q_n(t)$ is equal to

$$P_{Z(t)}(u) = \frac{Wu(L-\frac{W^2}{K})(1+erf(\frac{Wu}{\sqrt{K}}))}{\sqrt{KL}} e^{-(L-\frac{W^2}{K})u^2} + \frac{(L-\frac{W^2}{K})}{\sqrt{\pi L}} e^{-Lu^2}$$

Proof: Let $Z(t)$ be the magnitude of the peak of $Q_n(t)$ at t. Let us envisage a sample space, each point of which corresponds to a peak in $Q_n(t)$. Then we can define a probability measure for the peaks at t, which are smaller or equal to a specified level u. Thus the probability distribution function of the peak's magnitude at t is

$$F_{Z(t)}(u) = P(Z(t) \le u) = \frac{E(M_u(t))}{E(M_T(t))}, \tag{3.1}$$

Differentiating from relation (3.1), we find the probability density function of the peak's magnitude as

$$P_{Z(t)}(u) = \frac{1}{E(M_T(t))} \frac{\partial E(M_u(t))}{\partial u} = \frac{-\int_{-\infty}^{0} x p_t(u,0,x)dx}{E(M_T(t))} \tag{3.2}$$

Let

$$a_k(t) = t^k, \quad b_k(t) = kt^{k-1}, \quad c_k(t) = k(k-1)t^{k-2}.$$

By the assumptions of the theorem we find that

$$A^2 := \mathrm{Var}(Q_n(t)) = \sum_{k=0}^{n} a_k^2(t), \qquad B^2 := \mathrm{Var}(Q_n'(t)) = \sum_{k=0}^{n} b_k^2(t), \tag{3.3}$$

$$C^2 := \mathrm{Var}(Q_n''(t)) = \sum_{k=0}^{n} c_k^2(t), \qquad D := \mathrm{Cov}(Q_n(t), Q_n'(t)) = \sum_{k=0}^{n} a_k(t)b_k(t),$$

$$H := \mathrm{Cov}(Q_n(t), Q_n''(t)) = \sum_{k=0}^{n} a_k(t)c_k(t),$$

$$F := \mathrm{Cov}(Q_n'(t), Q_n''(t)) = \sum_{k=0}^{n} b_k(t)c_k(t).$$

Farahmand [4] showed that the three dimensional normal density of ($Q_n(t)$, $Q_n'(t)$, $Q_n''(t)$) is equal to

$$p_t(r,0,x) = \frac{\exp(-Lr^2 - 2Wrx - Kx^2)}{(2\pi)^{3/2} \det(\Sigma)^{1/2}}$$

in which Σ is the covariance matrix of ($Q_n(t)$, $Q_n'(t)$, $Q_n''(t)$), that its determinant

$$\det(\Sigma) = A^2B^2C^2 - A^2F^2 - B^2H^2 - C^2D^2 + 2DHF$$

and

$$K = \frac{A^2B^2 - D^2}{2\det(\Sigma)}, \qquad L = \frac{B^2C^2 - F^2}{2\det(\Sigma)}, \qquad W = \frac{DF - B^2H}{2\det(\Sigma)}, \tag{3.4}$$

where A, B, C, D, H and F defined by (3.3). By (3.2) and above calculation we obtain the probability density function of the peak's magnitude of $Q_n(t)$ as

$$P_{Z(t)}(u) = \frac{-\int_{-\infty}^{0} xp_t(u,0,x)dx}{E(M_T(t))},$$

in which

$$\begin{aligned} -\int_{-\infty}^{0} xp_t(u,0,x)dx &= -\int_{-\infty}^{0} x\frac{\exp(-Lu^2 - 2Wux - Kx^2)}{(2\pi)^{3/2}\det(\Sigma)^{1/2}}dx \\ &= \frac{Wu\sqrt{\pi}e^{-(L-\frac{W^2}{K})u^2}(1 + erf(\frac{Wu}{\sqrt{K}})) + \sqrt{K}e^{-Lu^2}}{2(2\pi K)^{3/2}(\det\Sigma)^{1/2}}, \end{aligned}$$

and

$$\begin{aligned} E(M_T(t)) &= -\int_{-\infty}^{\infty}\int_{-\infty}^{0} x\frac{\exp(-Lu^2 - 2Wux - Kx^2)}{(2\pi)^{3/2}\det(\Sigma)^{1/2}}dxdu \\ &= \frac{\sqrt{\pi L}}{2(KL - W^2)(2\pi)^{3/2}\det(\Sigma)^{1/2}}. \end{aligned}$$

Therefore

$$P_{Z(t)}(u) = \frac{Wu(L - \frac{W^2}{K})(1 + erf(\frac{Wu}{\sqrt{K}}))}{\sqrt{KL}}e^{-(L-\frac{W^2}{K})u^2} + \frac{(L - \frac{W^2}{K})}{\sqrt{\pi L}}e^{-Lu^2}.\diamond \tag{2.5}$$

Here we plot density of peak's magnitude of $Q_n(t)$ (DPM) for different points of t, which explain what magnitude has more chance to appear for each point, see Figure 1 and Figure 2. In other words these figures show that which magnitude are more likely to happen at each point. It also distinguish between positive and negative magnitudes.

We remind that peak's magnitude density $Z(t)$ is an even function of t. So we plot the densities only for positive t. As we find from the densities behavior of the peak's magnitude at different points, say $t = 0.4$, 0.8, 1, 1.1, 1.2, 1.3, two special features can be highlighted here.

- As the point t is taking far from zero the peak of the densities are moving from zero toward positive direction and this means that we could have peaks with larger magnitudes for points far from zero.
- The second interesting feature is that as t is taking far from zero the densities are going to concentrate on positive real numbers, and this means that as the point t is taking far from zero we have more peaks with positive magnitudes and the peaks with negative magnitudes are eliminated.

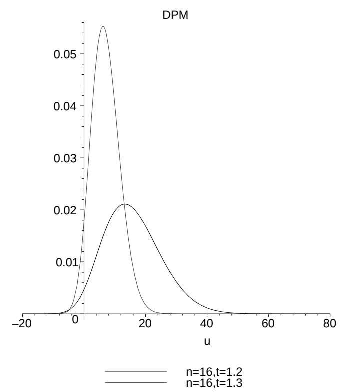

Figure 1: Density of peak's magnitude of $Q_n(t)$ for $n = 16$ at points $t = 1.2, 1.3$

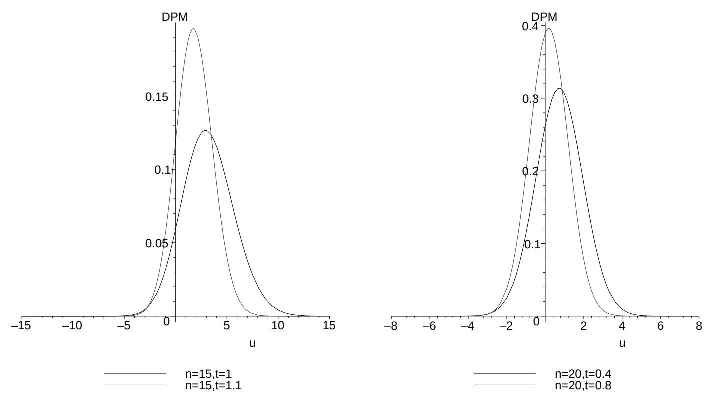

Figure 2: Density of peak's magnitude of $Q_n(t)$ for $n = 15, 20$ at points $t = 1,\ 1.1,\ 0.4,\ 0.8$

Mean and Variance of the Peak's Magnitude. Peak's magnitude and its properties are the main interests in the theory of vibration. Here we denote the magnitude of the peaks at each point t of the polynomials by $Z(t)$. As the effect of vibration to the dynamic structures, subject to such polynomials, can be evaluated by the magnitude of the peaks and as the vibrations could be upward or downward so we could have peaks with positive or negative magnitude. Thus we prefer to evaluate means and variances of such magnitudes at different points. We denote the mean of the magnitude of peaks at point x by $E(t) = \mathrm{E}(Z(t))$, and the variance of the magnitude of peaks by $V(t) = \mathrm{Var}(Z(t))$. Thus

$$E(t) = \int_{-\infty}^{\infty} u P_{Z(t)}(u) du = \frac{W\sqrt{\pi}}{2\sqrt{KL^2 - LW^2}},$$

$$E(Z(t)^2) = \int_{-\infty}^{\infty} u^2 P_{Z(t)}(u) du = \frac{W^2 + LK}{2L(KL - W^2)},$$

Then

$$V(t) = \frac{2W^2 + 2LK - W^2\pi}{4L(KL - W^2)}.$$

4. **Fatigue Failures**

Analysis of fatigue is typically based on the concept of a function $D(t)$ that represents the accumulated damage due to stresses and strains occurring prior to time t. This function is presumed to increase monotonically, and failure is expected when the accumulated damage reaches some critical level. Usually the damage function is normalized so that it reaches unity at failure $D(T) = 1$ if T is time of failure.

Current fatigue analysis methods are based on approximation of $D(t)$. The goal in formulating such approximations is to achieve compatibility with the results of experiments. These experiments are sometimes performed with quite complicated time histories of loading, but they more typically involve simple periodic loads in so-called *constant-amplitude* tests. It is presumed that each period of the motion contains only one peak and one valley.

In this situation, the number of cycles until failure is usually found to depend primarily on the amplitude of the cycles, although this is usually characterized with the alternative nomenclature of stress range, which is essentially the double amplitude, being equal to a peak value minus a valley value.

We will use the notation S_r to denote the stress range of a cycle and let N_f designate the number of cycle until failure in a constant amplitude. A typical experimental investigation of constant-amplitude fatigue for specimens of a given configuration and material involves performing a large number of tests including a number of values of S_r, then plotting the $(S_r, N_f(S_r))$ results. This is called an S/N curve and it forms the basis for most of our information and assumption about $D(t)$. We will emphasize the dependence of the fatigue life on stress range by writing $N_f(S_r)$ for the fatigue life observed for a given value of the stress range. In principle, the S/N curve of $N_f(S_r)$ versus S_r could be any non increasing curve.

Table 1: Mean and Variance of DMP

$n = 5$	$t = 0.8$	$t = 1.2$	$t = 2$	$t = 3$
$E(t)$	0.6884922672	1.516250836	6.024064731	24.25500411
$V(t)$	1.222674270	2.045427820	13.56715371	176.7972921

Table 2: Mean and Variance of DMP

$t = 2$	$n = 2$	$n = 3$	$n = 7$	$n = 10$
$E(t)$	1.215893310	2.143831089	17.80179311	100.5520012
$V(t)$	1.403956398	2.609403313	100.6521551	2943.76819

But experimental data commonly show that a large portion of that curve is well approximated by an equation of the form

$$N_f(S_r) = cS_r^{-b} \tag{4.1}$$

in which c, b are positive constant which are material properties. The constant c is positive since N and S are positive quantities. As the experimental fatigue data are typically characterized by the number of cycles to failure rather than time, we define the accumulated damage to be the sum of a number of discrete quantities

$$D(t) = \sum_{j=1}^{N(t)} \Delta D_j \tag{4.2}$$

in which ΔD_j, the increments of damage during cycle j, and $N(t)$ is the number of applied cycles of load up to time t. Furthermore, let T to be failure time. Thus $N(T) = N_f$ is the number of cycles to failure, this gives $D(T) = 1$. So

$$\sum_{j=1}^{N(T)} \Delta D_j = 1, \tag{4.3}$$

First we note that (4.3) indicates that the average value of ΔD_j over an entire $constant - amplitude$ fatigue test at constant stress level Sr is $\frac{1}{N_f(S_r)}$. Let us now assume that the conditional expected value of ΔD_j for all the cycles of stress level S_r within a stochastic time history will have this same average level of damage per cycle

$$E(\Delta D \mid S_r = u) = \frac{1}{N_f(u)}$$

This then gives the expected value of damage per cycle for any cycle, with random stress range S_r, within the time history as

$$E(\Delta D) = \int_0^\infty P_{S_r}(u) E(\Delta D \mid S_r = u) du = E\left(\frac{1}{N_f(S_r)}\right) \tag{4.4}$$

One of the most obvious cycle identification schemes is to consider the segment of a stress time history $Q_n(t)$ between any two subsequent local extrema (from a peak to a valley or from a valley to a peak) to be a half cycle. In this scheme the number of cycles is the same as the number of peaks. If the S/N curve is taken to have the power law form of (4.1), then by using (4.1), (4.4), (3.5) and by considering the stress range as $2u$, where u is the magnitude of peaks, we have that

$$\begin{aligned} E\left(\frac{1}{N_f(S_r)}\right) &= \frac{2^b}{c}\int_0^\infty u^b P_{S_r}(u)du \\ &= \frac{K^{-3/2}c^{-1}2^{b-1}}{\sqrt{\pi L}}\Bigg[\Gamma\left(\frac{b+1}{2}\right)\left(-\frac{W^2}{K}\right)^{-\frac{b+1}{2}}\left(\frac{W^2-KL}{W^2}\right)^{-\frac{3+b}{2}} \\ &\quad \times (b+1)\left(\sqrt{K}W^2 - K^{3/2}L\right) hypergeom\left([\frac{1}{2}, \frac{3+b}{2}], [\frac{3}{2}], \frac{W^2}{W^2-KL}\right) \end{aligned}$$

$$+\left(-\frac{W^2-KL}{K}\right)^{-1/2\,b}\Gamma\left(1+1/2\,b\right)\sqrt{\pi}KW$$
$$-\Gamma\left(\frac{b+1}{2}\right)\left(-L^{-\frac{b-1}{2}}K^{3/2}+L^{-\frac{b+1}{2}}W^2\sqrt{K}\right)\Bigg]$$

and the expected number of cycles to failure is given by

$$E(N(T))=\frac{1}{E\left(\frac{1}{N_f(S_r)}\right)}, \tag{4.5}$$

The results (4.4) and (4.5) are also equivalent to the common Palmgren-Miner hypothesis, Lin [6], that

$$\sum_{j=1}^{N(T)}(N_f(S_{r,j}))^{-1}=1 \tag{4.6}$$

The situation with $b=1$ is a special in which we can also exactly evaluate the prediction of the fatigue life. in particular (4.1) and (4.6) give the fatigue condition as

$$c^{-1}\sum_{j=1}^{N(T)}S_{r,j}=1$$

However, we can rewrite this summation of S_r values by noting the contributions to the summation from each time increment of length dt. In particular, there is an excursion $|Q^{'}(t)|dt$ during the time increment, this increment of excursion becomes a part of some $S_{r,j}$ stress range. Thus, it adds directly to the summation of all stress ranges, and we can say that

$$\sum_{j=1}^{N(T)}S_{r,j}=\frac{1}{2}\int_0^T|Q^{'}(t)|dt$$

in which the factor of 1/2 comes from the fact that a full cycle with range S_r corresponds to a total excursion of $2S_r$ substituting this relationship and taking the expected value gives

$$1=\frac{c^{-1}}{2}\int_0^T E(|Q^{'}(t)|)dt=\frac{c^{-1}}{2}\int_0^T\left(\frac{2}{\pi}\right)^{1/2}\sigma_{Q'}\,dt, \tag{4.7}$$

where probability density function of $|Q'(t)|$ equal to

$$P_{|Q'(t)|}(y)=\frac{2}{\sqrt{2\pi}\sigma_{Q'}}\exp\left\{-\frac{y^2}{2\sigma_{Q'}^2}\right\},$$

and

$$E(|Q'(t)|)=\left(\frac{2}{\pi}\right)^{1/2}\sigma_{Q'}$$

in which by (3.3) we have that

$$\sigma_{Q'}=(\mathrm{Var}(Q_n'(t)))^{\frac{1}{2}}=\sqrt{\frac{\left(t^4n^2-\left(2n^2+2n-1\right)t^2+\left(n+1\right)^2\right)\left(t^{2n+2}\right)-t^2-t^4}{t^2\left(t^2-1\right)^3}}$$

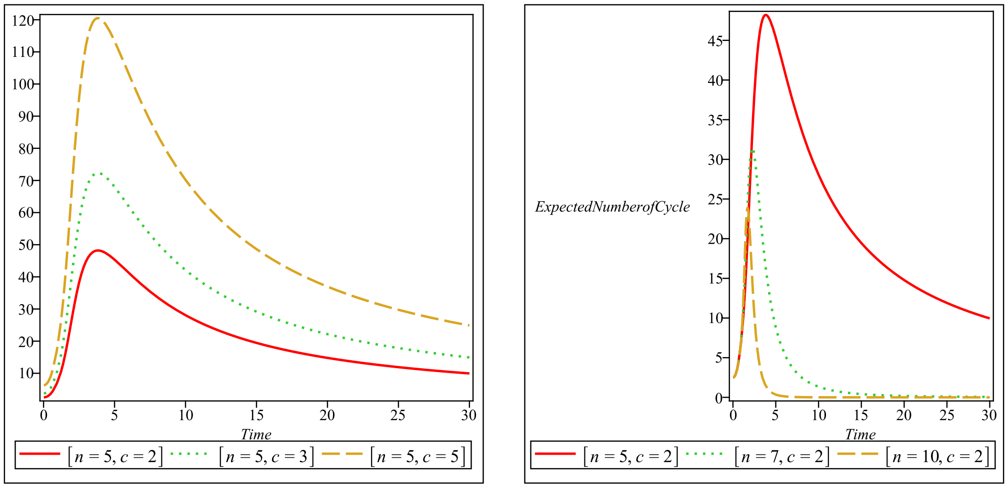

Figure 3: (a) Expected Number of Cycle (b) Expected Number of Cycle

Time to Failure T, and Expected Number of Cycles. Using (4.7), one can evaluate time to failure of systems under such stress process $Q_n(t)$. In table 3, this is done for the case $b = 1$. Using (4.5), we also evaluate the expected number of cycles to failure, $E(N(T))$, for different values c and n, for the case $b = 1$, which is recorded in table 3.

In figure 3(a), the parameter c is constant, so the structure of the systems are the same. For the case that n is 7 or 10, as the effect of the excitation of the system is based on the variance of the components of polynomial, so the failure time is less in compare with the cases with smaller n. This can be seen by table 3. As the cumulative number of cycles to the failure can be evaluated by the surface under the curve in figure 3(a) up to the failure time, so this is less as n is larger.

In figure 3(b) as the degree of polynomial $n = 5$ is fixed it can be seen by table 3 that as c is larger the structure of the systems are stronger and failure time are greater, and also as it can be seen by figure 3(b) the cumulative number of cycles to failure which is the surface under the curves up to the failure time is greater.

As our method is to provide a platform to study the effect of non-stationary stress process. One can use the classical methods to approximate the underlined stress process with some random polynomials, and then apply the method of this paper. In the following example we assume that such approximation by random polynomials has been done and so evaluate characteristics of the underlined process.

Example. We assume that the stress process has been evaluated with a random algebraic polynomial as $Q_n^*(t) = A_1 t + A_2 t^2 + A_3 t^3$, where the coefficients A_i are $i.i.d$ random variable with standard normal distribution. So we have by (3.3) that

$$A^2 = t^2 + t^4 + t^6, \quad B^2 = 1 + 4t^2 + 9t^4, \quad C^2 = 4 + 36t^2$$

Table 3: Expected Number of Cycles to Failure $E(N(T))$ and Time to Failure T for $b = 1$

c	n	2	5	7	10
0.5	T	0.8946932015	0.764136589	0.752569374	0.7482467534
	$E(N(T))$	1.177971022	1.324358126	1.309516770	1.262196083
1	T	1.395645645	1.0022981559	0.952824031	0.9240957382
	$E(N(T))$	3.581700163	3.763357000	3.634746927	3.425049612
2	T	2.0870963405	1.234613382	1.1255412145	1.0561167474
	$E(N(T))$	11.91232355	10.53928346	9.694139600	8.744537285
3	T	2.609103731	1.3736938635	1.2218261402	1.1242783208
	$E(N(T))$	24.92007552	19.29590011	17.20622447	15.10455626
5	T	3.428249425	1.5566961273	1.3427691936	1.2060980629
	$E(N(T))$	64.98443665	41.34518168	35.50692973	30.21206506

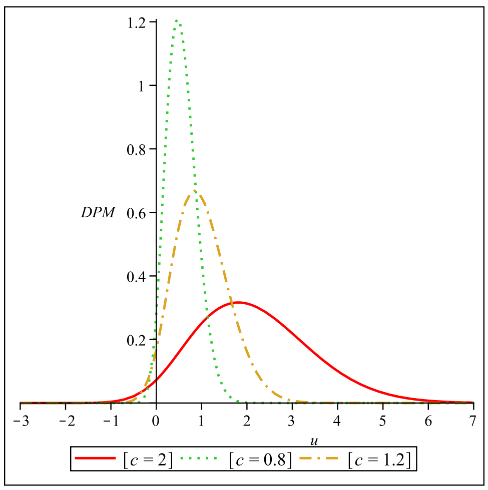

Figure 4: DPM at differen point $t = 0.8$, 1.2, 2

$$D = t + 2t^3 + 3t^5, \quad H = 2t^6 + 6t^4, \quad F = 4t + 18t^3$$

Using (3.4) and above relation we can obtain density of peaks magnitude of $Q_n^*(t)$ (DPM). Here we plot (DPM) by figure 4, at different points $t = 0.8$, 1.2, 2.

Using (4.5)–(4.7) we evaluate the time to failure and expected number of cycle for this stress process in table 4.

Conclusion

Many stress processes in real life are non-stationary and there is no method to handel non-stationary stress process in general. This paper provide a break through so that one can find an appropriate approximation for the non-stationary stress process

Table 4: Failure Time and Expected Number of Cycle at Failure Time

	$c = 0.5$	$c = 1$	$c = 2$	$c = 3$	$c = 5$
T	0.810829597	1.152700499	1.5539501985	1.8242292795	2.211597854
$E(N(T))$	98.21081777	98.77883741	134.2722994	176.9701853	270.0795861

by a random polynomial, which is still non-stationary. Then by the method of this paper one can evaluate statistical properties under study. To make more clarification for providing such approximation it is recommended to re-scale the time and consider the whole time duration of the study of the process as one, then largest term happens as the first term of the approximating random polynomial, $A_1 t$, and as we have further terms as $A_n t^n$, we provide a better approximation. Approximating the stress process by a random polynomial with independent Gaussian coefficients, one can apply results of this paper.

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