Neural, Parallel, and Scientific Computations 21 (2013) 293-304

A NUMERICAL STUDY OF EUROPEAN OPTIONS UNDER MERTON'S JUMP-DIFFUSION MODEL WITH RADIAL BASIS FUNCTION BASED FINITE DIFFERENCES METHOD

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ABSTRACT. The purpose of this paper is to design and describe the valuation of European options under Merton's jump-diffusion model by radial basis function approximation. The governing equation is discretized in space by radial basis function based finite difference method and in time by backward difference formula of order two. Numerical experiment for European call option is carried out to demonstrate the accuracy of the method. It is shown that method is second order accurate.

AMS (MOS) Subject Classification. 65L10

1. INTRODUCTION

There is evidence to suggest that the Black Scholes model for stock price behavior does not always model real stock behavior. Jump can appear at a random time and these jumps can not be captured by the log normal distributation characteristic of the stock price in the Black Scholes model. To overcome the above shortcoming, several models have been proposed in the literature. Among these, the jump diffusion model introduced by Merton[1] is one of the most used model.

The valuation of option under jump diffusion process requires to solve a partial integro differential equation. There are several numerical methods available in the literature to approximate the above equation. In [2] second order accurate time discretization is presented. An approach based on implicit-explicit Runge Kutta schemes in which integral term is treated explicitly has been proposed by Cont et al. [3] and Briani et al. [4]. Andersen et al. [5] proposed an unconditionally stable alternate direct implicit method for its solution. Song Wang et al. [6] developed a fitted finite volume method for jump diffusion process. Their method is based on fitted finite volume method spatial discretization and Crank Nicolson scheme for temporal discretization. More recently, Patidar et al. [7] developed an efficient method for pricing merton jump diffusion option, combining the spectral domain decomposition method and the Laplace transform method.

Received July 15, 2013

Lately a new method based on radial basis function (RBF) for approximation of spatial derivative in option pricing equation is under going active research. Application of RBF in one dimension European and American options is given by Hon et al. [8, 9]. Fasshauer et al.[10] solved multi-asset American option pricing model using penalty method. Pettersson [12] proposed a method for multidimensional optional pricing and Larsson et al. [11] used generalized fourier transform to reduced memory requirement and computation cost of RBF methods. In a recent work of Golbabai et al. [13], an algorithm based on global collocation for jump diffusion process has been proposed. Bhuruth et al. [16] proposed a radial basis function based differential quadrature rule for spatial descretization with exponential time integration.

To resolve issues related to stability and condition number of collocation matrix, many strategies have been developed in the literature, such as local RBF approach by Lee et al. [17], radial point interpolation method proposed by Liu et al. [18] and RBF based differential quadrature method proposed by Shu et al. [19]. Wright et al. [21, 20, 22] proposed radial basis function finite difference method; the idea is to use radial basis functions with a local collocation as in finite difference mode thereby reducing number of nodes and hence producing a sparse matrix. In this strategy, it is expected that the choice of the shape parameter will not be a critical issue, as in the case of global collocation method. The approach developed by Wright et al. [21] has been successfully extended by Kadalbajoo et al. [14] to solve multi asset exotic option and it was shown that radial basis function finite difference method are more accurate than classical finite difference method.

In the present work, we propose the radial basis function based finite difference method for spatial descretization with backward difference method for temporal descretization to solve partial integro differential equation governing jump diffusion model.

The rest of the paper is structured as follows, in section 2 we give description of the jump diffusion model in term of partial integro differential equation. The development of the scheme to solve the resulting equation is given in sections 3. Numerical results are presented in section 4. Finally, conclusions are given in section 5.

2. THE JUMP-DIFFUSION MODEL FORMULATION

For jump diffusion model, the movement of stock price is modeled by the following stochastic differential equation

$$\frac{dS}{S} = \nu d\tau + \sigma dz + (\eta - 1)dq \tag{2.1}$$

where S denote underlying asset price, ν is drift rate, σ is the volatilities, dz is an increment of standard Gauss-Wiener process, dq is a Poisson process with arrival rate (intensity) λ and $(\eta - 1)$ is impulse producing jump from S to $S\eta$. The average relative jump size $\mathbb{E}(\eta - 1)$ is denoted by κ. Here $\mathbb{E}$ is the expectation operator.

Let $V(S,\tau)$ represent the value of European contingent claim with strike price K on the underlying asset price S with current time τ. Merton [1] show that $V(S,\tau)$ satisfy following backward partial intgro differential equation

$$\frac{\partial V}{\partial \tau} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + (r - \lambda\kappa) S \frac{\partial V}{\partial S} - (r + \lambda)V + \lambda \int_0^\infty V(S\eta) g(\eta) d\eta = 0 \tag{2.2}$$

where σ is volatility, r is risk free interest rate and $g(\eta)$ is probability density function of the jump with amplitude η, where $g(\eta) \geq 0$ and is defined by

$$g(\eta) = \frac{1}{\sqrt{2\pi}\sigma_J \eta} \exp\left(\frac{-(\ln \eta - \mu_J)}{2\sigma_J^2}\right). \tag{2.3}$$

Hence, $\kappa = \mathbb{E}(\eta - 1) = \exp(\mu_J + \sigma_J^2/2) - 1$, where μ_J and σ_J are the mean and the variance of jump in return.

There are various types of asymptotic boundary conditions and initial condition depending on the types of the contingent contracts. In the case of call option, it is given by

$$\begin{aligned} V(0,\tau) &= 0, \\ V(S,\tau) &\to S - Ke^{-r(T-\tau)}, \ S \to \infty \\ V(S,T) &= \max(S - K, 0). \end{aligned} \tag{2.4}$$

For a put option, we have

$$\begin{aligned} V(0,\tau) &= Ke^{-r(T-\tau)}, \\ V(S,\tau) &\to 0, \ S \to \infty \\ V(S,T) &= \max(K - S, 0). \end{aligned} \tag{2.5}$$

Let us consider equation (2.2) and apply the change of variable $x = \ln(S/K)$ and $y = \ln(\eta)$, the time variable is transformed to $t = T - \tau$ to obtained problem forward in time. Under these transformation equation (2.2) can be written as

$$\frac{\partial u}{\partial t} - \frac{1}{2}\sigma^2 \frac{\partial^2 u}{\partial x^2} - (r - \frac{\sigma^2}{2} - \lambda\kappa)\frac{\partial u}{\partial x} + (r + \lambda)u - \lambda \int_{-\infty}^\infty u(x + y, t) f(y) dy = 0, \tag{2.6}$$

where $u(x,t) := V(Ke^x, T - t)$ and $f(y) = g(e^y)e^y$.

The boundary condition and initial condition under this transformation for European call option become

$$\begin{aligned} u(x,t) &\to 0, \quad x \to -\infty \\ u(x,t) &\to Ke^x - Ke^{-rt}, \quad x \to \infty \\ u(x,0) &= \max(Ke^x - K, 0). \end{aligned} \tag{2.7}$$

For a put option, we have

$$
\begin{aligned}
u(x,t) &\rightarrow Ke^{-rt}, \quad x \rightarrow -\infty \\
u(x,t) &\rightarrow 0, \quad x \rightarrow \infty \\
u(x,0) &= \max(K - Ke^{x}, 0).
\end{aligned} \tag{2.8}
$$

Other types of boundary conditions and payoff function can also be imposed.

Let us consider the truncated interval $\Omega = [x_{\min}, x_{\max}]$ and $\Omega^c = \mathbb{R} \setminus [x_{\min}, x_{\max}]$ is its complement in $\mathbb{R}$. To descretize the integral term, let us change the variables

$$
\begin{aligned}
\int_{-\infty}^{\infty} u(x+y,t)f(y)dy &= \int_{-\infty}^{\infty} u(z,t)f(z-x)dz \\
&= \int_{\Omega} u(z,t)f(z-x)dz + \int_{\Omega^c} u(z,t)f(z-x)dz.
\end{aligned} \tag{2.9}
$$

In the case of European style call option under given boundary condition we have[7]

$$
\begin{aligned}
R(t,x,x_{\max}) &:= \int_{\Omega^c} u(z,t)f(z-x)dz \\
&= Ke^{x+\mu_J+\frac{\sigma_J^2}{2}}\mathcal{N}\left(\frac{x - x_{\max} + \mu_J + \sigma_J^2}{\sigma_J}\right) - Ke^{-rt}\mathcal{N}\left(\frac{x - x_{\max} + \mu_J}{\sigma_J}\right),
\end{aligned}
$$

for European style put option, we have

$$
\begin{aligned}
R(t,x,x_{\min}) &:= \int_{\Omega^c} u(z,t)f(z-x)dz \\
&= Ke^{-rt}\mathcal{N}\left(\frac{x_{\min} - x - \mu_J}{\sigma_J}\right),
\end{aligned}
$$

where $\mathcal{N}(\cdot)$ is the cumulative normal distribution.

Let us consider M equispaced nodes $x_i = x_{\min} + (i-1)h$ with $h = (x_{\max} - x_{\min})/(M-1)$ and denote $u_i(t) \approx u(x_i,t)$, $f_{i,j} := f(x_j - x_i)$. By the application of trapezoidal rule on $[x_{\min}, x_{\max}]$, we have the following approximation for the integral term (2.9)

$$
\int_{\mathbb{R}} u(z,t)f(z-x_i)dz \approx \frac{h}{2}\left[u_1(t)f_{i,1} + u_M(t)f_{i,M} + 2\sum_{j=2}^{M-1} u_j(t)f_{i,j}\right] + R(t,x_i,x^*) \tag{2.10}
$$

for $i = 2, 3, \ldots, M-1$.

3. RBF APPROXIMATION AND TIME STEPPING

3.1. RBF-FD approximation of space operator. A function $\Phi : \mathbb{R}^d \rightarrow \mathbb{R}$ is called radial provided there exists a univariate function $\phi : [0,\infty) \rightarrow \mathbb{R}$ such that $\Phi(x) = \phi(r)$, where $r = \|x\|$ and $\|\cdot\|$ is some norm on $\mathbb{R}^d$. These functions can be broadly classified into two classes, infinitely smooth and piecewise smooth radial basis function. The former include a shape parameter ϵ, and upon varying this parameter the radial function can vary sharp peak to very flat one. Classical choices of RBF are given in Table-1 with their order, where for any $x \in \mathbb{R}$, the symbol $\lceil x \rceil$ denotes as usual the smallest integer greater than or equal to x. The Gaussian and inverse multiquadric are positive definite function where as thin plate spline and multiquadric are conditionally positive definite function of order $m > 0$.

TABLE 1. Examples of radial basis function and their order

R.B.F.	$\phi(r), r > 0$	Order
Multiquadric (MQ)	$(1+(\epsilon r)^2)^\upsilon, \upsilon > 0, \upsilon \notin \mathbb{N}$	$m = \lceil \upsilon \rceil$
Inverse multiquadric (IMQ)	$(1+(\epsilon r)^2)^\upsilon, \upsilon < 0, \upsilon \notin \mathbb{N}$	m=0
Gaussian (GA)	$e^{-(\epsilon r)^2}$	m=0
Polyharmonic spline	$\begin{cases} r^\upsilon, \upsilon > 0 \ if \ \upsilon \in 2\mathbb{N}-1 \\ r^\upsilon \log(r), \ if \ \upsilon \in 2\mathbb{N} \end{cases}$	$m = \begin{cases} \lceil \frac{\upsilon}{2} \rceil \\ \frac{\upsilon}{2}+1 \end{cases}$

For completeness, a brief discussion of RBF based finite difference method is presented. To derive local RBF-FD approximation of any linear differential operator $\mathscr{L} := \frac{d^k}{dx^k}$ of order k at a specific node point x_i, in the discretized domain $\Omega := \{x_1, x_2, \ldots, x_n\}$ containing n number of nodes, consider any subset Ω_i containing $n_i(<< n)$ nodes in the neighborhood of x_i. In RBF-FD approach we are required to compute weights w_j such that;

$$\mathscr{L}u(x_i) = \sum_{j=1}^{n_i} w_j u(x_j) \tag{3.1}$$

For each node $x_i \in \Omega$ we compute the weights w_j on each local support Ω_i. In traditional method, generally these nodes are equidistant and the weights are computed using classical polynomial interpolation. At the same time in radial basis function interpolation, a randomly distributed nodes are used.

Let $s(x)$ be radial basis function interpolant that interpolate function $u(x)$ at the interpolation points contained in Ω_i. Then $s(x)$ can be represented by

$$s(x) = \sum_{j=1}^{n_i} \lambda_j \phi(\|x - x_j\|) + \sum_{j=1}^{l} \gamma_j p_j(x) \tag{3.2}$$

where $\|\cdot\|$ is the Euclidian norm and $\{p_j(x)\}_{j=1}^{l}$ denote basis of $\prod_{m-1}^{d}$, which is space of d-variate polynomials of total degree $\leq m-1$, where m is order of ϕ. The coefficients λ_j and γ_j are evaluated by imposing the following conditions

$$s(x_i) = u(x_i), \quad 1 \leq i \leq n_i \tag{3.3}$$

$$\sum_{j=1}^{n_i} \lambda_j p_k(x_j) = 0, \quad 1 \leq k \leq l \tag{3.4}$$

Imposing condition (3.3–3.4) on $s(x)$ gives a linear system

$$\begin{pmatrix} \Phi & P \\ P^t & O \end{pmatrix} \begin{pmatrix} \lambda \\ \gamma \end{pmatrix} = \begin{pmatrix} u\,|_{\Omega_\mathrm{i}} \\ O \end{pmatrix} \tag{3.5}$$

where $\Phi := (\phi\|x_i - x_j\|)_{1\le i,j\le n_i} \in \mathbb{R}^{n_i\times n_i}$, $P := (p_j(x_i))_{1\le i\le n_i, 1\le j\le l} \in \mathbb{R}^{n_i\times l}$.

Suppose ϕ is conditionally positive definite function of order m on $\mathbb{R}^d$ and the points $\Omega_\mathrm{i} := \{x_i \in \mathbb{R}^d; i = 1, 2, \ldots, n_i\}$ form $(m-1)$ unisolvent set of centers. Then the system (3.5) is uniquely solvable. We will refer coefficient matrix in (3.5) by 'A' for future reference.

To derive RBF-FD approximation the interpolant is represented in Lagrangean form as

$$s(x) = \sum_{j=1}^{n_i} \psi_j(x) u(x_j) \tag{3.6}$$

where $\psi_j(x)$ are Lagrange functions that satisfy the cardinal conditions,

$$\psi_j(x_k) = \delta_{jk}, \quad j, k = 1, 2, \ldots, n_i \tag{3.7}$$

A closed form expression for each $\psi_j(x)$ can be obtained in term of corresponding radial basis functions by modeling another set of RBF interpolation problem and is given as [20];

$$\psi_j(x) = \frac{\det(A_j(x))}{\det(A)}, \quad j = 1, 2, \ldots, n_i \tag{3.8}$$

where matrix '$A_j(x)$' can be obtained from matrix 'A', by replacing j^{th} row vector by

$$B(x) = [\phi(\|x - x_1\|)\ \phi(\|x - x_2\|) \cdots \phi(\|x - x_{n_i}\|) \mid p_1(x)\ p_2(x) \cdots p_l(x)]. \tag{3.9}$$

Now application of operator $\mathscr{L}$ on the interpolant in (3.6) gives

$$\mathscr{L}u(x_i) \approx \mathscr{L}s(x_i) = \sum_{j=1}^{n_i} \mathscr{L}\psi_j(x_i) u(x_j) \tag{3.10}$$

From equation (3.1) and (3.10), the weights w_j can be written as,

$$w_j = \mathscr{L}\psi_j(x_i) = \frac{\mathscr{L}(\mid A_j(x) \mid)}{\mid A \mid}\Big|_{x=x_i}, \quad j = 1, 2, \ldots, n_i. \tag{3.11}$$

After some application of Cramer's rule to (3.10), and taking advantage of the nature of interpolation matrix, the weights can be given as;

$$\begin{pmatrix} \Phi & P \\ P^t & O \end{pmatrix} \begin{pmatrix} w \\ \xi \end{pmatrix} = (\mathscr{L}B(x))^t\Big|_{x=x_i} \tag{3.12}$$

where $B(x)$ is the vector defined by (3.9) ξ is a dummy vector corresponding to the vector γ in (3.2). It was shown by Wright et al. [21] that in the case of uniform points distribution, weights of RBF-FD formula converge to the weights of corresponding classical finite difference formula.

It is obvious from the linear system (3.12) that its size is only n_i+l, which is much smaller than the size $n+l$ of global RBF collocation. Thus the proposed method provides more stable system for a wide value of shape parameter ϵ.

3.2. **Temporal approximation.** The numerical solution of Merton model, using any implicit technique, requires the generation of a modified PDE operator through a finite difference approximation of time derivative, we will do this using second order backward difference formula which is L-stable method having a smoothing effect for the error. Let $\{0 = t_0 < t_1 < \cdots < t_N = T;\, t_n - t_{n-1} = \delta t,\, 1 \leq n \leq N\}$ be a partition of the interval $[0, T]$.

$$u_t(x_i, t_n) = \begin{cases} (\frac{3}{2}u_i^{n+1} - 2u_i^n + \frac{1}{2}u_i^{n-1})/\delta t & \text{for } n \geq 1, \\ (u_i^{n+1} - u_i^n)/\delta t & \text{for } n = 0, \end{cases} \tag{3.13}$$

where u_i^n approximate the exact solution $u(x,t)$ at (x_i, t_n).

Under this notation equation (2.4) can be written in matrix form as

$$(\omega_0 I + C + D)U^{n+1} = b^n + g, \tag{3.14}$$

where

$$\omega_0 = \begin{cases} 1 & \text{for } n = 0, \\ 3/2 & \text{for } n \geq 1, \end{cases} \tag{3.15}$$

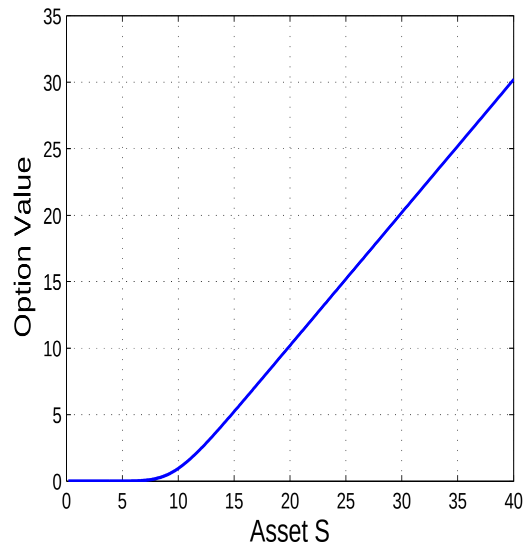
I is the identity matrix, C is the matrix of weights corresponding to differential operator obtained using radial basis function based finite difference method discussed through (3.1) to (3.12), matrix D is corresponding to integral operator given as $d_{i,j} = -\delta t h \lambda f(i,j)$ and g is the vector corresponding to boundary conditions. Finally the vector b^n is given by $b_i^n = \delta t \lambda R(t_n, x_i, x^*) + \omega_1 U_i^n + \omega_2 U_i^{n-1}$, where

$$\omega_1 = \begin{cases} 1 & \text{for } n = 0, \\ 2 & \text{for } n \geq 1, \end{cases} \tag{3.16}$$

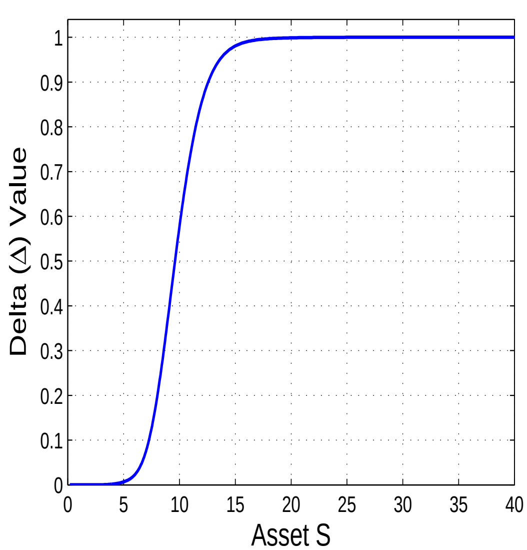
$$\omega_2 = \begin{cases} 0 & \text{for } n = 0, \\ -1/2 & \text{for } n \geq 1. \end{cases} \tag{3.17}$$

4. NUMERICAL EXPERIMENTS

In this section we are going to present some numerical result to illustrate the performance of proposed method. Although the scheme works for all radial basis function but we will use multi-quadric radial basis function on different experimental setup. By keeping the shape parameter ϵ fixed, the computational error produced by the numerical schemes was measured against the value of analytical solution or reference solution at specific asset price. The convergent rates are computed using standard technique. All numerical simulations are done in computational domain $[-4, 4]$.



(A)



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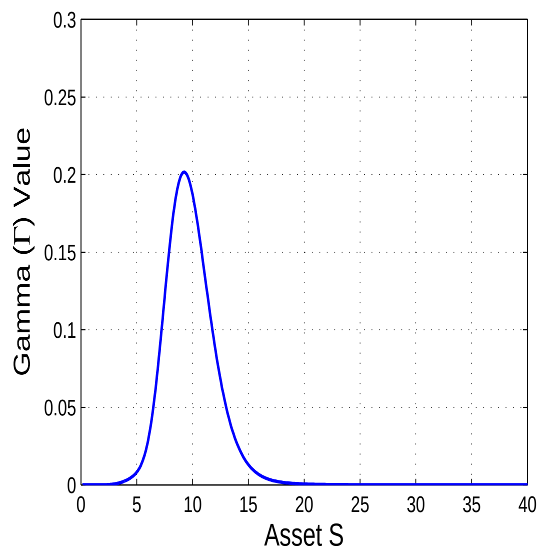

(C)

FIGURE 1. European call option value, Delta and Gamma at the last time step for example 4.1.

Example 4.1. In our first example, we perform a numerical test for European call option with parameters given by $\sigma = 0.2$, $\sigma_J = 0.3$, $\lambda = 0.1$, $r = 0.02$, $K = 10$ and maturity time $T = 1$.

The value of reference solution at strike price is 0.954987522325962 calculated using Merton's closed form solution. The numerical results obtained by the proposed

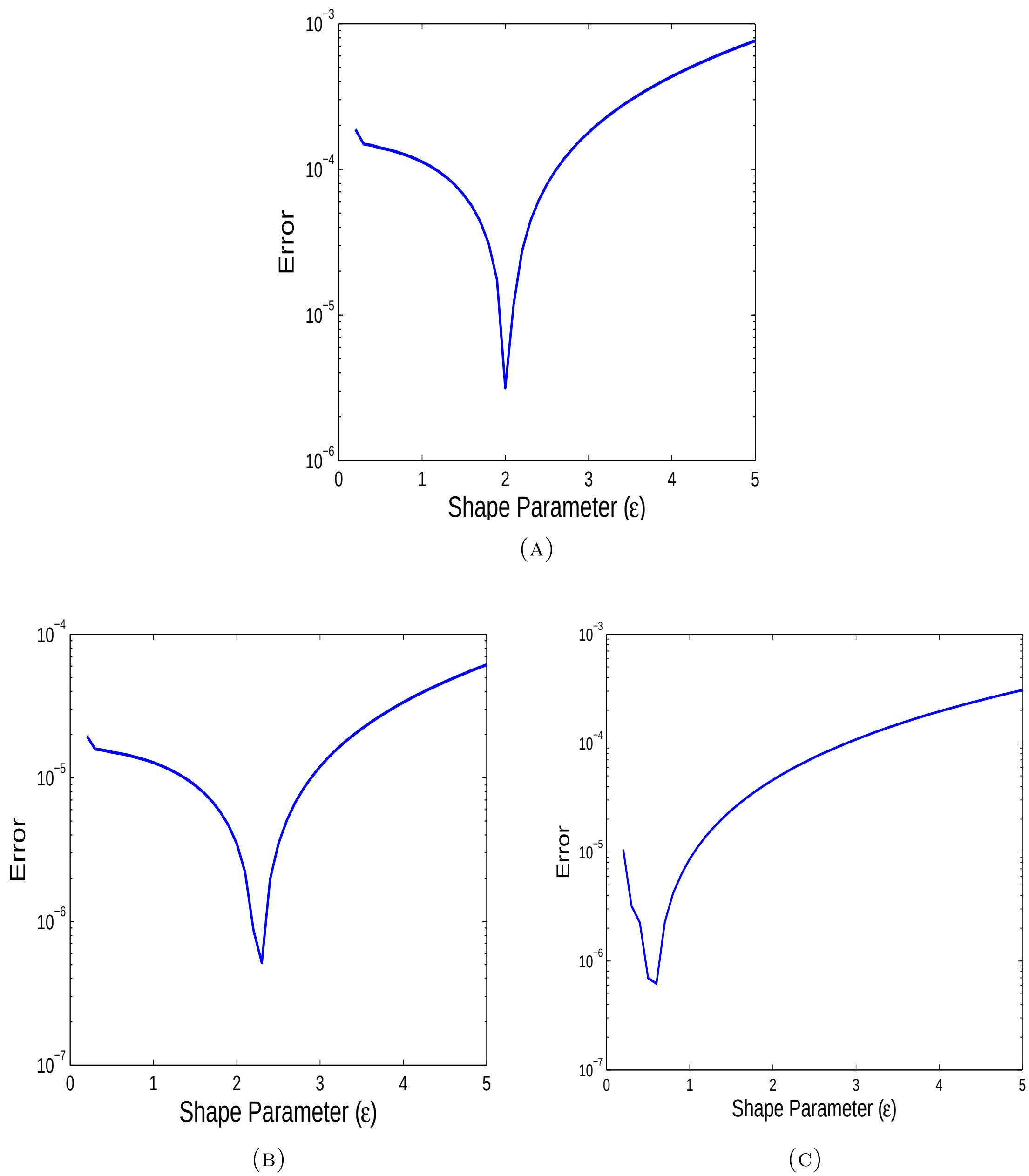

FIGURE 2. Plot of absolute error at strike price for various value of shape parameter ϵ; (a) example 4.1, (b) example 4.2 and (c) example 4.3.

method are reported in table 2. From the given table, we observe that the proposed method has second order convergence.

In figure 1 we plot the option values and its first and second derivatives, Delta and Gamma, at the last time step. From the figure we observe that the option values and Greeks are stable and no spurious oscillation occur.

TABLE 2. Absolute error and rate of convergence for European call option at asset price $S = K$.

M	N	Price	Error	rate
65	5	0.974445	1.9458e-02	-
129	10	0.962742	7.7549e-03	1.32
257	20	0.957112	2.1253e-03	1.87
513	40	0.955531	5.4431e-04	1.97

Example 4.2. In our next example we consider European call option with parameter $\sigma = 0.2$, $\sigma_J = 0.5$, $\lambda = 0.1$, $r = 0$, $K = 1.0$ and maturity time $T = 1$.

Using Merton's closed form solution, the reference value 0.094135525 at strike price is calculated and option value and computation error at strike price is calculated. We compare our results for different values of spatial node and temporal node with methods of Carry and Mayo [15] and Bhuruth et al. [16]. and the same is presented in table 3. We observe that the results obtained with the present method have nice agreement with other one.

TABLE 3. Comparison of European call option price and absolute error at asset price $S = K$.

		Carry et al. [15]		Bhuruth et al. [16]		Present Method	
M	N	Price	Error	Price	Error	Price	Error
65	5	0.09102	4.01e-03	0.09182	2.32e-03	0.093353	7.8162e-04
129	10	0.09320	9.32e-04	0.09356	5.70e-04	0.094117	1.7837e-05
257	20	0.09413	2.72e-04	0.09399	1.38e-04	0.094140	4.7182e-06
513	40	0.09408	5.39e-05	0.09410	3.02e-05	0.094137	1.8384e-06

Example 4.3. In our final example we consider European call option with parameter $\sigma = 0.2$, $\sigma_J = 0.35$, $\lambda = 0.1$, $r = 0.1$, $K = 1.0$ and maturity time $T = 3$.

The numerical computation are done with 513 spatial points and 121 temporal points and results obtained at different value of asset price are reported in table 4. The value of option price and respective error are compared with classical finite difference scheme and scheme of Bhuruth et al. [16]. From the table we observe that the present method is more accurate than the methods given in [16].

Finally, in figure 2 we plot the absolute error at strike price for different value of shape parameter.

TABLE 4. Comparison of European call option price and absolute error at different asset price.

S	Reference Solution	Computed Solution	Absolute Error: Present Method	Absolute Error: Classical Finite Difference	Absolute Error: Bhuruth et al. [16]
0.25	0.000553	0.000554	1.5985e-06	1.14e-04	1.39e-04
0.50	0.021135	0.021148	1.3325e-05	1.36e-06	1.72e-06
0.75	0.120108	0.120100	7.0734e-06	3.69e-05	4.08e-06
1.00	0.301392	0.301378	1.3654e-05	2.06e-04	2.67e-05
1.25	0.525354	0.525350	3.7211e-06	7.15e-04	9.82e-05
1.50	0.765832	0.765838	6.8525e-06	1.93e-03	2.73e-04
1.75	1.012184	1.012198	1.3945e-05	4.40e-03	6.40e-04

5. CONCLUSION

In this paper, we describes the valuation of European option under Merton's jump diffusion model using radial basis function based finite difference technique. The numerical scheme used the backward difference method for semi discrete system obtained after radial basis function based spatial discretization. Comparisons of solutions against existing scheme available in literature is carried out and it is found that the proposed method is more efficient and accurate.

ACKNOWLEDGMENTS

The work of authors [Alpesh Kumar, Lok Pati Tripathi] is supported by Council of Scientific and Industrial Research, India.

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