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RATIONAL ARNOLDI & ADAPTIVE ORDER RATIONAL ARNOLDI FOR SWITCHED LINEAR SYSTEMS

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ABSTRACT. This paper is concerned with the problem of model reduction for the large-scale switched linear system, which is characterized by high order mathematical models. Two model reduction algorithms are present. We present at first the rational Arnodli for Switched linear systems (RASLS) method. It is based on generation of orthonormal basis by the use of the Krylov subspace technique for each sub-system. In the second part we present the adaptive order rational arnoldi for switched linear systems (AORASLS). Is Also an improvement of the RASLS method, but it is based on automatic choice of matching moment. A simulation example is considered in order to take a performance study of the proposed approaches.

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1. Introduction

Control systems are now increasingly complex. Including many corresponding application has called embedded systems, with applications in several fields aeronautics, the automobile industry, metallurgical industry, and the management of energy. These types of structure can be assembled and modeled by the generic category of dynamic hybrid systems. these systems have an interaction between dynamic processes in continuous or discrete time and logical process. A dynamic switched system is an association of a finite set of differential sub-systems or differences and a switching law that says each time the active system. Also the modeling is an important tool in the description and the characterization of Large-scale switched systems. However, these high order models are difficult to manipulate, the resolution of such models is indeed very demanding in computational resources, storage space, and mainely in CPU time, especially when applying a control strategy which become very difficult to determine. Model order reduction presents a good solution in the analysis and simulation of large switched system, the reduced model capture the main advantages of the original complex one. Hence, the use of model order reduction techniques generate reduced order systems that capture the essential dynamic behavior of the

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original systems (stability, passivity, balanced) and increase the performance during the numerical simulation procedure. In the last decade, several approaches exist in the literature are studied the problem of linear and linear switched switched systems(Antoulas et al. in Approximation of large-scale Dynamical systems, Huijun et al. in the Model simplification for switched hybrid systems, Domgmei et al. in the LMI approach to H_2 Reduction Model of Switched Systems, Nima et al. in A Simultaneous Balanced Truncation Approach to Model Reduction of Switched Linear Systems, Klaus and al. in the Guard-based Model Order Reduction for Switched Linear Systems). However, these methods have remained very limited on systems of medium order, for this reason, we present in this paper the rational Arnoldi approach and the adaptive order rational Arnoldi approach applicable in the linear switched linear systems. these new approaches presents better results than other approach, which guaranteeing stability and minimizing the error between the original system and reduced one. The model reduction problem we are interested in can be stated as follows.

Given a linear Switched dynamical system in state space form [1, 2, 3]:

$$\Sigma_q = \begin{cases} E_q x(t+1) = A_q x(t) + B_q u(t) \\ y(t) = C_q x(t) + D_q u(t) \end{cases} \tag{1}$$

In which E_q and $A_q \in \mathbb{R}^{n\times n}$, $B_q \in \mathbb{R}^{n\times p}$, $C_q \in \mathbb{R}^{p\times n}$, $D_q \in \mathbb{R}^{p\times p}$, $u(t) \in \mathbb{R}^{n\times p}$, $y(t) \in \mathbb{R}^{p\times n}$ and $q(t)$ is a function piecewise constant over time called a switching signal, for simplicity we write q. we assume that evaluate to $q(t)$ is unknown, but its instantaneous value is available in real time.

The transfer function of the original system for each sub-system is given by [3, 4]:

$$f_q(s) = C_q(sE_q - A_q)^{-1}B_q + D_q \tag{2}$$

The problem consist in approximating the matrices of each sub-system of order $k \ll n$ In which $\hat{E}_q$ and $\hat{A}_q \in \mathbb{R}^{k\times k}$, $\hat{B}_q \in \mathbb{R}^{k\times p}$, $\hat{C}_q \in \mathbb{R}^{p\times k}$, $\hat{D}_q \in \mathbb{R}^{k\times k}$ and $\hat{y}(t) \in \mathbb{R}^{p\times k}$. Then, the state space of the reduced sub-system is as follow [5, 6, 7]:

$$\hat{\Sigma}_q = \begin{cases} \hat{E}_q \hat{x}(t+1) = \hat{A}_q \hat{x}(t) + \hat{B}_q u(t) \\ \hat{y}(t) = \hat{C}_q \hat{x}(t) + \hat{D}_q u(t) \end{cases} \tag{3}$$

The transfer function of the reduced system for each sub-system is given by:

$$\hat{f}_q(s) = \hat{C}_q(s\hat{E}_q - \hat{A}_q)^{-1}\hat{B}_q + \hat{D}_q \tag{4}$$

This paper is organized as follows: in section 2, the Krylov subspace for linear switched system are given. Section 3, the Rational Arnoldi method for switched linear system, will be presented with application on the numerical example (composed of two sub-system). In section 4, we present the Adaptive Rational Arnodli method for switched linear systems and we give a numerical example(composed of two sub-system) to evaluate this approach. The last section is dedicated to conclude this paper.

2. **Krylov Subspace for Switched Linear System**

Given a linear switched system in the states space form (1), [3, 4, 8]. Applying the Laplace transform of equation (1), we obtain the transfer functions under the form (2) of state variables and those of the outputs are $X_q(s) = (sE_q - A_q)^{-1}B_q$ and $Y(s) = C_qX(s) + D_qu(s)$, respectively.

Let us define two matrices $\psi_{i_q} = -(s_{i_q}E_q - A_q)^{-1}E_q$ and $\xi_{i_q} = (s_{i_q}E_q - A_q)^{-1}B_q$, where $(s_{i_q}E_q - A_q)$ is assumed to be nonsingular and an expansion frequency $s_{i_q} \in S_q$, for $i_q = 1, 2, \ldots, \hat{i}_q$. Applying the Taylor expansion of $X_q(s)$ at expansion points s_{i_q}, we obtain [4, 9, 8]:

$$X_q(s) = \sum_{j=0}^{\infty} X_q^{j_q}(s_{i_q})(s - s_{i_q})^i \tag{5}$$

where $X_q^{j_q}(s_{i_q}) = \psi_{i\,q}^{j}\xi_{i_q}$ and $Y_q^{j_q}(s_{i_q}) = C_qX_q^{j_q}(s_{i_q})$ are the jth order system moment and the jth order output moment at s_{i_q}, respectively, for $j_q = 0, 1, \hat{j}_q$. We use the modified Gram-Schmidt orthogonalization technique to generate the Krylov subspace [4, 9, 10, 11, 12, 13] $K_{r_q}(\psi_q, \xi_q) = \{\xi_{i_q}, \psi_{i_q}\xi_{i_q}, \ldots, \psi_{i_q}^{j-1}\xi_{i_q}\}$. Let $V_{r_q} \in K_{r_q}$ be the orthonormal basis.

3. **Rational Arnoldi for Linear Switched Systems**

In this section, we present a Rational Arnoldi model order reduction for Switched Linear System. This method is a generalization of the shifted-and-inverted Arnoldi method for each sub-systems. This method is based on the determination of the frequency range and the number of matched moments at each expansion point for each sub-system. Let $S_q = \{s_{1_q}, s_{2_q}, \ldots, s_{\hat{i}_q}\}$ represent the range of expansion frequencies and fixed the number of matched moments at each corresponding frequency, then $J_q = \{\hat{j}_{1_q}, \hat{j}_{1_q}, \ldots, \hat{j}_{\hat{i}_q}\}$. During the a iteration process of the Rational Arnoldi Algorithm for switched linear system, a Krylov sub-space can be generate V_{r_q} and an upper-Hessenberg matrix H_{r_q} for each sub-system, that are satisfies the recursive relationship [13, 14]:

$$\psi_q V_{r_q} = V_{r_q}H_{r_q} + h_{(r+1,r)_q}v_{(r+1)_q}e_{r_q}^T \tag{6}$$

and

$$v_{r_q} = \xi_q / \|\xi_q\| \tag{7}$$

where $e_{r_q} \in \mathbb{R}^n$ is the rth unit vector. The vector $v_{(r+1)_q}$ satisfies a $(r+1)_q$ term recurrence relation, involving itself and the preceding Krylov vectors for each sub systems.

The reduced-order system transfer function $\hat{Y}_{r_q}$ for each sub-system is obtained by the use of the orthogonal projection $x(t) = V_{r_q}\hat{x}_{r_q}(t)$. From this relationship, the reduced system parameters in (2) can be defined by the congruence transformation [3, 15, 16]:

$$\hat{E}_{r_q} = V_{r_q}^T E_q V_{r_q}, \;\; \hat{A}_{r_q} = V_{r_q}^T A_q V_{r_q}, \;\; \hat{B}_q = V_{r_q}^T B_q, \;\; \hat{C}_q = V_{r_q}^T C_q,$$

and

$$\hat{D}_q = D_q. \tag{8}$$

The order of each sub-system is equal to $r_q = \sum_{i_q=1}^{\hat{i_q}} \hat{j}_{i_q}$, and the moments number of the original system and reduced one for each sub-system is matched for each expansion point [3, 13, 8]:

$$Y_q^j(s_{i_q}) = \hat{Y}_q^j(s_{i_q}), \quad \text{for } j_q = 0, 1, \ldots, r_q - 1 \tag{9}$$

The details of the Rational Arnoldi Algorithm for Switched Linear Systems can be found in Table 1 [4, 17, 8]:

The principle of the Rational Algorithm for the Switched linear systems is based on implementation of the Arnoldi Algorithm around single frequency s_{i_q} for each $\hat{J}_{i_q}$. During the iteration process an orthonormal basis V_{r_q} is generate from the a union krylov subspace at various expansion points:

$$K_{r_q} = [X_q^{(0)}(s_{1_q}), \ldots, X_q^{(\hat{j}_{1_q}-1)}(s_{1_q}), \ldots, X_q^{(0)}(s_{\hat{i}_q}), \ldots, X_q^{(\hat{j}_{\hat{i}_q}-1)}(s_{\hat{i}_q})] \tag{10}$$

The main steps of this method are:

Step 1: In the first step we set the expansion point use the criteria of the dominant pole, the number of moments matched for each frequency and initialize the first vector of residue $z0_q$ for each sub-system.

Step 1: Determine the orthonormal basis V_{r_q}, where satisfied the condition of the orthogonal projection $x(t) = V_{r_q}\hat{x}_{r_q}(t)$ and for each expansion point update the residue vector r_{k_q}, also an upper-Hessenberg matrix is generate and satisfied the recursive relationship in (6). Theorem 1 summarizes this result.

Theorem 1 ([4, 9])**.** *Take an initial matrix $\psi_{i_q} = -(s_{i_q}E_q - A_q)^{-1}E_q$ and an initial vector $\xi_{i_q} = (s_{i_q}E - A)^{-1}B$, and let $S_1 = [s_1, s_2, \ldots, s_{\hat{i}_1}] \in \mathbb{C}$ and $S_2 = [s_1, s_2, \ldots, s_{\hat{i}_1}] \in \mathbb{C}$, which the expansion point are distinct of two sets, also give the two set of moment matching for each expansion point range, $J_1 = [j_1, j_2, \ldots, j_{\hat{i}_1}] \in \mathbb{R}$ and $J_2 = [j_1, j_2, \ldots, j_{\hat{i}_2}] \in \mathbb{R}$. Generate the orthonormal basis V_{r_q} for each sub-system with $r_q = \sum J_q$ iteration from the Krylov subspace $K_{j_q}(\psi_{i_q}, \xi_{i_q}) = span\{\xi_{i_q}, \psi_{i_q}\xi_{i_q}, \ldots, \psi_{i_q}^{j-1}\xi_{i_q}\}$. Since $X_q^j(s_{i_q}) \in colspan\{V_{r_q}\}$ for $j_q = 0, 1, \ldots, \hat{j}_{i_q}$ and $i_q = 1, 2, \ldots, \hat{i}_q$, we obtain*

$$X_q^j(s_{i_q}) = V_{r_q}\hat{X}_q^j(s_{i_q}) \;\; \text{and} \;\; Y_q^j(s_{i_q}) = \hat{Y}_q^j(s_{i_q}) \tag{11}$$

TABLE 1. RASLS Algorithm

RASLS Algorithm:($input: E_q, A_q, B_q, C_q, D_q, S_q, J_q; output: V_{r_q}$)

Switch q

1/*Initialization of the first Residue Vector for Each Sub-system*/

$z_{0_q} := (s_{1_q} E_q - A_q)^{-1} B_q$

2/*Construction of the Matrix V_{r_q}*/

for $i_q = 1, 2, \ldots, \hat{i}_q$ **do**

  for $j_q = 1, 2, \ldots, \hat{j}_{i_q}$ **do**

   $k_q := (i_q - 1)\hat{j}_{i_q} + j_q$

   (2.1):/*Generate the New Orthonormal Vector v_{k_q}*/

   $h_{(k,k-1)_q} := \|z_{(k-1)_q}\|$

   $v_{(k)_q} := z_{(k-1)_q} / h_{(k,k-1)_q}$

   (2.2):/*Update the Residue z_{k_q} for the Next Iteration*/

   if $j_q = \hat{j}_{i_q}$ and $i_q <= \hat{i}_q$ **then**

   $z_{k_q} := (s_{(i+1)_q} E_q - A_q)^{-1} B_q$

   else $z_{k_q} := -(s_{i-q} E_q - A_q)^{-1} E_q v_{k_q}$

   end if

   for $t_q = 1, 2, \ldots, k_q$

   $h_{(t,k)_q} := v_{t_q}^H$, $z_{k_q} := z_{k_q} - h_{(t,k)_q} v_{t_q}$

   end for

  end for

end for

end Switch

Numerical Example. To evaluate this approach, we present the largest singular value of the frequency response, the poles and the absolute error for each sub-system. For these we take a FOM model of order 1006, which parameters of states representation are as follows [18]:

$A_1 = diag(\gamma_1, \gamma_2, \gamma_3, \gamma_4)$ with,

$$\gamma_1 = \begin{pmatrix} -1 & -100 \\ -100 & -1 \end{pmatrix},$$

$$\gamma_2 = \begin{pmatrix} -1 & -200 \\ -200 & -1 \end{pmatrix},$$

$$\gamma_3 = \begin{pmatrix} -1 & -400 \\ -400 & -1 \end{pmatrix},$$

$\gamma_4 = diag(-1, \ldots, -1000)$,

$B_1 = [10 * ones(6,1); ones(1000,1)]$, $C_1 = B_1^T$, $D_1 = 0$.

$A_2 = A_1 - 5 * I$, $B_2 = B_1$, $C_2 = C_1$, $D_2 = D_1$.

$S_1 = [1 \pm 400 * i, 1 \pm 200 * i, 1 \pm 100 * i]$,
$S_2 = [6 \pm 400 * i, 6 \pm 200 * i, 6 \pm 100 * i]$.

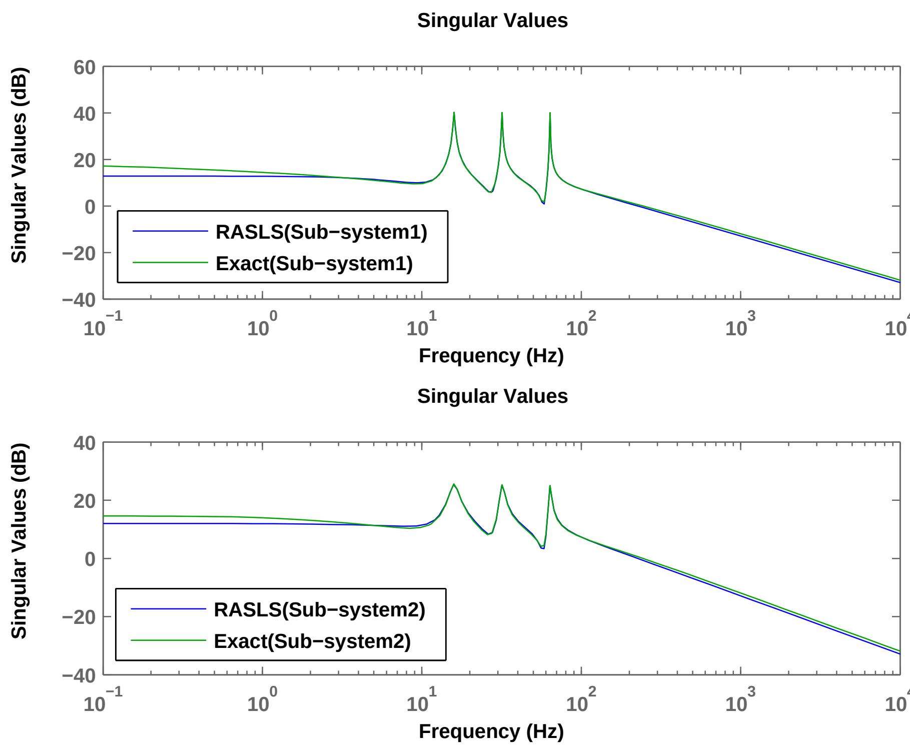

FIGURE 1. Largest singular value of the frequency response of the original system (sub-system 1) of order 1006 and reduced one of order 10 to a frequency range with RASLS method

The figure 1 presents the largest singular value of the frequency response of the original sub-systems (order 1006) and reduced one (order 10) to a frequency range. We can see when a correlation over the entire frequency range shape with a low error rate. The figure 2 shows the variation of the singular value of the absolute error between the original sub-systems and the reduced one, we see that the error is small over the entire frequency range. The distribution poles in the complex plane of each sub-systems is depicts in figure 3, all poles are negative real part, then the system is stable.

4. **Adaptive Order Rational Arnoldi for Switched Linear Systems**

The adaptive order rational arnoldi for switched linear systems method, is an improvement of the arnoldi [19, 3, 13, 11] and rational arnoldi for switched linear systems method. AORASLS Algorithm generate a reduced model around a frequency range with an automatic choice of matching moments of each sub-system. However, the expansion frequency s_{i_q} and the number of matched moment $\hat{j}_{i_q}$ must be

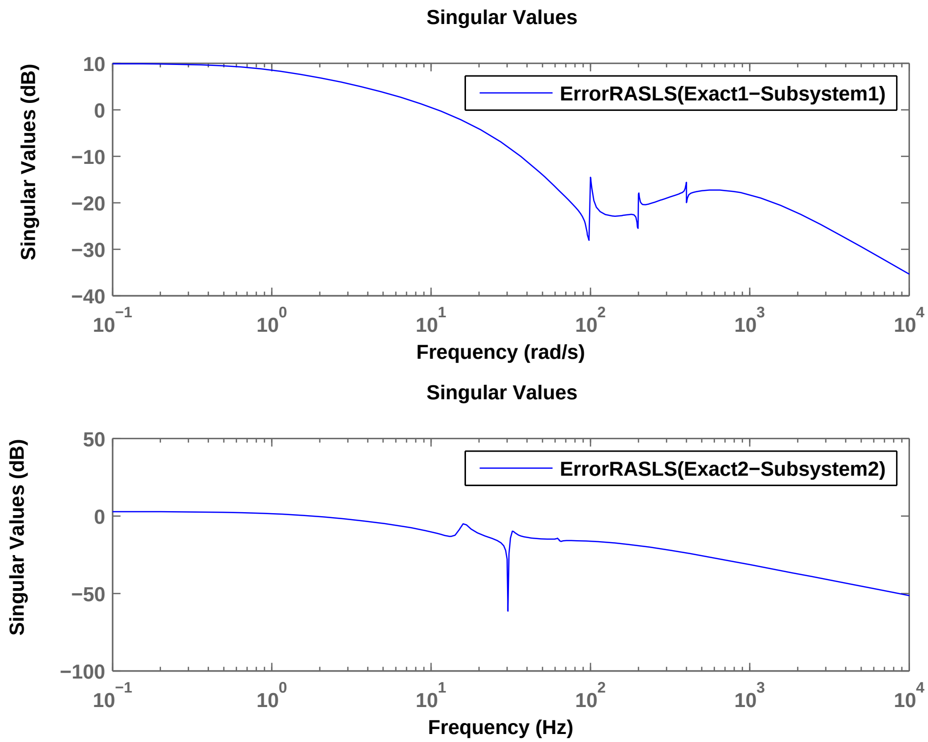

FIGURE 2. Absolute error system between original system of order (1006) and the reduced one of order (10) with RASLS method

given. For simplicity, the expansion points s_{i_q} for $i_q = 1, \ldots, \hat{i}_q$ are determined in advance using the technique of eigenvalues. Given a fixed set of expansion points $S_q = [s_{1_q}, s_{2_q}, \ldots, s_{r_q}]$ and the number of matched moments for each sub-system $r_q = \sum \hat{j}_{i_q}$, where r presents the order of the reduced systems. The method can generate an orthonormal matrix for each sub-system V_{r_q} from the successive Krylov subspace $K_{r_q}(\psi_q, \xi_q) = span[V_{1_q}, V_{2_q}, \ldots, V_{r_q}]$, where $\psi_q(s_{i_q}) = -(s_{i_q}E_q - A_q)^{-1}E_q$ and $\xi_q(s_{i_q}E_q - A_q)^{-1}B_q$ satisfying the following orthogonality relation [4, 14]:

$$V_{r_q}^T V_{r_q} = I_{r_q} \tag{12}$$

The transfer function error $E_{r_q} = Y_{r_q}(s) - \hat{Y}_{r_q}(s)$ can be represented as:

$$\begin{aligned} E_{r_q}(s) = & \sum_{j_q=0}^{\hat{j}_{(i-1)_q}} (Y_q^{(j)}(s_{i_q}) - \hat{Y}_{r_q}^{(j)}(s_{i_q})(s - s_{i_q})^j \\ & + (Y_q^{\hat{j}_{i_q}}(s_{i_q}) - \hat{Y}_q^{\hat{j}_{i_q}}(s_{i_q}(s_{i_q})))(s - s_{i_q})^{\hat{j}} + o(s - s_{i_q})^{\hat{j}_{(i+1)_q}} \\ = & \sum_{j_q=0}^{\hat{j}_{(i-1)_q}} 0.(s - s_{i_q})^j + (Y_q^{\hat{j}}(s_{i_q}) - \hat{Y}_{r_q}^{\hat{j}}(s_{i_q}))(s - s_{i_q})^{\hat{j}} \\ & + o(s - s_{i_q})^{\hat{j}_{(i+1)_q}} \end{aligned}$$

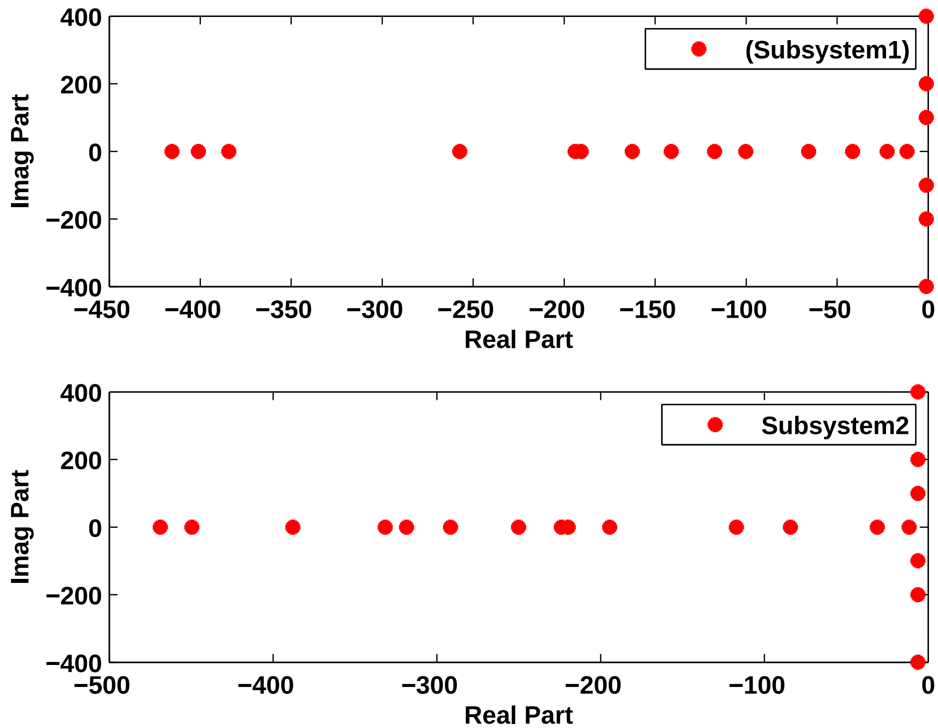

FIGURE 3. Pole Distribution of FOM reduced system (order 10) with RASLS method

$$= E_q^{\hat{j}_i}(s_{i_q})(s - s_{i_q})^{\hat{j}_{i_j}} + o(s - s_{i_q})^{\hat{j}_{(i+1)_q}} \tag{13}$$

Theorem 2 [4, 9]. *Let $S_q = [s_1, s_2, \ldots, s_{\hat{i}_q}]$ and r_q present the order of model reduction of each sub-system. Suppose that the V_{r_q} the orthonormal basis for each sub-system generate automatically by the use of the moments matching for each expansion point, which are matched with that of the original system. That is $Y_q^{j_q}(s_{i_q}) = \hat{Y}_q^{j_q}(s_{i_q})$ for $j_q = 0, 1, 2, \ldots, \hat{j}_{i_q} - 1$ and $i_q = 1, 2, \ldots, \hat{i}_q$. then the expression of error between the j_{i_q}th-order moments $Y^{\hat{j}_{i_q}}(s_{i_q})$ and $\hat{Y}^{\hat{j}_{i_q}}(s_{i_q})$ at each expansion point s_{i_q} and each sub-system can be expressed as follows:*

$$|E_{r_q}^{\hat{j}_i}| = |Y^{\hat{j}_{i_q}}(s_{i_q}) - \hat{Y}^{\hat{j}_{i_q}}(s_{i_q})| = |C_q^T h_{\Pi_q}(s_{i_q}) z_q^r(s_{i_q})| \tag{14}$$

The details of the Adaptive Order Rational Arnoldi for Switched Linear Systems Algorithm is found in table 2.

The Adaptive Order Rational Arnodli for Switched Linear Systems Algorithm includes the following main steps:

Step 1: Initialize of the two vectors $k_q^{(0)}(s_{i_q})$ and $z_q^{(0)}(s_{i_q})$ of the Krylov sequence for each expansion point s_{i_q} where $i_q = 1, \ldots, \hat{i}_q$, knowing that $k_q^{(0)}(s_{i_q}) = z_q^{(0)}(s_{i_q})$. Initialization of the normalization coefficient $h_{\Pi_q}(s_{i_q})$ for all s_{i_q} of each sub-system.

TABLE 2. AORASLS Algorithm

AORASLS Algorithm:($input : E_q, A_q, B_q, C_q, D_q, S_q, r_q; output : V_{r_q}$)

Switch q

(1)/*Initialization of the Expansion Frequencies, Matched Moment and the Initial Residue Vector for Each Sub-system*/

for each $s_{i_q} \in S$ **do**

$k_q^{(0)}(s_{i_q}) := (s_{i_q}E_q - A_q)^{-1}B_q,$

$z_q^{(0)} := k_q^{(0)}(s_{i_q}),$

$h_{\Pi_q}(s_{i_q}) := 1$/*the normalization coefficient*/

end for

(2):/*Begin AORASLS Iterations*/

forj=1,2,...,q **do**

(2.1):/*select the expansion frequency with the maximum output moment error*/

Choose $s_{i_q} \in S_q$ as the i_q giving $\max_{i_q}(|h_{\Pi_q}(s_{i_q}C_q^T z_q^{j-1}(s_{i_q})|)$

Set $s_{i^*_{jq}}$ be the expansion frequency in the jth iteration

(2.2):/*Generate the orthonormal vector at $s_{i^*_{jq}}$ */

$h_{(j,j-1)_q}(s_{i^*_{jq}}) := \|z_q^{j-1}(s_{i^*_{jq}})\|$

$v_{j_q} = z_q^{j-1}(s_{i^*_{jq}})/h_{(j,j-1)_q}(s_{i^*_{jq}})$

$h_{\Pi_q}(s_{i^*_{jq}}) := h_{\Pi_q}(s_{i^*_{jq}})h_{(j,j-1)_q}(s_{i^*_{jq}})$

(2.3):*/Update the residue $z_q^j(s_{i_q})$ for the next iteration*/

for each $s_{i_q} \in S_q$ **do**

if ($s_{i_q} == s_{i^*_{jq}}$) **then**

$k_q^j(s_{i^*_{jq}}) := -(s_{i_q}E_q - A_q)^{-1}E_q v_{j_q}$

else

$k_q^{(j)}(s_{i_q}) = k_q^{(j-1)}(s_{i_q})$

end if

$z^{(j_q)}(s_{i_q}) := k_q^{(j)}(s_{i_q})$

for $t_q = 1, 2, \ldots, j_q$ **do**

$h_{(t,j)_q}(s_{i_q}) := v_{t_q}^H z_q^{j_q}(s_{i_q})$

$z_q^{(j)_q}(s_{i_q}) := z_q^{(j)_q}(s_{i_q}) - h_{(t,j)_q}(s_{i_q})v_{t_q}.$

end for

end for

end for

end Switch

$V_{r_q} = [v_{1_q} v_{2_q} \ldots v_{r_q}]$

(3):/*Generate Real V_{r_q} for Complex Expansion points*/
if there exists any $s_{i_q} \in S_q$ such that s_{i_q} is not a real number
then $V_{real_q} := real(V_{r_q})$,$V_{imag_q} := imag(V_{r_q})$,
$[V_{r_q}, rr] = qr([V_{real_q} V_{imag_q}])$.
end if

Step 2: Choose the expansion frequency s_{i_q} knowing that s_{i_q} gives the greatest difference between the $j_{i_q}th$-order moments of the ooriginal system $Y_q(s)$ and the reduced one $\hat{Y}_q$, the chosen expansion frequency is called $s_{i^*_{j_q}}$ from this criterion:

$$\max |Y^{\hat{j}_{i_q}}(s_{i_q}) - \hat{Y}^{\hat{j}_{i_q}}(s_{i_q})| = \max |h_{\Pi_q}(s_{i_q} C_q^T z_q^{(j-1)_q}(s_{i_q}))| \tag{15}$$

Step 2.1: Select the expansion point $s^*_{i_{j_q}}$, and apply the Arnoldi algorithm around this point for each sub-system. The new orthonormal vector v_{j_q} is incorporated into the orthonormal matrix $V_{(j-1)_q}$. The normalization coefficient $h_{\Pi_q}(s_{i_q})$, according to $s^*_{i_{j_q}}$.

Step 2.2: Update the residue vector $Z_q^{(j)_q}(s_{i_q})$ and generate the Hessenberg matrix H using the modified Gram-Schmidt orthogonalization technique.

Numerical Example. To evaluate the Algorithm, we take the model and the frequencies range for each sub-systems used previously. We fix the largest singular value of the frequency response of the original system and reduced one, we present the variation of absolute error between the original and reduced one and we give the poles distribution of the reduced system.

The figure 4 shows the largest singular value of the frequency response of the original sub-systems of order 1006 and the reduced one of order 12, we see a good correlation between the two systems over the entire frequency range of the original system, even when comparing these results with those obtained with RASLS algorithm. The figure 5 presents the variation of the absolute error between the original sub-systems and the reduced one. Note that the variation of error is low compared to previous method. The figure 6 depicts the poles distribution of the reduced system in the complex plane, we see that all poles are negative real part, which explains the stability of the reduced system.

5. **Conclusion**

We have proposed a new two model reduction methods for large-scale switched linear systems. The proposed methods are based on generation of Krylov subspaces by the use of the moments matching technique for each sub-system. the two methods is an extension of the Arnoldi Algorithm of the switched linear systems around a single frequency. However, The RASLS is based on the fixing of the frequency range and the

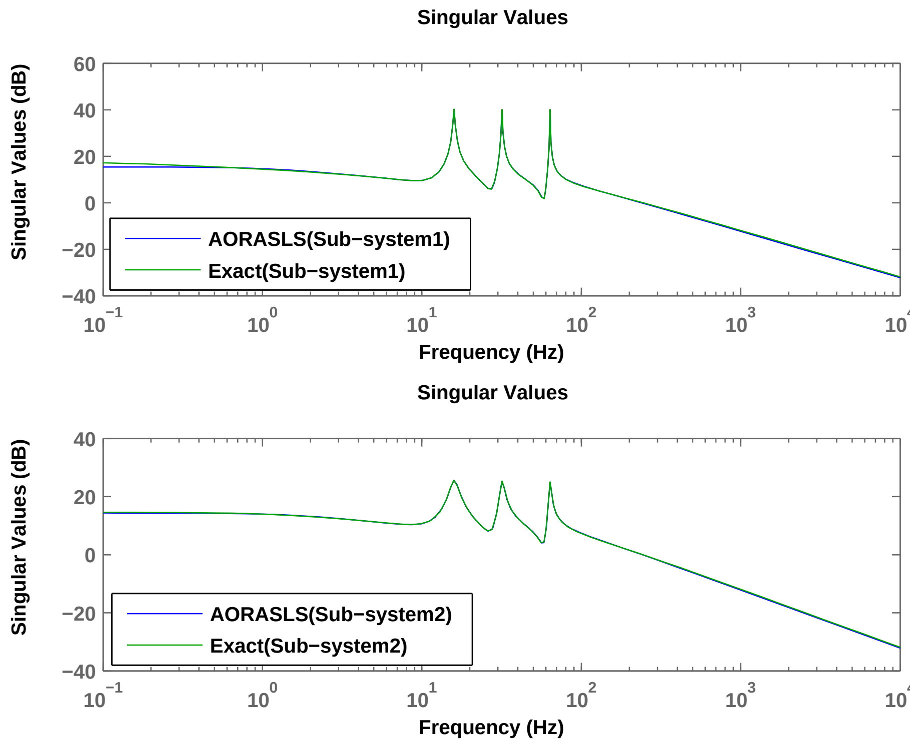

FIGURE 4. Largest singular value of the frequency response of the original system (sub-system 1) of order 1006 and reduced one of order 10 to a frequency range with AORASLS method

moment matching for each frequency from the beginning. But the AORASLS is based on automatically choice of the moment for each frequency. the advantages of these methods are, the stability of the reduced systems is guaranteed and the minimizing of absolute error is established. to evaluate and demonstrate the accuracy and efficient of these methods. From simulation results we noted that the best results is obtained by Adaptive Order Rational Arnoldi Algorithm for Switched Linear Systems.

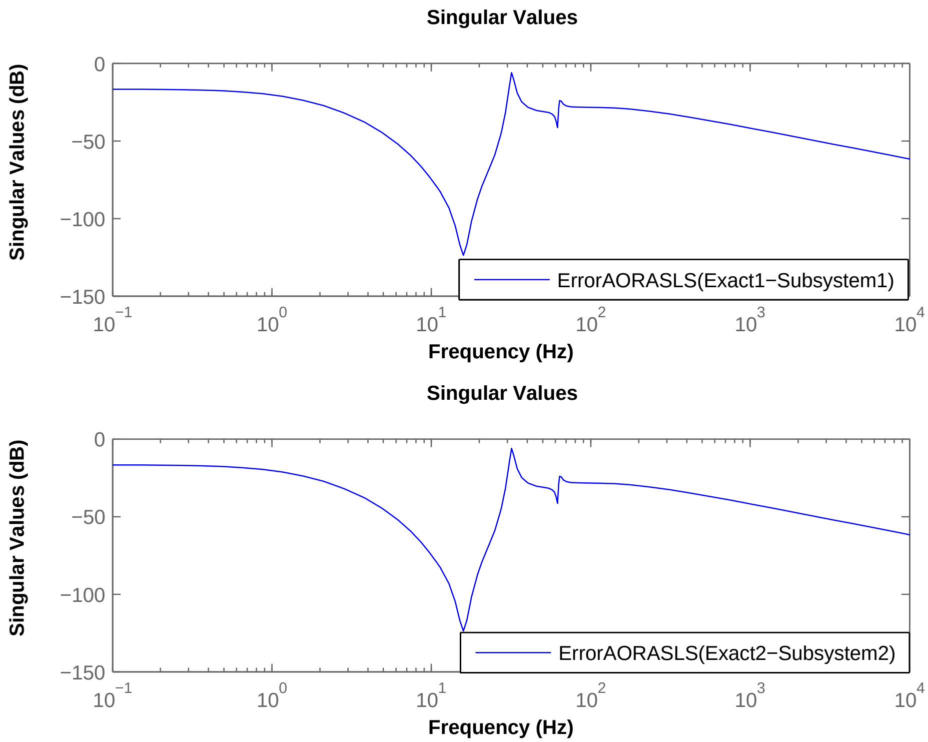

FIGURE 5. Absolute error system between original system of order (1006) and the reduced one of order (10) with AORASLS method

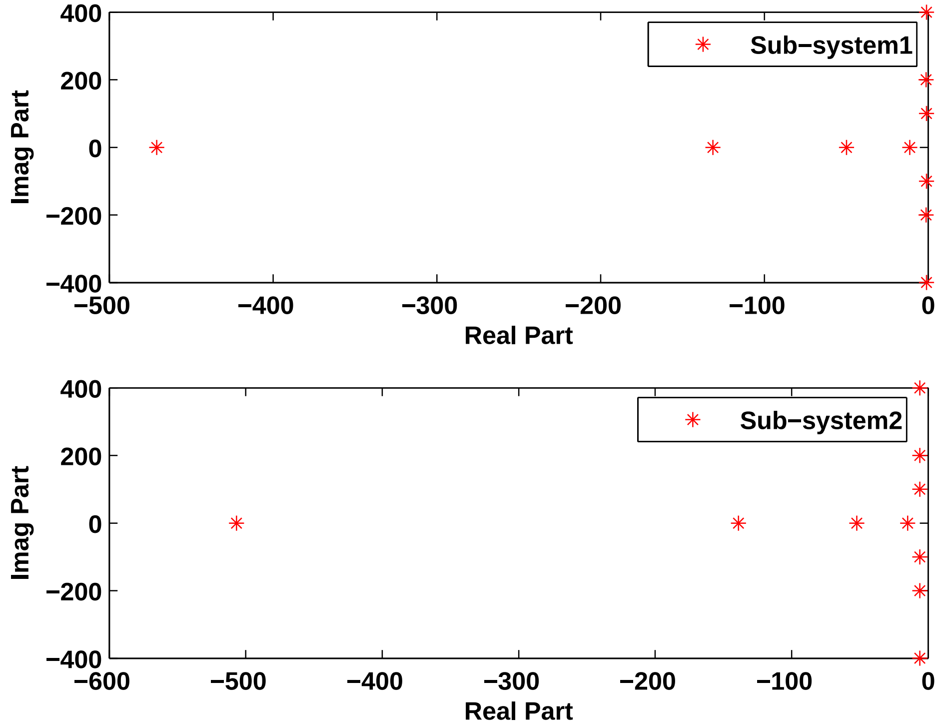

Figure 6. Pole Distribution of FOM reduced system (order 10) with AORASLS method

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