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NOVEL CRITERIA FOR GLOBAL STABILIZATION OF DISCRETE-TIME NEURAL NETWORKS WITH INTERVAL TIME-VARYING DELAYS

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ABSTRACT. This paper deals with the problem global stabilization of discrete-time neural networks with interval time-varying delays. The problem is solved by applying a novel set of Lyapunov functionals, and an improved delay-dependent stability criterion is obtained in terms of a linear matrix inequality. The stabilizing state feedback controller can be constructed by using the corresponding feasible solution of the linear matrix inequality. An examples is presented to demonstrate the effectiveness of the proposed approach.

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1. Introduction

In the past few decades, stability of neural networks have been extensively studied since their wide applications, such as optimization, classification, signal and image processing, solving nonlinear algebraic equations, pattern recognition, associative memories, automatic control, and so on [1,2]. Problem of stability and control of neural networks is a very important issue, and many results have been developed in the literature (see, e.g. [3–8] and the references cited therein). Although most neural networks are concerned with continuous-time cases, discrete-time neural networks (DNNs) have gradually attracted much attention. DNNs are important in formulating discrete-time systems that are analogues of the continuous-time neural networks in order to provide convenient ways in simulating and computing the continuous-time systems. Therefore, both analysis and synthesis problems for DNNs have been extensively studied and many nice results have been reported [9–11]. Based on constructing a Lyapunov functional which divide delay interval into two subintervals, an improved stability criterion was proposed in [12] for DDNs with time-varying delays. However, when the number of delay-partitioning number increases, the matrix formulation becomes more complex, and the computational burden and time-consuming

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grow bigger. Recently, by construction of an augmented Lyapunov-Krasovskii functional and utilization of discrete version of Jensen inequality, new stability criteria are derived in [13, 14] for DDNs with time-varying delays. There have been several papers [15–17] reported on stabilization criteria for delayed neural networks. Nevertheless, the results reported therein either for continuous-time case or neural networks with constant delays. Motivated by this discussed above, the problem of global stabilization of discrete-time neural networks with time-varying delays is considered in this paper.

In this paper, our objective is to obtain new delay-dependent stabilizability criteria for a class of DNNs with interval time-varying delays by choosing a new set of augmented Lyapunov-Krasovski functionals and estimating its derivative tightly from a novel viewpoint. Different from the previous investigations, this paper focuses on the global stabilization of DNNs by using a combination of Lyapunov functional method with the linear matrix inequality (LMI) technique. The main advantages of the present approach include: (i) it leads to less conservation and less restriction; (ii) it can be efficiently verified via numerically solving the LMI using interior-point algorithms or just the LMI-toolbox in Matlab [18].

The paper is organized as follows. The problem statement is given in Section 2. Sufficient delay-dependent conditions for global stabilization of DDNs with time-varying delays and a numerical example showing the effectiveness of the proposed result are given in Section 3.

Notation: The following notations will be used throughout this paper: $\mathbb{Z}^+$ denotes the set of all nonnegative integers; $\mathbb{R}^n$ denotes the n-dimensional Euclidean space; $\mathbb{R}^{n\times m}$ is the set of all $n \times m$ real matrices; $\mathbb{I}$ is the identity matrix of appropriate dimensions; an asterisk $*$ denotes the symmetric part; A^T denotes the transpose of A; $\lambda(A)$ denotes the set of all eigenvalues of A; $\lambda_{\max}(A) = \max\{Re\lambda : \lambda \in \lambda(A)\}$; $\lambda_{\min}(A) = \min\{Re\lambda : \lambda \in \lambda(A)\}$; Matrix A is semi-positive definite ($A \geq 0$) if $\langle x^T Ax\rangle \geq 0$ for all $x \in \mathbb{R}^n$; A is positive definite ($A > 0$) if $\langle x^T Ax\rangle > 0$ for all $x \neq 0$; Matrices X and Y, the notation $X > Y$ (respectively, $X \geq Y$) means that the matrix $X - Y$ is positive definite (respectively, semi-positive definite).

2. **Problem Statetment**

Consider the discrete-time neural networks with time-varying delay:

(2.1)
$$\begin{cases} x(k+1) = -Ax(k) + W_0 f(x(k)) + W_1 g(x(k-h(k))) + Bu(k), & k \in \mathbb{Z}^+ \\ x(k) = \phi(k), & k \in [-\overline{h}, \ldots, 0], \end{cases}$$

where $x(k) = [x_1(k), x_2(k), \ldots, x_n(k)]^T \in \mathbb{R}^n$ is the state; $u(k)$ is the control; n is the number of neurals; $f(x(k)) = [f_1(x_1(k)), f_2(x_2(k)), \ldots, f_n(x_n(k))]^T$ and $g(x(k)) =$

$[g_1(x_1(k)), g_2(x_2(k)), \ldots, g_n(x_n(k))]^T$ are the activation functions; the diagonal matrix $A = diag(\lambda_1, \lambda_2, \ldots, \lambda_n)$ represents the self-feedback term and the matrices $W_0, W_1 \in \mathbb{R}^{n\times n}, B \in \mathbb{R}^{n\times m}$ denote the distributively delayed connection weights and the control input weights; the time-varying delay funtion $h(k)$ satisfies the condition:

$$0 \le \underline{h} \le h(k) \le \overline{h}, \quad k \in \mathbb{Z}^+.$$

In this paper, we consider various activation functions and assume that the activation functions $f(x), g(x)$ satisfy the following growth conditions:

$$\begin{aligned} \exists a_i > 0, b_i > 0 : |f_i(\xi)| &\le a_i|\xi|, \quad i = 1, 2, \ldots, n, \forall \xi \in \mathbb{R}, \\ |g_i(\xi)| &\le b_i|\xi|, \quad i = 1, 2, \ldots, n, \forall \xi \in \mathbb{R}. \end{aligned} \tag{2.2}$$

Definition 2.1. The system (2.1) is stabilizable if there is a state feedback control $u(k) = Kx(k)$ such that the zero solution of the closed-loop system

$$x(k+1) = -[A + BK]x(k) + W_0 f(x(k)) + W_1 g(x(k - h(k))), \quad k \in \mathcal{Z}^+,$$

is asymptotically stable.

The following well-known propositions will be used for the proofs in the subsequent section.

Proposition 2.1 (Schur Complement lemma [19])**.** *Given constant matrices X, Y, Z where $Y^T = Y > 0$, $X = X^T$. Then $X + ZY^{-1}Z^T < 0$ if and only if*

$$\begin{pmatrix} X & Z \\ Z^T & -Y \end{pmatrix} < 0.$$

Proposition 2.2. *For any given vectors $v_i \in \mathbb{R}^n$, $i = 1, 2, \ldots, n$, the following inequality holds:*

$$\left[\sum_{i=1}^n v_i\right]^T \left[\sum_{i=1}^n v_i\right] \le n \sum_{i=1}^n v_i^T v_i.$$

3. Main Result

In this section, we present stabilizability criteria for neurals networks (2.1). Before stating main result, the following notations of several matrices variables are defined for simplicity.

$$\begin{aligned} F &= diag\{a_1, a_2, \ldots, a_n\}, \quad H = diag\{b_1, b_2, \ldots, b_n\}, \\ T_{11} &= -P + Q^T A + A^T Q + (\overline{h} - \underline{h} + 1)R + M + N \\ &\quad + (1 + \underline{h})U + (1 + \overline{h})S + P_1 + P_1^T + FF + HH, \\ T_{12} &= Q^T + A^T Q, \quad T_{13} = -Q^T W_0 + P_1^T; \quad T_{14} = -Q^T W_1 + P_1^T, \\ T_{15} &= P_2^T + P_1^T - P_1; \quad T_{16} = P_1^T - P_1; \end{aligned}$$

$$T_{22} = P + Q^T + Q - Q^T BB^T - BB^T Q + BB^T,$$
$$T_{55} = -R - P_2 - P_2^T - P_1 - P_1^T; \quad T_{56} = -P_2^T - P_2 - P_1 - P_1^T,$$
$$T_{66} = -P_2^T - P_2 - P_1 - P_1^T,$$

$$\Sigma_{11} = \begin{pmatrix} T_{11} & T_{12} & T_{13} & T_{14} & T_{15} & T_{16} \\ * & T_{22} & Q^T W_0 & Q^T W_1 & -P_1 & P_2^T - P_1 \\ * & * & -I & 0 & -P_1 & -P_1 \\ * & * & * & -I & -P_1 & -P_1 \\ * & * & * & * & T_{55} & T_{56} \\ * & * & * & * & * & T_{66} \end{pmatrix},$$

$$\Sigma_{12} = \begin{pmatrix} P_1 & P_1 & P_1 & P_1 & QB & QB \\ 0 & 0 & 0 & 0 & QB & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -P_1 & -P_1 & -P_1 & -P_1 & 0 & 0 \\ -P_1 & -P_1 & -P_1 & -P_1 & 0 & 0 \end{pmatrix},$$

$$\Sigma_{22} = diag\left\{-N, -M, -\frac{1}{1+\underline{h}}U, -\frac{1}{1+\overline{h}}S, -I, -I\right\}.$$

Theorem 3.1. *System* (2.1) *is stabilizable if there exist symmetric positive definite matrices* P, R, U, S, M, N *and any matrices* P_1, P_2, Q *such that the following LMI holds:*

$$\begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ * & \Sigma_{22} \end{pmatrix} < 0. \tag{3.1}$$

The state-feedback controller is given by

$$u(k) = -B^T Q x(k), \quad k \in \mathbb{Z}^+.$$

Proof. With the state feedback control $u(k) = -B^T Q x(k)$ the closed-loop system becomes:

(3.2)
$$\begin{cases} x(k+1) = -\overline{A}x(k) + W_0 f(x(k)) + W_1 g(x(k-h(k))), & k \in \mathbb{Z}^+ \\ x(k) = \phi(k), & k \in [-h, -h+1, \dots, 0], \end{cases}$$

where $\overline{A} = A + BB^T Q$. Consider Lyapunov-Krasovskii functional for system (3.2) $V(x(k)) = \sum_{i=1}^5 V_i(x(k))$, where

$$V_1(x(k)) = x^T(k) P x(k),$$
$$V_2(x(k)) = \sum_{i=k-h(k)}^{k-1} x^T(i) R x(k),$$

$$V_3(x(k)) = \sum_{l=-\overline{h}+2}^{-\underline{h}+1} \sum_{i=k+l-1}^{k-1} x^T(i)Rx(i),$$

$$V_4(x(k)) = \sum_{i=k-\overline{h}}^{k-1} x^T(i)Mx(i) + \sum_{i=k-\underline{h}}^{k-1} x^T(i)Nx(i),$$

$$V_5(x(k)) = \sum_{j=k-\overline{h}}^{k} \sum_{i=j}^{k-1} x^T(i)Sx(i) + \sum_{j=k-\underline{h}}^{k} \sum_{i=j}^{k-1} x^T(i)Ux(i).$$

The difference of $V_1(x(k))$ gives

$$\begin{aligned}\Delta V_1(x(k)) &= V_1(x(k+1)) - V_1(x(k))\\ &= x^T(k+1)Px(k+1) - x^T(k)Px(k).\end{aligned}$$

Let us denote $x(k+1) = y(k)$, $z(k) = [x(k), y(k), f(\cdot), g(\cdot)]$, $\Gamma = \begin{pmatrix} P & 0 & 0 & 0 \\ Q & Q & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{pmatrix}$.

Since

$$0 = -y(k) - \overline{A}x(k) + W_0 f(x(k)) + W_1 g(x(k-h(k))),$$

we have

$$\Delta V_1(x(k)) = z^T(k)\Big[\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & P & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} - \Gamma^T \begin{pmatrix} \frac{1}{2}I & 0 & 0 & 0 \\ -\overline{A} & -I & W_0 & W_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} - \begin{pmatrix} \frac{1}{2}I & 0 & 0 & 0 \\ -\overline{A} & -I & W_0 & W_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}^T \Gamma \Big] z(k). \tag{3.3}$$

The difference of $\Delta V_2(x(k))$ gives

$$\begin{aligned}\Delta V_2(x(k)) &= V_2(x(k+1)) - V_2(x(k))\\ &= \sum_{i=k+1-h(k+1)}^{k} x^T(i)Rx(i) - \sum_{i=k-h(k)}^{k-1} x^T(i)Rx(i)\\ &= x^T(k)Rx(k) - x^T(k-h(k))Rx(k-h(k))\\ &\quad + \sum_{i=k+1-h(k+1)}^{k-\underline{h}} x^T(i)Rx(i)\\ &\quad + \sum_{i=k+1-\underline{h}}^{k-1} x^T(i)Rx(i) - \sum_{i=k+1-h(k)}^{k-1} x^T(i)Rx(i).\end{aligned} \tag{3.4}$$

The difference of $\Delta V_3(x(k))$ gives

$$\begin{aligned}
\Delta V_3(x(k)) &= \sum_{l=-\overline{h}+2}^{-\underline{h}+1} \sum_{i=k+l}^{k} x^T(i)Rx(i) - \sum_{l=-\overline{h}+2}^{-\underline{h}+1} \sum_{i=k+l-1}^{k-1} x^T(i)Rx(i) \\
&= \sum_{l=-\overline{h}+2}^{-\underline{h}+1} \Big[\sum_{i=k+l}^{k-1} x^T(i)Rx(i) + x^T(k)Rx(k) - \sum_{i=k+l}^{k-1} x^T(i)Rx(i) \\
&\quad - x^T(k+l-1)Rx(k+l-1)\Big] \\
&= \sum_{l=-\overline{h}+2}^{-\underline{h}+1} (x^T(k)Rx(k) - x^T(k+l-1)Rx(k+l-1)) \\
&= (\overline{h} - \underline{h})x^T(k)Rx(k) - \sum_{l=-\overline{h}+2}^{-\underline{h}+1} x^T(k+l-1)Rx(k+l-1) \\
&= (\overline{h} - \underline{h})x^T(k)Rx(k) - \sum_{i=k+1-\overline{h}}^{k-\underline{h}} x^T(i)Rx(i).
\end{aligned} \tag{3.5}$$

The difference of $\Delta V_4(x(k))$ gives

$$\Delta V_4(x(k)) = x^T(k)(M+N)x(k) - x^T(k-\underline{h})Nx(k-\underline{h}) - x^T(k-\overline{h})Mx(k-\overline{h}). \tag{3.6}$$

Using Proposition 2.2, the estimation of difference of $\Delta V_5(x(k))$ gives

(3.7)

$$\begin{aligned}
\Delta V_5 x(k) &= \sum_{j=k+1-\underline{h}}^{k+1} \sum_{i=j}^{k} x^T(i)Ux(i) - \sum_{j=k-\underline{h}}^{k} \sum_{i=j}^{k-1} x^T(i)Ux(i) \\
&\quad + \sum_{j=k+1-\overline{h}}^{k+1} \sum_{i=j}^{k} x^T(i)Sx(i) - \sum_{j=k-\overline{h}}^{k} \sum_{i=j}^{k-1} x^T(i)Sx(i) \\
&= \sum_{j=k-\underline{h}}^{k} \sum_{i=j+1}^{k} x^T(i)Ux(i) - \sum_{j=k-\underline{h}}^{k} \sum_{i=j}^{k-1} x^T(i)Ux(i) \\
&\quad + \sum_{j=k-\overline{h}}^{k} \sum_{i=j+1}^{k} x^T(i)Sx(i) - \sum_{j=k-\overline{h}}^{k} \sum_{i=j}^{k-1} x^T(i)Sx(i) \\
&= \sum_{j=k-\underline{h}}^{k} (x^T(k)Ux(k) - x^T(j)Ux(j)) + \sum_{j=k-\overline{h}}^{k} (x^T(k)Sx(k) - x^T(j)Sx(j)) \\
&= (1+\underline{h})x^T(k)Ux(k) - \sum_{j=k-\underline{h}}^{k} x^T(j)Ux(j) \\
&\quad + (1+\overline{h})x^T(k)Sx(k) - \sum_{j=k-\overline{h}}^{k} x^T(j)Sx(j)
\end{aligned}$$

$$\leq (1+\underline{h})x^T(k)Ux(k) - \frac{1}{1+\underline{h}}\left(\sum_{j=k-\underline{h}}^{k} x(j)\right)^T U \left(\sum_{j=k-\underline{h}}^{k} x(j)\right)$$

$$+ (1+\overline{h})x^T(k)Sx(k) - \frac{1}{1+\overline{h}}\left(\sum_{j=k-\overline{h}}^{k} x(j)\right)^T S \left(\sum_{j=k-\overline{h}}^{k} x(j)\right).$$

Since $0 \leq \underline{h} \leq h(k) \leq \overline{h}, \forall k \in \mathbb{Z}^+$, we have:

$$\sum_{i=k+1-\underline{h}}^{k-1} x^T(i)Rx(i) \leq \sum_{i=k+1-h(k)}^{k-1} x^T(i)Rx(i); \tag{3.8}$$

$$\sum_{i=k+1-h(k+1)}^{k-\underline{h}} x^T(i)Rx(i) \leq \sum_{i=k+1-\overline{h}}^{k-\underline{h}} x^T(i)Rx(i).$$

Let $\nu(k) = x(k+1) - x(k)$, we obtain $x(k) - \sum\limits_{i=k-h(k)}^{k-1} \nu(i) - x(k-h(k)) = 0$, then for arbitrary matrices P_1, P_2 we have

$$X^T \begin{pmatrix} 0 & P_1 \\ 0 & P_2 \end{pmatrix} Y = 0, \tag{3.9}$$

where

$$X^T = \left(\xi^T(k), \sum_{i=k-h(k)}^{k-1} \nu^T(i) + x^T(k-h(k))\right),$$

$$Y^T = \left(y^T(k), x^T(k) - \sum_{i=k-h(k)}^{k-1} \nu^T(i) - x^T(k-h(k))\right),$$

$$\xi(k) = (x(k) + y(k) + x(k-h(k)) + x(k-\underline{h}) + x(k-\overline{h}) + \sum_{i=k-\underline{h}}^{k} x(i)$$

$$+ \sum_{i=h+\overline{h}}^{k} x(i) + \sum_{i=k-h_j(k)}^{k-1} \nu(i) + f(\cdot)).$$

We note that the condition (2.2) is equivalent to

$$z_i^T(k) \begin{pmatrix} -FF-HH & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{pmatrix} z_i(k) \leq 0. \tag{3.10}$$

Therefore, from (3.3)-(3.10) it follows that

(3.11)

$$\Delta V(x_k) \leq x^T(k)\Phi_{11}x(k) + 2x^T(k)\Phi_{12}y(k) + 2x^T(k)T_{13}f(x(k)) + 2x^T(k)T_{14}g(x(k))$$

$$
\begin{aligned}
&+ 2x^T(k)T_{15}x(k-h(k)) + 2x^T(k)T_{16}\sum_{i=k-h_j(k)}^{k-1}\nu(i) + 2x^T(k)P_1^T x(k-\overline{h})\\
&+ 2x^T(k)P_1^T x(k-\overline{h}) + 2x^T(k)P_1^T\sum_{i=k-\underline{h}}^{k}x(i) + 2x^T(k)P_1^T\sum_{i=k-\overline{h}}^{k}x(i)\\
&+ 2y^T(k)T_{22}y(k)x + 2y^T(k)Q^TW_0f(x(k)) + 2y^T(k)Q^TW_1g(x(k))\\
&- 2y^T(k)P_1x(k-h(k)) + 2y^T(k)(P_2^T-P_1)\sum_{i=k-h(k)}^{k-1}\nu(i)\\
&- f^T(x(k))If(x(k)) - f^T(x(k))P_1x(k-h(k)) - f^T(x(k))P_1\sum_{i=k-h(k)}^{k-1}\nu(i)\\
&- g^T(x(k))Ig(x(k-h(k))) - g^T(x(k))P_1x(k-h(k))\\
&- g^T(x(k))P_1\sum_{i=k-h(k)}^{k-1}\nu(i) + x(k-h(k))^TT_{55}x(k-h(k))\\
&+ x(k-h(k))^TT_{56}\sum_{i=k-h(k)}^{k-1}\nu(i) - x(k-h(k))^TP_1^Tx(k-\underline{h})\\
&- x(k-h(k))^TP_1^Tx(k-\overline{h}) - x(k-h(k))^TP_1^T\sum_{i=k-\underline{h}}^{k}x(i)\\
&- x(k-h(k))^TP_1^T\sum_{i=k-\overline{h}}^{k}x(i) + \sum_{i=k-h_j(k)}^{k-1}\nu(i)^TT_{66}\sum_{i=k-h_j(k)}^{k-1}\nu(i)\\
&- \sum_{i=k-h_j(k)}^{k-1}\nu(i)^TP_1^Tx(k-\underline{k}) - \sum_{i=k-h_j(k)}^{k-1}\nu(i)^TP_1^Tx(k\overline{k})\\
&- \sum_{i=k-h_j(k)}^{k-1}\nu(i)^TP_1^T\sum_{i=k-\underline{h}}^{k}x(i) - \sum_{i=k-h_j(k)}^{k-1}\nu(i)^TP_1^T\sum_{i=k-\overline{h}}^{k}x(i)\\
&- x^T(k-\underline{h})Nx(k-\underline{h}) - x^T(k-\overline{h})Mx(k-\overline{h})\\
&- \sum_{i=k-\underline{h}}^{k}x(i)\frac{1}{1+\underline{h}}U\sum_{i=k-\underline{h}}^{k}x(i) - \sum_{i=k-\overline{h}}^{k}x(i)\frac{1}{1+\overline{h}}S\sum_{i=k-\overline{h}}^{k}x(i).
\end{aligned}
$$

Let

$$
\varphi(x(k)) = [x(k), y(k), f(x(k)), g(x(k-h(k))), x(k-h(k)), \sum_{i=k-h(k)}^{k-1}\nu(i),
$$

$$
x(k-\underline{h}), x(k-\overline{h}), \sum_{i=k-\underline{h}}x(i), \sum_{i=k-\overline{h}}x(i)];
$$

$$
\Phi_{11} = -P + Q^TA + A^TQ + 2Q^TBB^TQ + (\overline{h} - \underline{h} + 1)R
$$

$$+ M + N + (1+\underline{h})U + (1+\overline{h})S + P_1 + P_1^T + FF + HH;$$

$$\Phi_{12} = Q^T + A^T Q + QBB^T Q.$$

$$\Phi_{22} = P + Q^T + Q$$

Thus, from the difference inequality (3.11) we obtain

$$\Delta V(x(k)) \le \varphi(x(k))^T \Sigma' \varphi(x(k)),$$

where

$$\Sigma' = \begin{pmatrix} \Phi_{11} & \Phi_{12} & T_{13} & T_{14} & T_{15} & T_{16} & P_1^T & P_1^T & P_1^T & P_1^T \\ * & \Phi_{22} & Q^T W_0 & Q^T W_1 & -P_1 & P_2^T - P_1 & 0 & 0 & 0 & 0 \\ * & * & -I & 0 & -P_1 & -P_1 & 0 & 0 & 0 & 0 \\ * & * & * & -I & -P_1 & -P_1 & 0 & 0 & 0 & 0 \\ * & * & * & * & T_{55} & T_{56} & -P_1^T & -P_1^T & -P_1^T & -P_1^T \\ * & * & * & * & * & T_{66} & -P_1^T & -P_1^T & -P_1^T & -P_1^T \\ * & * & * & * & * & * & -N & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -M & 0 & 0 \\ * & * & * & * & * & * & * & * & -\frac{1}{1+\underline{h}}U & 0 \\ * & * & * & * & * & * & * & * & * & -\frac{1}{1+\overline{h}}S \end{pmatrix}.$$

Using Shur complement lemma Proposition 2.1, the condition $\Sigma' < 0$ is equivalent to

$$\Xi = \begin{pmatrix} \Xi_{11} & \Xi_{12} \\ * & \Xi_{22} \end{pmatrix} < 0, \tag{3.12}$$

where

$$\Xi_{11} = \begin{pmatrix} T_{11} & T_{12} & T_{13} & T_{14} & T_{15} & T_{16} \\ * & \Psi_{22} & Q^T W_0 & Q^T W_1 & -P_1 & P_2^T - P_1 \\ * & * & -I & 0 & -P_1 & -P_1 \\ * & * & * & -I & -P_1 & -P_1 \\ * & * & * & * & T_{55} & T_{56} \\ * & * & * & * & * & T_{66} \end{pmatrix},$$

$$\Xi_{12} = \begin{pmatrix} P_1 & P_1 & P_1 & P_1 & Q^T B & Q^T B \\ 0 & 0 & 0 & 0 & Q^T B & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -P_1 & -P_1 & -P_1 & -P_1 & 0 & 0 \\ -P_1 & -P_1 & -P_1 & -P_1 & 0 & 0 \end{pmatrix},$$

$$\Xi_{22} = diag\left\{-N, -M, -\frac{1}{1+\underline{h}}U, -\frac{1}{1+\overline{h}}S, -I, -I\right\}.$$

$$T_{11} = -P + Q^T A + A^T Q + (\overline{h} - \underline{h} + 1)R + M + N$$

$$+ (1 + \underline{h})U + (1 + \overline{h})S + P_1 + P_1^T + FF + HH$$

$$T_{12} = Q^T + A^T Q, .$$

Note that $(Q^T - I)BB^T(Q - I) \geq 0$, and then

$$Q^T BB^T Q \leq -Q^T BB^T - BB^T Q + BB^T,$$

we obtain that the condition $\Xi < 0$ holds if LMI (3.1) holds, which gives

$$\Delta V(x(k) < 0, \quad \forall k \in \mathcal{Z}^+.$$

The proof of the theorem is complete. □

Remark 3.1. The designed state feedback controller can ensure asymptotical stability of the closed-loop system which is expressed in the solutions of LMI. The result in this paper advances recent findings state feedback controller reported in [9, 11, 15] in that time delays considered are interval time-varying as oppose constant delays. Moreover, we construct Lyapunov-like functionals different from the ones in [10, 12-14] and estimate the derivative of $V(x(k)$ by new summation inequality, which leads to a less conservative LMI condition and reduced numerical complexity, and also as shown in the numerical example below, the proposed LMI condition in this paper can be solved with less free weighting matrix unknowns comparatively.

Example 3.1. Consider system (2.1), where

$$A = \begin{bmatrix} 0.25 & 0 \\ 0 & 0.1 \end{bmatrix}, \quad W_0 = \begin{bmatrix} 0.12 & 0.24 \\ -0.15 & 0.2 \end{bmatrix}, \quad W_1 = \begin{bmatrix} -0.25 & 0.1 \\ 0.02 & 0.09 \end{bmatrix}, \quad F = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.3 \end{bmatrix},$$

$$H = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.3 \end{bmatrix}, \quad B = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}, \quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$h(k) = 1 + 5 \sin \frac{k\pi}{2}, \quad k \in \mathcal{Z}^+.$$

Note that the functions $h(k)$ are interval time-varying, therefore, the methods proposed in [10, 12-14, 17] are not applicable to this system. Given $\underline{h} = 1$ and $\overline{h} = 6$, by using the Matlab LMI toolbox, we find that the LMI (3.1) of Theorem 3.1 is feasible with

$$P = \begin{bmatrix} 1.9106 & 0.0808 \\ 0.0808 & 2.1116 \end{bmatrix}, \quad R = \begin{bmatrix} 0.0400 & -0.0030 \\ -0.0030 & 0.0375 \end{bmatrix},$$

$$U = \begin{bmatrix} 0.0515 & -0.0045 \\ -0.0045 & 0.0476 \end{bmatrix}, \quad S = \begin{bmatrix} 0.0235 & -0.0014 \\ -0.0014 & 0.0222 \end{bmatrix},$$

$$M = \begin{bmatrix} 0.1673 & -0.0150 \\ -0.0150 & 0.1566 \end{bmatrix}, \quad N = \begin{bmatrix} 0.1673 & -0.0150 \\ -0.0150 & 0.1566 \end{bmatrix},$$

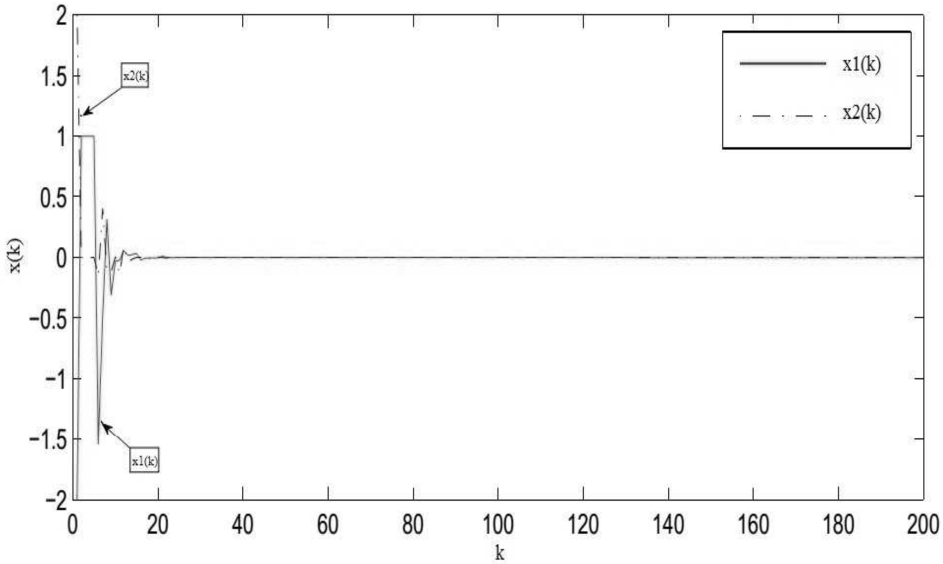

FIGURE 1. Response solution of the closed-loop system

$$P_1 = \begin{bmatrix} 0.0178 & -0.0106 \\ 0.0085 & 0.0158 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 0.0064 & -0.0028 \\ 0.0012 & 0.0062 \end{bmatrix},$$

$$Q = \begin{bmatrix} -2.0695 & -0.4646 \\ 0.2137 & -2.0971 \end{bmatrix}.$$

The state feedback controller is:

$$u(k) = 0.1856x_1(k) + 0.2562x_2(k), \quad k \in \mathcal{Z}^+.$$

Fig. 1 shows the simulation of the state response of the closed-loop systems with initial condition $\phi(k) = [-2 \quad 2]^T$.

4. Conclusions

This paper has investigated the global stabilzation via state feedback control for discrete-time neural networks with interval time-varying delays. Based on constructing the improved Lyapunov functionals and by utilizing a new estimation techniqueas, a new LMI-based sufficient condition for designing state feedback controller is derived for the considered system. A numerical example is given to illustrate the effectiveness of the proposed main result.

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REFERENCES

[1] K. J. Hunt, D. Sbarbaro, R. Zbikowski, and P. J. Gawthrop, Neural networks for control systems: A survey, *Automatica,* 28: 1083–1112, 1992

[2] M. M. Gupta, L. Jin, and N. Homma, Static and Dynamic Neural Networks. From Fundamentals to Advanced Theory. New York: Wiley, 2003.

[3] X. Liao, G. Chen, and E. N. Sanchez, Delay-dependent exponential stability of delayed neural networks: An LMI approach, *Neural Networks,*, 15: 855–866, 2002.

[4] C. Sun, C. Bo, Exponential periodicity of continuous-time and discrete-time neural networks with delays, *Neural Process. Lett.* 28: 131–146, 2004.

[5] D. Yue, Y. Zhang, E. Tian, C. Peng, Delay-distribution-dependent exponential stability criteria for discrete-time recurrent neural networks with stochastic delay, *IEEE Trans. Neural Networks,* 19: 1299–1306, 2008.

[6] K. Ratchagit, V. N. Phat, Robust stability and stabilization of linear polytopic delay-difference equations, *Neural Parallel and Scientific Computations,* 19: 361–372, 2011.

[7] R. Rakkiyappan, P. Balasubramaniam, New global exponential stability results for neutral type neural networks with distributed delays. *Neurocomputing,* 71: 1039–1045, 2008.

[8] M. V. Thuan, V. N. Phat, New criteria for stability and stabilization of neural networks with mixed interval time-varying delays, *Vietnam J. Math.,* 40: 79–93, 2012.

[9] S. Mohamad, K. Gopalsamy, Exponential stability of continuous-time and discrete-time cellular neural networks, *Appl. Math. Computation,* 135: 17–38, 2003.

[10] Z. Wu, H. Su, J. Chu, W. Zhou, Improved delay-dependent stability condition of discrete recurrent neural networks with time-varying delays, *IEEE Trans. Neural Networks,* 21: 692–697, 2010.

[11] C. Li, S. Wu, G. G. Feng, X. Liao, Stabilizing effects of impulses in discrete-time delayed neural networks, *IEEE Trans. NN,* 22: 323–329, 2011.

[12] S. L. Wu, K. L. Li, T. Z. Huang, Global exponential stability of static neural networks with delay and impulses: Discrete-time case, *Commun Nonlinear Sci Numer Simulat.,* 17: 3947–3960, 2012.

[13] O. M. Kwon, M. J. Park, J. H. Park, S. M. Lee, E. J. Cha, New criteria on delay-dependent stability for discrete-time neural networks with time-varying delays, *Neurocomputing,* 121: 185–194, 2013.

[14] T. Wang, C. Zhang, S. Fei, T. Li, Further stability criteria on discrete-time delayed neural networks with distributed delay, *Neurocomputing,* 111: 195–203, 2013.

[15] K. Patan, Stability analysis and the stabilization of a class of discrete-time dynamic neural networks,*IEEE Trans. Neural Netw.*, 18: 660–673, 2007.

[16] V. N. Phat, H. Trinh, Exponential stabilization of neural networks with various activation functions and mixed time-varying delays, *IEEE Trans. Neural Networks,* 21: 1180–1185, 2010.

[17] H. Huang, T. Huang, X. Chn, C. Qian, Exponential stabilization of delayed recurrent neural networks: A state estimation based approach, *Neural Networks,* 8: 153–157, 2013.

[18] P. Gahinet, A. Nemirovskii, A. J. Laub, M. Chilali, LMI Control Toolbox For use with MATLAB, The MathWorks, Inc., 1995.

[19] S. Boyd, H. Ghaoui, El, E. Feron., V. Balakrishnan, Linear Matrix Inequalities in System and Control Theory, Philadelphia: SIAM, 1994.