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# TRANSIENT ANALYSIS OF PARALLEL AND SERIES QUEUEING MODEL WITH NON-HOMOGENEOUS COMPOUND POISSON BINOMIAL BULK ARRIVALS AND LOAD DEPENDENT SERVICE RATES

A. V. S. SUHASINI[1], K. SRINIVASA RAO[2], AND P. R. S. REDDY[3]

[1]Dept. of Statistics, Sri Venkateswara University, Tirupati, avssuhasini@gmail.com
[2]Dept. of Statistics, Andhra University, Visakhapatnam, ksraoau@yahoo.co.in
[3]Dept. of Statistics, Sri Venkateswara University, Tirupati, putharsr@yahoo.co.in

**ABSTRACT**. In this paper the parallel and series queueing model with non-homogeneous compound Poisson binomial bulk arrivals and load dependent service rates is introduced and analyzed. The model performance is analysed. The explicit expressions for the performance measures of the model are derived. A set of 15 queueing models which are particular cases of the proposed model are derived as special cases. The sensitivity analysis of the model revealed that the bulk size distribution parameters and load dependent service can reduce the congestion in buffer and mean delay. A comparative study of the non-homogeneous and homogeneous queueing models is given.

Keywords: Non-homogeneous compound Poisson binomial arrivals, Performance evaluation, Transient Analysis, Tandem queueing model.

## 1. INTRODUCTION:

The single server queueing system with bulk arrivals is known for the last several decades Jaiswal (1960) wrote a paper on bulk queueing models and their applications. Thereafter, several authors studied bulk queueing models with various assumptions on arrival and service processes due to their ready applicability at various places like production processes, transportation systems, communication networks, cargo handling, reservation systems, etc. Recently, Chaudhry and Chang (2004) developed a discrete time bulk–service queueing model known as the Geo/$G^Y$/1/N+B model. Madan et al. (2004) studied a single-server queue with batch arrivals having two types of heterogeneous service with a general service time distribution. Juan (2005) developed M/$G^Y$/1, a queueing model with a discretized service time distribution. Srinivasa Rao et al. (2006) developed an interdependent communication network with bulk arrivals. Ahmed (2007) considered a multi- channel bi-level heterogeneous-server bulk arrival queueing system with an Erlangian service time. Chen A., et al. (2010) studied a modified Markovian bulk arrival and bulk service queue incorporating state-dependent control. Dieter Claeys et al. (2010) studied a threshold-based service system with batch arrivals and general service times. Charan Jeet Singh et al. (2011) studied a single-server bulk queueing system with state-dependent rates and a second optional service. Arumuganathan et al. (2011) studied two-node tandem communication network models with bulk arrivals using queueing theory. In all these models the authors considered the arrivals are homogeneous and characterized by compound Poisson process.

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However, in many practical situations arising at communication networks or production processes the arrivals may not be homogeneous and exhibit a time dependent nature (Leland et al. (1994), Feldmann A.(2000), Murali Krishna et al.(2003), Dinda et al. (2006), Rakesh Singhai et al. (2007)). The time dependent arrivals can be well characterized by non-homogeneous Poisson process (Trinatha Rao et al. (2011)). They assumed the customers arrive one after another. But in several practical situations the arrivals one in batches and time dependent. For example in store and forward communication networks such as LAN, MAN, WAN the messages arrive to the source are converted into a random number of packets and stored in buffers for forward transmission. Here, the number of packets a message can be converted into packets is random and follows a probability distribution. In general it is customary to consider the batch size distribution is left truncated geometric (Titiobilade (2002)). The geometric distribution has certain draw backs in approximating the batch size random variable, since the geometric distribution has infinite range and left skewed with long upper tail. In communication networks and container cargo handling the batch size may not be infinite and have finite bounds with different types of skewness. Due to this geometric distribution may not be appropriate for characterizing the batch size random variable for all situations and it is true when the batch size is symmetric or right skewed. Hence in this paper a left truncated binomial distribution is considered for approximating the batch size distribution. The binomial distribution is having finite range and includes positively/ negatively skewed or symmetric distributions for specific values of the parameter.

In addition to this the congestion in queues and delay can be reduced by adopting load dependent service strategy. Much work has been reported in the literature regarding load dependent service queueing models (Choi, .B.D and Choi,.D.I. (1996), Parthasarathy, P.R. and Selvaraju, N.(2001), Kin. K. Leung (2002), Suresh Varma, P. et al. (2007), Padmavathi, G. et al. (2009)). Very little work has been reported regarding queueing models with non-homogeneous compound Poisson binomial bulk arrivals with load dependent service rates. With this motivation, we develop and analyse a two node parallel and series queueing model with non-homogeneous compound Poisson binomial bulk arrivals and load dependent service rates. This queueing model also includes various queueing models as particular cases for specific values of the parameters. For example two node tandem queueing model with non-homogeneous compound Poisson binomial arrivals with load dependent service rates, single node queueing model with non-homogeneous compound Poisson binomial bulk arrivals with load dependent service rate etc., are particular cases of it. These models are useful for evaluating the performance of satellite communications, tele communications, personal mobile communications, intranet etc, under variable load conditions.

The rest of the paper is organized as follows Secton-2 deals with the derivation of joint probability generating function of two node parallel series queueing model with compound Poisson binomial bulk arrivals and load dependent service rates. Assuming the customers arrive in batches of random size and the number of customer in a batch follows a left truncated binomial distribution, the difference-differential equations of the model are derived and solved them for the joint probability generating function of the number of customers in each queue. Section -3 deals with the derivation of performance measures of the queueing model. The explicit expressions for average number of customers in each queue, probability of emptiness of each queue, the waiting time of a customer in each queue, the throughput of nodes, the utility of the server etc., are derived. In Section – 4 the particular cases of the queueing model namely, two node queueing model with non-homogenous compound Poisson binomial bulk arrivals, along with other 13 models are discussed. The performance measures of these models are also derived. Section-5 deals with the numerical illustration and sensitivity analysis of the model. The effect of the changes in input parameter on system performance measures is studied. In section- 6 the comparative study of the queueing models with non-homogeneous compound Poisson binomial bulk arrivals and homogeneous compound Poisson binomial bulk arrivals and other models is presented. The conclusions and scope for further research in this area are given in section 7.

## 2. QUEUING MODEL

In this section, we consider three queues $Q_1$, $Q_2$, $Q_3$ and three service stations $S_1$, $S_2$ and $S_3$ which are connected as parallel and series in order. We assume that the customers after getting service through first or second service station they join the third queue which is connected in series to $S_1$ and $S_2$ service stations. That is the customers after getting service from $S_1$ or $S_2$, join the third queue $Q_3$. It is further assumed the customers arrive to the first two queues in batches of random size and the compound mean arrival rates are time dependent. This means that the actual number of customers in any arriving module is an random variable X and follows a truncated binomial distribution with parameter n and p. Therefore the arrival of customers to both the queues follow non-homogeneous compound Poisson binomial processes with mean composite arrival rates $\lambda_1(t)$ and $\lambda_2(t)$ respectively for $Q_1$ and $Q_2$. The number of service completions in all three service stations follow Poisson processes with parameters $\mu_1$, $\mu_2$ and $\mu_3$ for the first, second and third service stations respectively. It is further assumed the mean service rate in each service station is linearly dependent on the content of the queue connected to it. The queue discipline is first in first out.

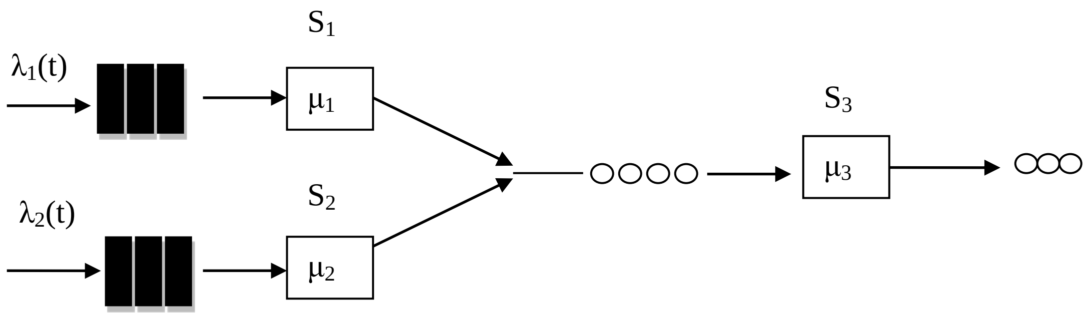



**Figure 1**: Schematic diagram of the queueing model.

Let $n_1$, $n_2$ and $n_3$ denote the number of customers in first queue, second queue and third queue. Let $P_{n_1,n_2,n_3}(t)$ be the probability that there are $n_1$ customers in the first queue, $n_2$ customers in the second queue and $n_3$ customers in the third queue at time t. The difference differential equations governing the model as follows:

$$\begin{aligned}\frac{\partial P_{n_1,n_2,n_3}(t)}{\partial t} &= -(\lambda_1(t)+\lambda_2(t)+n_1\mu_1+n_2\mu_2+n_3\mu_3)P_{n_1,n_2,n_3}(t)\\ &+\lambda_1(t)\sum_{k=1}^{n_1}P_{n_1-k,n_2,n_3}(t)C_k+\lambda_2(t)\sum_{s=1}^{n_2}P_{n_1,n_2-s,n_3}(t)C_s\\ &+(n_1+1)\mu_1P_{n_1+1,n_2,n_3}(t)+(n_2+1)\mu_2P_{n_1,n_2+1,n_3-1}(t)\\ &+(n_3+1)\mu_3P_{n_1,n_2,n_3+1}(t), \qquad for\ n_1,n_2,n_3>0\end{aligned}$$

$$\begin{aligned}\frac{\partial P_{n_1,n_2,0}(t)}{\partial t} &= -(\lambda_1(t)+\lambda_2(t)+n_1\mu_1+n_2\mu_2)P_{n_1,n_2,0}(t)+\lambda_1(t)\left[\sum_{k=1}^{n_1}P_{n_1-k,n_2,0}(t)C_k\right]\\ &+\lambda_2(t)\left[\sum_{s=1}^{n_2}P_{n_1,n_2-s,0}(t)C_s\right]+\mu_3P_{n_1,n_2,1}(t), \quad for\ n_1,n_2>0,n_3=0\end{aligned}$$

$$\begin{aligned}\frac{\partial P_{n_1,0,n_3}(t)}{\partial t} &= -(\lambda_1(t)+\lambda_2(t)+n_1\mu_1+n_3\mu_3)P_{n_1,n_2,n_3}(t)\\ &+\lambda_1(t)\left[\sum_{k=1}^{n_1}P_{n_1-k,0,n_3}(t)C_k\right]+(n_1+1)\mu_1P_{n_1+1,0,n_3-1}(t)\\ &+\mu_2P_{n_1,1,n_3-1}(t)+(n_3+1)\mu_3P_{n_1,0,n_3+1}(t),\end{aligned}$$

$$for\ n_1,n_3>0,n_2=0$$

$$\frac{\partial P_{0,n_2,n_3}(t)}{\partial t} = -(\lambda_1(t) + \lambda_2(t) + n_2\mu_2 + n_3\mu_3)P_{0,n_2,n_3}(t) + \lambda_2(t)\left[\sum_{s=1}^{n_2} P_{0,n_2-s,n_3}(t)C_s\right] + \mu_1 P_{1,n_2+1,n_3-1}(t) + (n_2+1)\mu_2 P_{0,n_2+1,n_3-1}(t) + (n_3+1)\mu_3 P_{0,n_2,n_3+1}(t),$$

$$for\ n_2, n_3 > 0, n_1 = 0$$

$$\frac{\partial P_{n_1,0,0}(t)}{\partial t} = -(\lambda_1(t) + \lambda_2(t) + n_1\mu_1)P_{n_1,0,0}(t) + \lambda_1(t)\left[\sum_{k=1}^{n_1} P_{n_1-k,0,0}(t)C_k\right] + \mu_3 P_{n_1,0,1}(t),$$

$$for\ n_1 > 0, n_2, n_3 = 0$$

$$\frac{\partial P_{0,n_2,0}(t)}{\partial t} = -(\lambda_1(t) + \lambda_2(t) + n_2\mu_2)P_{0,n_2,0}(t) + \lambda_2(t)\left[\sum_{s=1}^{n_2} P_{0,n_2-s,0}(t)C_s\right] + \mu_3 P_{0,n_2,1}(t),$$

$$for\ n_2 > 0, n_1, n_3 = 0$$

$$\frac{\partial P_{0,0,n_3}(t)}{\partial t} = -(\lambda_1(t) + \lambda_2(t) + n_3\mu_3)P_{0,0,n_3}(t) + \mu_1 P_{1,0,n_3-1}(t) + \mu_2 P_{0,1,n_3-1}(t) + (n_3+1)\mu_3 P_{0,0,n_3+1}(t), \qquad for\ n_3 > 0, n_1, n_2 = 0$$

$$\frac{\partial P_{0,0,0}(t)}{\partial t} = -\big(\lambda_1(t) + \lambda_2(t)\big)P_{0,0,0}(t) + \mu_3 P_{0,0,1}(t), \qquad for\ n_1, n_2, n_3 = 0 \tag{1}$$

Let be the joint probability generating function of $P_{n_1,n_2,n_3}(t)$ is be

$$P(Z_1, Z_2, Z_3, t) = \sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\sum_{n_3=0}^{\infty} {Z_1}^{n_1}{Z_2}^{n_2}{Z_3}^{n_3}\, P_{n_1,n_2,n_3}(t) \tag{2}$$

Multiplying the equation (1) by $Z_1$, $Z_2$, $Z_3$ and summing over all $n_1$, $n_2$ and $n_3$ we obtain

$$\frac{\partial P(Z_1,Z_2,Z_3,t)}{\partial t} = [-\mu_1(Z_1-Z_3)]\frac{\partial P(Z_1,Z_2,Z_3,t)}{\partial Z_1} + [\mu_2(Z_2-Z_3)]\frac{\partial P(Z_1,Z_2,Z_3,t)}{\partial Z_2} + [-\mu_3(Z_3-1)]\frac{\partial P(Z_1,Z_2,Z_3,t)}{\partial Z_3} + P(Z_1,Z_2,Z_3,t)[\lambda_1(t)\big(C(Z_1-1)\big) + \lambda_2(t)\big(C(Z_2-1)\big)]$$

where, $C(Z_1) = \sum_{k=1}^{n} \frac{\binom{n}{k}p^k (1-p)^{n-k}}{1-(1-p)^n} {Z_1}^k$ and $C(Z_2) = \sum_{s=1}^{n} \frac{\binom{n}{k}p^k (1-p)^{n-k}}{1-(1-p)^n} {Z_2}^s$ (3)

are the probability generating functions for the arriving batch size distributions of $Q_1$ and $Q_2$ respectively.

Solving the equation (3) by the Lagrangian method, the auxiliary equations are

$$\frac{\partial t}{1} = \frac{\partial Z_1}{\mu_1(Z_1-Z_3)} = \frac{\partial Z_2}{\mu_2(Z_2-Z_3)} = \frac{\partial Z_3}{\mu_3(Z_3-1)} = \frac{\partial P(Z_1,Z_2,Z_3,t)}{P(Z_1,Z_2,Z_3,t)[\lambda_1(t)\big(C(Z_1-1)\big) + \lambda_2(t)\big(C(Z_2-1)\big)]} \tag{4}$$

Solving the first and fourth terms in equation (4), we obtain

$$a = (Z_3-1)e^{-\mu_3 t}$$

Solving the first and third terms in equation (4), we obtain

$$b = (Z_2-1)e^{-\mu_2 t} + a\mu_2\frac{e^{(\mu_3-\mu_2)t}}{(\mu_3-\mu_2)}$$

Solving the first and second terms in equation (4), we obtain

$$c = (Z_1-1)e^{-\mu_1 t} + a\mu_1\frac{e^{(\mu_3-\mu_1)t}}{(\mu_3-\mu_1)}$$

Taking $\lambda_1(t) = \lambda_1 + \alpha_1 t$ and $\lambda_2(t) = \lambda_2 + \alpha_2 t$ and solving the first and fifth terms in equation (4), we obtain

$$d = P(Z_1, Z_2, Z_3, t) exp\left\{\frac{\sum_{r=1}^{n}\sum_{i=1}^{r}(-1)^i\binom{n}{r}\binom{r}{i}(pc)^{r-i}\left[\frac{pa\mu_1}{\mu_3-\mu_1}\right]^i}{1-(1-p)^n}\left[\frac{-\lambda_1}{i\mu_3+(r-i)\mu_1} + \frac{\alpha_1}{[i\mu_3+(r-i)\mu_1]^2}\right] + \frac{\sum_{l=1}^{n}\sum_{j=1}^{l}(-1)^i\binom{n}{l}\binom{l}{j}(pb)^{l-j}\left[\frac{pa\mu_2}{\mu_3-\mu_2}\right]^j}{1-(1-p)^n}\left[\frac{-\lambda_2}{j\mu_3+(l-j)\mu_2} + \frac{\alpha_2}{[j\mu_3+(l-j)\mu_2]^2}\right]\right\} \tag{5}$$

where, a, b, c and d are arbitrary constants derived using the initial conditions $P_{000}(0) = 1$, $P_{000}(t)$ 0 $\forall, t > 0$

The general solution of (4) gives the probability generating function of the number of customers in each queue at time 't' as

$$P(Z_1, Z_2, Z_3, t) = \exp\left\{\frac{\sum_{r=1}^{n}\sum_{i=1}^{r}(-1)^i\binom{n}{r}\binom{r}{i}p^r\left[(Z_1-1)+\frac{(Z_3-1)\mu_1}{\mu_3-\mu_1}\right]^{r-i}\left[\frac{(Z_3-1)\mu_1}{\mu_3-\mu_1}\right]^i}{1-(1-p)^n}\left[\lambda_1\left[\frac{1-e^{-[i\mu_3+(r-i)\mu_1]t}}{i\mu_3+(r-i)\mu_1}\right] + \alpha_1\left[\frac{t[i\mu_3+(r-i)\mu_1]-1+e^{-[i\mu_3+(r-i)\mu_1]t}}{[i\mu_3+(r-i)\mu_1]^2}\right]\right] + \frac{\sum_{l=1}^{n}\sum_{j=1}^{l}(-1)^j\binom{n}{l}\binom{l}{j}p^l\left[(Z_2-1)+\frac{(Z_3-1)\mu_2}{\mu_3-\mu_2}\right]^{l-j}\left[\frac{(Z_3-1)\mu_2}{\mu_3-\mu_2}\right]^j}{1-(1-p)^n}\left[\lambda_2\left[\frac{1-e^{-[j\mu_3+(l-j)\mu_2]t}}{j\mu_3+(l-j)\mu_2}\right] + \alpha_2\left[\frac{t[j\mu_3+(l-j)\mu_2]-1+e^{-[j\mu_3+(l-j)\mu_2]t}}{[j\mu_3+(l-j)\mu_2]^2}\right]\right]\right\}$$

$$|Z_1| < 1, |Z_2| < 1, |Z_3| < 1 \tag{6}$$

## 3. CHARACTERISTICS OF THE QUEUING MODEL:

In this section we briefly discuss the various performance measures of the model.

Expanding $P(Z_1, Z_2, Z_3, t)$ given in equation (6) and collecting the constant terms, we obtain the probability that the system is empty as

$$P_{000}(t)$$
$$= \exp\left\{\lambda_1 \frac{\sum_{r=1}^{n}\sum_{i=1}^{r}(-1)^i \binom{n}{r}\binom{r}{i} p^r \left[\frac{{\mu_3}^{r-i}{\mu_1}^i}{(\mu_3-\mu_1)^r}\right]}{1-(1-p)^n}\left[\frac{1-\mathrm{e}^{-[i\mu_3+(r-i)\mu_1]t}}{i\mu_3+(r-i)\mu_1}\right]\right.$$
$$+\lambda_2 \frac{\sum_{l=1}^{n}\sum_{j=1}^{l}(-1)^j \binom{n}{l}\binom{l}{j} p^l \left[\frac{{\mu_3}^{l-j}{\mu_2}^j}{(\mu_3-\mu_2)^l}\right]}{1-(1-p)^n}\left[\frac{1-\mathrm{e}^{-[j\mu_3+(l-j)\mu_2]t}}{j\mu_3+(l-j)\mu_2}\right]$$
$$+\alpha_1 \frac{\sum_{r=1}^{n}\sum_{i=1}^{r}(-1)^i \binom{n}{r}\binom{r}{i} p^r \left[\frac{{\mu_3}^{r-i}{\mu_1}^i}{(\mu_3-\mu_1)^r}\right]}{1-(1-p)^n}\left[\frac{\mathrm{t}[i\mu_3+(r-i)\mu_1]-1+e^{-[i\mu_3+(r-i)\mu_1]t}}{[i\mu_3+(r-i)\mu_1]^2}\right]$$
$$\left.+\alpha_2 \frac{\sum_{l=1}^{n}\sum_{j=1}^{l}(-1)^j \binom{n}{l}\binom{l}{j} p^l \left[\frac{{\mu_3}^{l-j}{\mu_2}^j}{(\mu_3-\mu_2)^l}\right]}{1-(1-p)^n}\left[\frac{\mathrm{t}[j\mu_3+(l-j)\mu_2]-1+e^{-[j\mu_3+(l-j)\mu_2]t}}{[j\mu_3+(l-j)\mu_2]^2}\right]\right\} \tag{7}$$

Using P($Z_1$, $Z_2$, $Z_3$, t), we obtain the probability generating functions of the first, second, and third queue size distributions as

$$P(Z_1,t) = exp\left\{\lambda_1 \frac{\sum_{r=1}^{n}\binom{n}{r} p^r (Z_1-1)^r}{1-(1-p)^n}\left[\frac{1-\mathrm{e}^{-r\mu_1 t}}{r\mu_1}\right]\right.$$
$$\left.+\alpha_1 \frac{\sum_{r=1}^{n}\binom{n}{r} p^r (Z_1-1)^r}{1-(1-p)^n}\left[\frac{\mathrm{t}r\mu_1-1+e^{-r\mu_1 t}}{[r\mu_1]^2}\right]\right\}, |Z_1|<1 \tag{8}$$

$$P(Z_2,t) = exp\left\{\lambda_2 \frac{\sum_{l=1}^{n}\binom{n}{l} p^l (Z_2-1)^l}{1-(1-p)^n}\left[\frac{1-\mathrm{e}^{-l\mu_2 t}}{l\mu_2}\right]\right.$$
$$\left.+\alpha_2 \frac{\sum_{l=1}^{n}\binom{n}{l} p^l (Z_2-1)^l}{1-(1-p)^n}\left[\frac{\mathrm{t}l\mu_2-1+e^{-l\mu_2 t}}{[l\mu_2]^2}\right]\right\}, \qquad |Z_2|<1 \tag{9}$$

$$P(Z_3,t)$$
$$= exp\left\{\lambda_1 \frac{\sum_{r=1}^{n}\sum_{i=0}^{r}(-1)^i \binom{n}{r}\binom{r}{i} p^r \left[\frac{(Z_3-1)\mu_1}{\mu_3-\mu_1}\right]^r}{1-(1-p)^n}\left[\frac{1-\mathrm{e}^{-[i\mu_3+(r-i)\mu_1]t}}{i\mu_3+(r-i)\mu_1}\right]\right.$$
$$+\alpha_1 \frac{\sum_{r=1}^{n}\sum_{i=0}^{r}(-1)^i \binom{n}{r}\binom{r}{i} p^r \left[\frac{(Z_3-1)\mu_1}{\mu_3-\mu_1}\right]^r}{1-(1-p)^n}\left[\frac{\mathrm{t}[i\mu_3+(r-i)\mu_1]-1+e^{-[i\mu_3+(r-i)\mu_1]t}}{[i\mu_3+(r-i)\mu_1]^2}\right]$$
$$+\lambda_2 \frac{\sum_{l=1}^{n}\sum_{j=0}^{l}(-1)^j \binom{n}{l}\binom{l}{j} p^l \left[\frac{(Z_3-1)\mu_2}{\mu_3-\mu_2}\right]^l}{1-(1-p)^n}\left[\frac{1-\mathrm{e}^{-[j\mu_3+(l-j)\mu_2]t}}{j\mu_3+(l-j)\mu_2}\right]$$
$$\left.+\alpha_2 \frac{\sum_{l=1}^{n}\sum_{j=0}^{l}(-1)^j \binom{n}{l}\binom{l}{j} p^l \left[\frac{(Z_3-1)\mu_2}{\mu_3-\mu_2}\right]^l}{1-(1-p)^n}\left[\frac{\mathrm{t}[j\mu_3+(l-j)\mu_2]-1+e^{-[j\mu_3+(l-j)\mu_2]t}}{[j\mu_3+(l-j)\mu_2]^2}\right]\right\},$$
$$|Z_3|<1 \tag{10}$$

The probability that the first, second and third queues are empty are

$$P_{0..}(t) = exp\{\mathrm{D}_1(\mathrm{t})\}$$

$$\text{where, } \mathrm{D}_1(\mathrm{t}) = \lambda_1 \frac{\sum_{r=1}^{n}\binom{n}{r} p^r (-1)^r}{1-(1-p)^n}\left[\frac{1-\mathrm{e}^{-r\mu_1 t}}{r\mu_1}\right] + \alpha_1 \frac{\sum_{r=1}^{n}\binom{n}{r} p^r (-1)^r}{1-(1-p)^n}\left[\frac{\mathrm{t}r\mu_1 - 1 + e^{-r\mu_1 t}}{[r\mu_1]^2}\right] \tag{11}$$

$$P_{.0.}(t) = exp\{\mathrm{D}_2(\mathrm{t})\}$$

$$\text{where, } \mathrm{D}_2(\mathrm{t}) = \lambda_2 \frac{\sum_{l=1}^{n}\binom{n}{l} p^l (-1)^l}{1-(1-p)^n}\left[\frac{1-\mathrm{e}^{-l\mu_2 t}}{l\mu_2}\right] + \alpha_2 \frac{\sum_{l=1}^{n}\binom{n}{l} p^l (-1)^l}{1-(1-p)^n}\left[\frac{\mathrm{t}l\mu_2 - 1 + e^{-l\mu_2 t}}{[l\mu_2]^2}\right] \tag{12}$$

$$P_{..0}(t) = exp\{\mathrm{D}_3(\mathrm{t})\}$$

where, $\mathrm{D}_3(\mathrm{t})$

$$= \lambda_1 \frac{\sum_{r=1}^{n}\sum_{i=0}^{r}(-1)^{i+r}\binom{n}{r}\binom{r}{i}p^r\left[\frac{\mu_1}{\mu_3-\mu_1}\right]^r}{1-(1-p)^n}\left[\frac{1-\mathrm{e}^{-[i\mu_3+(r-i)\mu_1]t}}{i\mu_3+(r-i)\mu_1}\right]$$
$$+ \alpha_1 \frac{\sum_{r=1}^{n}\sum_{i=0}^{r}(-1)^{i+r}\binom{n}{r}\binom{r}{i}p^r\left[\frac{\mu_1}{\mu_3-\mu_1}\right]^r}{1-(1-p)^n}\left[\frac{\mathrm{t}[i\mu_3+(r-i)\mu_1]-1+e^{-[i\mu_3+(r-i)\mu_1]t}}{[i\mu_3+(r-i)\mu_1]^2}\right]$$
$$+ \lambda_2 \frac{\sum_{l=1}^{n}\sum_{j=0}^{l}(-1)^{j+l}\binom{n}{l}\binom{l}{j}p^l\left[\frac{\mu_2}{\mu_3-\mu_2}\right]^l}{1-(1-p)^n}\left[\frac{1-\mathrm{e}^{-[j\mu_3+(l-j)\mu_2]t}}{j\mu_3+(l-j)\mu_2}\right]$$
$$+ \alpha_2 \frac{\sum_{l=1}^{n}\sum_{j=0}^{l}(-1)^{j+l}\binom{n}{l}\binom{l}{j}p^l\left[\frac{\mu_2}{\mu_3-\mu_2}\right]^l}{1-(1-p)^n}\left[\frac{\mathrm{t}[j\mu_3+(l-j)\mu_2]-1+e^{-[j\mu_3+(l-j)\mu_2]t}}{[j\mu_3+(l-j)\mu_2]^2}\right] \tag{13}$$

The mean number of customers in the first, second and third queues are

$$L_1(t) = \frac{np}{1-(1-p)^n}\left[\frac{\lambda_1}{\mu_1}[1-\mathrm{e}^{-\mu_1 t}] + \frac{\alpha_1}{{\mu_1}^2}[\mathrm{t}\mu_1 - 1 + e^{-\mu_1 t}]\right] \tag{14}$$

$$L_2(t) = \frac{np}{1-(1-p)^n}\left[\frac{\lambda_2}{\mu_2}[1-\mathrm{e}^{-\mu_2 t}] + \frac{\alpha_2}{{\mu_2}^2}[\mathrm{t}\mu_2 - 1 + e^{-\mu_2 t}]\right] \tag{15}$$

$$L_3(t) = \frac{np}{1-(1-p)^n}\left[\left[\frac{\mu_1}{\mu_3-\mu_1}\right]\left[\frac{\lambda_1}{\mu_1}[1-\mathrm{e}^{-\mu_1 t}] + \frac{\alpha_1}{{\mu_1}^2}[\mathrm{t}\mu_1 - 1 + e^{-\mu_1 t}]\right]\right.$$
$$\left. + \left[\frac{\mu_2}{\mu_3-\mu_2}\right]\left[\frac{\lambda_2}{\mu_2}[1-\mathrm{e}^{-\mu_2 t}] + \frac{\alpha_1}{{\mu_2}^2}[\mathrm{t}\mu_2 - 1 + e^{-\mu_2 t}]\right]\right] \tag{16}$$

The utilization of the first, second and third service stations are

$$U_1(t) = 1 - exp\{\mathrm{D}_1(\mathrm{t})\} \tag{17}$$

$$U_2(t) = 1 - exp\{\mathrm{D}_2(\mathrm{t})\} \tag{18}$$

$$U_3(t) = 1 - exp\{\mathrm{D}_3(\mathrm{t})\} \tag{19}$$

The throughput of the first, second and third service stations are

$$T_1(t) = \mu_1[1 - exp\{\mathrm{D}_1(\mathrm{t})\}] \tag{20}$$

$$T_2(t) = \mu_2[1 - exp\{\mathrm{D}_2(\mathrm{t})\}] \tag{21}$$

$$T_3(t) = \mu_3[1 - exp\{\mathrm{D}_3(\mathrm{t})\}] \tag{22}$$

The average waiting time for the customer in the first, second and third queues are

$$W_1(t) = \frac{\frac{np}{1-(1-p)^n}\left[\frac{\lambda_1}{\mu_1}[1-\mathrm{e}^{-\mu_1 t}] + \frac{\alpha_1}{{\mu_1}^2}[\mathrm{t}\mu_1 - 1 + e^{-\mu_1 t}]\right]}{\mu_1[1 - exp\{\mathrm{D}_1(\mathrm{t})\}]} \tag{23}$$

$$W_2(t) = \frac{\frac{np}{1-(1-p)^n}\left[\frac{\lambda_2}{\mu_2}[1-\mathrm{e}^{-\mu_2 t}] + \frac{\alpha_2}{{\mu_2}^2}[\mathrm{t}\mu_2 - 1 + e^{-\mu_2 t}]\right]}{\mu_2[1 - exp\{\mathrm{D}_2(\mathrm{t})\}]} \tag{24}$$

$$W_3(t) = \frac{L_3(t)}{\mu_3[1 - exp\{\mathrm{D}_3(\mathrm{t})\}]} \tag{25}$$

The variance of the number of customers in the first, second and third queues are

$$V_1(t) = \frac{\sum_{r=1}^{n}\binom{n}{r}p^r}{1-(1-p)^n}\left[\frac{\lambda_1}{\mu_1}\left[\frac{(n-1)p}{2}(1-e^{-2\mu_1 t}) + (1-e^{-\mu_1 t})\right]\right.$$
$$\left. + \alpha_1\left[\frac{(n-1)p}{2}\left(\frac{2\mu_1 t - 1 + e^{-2\mu_1 t}}{2}\right) + (\mathrm{t}\mu_1 - 1 + e^{-\mu_1 t})\right]\right] \tag{26}$$

$$V_2(t) = \frac{\sum_{l=1}^{n}\binom{n}{l} p^l}{1-(1-p)^n}\left[\frac{\lambda_2}{\mu_2}\left[\frac{(n-1)p}{2}(1-e^{-2\mu_2 t}) + (1-e^{-\mu_1 t})\right] + \frac{\alpha_2}{\mu_2{}^2}\left[\frac{(n-1)p}{2}\left(\frac{2\mu_2 t - 1 + e^{-2\mu_2 t}}{2}\right) + (\mathrm{t}\mu_2 - 1 + e^{-\mu_2 t})\right]\right] \quad (27)$$

$$\begin{aligned} V_3(t) = {} & \frac{\sum_{r=1}^{n}\binom{n}{r} p^r}{1-(1-p)^n}\frac{(n-1)p}{2}\left[\frac{\mu_1}{\mu_3-\mu_1}\right]^2 \Bigg[\lambda_1\left[\left(\frac{1-e^{-2\mu_1 t}}{\mu_1}\right) - 4\left(\frac{1-e^{-(\mu_3+\mu_1)t}}{\mu_3+\mu_1}\right) + \left(\frac{1-e^{-2\mu_3 t}}{\mu_3}\right)\right] \\ & + \alpha_1\left[\left(\frac{2\mu_1 t - 1 + e^{-2\mu_1 t}}{2\mu_1{}^2}\right) - 4\left(\frac{t[\mu_3+\mu_1] - 1 + e^{-[\mu_3+\mu_1]t}}{[\mu_3+\mu_1]^2}\right) \right. \\ & \left. + \left(\frac{2\mu_3 t - 1 + e^{-2\mu_3 t}}{2\mu_3{}^2}\right)\right]\Bigg] \\ & + \frac{\sum_{l=1}^{n}\binom{n}{l} p^l}{1-(1-p)^n}\frac{(n-1)p}{2}\left[\frac{\mu_2}{\mu_3-\mu_2}\right]^2 \Bigg[\lambda_2\left[\left(\frac{1-e^{-2\mu_2 t}}{\mu_2}\right) - 4\left(\frac{1-e^{-(\mu_3+\mu_2)t}}{\mu_3+\mu_2}\right) \right. \\ & \left. + \left(\frac{1-e^{-2\mu_3 t}}{\mu_3}\right)\right] \\ & + \alpha_2\left[\left(\frac{2\mu_2 t - 1 + e^{-2\mu_2 t}}{2\mu_2{}^2}\right) - 4\left(\frac{t[\mu_3+\mu_2] - 1 + e^{-[\mu_3+\mu_2]t}}{[\mu_3+\mu_2]^2}\right) \right. \\ & \left. + \left(\frac{2\mu_3 t - 1 + e^{-2\mu_3 t}}{2\mu_3{}^2}\right)\right]\Bigg] \\ & + \frac{\sum_{r=1}^{n}\binom{n}{r} p^r}{1-(1-p)^n}\left[\frac{\mu_1}{\mu_3-\mu_1}\right]\left[\frac{\lambda_1}{\mu_1}[1-\mathrm{e}^{-\mu_1 t}] + \frac{\alpha_1}{\mu_1{}^2}[\mathrm{t}\mu_1 - 1 + e^{-\mu_1 t}]\right] \\ & + \frac{\sum_{l=1}^{n}\binom{n}{l} p^l}{1-(1-p)^n}\left[\frac{\mu_2}{\mu_3-\mu_2}\right]\left[\frac{\lambda_2}{\mu_2}[1-\mathrm{e}^{-\mu_2 t}] + \frac{\alpha_1}{\mu_2{}^2}[\mathrm{t}\mu_2 - 1 + e^{-\mu_2 t}]\right] \end{aligned} \quad (28)$$

## 4. PARTICULAR CASES OF THE MODEL

It is interesting to note that the queueing model proposed in this paper includes various queueing models as particular cases for specific values of the parameters. The particular cases are as follows

**Case – 1:** When $\alpha_1 = 0$ and $\alpha_2 = 0$ then this model includes the parallel and series two node queueing model with compound Poisson binomial bulk arrivals and load dependent service rates. Its joint probability generating function of the number of customers in each queue is

$$P(Z_1, Z_2, Z_3, t)$$
$$= \exp\left\{\lambda_1 \frac{\sum_{r=1}^{n}\sum_{i=1}^{r}(-1)^i\binom{n}{r}\binom{r}{i}p^r\left[(Z_1-1)+\frac{(Z_3-1)\mu_1}{\mu_3-\mu_1}\right]^{r-i}\left[\frac{(Z_3-1)\mu_1}{\mu_3-\mu_1}\right]^i}{1-(1-p)^n}\left[\frac{1-\mathrm{e}^{-[i\mu_3+(r-i)\mu_1]t}}{i\mu_3+(r-i)\mu_1}\right]\right.$$
$$\left.+\lambda_2 \frac{\sum_{l=1}^{n}\sum_{j=1}^{l}(-1)^j\binom{n}{l}\binom{l}{j}p^l\left[(Z_2-1)+\frac{(Z_3-1)\mu_2}{\mu_3-\mu_2}\right]^{l-j}\left[\frac{(Z_3-1)\mu_2}{\mu_3-\mu_2}\right]^j}{1-(1-p)^n}\left[\frac{1-\mathrm{e}^{-[j\mu_3+(l-j)\mu_2]t}}{j\mu_3+(l-j)\mu_2}\right]\right\}$$
$$|Z_1|<1, |Z_2|<1, |Z_3|<1 \quad (29)$$

**Case – 2:** When $\mu_2 = 0$ and $\lambda_2(t) = 0$ then this model becomes two node tandem queueing model with non-homogeneous compound Poisson binomial bulk arrivals and load dependent service rates. Its joint probability generating function of the number of customers in the first and second queues is

$$P(Z_1, Z_3 t)$$
$$= \exp\left\{\lambda_1 \frac{\sum_{r=1}^{n}\sum_{i=1}^{r}(-1)^i\binom{n}{r}\binom{r}{i}p^r\left[(Z_1-1)+\frac{(Z_3-1)\mu_1}{\mu_3-\mu_1}\right]^{r-i}\left[\frac{(Z_3-1)\mu_1}{\mu_3-\mu_1}\right]^i}{1-(1-p)^n}\left[\frac{1-\mathrm{e}^{-[i\mu_3+(r-i)\mu_1]t}}{i\mu_3+(r-i)\mu_1}\right]\right.$$
$$\left.+\alpha_1 \frac{\sum_{r=1}^{n}\sum_{i=1}^{r}(-1)^i\binom{n}{r}\binom{r}{i}p^r\left[(Z_1-1)+\frac{(Z_3-1)\mu_1}{\mu_3-\mu_1}\right]^{r-i}\left[\frac{(Z_3-1)\mu_1}{\mu_3-\mu_1}\right]^i}{1-(1-p)^n}\left[\frac{\mathrm{t}[i\mu_3+(r-i)\mu_1]-1+e^{-[i\mu_3+(r-i)\mu_1]t}}{[i\mu_3+(r-i)\mu_1]^2}\right]\right\}$$
$$|Z_1|<1, |Z_3|<1 \quad (30)$$

**Case -3 :** When $\mu_2 = 0$, $\lambda_2(t) = 0$ and $\alpha_1 = 0$ then this model become two node tandem queueing model with compound Poisson binomial bulk arrivals and load dependent service rates. Its joint probability generating function of the number of customers in the first and second queues is

$$P(Z_1, Z_3, t)$$
$$= \exp\left\{\lambda_1 \frac{\sum_{r=1}^{n}\sum_{i=1}^{r}(-1)^i\binom{n}{r}\binom{r}{i}p^r\left[(Z_1-1)+\frac{(Z_3-1)\mu_1}{\mu_3-\mu_1}\right]^{r-i}\left[\frac{(Z_3-1)\mu_1}{\mu_3-\mu_1}\right]^i}{1-(1-p)^n}\left[\frac{1-\mathrm{e}^{-[i\mu_3+(r-i)\mu_1]t}}{i\mu_3+(r-i)\mu_1}\right]\right\}$$
$$|Z_1|<1, \ |Z_3|<1 \quad (31)$$

**Case – 4**: If $\mu_3 = 0$ then this queueing model become two queues in parallel with non-homogeneous compound Poisson binomial bulk arrivals and load dependent service rates. Its joint probability generating function of the number of customers in the first and second queues is

$$P(Z_1, Z_2, t) = \frac{\sum_{r=1}^{n}\binom{n}{r} p^r [Z_1 - 1]^r}{1-(1-p)^n}\left[\lambda_1\left[\frac{1-e^{-r\mu_1 t}}{r\mu_1}\right] + \alpha_1\left[\frac{tr\mu_1 - 1 + e^{-r\mu_1 t}}{(r\mu_1)^2}\right]\right]$$

$$+ \frac{\sum_{l=1}^{n}\binom{n}{l} p^l [Z_2 - 1]^l}{1-(1-p)^n}\left[\lambda_2\left[\frac{1-e^{-l\mu_2 t}}{l\mu_2}\right] + \alpha_1\left[\frac{tl\mu_2 - 1 + e^{-l\mu_2 t}}{(l\mu_2)^2}\right]\right]$$

$$|Z_1| < 1, |Z_2| < 1 \quad (32)$$

**Case – 5:** If $\mu_3 = 0$, $\alpha_1 = 0$, $\alpha_2 = 0$ then this model becomes two queues in series with compound Poisson binomial bulk arrivals and load dependent service rates. Its joint probability generating function of the number of customers in the first and second queues is

$$P(Z_1, Z_2, t) = \lambda_1 \frac{\sum_{r=1}^{n}\binom{n}{r} p^r [Z_1 - 1]^r}{1-(1-p)^n}\left[\frac{1-e^{-r\mu_1 t}}{r\mu_1}\right]$$

$$+ \lambda_2 \frac{\sum_{l=1}^{n}\binom{n}{l} p^l [Z_2 - 1]^l}{1-(1-p)^n}\left[\frac{1-e^{-l\mu_2 t}}{l\mu_2}\right]$$

$$|Z_1| < 1, |Z_2| < 1 \quad (33)$$

**Case – 6:** If $\mu_2 = 0$, $\mu_3 = 0$, $\lambda_2(t) = 0$ then this model becomes single node queueing model with non-homogeneous compound binomial bulk arrivals and load dependent service rates. Its probability generating function of the number of customers in the queue is

$$P(Z_1, t) = \exp\left\{\frac{\sum_{r=1}^{n}\binom{n}{r} p^r [Z_1 - 1]^r}{1-(1-p)^n}\left[\lambda_1\left[\frac{1-e^{-r\mu_1 t}}{r\mu_1}\right] + \alpha_1\left[\frac{tr\mu_1 - 1 + e^{-r\mu_1 t}}{(r\mu_1)^2}\right]\right]\right\}, |Z_1| < 1 \quad (34)$$

**Case -7:** If $\mu_2 = 0$, $\mu_3 = 0$, $\lambda_2(t) = 0$, $\alpha_1 = 0$ then this model becomes single node queueing model with compound Poisson binomial bulk arrivals and load dependent service rates Its probability generating function of the number of customers in the queue is

$$P(Z_1, t) = \exp\left\{\lambda_1 \frac{\sum_{r=1}^{n}\binom{n}{r} p^r [Z_1 - 1]^r}{1-(1-p)^n}\left[\frac{1-e^{-r\mu t}}{r\mu}\right]\right\}, |Z_1| < 1 \quad (35)$$

**Case – 8:** If n = 1, p =1 then this model includes the parallel and series two node queueing model with non-homogeneous compound Poisson binomial single arrivals and load dependent service rates. Its joint probability generating function of the number of customers in each queue is

$$\begin{aligned}P(Z_1,Z_2,Z_3,t) = \exp\Bigg\{&\lambda_1\left[\frac{Z_1-1}{\mu_1}(1-e^{-\mu_1 t})+\frac{Z_3-1}{\mu_3}(1-e^{-\mu_3 t})+\frac{Z_3-1}{\mu_1-\mu_3}(e^{-\mu_1 t}-e^{-\mu_3 t})\right]\\ &+\alpha_1 t\left(\frac{Z_1-1}{\mu_1}+\frac{Z_3-1}{\mu_3}\right)\\ &-\alpha_1\left[\frac{Z_1-1}{{\mu_1}^2}(1-e^{-\mu_1 t})+\frac{Z_3-1}{{\mu_3}^2}(1-e^{-\mu_3 t})+\frac{Z_3-1}{\mu_1(\mu_1-\mu_3)}(e^{-\mu_1 t}-e^{-\mu_3 t})\right.\\ &\left.+\frac{Z_3-1}{\mu_1\mu_3}(1-e^{-\mu_3 t})\right]\\ &+\lambda_2\left[\frac{Z_2-1}{\mu_2}(1-e^{-\mu_2 t})+\frac{Z_3-1}{\mu_3}(1-e^{-\mu_3 t})+\frac{Z_3-1}{\mu_2-\mu_3}(e^{-\mu_1 t}-e^{-\mu_3 t})\right]\\ &+\alpha_2 t\left(\frac{Z_2-1}{\mu_2}+\frac{Z_3-1}{\mu_3}\right)\\ &-\alpha_2\left[\frac{Z_2-1}{{\mu_2}^2}(1-e^{-\mu_2 t})+\frac{Z_3-1}{{\mu_3}^2}(1-e^{-\mu_3 t})+\frac{Z_3-1}{\mu_2(\mu_2-\mu_3)}(e^{-\mu_2 t}-e^{-\mu_3 t})\right.\\ &\left.+\frac{Z_3-1}{\mu_2\mu_3}(1-e^{-\mu_3 t})\right]\Bigg\}\end{aligned}$$

$$|Z_1|<1, |Z_2|<1, |Z_3|<1 \quad (36)$$

**Case – 9:** If n = 1, p =1 $\alpha_1 = 0$ and $\alpha_2 = 0$ then this model includes the parallel and series two node queueing model with compound Poisson binomial arrivals and load dependent service rates. Its joint probability generating function of the number of customers in each queue is

$$\begin{aligned}P(Z_1,Z_2,Z_3,t) = \exp\Bigg\{&\lambda_1\left[\frac{Z_1-1}{\mu_1}(1-e^{-\mu_1 t})+\frac{Z_3-1}{\mu_3}(1-e^{-\mu_3 t})+\frac{Z_3-1}{\mu_1-\mu_3}(e^{-\mu_1 t}-e^{-\mu_3 t})\right]\\ &+\lambda_2\left[\frac{Z_2-1}{\mu_2}(1-e^{-\mu_2 t})+\frac{Z_3-1}{\mu_3}(1-e^{-\mu_3 t})+\frac{Z_3-1}{\mu_2-\mu_3}(e^{-\mu_1 t}-e^{-\mu_3 t})\right]\Bigg\}\end{aligned}$$

$$|Z_1|<1, |Z_2|<1, |Z_3|<1 \quad (37)$$

**Case - 10:** If n = 1, p =1, $\mu_2 = 0$ and $\lambda_2(t) = 0$ then this model becomes two node tandem queueing model with non-homogeneous compound Poisson binomial arrivals and load dependent service rates. Its joint probability generating function of the number of customers in the first and second queues is

$$P(Z_1, Z_3, t) = \exp\left\{\lambda_1\left[\frac{Z_1-1}{\mu_1}(1-e^{-\mu_1 t}) + \frac{Z_3-1}{\mu_3}(1-e^{-\mu_3 t}) + \frac{Z_3-1}{\mu_1-\mu_3}(e^{-\mu_1 t}-e^{-\mu_3 t})\right] + \alpha_1 t\left(\frac{Z_1-1}{\mu_1}+\frac{Z_3-1}{\mu_3}\right) - \alpha_1\left[\frac{Z_1-1}{{\mu_1}^2}(1-e^{-\mu_1 t}) + \frac{Z_3-1}{{\mu_3}^2}(1-e^{-\mu_3 t}) + \frac{Z_3-1}{\mu_1(\mu_1-\mu_3)}(e^{-\mu_1 t}-e^{-\mu_3 t}) + \frac{Z_3-1}{\mu_1\mu_3}(1-e^{-\mu_3 t})\right]\right\}, |Z_1|<1, |Z_3|<1 \tag{38}$$

**Case -11 :** If n = 1, p =1, $\mu_2 = 0$, $\lambda_2(t) = 0$ and $\alpha_1 = 0$ then this model become two node tandem queueing model with compound Poisson binomial arrivals and load dependent service rates. Its joint probability generating function of the number of customers in the first and second queues is

$$P(Z_1, Z_3, t) = \exp\left\{\lambda_1\left[\frac{Z_1-1}{\mu_1}(1-e^{-\mu_1 t}) + \frac{Z_3-1}{\mu_3}(1-e^{-\mu_3 t}) + \frac{Z_3-1}{\mu_1-\mu_3}(e^{-\mu_1 t}-e^{-\mu_3 t})\right]\right\},$$

$$|Z_1|<1, |Z_3|<1 \tag{39}$$

**Case – 12:** If n = 1, p =1, $\mu_3 = 0$ then this queueing model become two queues in parallel with non-homogeneous compound Poisson binomial bulk arrivals and load dependent service rates. Its joint probability generating function of the number of customers in the first and second queues is

$$P(Z_1, Z_2, t) = \exp\left\{\lambda_1\left[\frac{Z_1-1}{\mu_1}(1-e^{-\mu_1 t})\right] + \alpha_1 t\left(\frac{Z_1-1}{\mu_1}\right) - \alpha_1\left[\frac{Z_1-1}{{\mu_1}^2}(1-e^{-\mu_1 t})\right] + \lambda_2\left[\frac{Z_2-1}{\mu_2}(1-e^{-\mu_2 t})\right] + \alpha_2 t\left(\frac{Z_2-1}{\mu_2}\right) - \alpha_2\left[\frac{Z_2-1}{{\mu_2}^2}(1-e^{-\mu_2 t})\right]\right\}, |Z_1| < 1, |Z_2| < 1 \tag{40}$$

**Case – 13:** If n = 1, p =1 $\mu_3 = 0$, $\alpha_1 = 0$, $\alpha_2 = 0$ then this model becomes two queues in series with compound Poisson binomial arrivals and load dependent service rates. Its joint probability generating function of the number of customers in the first and second queues is

$$P(Z_1, Z_2, t) = \exp\left\{\lambda_1 \left[\frac{Z_1 - 1}{\mu_1}(1 - e^{-\mu_1 t})\right] + \lambda_2 \left[\frac{Z_2 - 1}{\mu_2}(1 - e^{-\mu_2 t})\right]\right\}, |Z_1| < 1, |Z_2| < 1 \tag{41}$$

**Case – 14:** If n = 1, p =1 $\mu_2 = 0$, $\mu_3 = 0$, $\lambda_2(t) = 0$ then this model becomes single node queueing model with non-homogeneous compound binomial arrivals and load dependent service rates. Its probability generating function of the number of customers in the queue is

$$P(Z_1, t) = \exp\left\{\lambda_1 \left[\frac{Z_1 - 1}{\mu_1}(1 - e^{-\mu_1 t})\right] + \alpha_1 t\left(\frac{Z_1 - 1}{\mu_1}\right) - \alpha_1 \left[\frac{Z_1 - 1}{\mu_1{}^2}(1 - e^{-\mu_1 t})\right]\right\}, |Z_1| < 1 \tag{42}$$

**Case -15:** If n = 1, p =1,$\mu_2 = 0$, $\mu_3 = 0$, $\lambda_2(t) = 0$, $\alpha_1 = 0$ then this model becomes single node queueing model with compound Poisson binomial arrivals and load dependent service rates Its probability generating function of the number of customers in the queue is

$$P(Z_1, t) = \exp\left\{\lambda_1 \left[\frac{Z_1 - 1}{\mu_1}(1 - e^{-\mu_1 t})\right]\right\}, |Z_1| < 1 \tag{43}$$

## 5. NUMERICAL DEMONSTRATION AND SENSITIVITY ANALYSIS:

In this section, the performance of the proposed queueing model is discussed through a numerical illustration. The customers arrive in batches to the queue, and the arrival process is a non-homogeneous compound Poisson binomial bulk processes. The composite mean arrival rate is $\lambda + \alpha$ t. Each arriving module represents a batch of customers. The number of customers in each arriving module follows a binomial distribution with parameters (n,p). Because the characteristics of the queueing model are highly sensitive with respect to time, the transient behaviour of the model is studied by computing the performance measures with the following set of values for the model parameters:

$t = 0.2, 0.6, 1$; $n = 15, 25, 35$; $p = 0.5, 0.7, 0.9$; $\lambda_1 = 1, 2, 3$; $\lambda_2 = 1.5, 2.5, 3.5$;

$\alpha_1 = 1.5, 2.5, 3.5$; $\alpha_2 = 2, 3, 4$;

$\mu_1 = 16, 18, 20$; $\mu_2 = 22, 24, 26$; $\mu_3 = 31, 33, 35$;

The probability that the system is empty and the emptiness of the marginal queues, the expected number of customers, and the utilization of servers are computed for different values of the parameters t, a, b, $\lambda_1$, $\lambda_2$, $\alpha_1$, $\alpha_2$, $\mu_1$, $\mu_2$, $\mu_3$ and presented in Table 1. The relationships between the parameters and performance measures are shown in Figure. 2.

It can be observed that the probability of emptiness of the queueing system and three marginal queues are highly sensitive with respect to time. As time (t) increases the probability that the queue is empty decreases; the probability that the marginal queues is empty decreases; the expected number of customers in each queue and the system increases; and the utilization of the service station increases, when all other parameters are fixed. As the parameter n increases, the probability that the queue is empty decreases; the probability that the marginal queues is empty decreases; the expected number of customers in each queue and in the system increases; and the utilization of the service station increases, when all other parameters are fixed. As the parameter p increases, the probability that the queue is empty decreases; the probability that the marginal queues is empty decreases; the expected number of customers in each queue and the system increases; and the utilization of the service station increases, when all other parameters are fixed.

It can be further observed that as the parameter ($\lambda_1$) increases, the probability that the queue is empty decreases; the probability that the first and third marginal queues is empty decreases, but the second queue remains constant; the expected number of customers in first and third queue increases, but the second queue remains constant; the expected number of customers in the system increases; and the utilization of the service station in the first and third queues increases, but second queue it remains constant, when all other parameters are fixed. The same patterns hold with respect to the parameter ($\alpha_1$).
It can be further observed that as the parameter ($\lambda_2$) increases, the probability that the queue is empty decreases; the probability that the second and third marginal queues is empty decreases, but the first queue remains constant; the expected number of customers in second and third queues increases, but the first queue remains constant; the expected number of customers in the system increases; and the utilization of the service station in

the second and third queues increases, but first queue it remains constant, when all other parameters are fixed. The same patterns hold with respect to the parameter ($\alpha_2$).

It can also be observed that as the service rate ($\mu_1$) increases, the probability that the queue is empty increases; the probability that the first and third queues increases, but the second queue remains constant; the expected number of customers in the first queue decreases, but the second queue remains constant; the expected number of customers in the third queue and the system increases; and the utilisation of the service station in the first and third queue decreases, but the second queue remains constant, when all other parameters are fixed.

It can also be observed that as the service rate ($\mu_2$) increases, the probability that the queue is empty increases; the probability that the second and third queues increases, but the first queue remains constant; the expected number of customers in the second queue decreases, but the first queue remains constant; the expected number of customers in the third queue and the system increases; and the utilisation of the service station in the second and third queue decreases, but the first queue remains constant, when all other parameters are fixed.

It can also be observed that as the service rate ($\mu_3$) increases, the probability that the queue is empty increases; the probability that the third queues increases, but the first and second queue remains constant; the expected number of customers in the third queue decreases, but the first and second queues remains constant; and the utilisation of the service station in the third queue decreases, but the first and second queues remains constant, when all other parameters are fixed.

The throughput of the service stations, the average waiting time, the variance of the number of customers in each queue are computed for different values of t, n, p, $\lambda_1$, $\alpha_1$, $\lambda_2$, $\alpha_2$ $\mu_1$, $\mu_2$, $\mu_3$ and presented in Table 2. The relationships between parameters and the performance measures are shown in Figure. 3.

From Table 2, it can be observed that the throughput of the service stations, waiting time of a customer in each queue, variance of the number of customers in each queue are highly sensitive with respect to time. As time (t) increases, the throughput of the service station, the average waiting time of a customer, and the variance of the number of customers in each queue increases; when all other parameters are fixed. As the parameter n increases, the throughput of the service station, the waiting time of a customer in each queue, and the variance of the number of customers in each queue increases. The same patterns hold with respect to the parameter p.

It can be further observed that as the parameter ($\lambda_1$) increases, the throughput of the service station, the waiting time of the customer in each queue, and the variance of the number of customers in first and third queues increases, but the second queue remains constant, when all other parameters are fixed. The same patterns hold with respect to the parameter ($\alpha_1$). It can be further observed that as the parameter ($\lambda_2$) increases, the throughput of the service station, the waiting time of the customer in each queue, and the variance of the number of customers in second and third queues increases, but the first queue remains constant, when all other parameters are fixed. The same patterns hold with respect to the parameter ($\alpha_1$).

It can also be observed that as the service rate ($\mu_1$) increases, the throughput of the first service station increases, and third queue decreases, but the second queue remains constant; the average waiting time, and the variance of the number of customers in the first queue decreases, but the third queues increases; the average waiting time, and the variance of the number of customers in the second queue remains constant, when all other parameters are constant. It can also be observed that as the service rate ($\mu_2$) increases the throughput of the second and third service stations increases, but the first service station remains constant; the average waiting time, and the variance of the number of customers in the second queue decreases, but the first queue remains constant; and average waiting time, and the variance of the number of customers in the third queue increases, when all other parameters are constant. It can also be observed that as the service rate ($\mu_3$) increases the throughput of the first and second service stations remains constant, but the third service station increases; the average waiting time, and the variance of the number of customers in the third queue decreases, but the first and second queues remains constant, when all other parameters are constant.

## 6. COMPARATIVE STUDY

The comparative study of the developed model with that of homogeneous compound Poisson binomial bulk arrivals is carried by taking $\alpha_1 = 0$, $\alpha_2 = 0$ in the model and different values of t. It is also further a comparative study with non-homogeneous and homogeneous compound Poisson binomial single arrivals. Table 3, shows comparative study models with homogeneous and non-homogeneous compound Poisson binomial bulk arrivals.

From Table 3, it can be observed that as time increases the percentage variation between the performance measures of the models increases. The model with non-homogeneous compound Poisson binomial bulk arrivals has higher utilization than the model with homogeneous compound Poisson binomial bulk arrivals. It is also observed that the assumption of non-homogeneous compound Poisson binomial bulk arrivals has significant influence on all the performance measures of the queueing model. Time has significant effect on the system performance measures, and the proposed model can predict performance measures more accurately.

## 7. CONCLUSIONS:

This paper address a generalized non-homogeneous compound Poisson binomial bulk queueing model for parallel and series configuration and load dependent service rates. It is assumed the customers arrive in groups of random size depending on time. The batch size of each arriving module follows a left truncates binomial distribution. The left truncated binomial distribution is capable of representing positive/negative skewed or symmetric distributions for specific values of the parameters. The arrival processes of the queueing is characterized by non-homogeneous compound Poisson binomial process with compound mean arrival rate $\lambda(t) = \lambda+\alpha t$, where $\lambda$ and $\alpha$ are parameters. The explicit expressions for the system characteristics such as average number of customers in the queue, the probability that the queue is empty, the average waiting time of customer of the queue, the throughput of the each service station are derived. This model can be viewed as the generalization of Poisson arrival queueing model, since it includes various types of queueing models for specific values of the parameters. The particular queueing models which can be generated from this model are capable of evaluating various systems. The sensitivity analysis of the model revealed that the bulk size distribution parameters have a significant influence on the performance measures of the system. By regulating the input parameters of the arrival process one can control the congestion in queues and mean delay. It is also observed the load dependent service strategy have a tremendous influence in reducing the burstiness of the buffers. A comparative study of the developed model with model using homogeneous compound Poisson binomial bulk arrivals revealed that the time has significant effect on system performance measures and transient analysis can predict more accurately and realistically with the developed model. This model can also be extended by obtaining the optimal values of the parameters under cost considerations and starting the inferential aspects of the model which will be pursued elsewhere.

# Appendix

**Table – 1, Values of $P_{0..}(t)$, $P_{.0.}(t)$, $P_{..0}(t)$, $P_{000}(t)$, $L_1(t)$, $L_2(t)$, $L_3(t)$, $L(t)$, $U_1(t)$, $U_2(t)$, $U_3(t)$ for different values of parameters**

| t | n | p | $\lambda_1$ | $\lambda_2$ | $\alpha_1$ | $\alpha_2$ | $\mu_1$ | $\mu_2$ | $\mu_2$ | $P_{0..}(t)$ | $P_{.0.}(t)$ | $P_{..0}(t)$ | $P_{000}(t)$ | $L_1(t)$ | $L_2(t)$ | $L_3(t)$ | $L(t)$ | $U_1(t)$ | $U_2(t)$ | $U_3(t)$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.2 | 10 | 0.4 | 0.5 | 1 | 1 | 1.5 | 15 | 21 | 30 | 0.9264 | 0.8930 | 0.7903 | 0.7273 | 0.1641 | 0.2328 | 0.7073 | 1.1042 | 0.0736 | 0.1070 | 0.2097 |
| 0.6 | | | | | | | | | | 0.8728 | 0.8406 | 0.7073 | 0.6092 | 0.2772 | 0.3504 | 1.0949 | 1.7225 | 0.1272 | 0.1594 | 0.2927 |
| 1 | | | | | | | | | | 0.8269 | 0.7933 | 0.6414 | 0.5216 | 0.3845 | 0.4654 | 1.4705 | 2.3204 | 0.1731 | 0.2067 | 0.3586 |
| | 15 | | | | | | | | | 0.9160 | 0.8760 | 0.7540 | 0.7046 | 0.2448 | 0.3472 | 1.0550 | 1.6470 | 0.0840 | 0.1240 | 0.2460 |
| | 25 | | | | | | | | | 0.9045 | 0.8555 | 0.7141 | 0.6810 | 0.4078 | 0.5784 | 1.7575 | 2.7437 | 0.0955 | 0.1445 | 0.2859 |
| | 35 | | | | | | | | | 0.8985 | 0.8431 | 0.6935 | 0.6627 | 0.5710 | 0.8098 | 2.4604 | 3.8412 | 0.1015 | 0.1569 | 0.3065 |
| | | 0.5 | | | | | | | | 0.9203 | 0.8833 | 0.7691 | 0.7138 | 0.2041 | 0.2895 | 0.8796 | 1.3732 | 0.0797 | 0.1167 | 0.2309 |
| | | 0.7 | | | | | | | | 0.9115 | 0.8686 | 0.7386 | 0.6951 | 0.2855 | 0.4049 | 1.2302 | 1.9206 | 0.0885 | 0.1314 | 0.2614 |
| | | 0.9 | | | | | | | | 0.9058 | 0.8583 | 0.7185 | 0.6834 | 0.3671 | 0.5206 | 1.5817 | 2.4694 | 0.0942 | 0.1417 | 0.2815 |
| | | | 1 | | | | | | | 0.8714 | 0.8930 | 0.7560 | 0.6739 | 0.2916 | 0.2328 | 0.8347 | 1.3591 | 0.1286 | 0.1070 | 0.2440 |
| | | | 2 | | | | | | | 0.7711 | 0.8930 | 0.6917 | 0.5786 | 0.5465 | 0.2328 | 1.0897 | 1.8690 | 0.2289 | 0.1070 | 0.3083 |
| | | | 3 | | | | | | | 0.6824 | 0.8930 | 0.6329 | 0.4968 | 0.8015 | 0.2328 | 1.3446 | 2.3789 | 0.3176 | 0.1070 | 0.3671 |
| | | | | 1.5 | | | | | | 0.9264 | 0.8521 | 0.7279 | 0.6582 | 0.1641 | 0.3271 | 0.9275 | 1.4187 | 0.0736 | 0.1479 | 0.2721 |
| | | | | 2.5 | | | | | | 0.9264 | 0.7760 | 0.6176 | 0.5390 | 0.1641 | 0.5159 | 1.3679 | 2.0479 | 0.0736 | 0.2240 | 0.3824 |
| | | | | 3.5 | | | | | | 0.9264 | 0.7066 | 0.5239 | 0.4414 | 0.1641 | 0.7047 | 1.8084 | 2.6772 | 0.0736 | 0.2934 | 0.4761 |
| | | | | | 1.5 | | | | | 0.9193 | 0.8930 | 0.7863 | 0.7209 | 0.1825 | 0.2328 | 0.7256 | 1.1409 | 0.0807 | 0.1070 | 0.2137 |
| | | | | | 2.5 | | | | | 0.9053 | 0.8930 | 0.7784 | 0.7082 | 0.2191 | 0.2328 | 0.7622 | 1.2141 | 0.0947 | 0.1070 | 0.2216 |
| | | | | | 3.5 | | | | | 0.8915 | 0.8930 | 0.7705 | 0.6958 | 0.2558 | 0.2328 | 0.7989 | 1.2875 | 0.1085 | 0.1070 | 0.2295 |
| | | | | | | 2 | | | | 0.9264 | 0.8872 | 0.7860 | 0.7213 | 0.1641 | 0.2474 | 0.7415 | 1.1530 | 0.0736 | 0.1128 | 0.2140 |
| | | | | | | 3 | | | | 0.9264 | 0.8757 | 0.7775 | 0.7095 | 0.1641 | 0.2768 | 0.8099 | 1.2508 | 0.0736 | 0.1243 | 0.2225 |
| | | | | | | 4 | | | | 0.9264 | 0.8643 | 0.7691 | 0.6978 | 0.1641 | 0.3061 | 0.8784 | 1.3486 | 0.0736 | 0.1357 | 0.2309 |
| | | | | | | | 16 | | | 0.9293 | 0.8930 | 0.8015 | 0.7389 | 0.1559 | 0.2328 | 0.7213 | 1.1100 | 0.0707 | 0.1070 | 0.1985 |
| | | | | | | | 18 | | | 0.9348 | 0.8930 | 0.8221 | 0.7597 | 0.1414 | 0.2328 | 0.7552 | 1.1294 | 0.0652 | 0.1070 | 0.1779 |
| | | | | | | | 20 | | | 0.9397 | 0.8930 | 0.8415 | 0.7781 | 0.1291 | 0.2328 | 0.8014 | 1.1633 | 0.0603 | 0.1070 | 0.1585 |
| | | | | | | | | 22 | | 0.9264 | 0.8968 | 0.7786 | 0.7207 | 0.1641 | 0.2232 | 0.7780 | 1.1653 | 0.0736 | 0.1032 | 0.2214 |
| | | | | | | | | 24 | | 0.9264 | 0.9038 | 0.7541 | 0.7056 | 0.1641 | 0.2062 | 0.9890 | 1.3593 | 0.0736 | 0.0962 | 0.2459 |
| | | | | | | | | 26 | | 0.9264 | 0.9101 | 0.7267 | 0.6859 | 0.1641 | 0.1915 | 1.4088 | 1.7644 | 0.0736 | 0.0899 | 0.2733 |
| | | | | | | | | | 31 | 0.9264 | 0.8930 | 0.7967 | 0.7305 | 0.1641 | 0.2328 | 0.6427 | 1.0396 | 0.0736 | 0.1070 | 0.2033 |
| | | | | | | | | | 33 | 0.9264 | 0.8930 | 0.8080 | 0.7361 | 0.1641 | 0.2328 | 0.5441 | 0.9410 | 0.0736 | 0.1070 | 0.1920 |
| | | | | | | | | | 35 | 0.9264 | 0.8930 | 0.8177 | 0.7407 | 0.1641 | 0.2328 | 0.4722 | 0.8691 | 0.0736 | 0.1070 | 0.1823 |

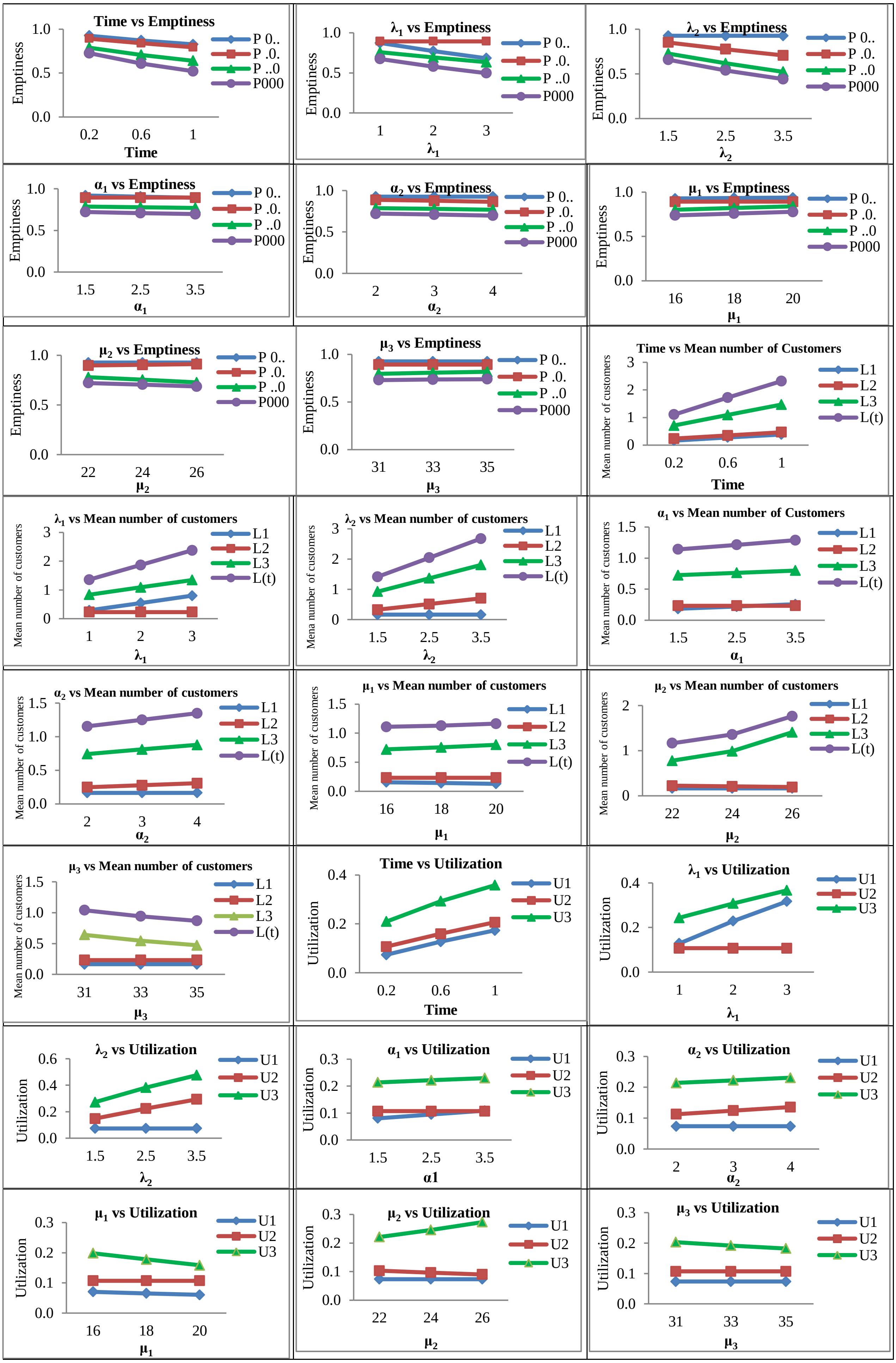
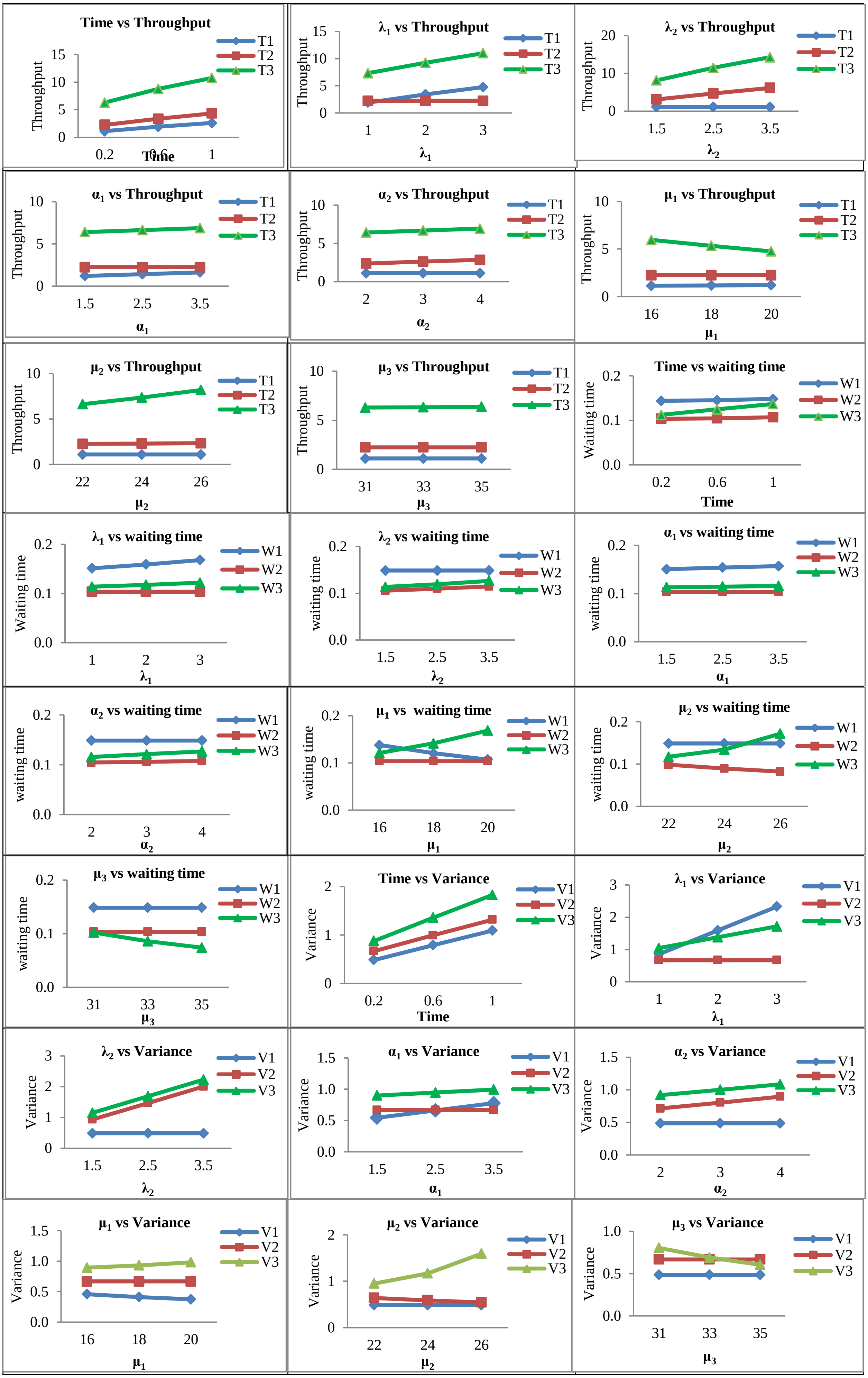



Figure.2 , Relationship between parameters and performance measures

**Table – 2, Values of $Th_1(t)$, $Th_2(t)$, $Th_3(t)$, $W_1(t)$, $W_2(t)$, $W_3(t)$, $V_1(t)$, $V_2(t)$, $V_3(t)$, $CV_1(t)$, $CV_2(t)$, $CV_3(t)$ for different values of parameter**

| t | n | p | $\lambda_1$ | $\lambda_2$ | $\alpha_1$ | $\alpha_2$ | $\mu_1$ | $\mu_2$ | $\mu_3$ | $Th_1(t)$ | $Th_2(t)$ | $Th_3(t)$ | $W_1(t)$ | $W_2(t)$ | $W_3(t)$ | $V_1(t)$ | $V_2(t)$ | $V_3(t)$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.2 | 10 | 0.4 | 0.5 | 1 | 1 | 1.5 | 15 | 21 | 30 | 1.1046 | 2.2473 | 6.2907 | 0.1436 | 0.1035 | 0.1124 | 0.4855 | 0.6688 | 0.8772 |
| 0.6 | | | | | | | | | | 1.9075 | 3.3483 | 8.7824 | 0.1453 | 0.1047 | 0.1247 | 0.7923 | 0.9935 | 1.3575 |
| 1 | | | | | | | | | | 2.5962 | 4.3409 | 10.7580 | 0.1481 | 0.1072 | 0.1367 | 1.0928 | 1.3154 | 1.8250 |
| | 15 | | | | | | | | | 1.2606 | 2.6035 | 7.3809 | 0.1942 | 0.1334 | 0.1429 | 0.9905 | 1.3589 | 1.4494 |
| | 25 | | | | | | | | | 1.4323 | 3.0347 | 8.5780 | 0.2848 | 0.1906 | 0.2049 | 2.5375 | 3.4677 | 2.8839 |
| | 35 | | | | | | | | | 1.5224 | 3.2949 | 9.1942 | 0.3751 | 0.2458 | 0.2676 | 4.7947 | 6.5402 | 4.6945 |
| | | 0.5 | | | | | | | | 1.1960 | 2.4516 | 6.9262 | 0.1707 | 0.1181 | 0.1270 | 0.7037 | 0.9673 | 1.1439 |
| | | 0.7 | | | | | | | | 1.3270 | 2.7584 | 7.8413 | 0.2151 | 0.1468 | 0.1569 | 1.2638 | 1.7322 | 1.7477 |
| | | 0.9 | | | | | | | | 1.4133 | 2.9760 | 8.4438 | 0.2597 | 0.1749 | 0.1873 | 1.9842 | 2.7146 | 2.4371 |
| | | | 1 | | | | | | | 1.9286 | 2.2473 | 7.3208 | 0.1512 | 0.1036 | 0.1140 | 0.8538 | 0.6688 | 1.0444 |
| | | | 2 | | | | | | | 3.4329 | 2.2473 | 9.2488 | 0.1592 | 0.1036 | 0.1178 | 1.5905 | 0.6688 | 1.3787 |
| | | | 3 | | | | | | | 4.7641 | 2.2473 | 11.0129 | 0.1682 | 0.1036 | 0.1221 | 2.3272 | 0.6688 | 1.7130 |
| | | | | 1.5 | | | | | | 1.1046 | 3.1049 | 8.1619 | 0.1486 | 0.1054 | 0.1136 | 0.4855 | 0.9356 | 1.1470 |
| | | | | 2.5 | | | | | | 1.1046 | 4.7043 | 11.4729 | 0.1486 | 0.1097 | 0.1192 | 0.4855 | 1.4692 | 1.6866 |
| | | | | 3.5 | | | | | | 1.1046 | 6.1608 | 14.2819 | 0.1486 | 0.1144 | 0.1266 | 0.4855 | 2.0029 | 2.2261 |
| | | | | | 1.5 | | | | | 1.2109 | 2.2473 | 6.4106 | 0.1507 | 0.1036 | 0.1132 | 0.5441 | 0.6688 | 0.9007 |
| | | | | | 2.5 | | | | | 1.4210 | 2.2473 | 6.6487 | 0.1542 | 0.1036 | 0.1146 | 0.6613 | 0.6688 | 0.9477 |
| | | | | | 3.5 | | | | | 1.6280 | 2.2473 | 6.8843 | 0.1571 | 0.1036 | 0.1160 | 0.7785 | 0.6688 | 0.9947 |
| | | | | | | 2 | | | | 1.1046 | 2.3692 | 6.4195 | 0.1486 | 0.1044 | 0.1155 | 0.4855 | 0.7139 | 0.9184 |
| | | | | | | 3 | | | | 1.1046 | 2.6105 | 6.6750 | 0.1486 | 0.1060 | 0.1213 | 0.4855 | 0.8040 | 1.0008 |
| | | | | | | 4 | | | | 1.1046 | 2.8488 | 6.9278 | 0.1486 | 0.1075 | 0.1268 | 0.4855 | 0.8941 | 1.0832 |
| | | | | | | | 16 | | | 1.1309 | 2.2473 | 5.9560 | 0.1378 | 0.1036 | 0.1211 | 0.4583 | 0.6688 | 0.8938 |
| | | | | | | | 18 | | | 1.1738 | 2.2473 | 5.3356 | 0.1204 | 0.1036 | 0.1415 | 0.4117 | 0.6688 | 0.9324 |
| | | | | | | | 20 | | | 1.2066 | 2.2473 | 4.7563 | 0.1070 | 0.1036 | 0.1685 | 0.3736 | 0.6688 | 0.9828 |
| | | | | | | | | 22 | | 1.1046 | 2.2699 | 6.6414 | 0.1486 | 0.0983 | 0.1171 | 0.4855 | 0.6400 | 0.9516 |
| | | | | | | | | 24 | | 1.1046 | 2.3078 | 7.3766 | 0.1486 | 0.0894 | 0.1341 | 0.4855 | 0.5891 | 1.1693 |
| | | | | | | | | 26 | | 1.1046 | 2.3381 | 8.2005 | 0.1486 | 0.0819 | 0.1718 | 0.4855 | 0.5456 | 1.5954 |
| | | | | | | | | | 31 | 1.1046 | 2.2473 | 6.3020 | 0.1486 | 0.1036 | 0.1020 | 0.4855 | 0.6688 | 0.8042 |
| | | | | | | | | | 33 | 1.1046 | 2.2473 | 6.3368 | 0.1486 | 0.1036 | 0.0859 | 0.4855 | 0.6688 | 0.6904 |
| | | | | | | | | | 35 | 1.1046 | 2.2473 | 6.3813 | 0.1486 | 0.1036 | 0.0740 | 0.4855 | 0.6688 | 0.6054 |



Time vs Throughput
Throughput
Time
T1
T2
T3
λ1 vs Throughput
λ1
λ2 vs Throughput
λ2
α1 vs Throughput
α1
α2 vs Throughput
α2
μ1 vs Throughput
μ1
μ2 vs Throughput
μ2
μ3 vs Throughput
μ3
Time vs waiting time
Waiting time
W1
W2
W3
λ1 vs waiting time
λ2 vs waiting time
waiting time
α1 vs waiting time
α2 vs waiting time
μ1 vs waiting time
μ2 vs waiting time
μ3 vs waiting time
Time vs Variance
Variance
V1
V2
V3
λ1 vs Variance
λ2 vs Variance
α1 vs Variance
α2 vs Variance
μ1 vs Variance
μ2 vs Variance
μ3 vs Variance

Figure. 3, Relationship between parameters and performance measures



**Table 3: Comparative study of models with non- homogeneous and homogeneous Poisson binomial bulk arrivals**

| t | Parameters Measured | $\alpha_1, \alpha_2 = 1$ | $\alpha_1, \alpha_2 = 0$ | Difference | Percentage of variation |
|---|---|---|---|---|---|
| 0.2 | $L_1(t)$ | 0.1641 | 0.1275 | 0.0366 | 28.7059 |
| | $L_2(t)$ | 0.2181 | 0.1888 | 0.0293 | 15.5191 |
| | $L_3(t)$ | 0.6730 | 0.5679 | 0.1051 | 18.5068 |
| | $U_1(t)$ | 0.0736 | 0.0593 | 0.0143 | 24.1147 |
| | $U_2(t)$ | 0.1012 | 0.0894 | 0.0118 | 13.1991 |
| | $U_3(t)$ | 0.2054 | 0.1885 | 0.0169 | 8.9655 |
| | $Thp_1(t)$ | 1.1046 | 0.8895 | 0.2151 | 24.1821 |
| | $Thp_2(t)$ | 2.1246 | 1.8769 | 0.2477 | 13.1973 |
| | $Thp_3(t)$ | 6.1612 | 5.6544 | 0.5068 | 8.9629 |
| | $W_1(t)$ | 0.1486 | 0.1433 | 0.0053 | 3.6985 |
| | $W_2(t)$ | 0.1027 | 0.1006 | 0.0021 | 2.0875 |
| | $W_3(t)$ | 0.1092 | 0.1004 | 0.0088 | 8.7649 |
| 0.6 | $L_1(t)$ | 0.2772 | 0.1341 | 0.1431 | 106.7114 |
| | $L_2(t)$ | 0.2975 | 0.1916 | 0.1059 | 55.2714 |
| | $L_3(t)$ | 0.9714 | 0.5813 | 0.3901 | 67.1082 |
| | $U_1(t)$ | 0.1272 | 0.0653 | 0.0619 | 94.7933 |
| | $U_2(t)$ | 0.1375 | 0.0920 | 0.0455 | 49.4565 |
| | $U_3(t)$ | 0.2753 | 0.2002 | 0.0751 | 37.5125 |
| | $Thp_1(t)$ | 1.9075 | 0.9792 | 0.9283 | 94.8019 |
| | $Thp_2(t)$ | 2.8880 | 1.9310 | 0.9570 | 49.5598 |
| | $Thp_3(t)$ | 8.2590 | 6.0073 | 2.2517 | 37.4827 |
| | $W_1(t)$ | 0.1453 | 0.1370 | 0.0083 | 6.0584 |
| | $W_2(t)$ | 0.1030 | 0.0992 | 0.0038 | 3.8306 |
| | $W_3(t)$ | 0.1176 | 0.0968 | 0.0208 | 21.4876 |
| 1 | $L_1(t)$ | 0.3845 | 0.1341 | 0.2504 | 186.7263 |
| | $L_2(t)$ | 0.3741 | 0.1916 | 0.1825 | 95.2505 |
| | $L_3(t)$ | 1.2576 | 0.5813 | 0.6763 | 116.3427 |
| | $U_1(t)$ | 0.1731 | 0.0653 | 0.1078 | 165.0842 |
| | $U_2(t)$ | 0.1702 | 0.0920 | 0.0782 | 85.0000 |
| | $U_3(t)$ | 0.3301 | 0.2003 | 0.1298 | 64.8028 |
| | $Thp_1(t)$ | 2.5962 | 0.9794 | 1.6168 | 165.0807 |
| | $Thp_2(t)$ | 3.5735 | 1.9310 | 1.6425 | 85.0596 |
| | $Thp_3(t)$ | 9.9039 | 6.0078 | 3.8961 | 64.8507 |
| | $W_1(t)$ | 0.1481 | 0.1370 | 0.0111 | 8.1022 |
| | $W_2(t)$ | 0.1047 | 0.0992 | 0.0055 | 5.5444 |
| | $W_3(t)$ | 0.1270 | 0.0968 | 0.0302 | 31.1983 |

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