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EARLIEST ARRIVAL CONTRAFLOW MODEL FOR EVACUATION PLANNING

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ABSTRACT. The transportation network problem was modeled as a non-linear problem in the continuous time that makes complication during evacuation planning. To solve the evacuation problem approximately as quickly as possible we adopt the model of simple graph in discrete time setting. The earliest arrival flow (EAF) and the contraflow problems that have been highly focused in evacuation planning are considered. The EAF problem obtains the maximum amount of flow for every time steps from the sources to the sinks. In general, no polynomial algorithm has been found. A polynomial algorithm for the EAF problem has been presented on series-parallel graph. Contraflow reduces the traffic jam by increasing the outbound evacuation route capacity. A polynomial time algorithm for single source single sink maximum dynamic contraflow has been presented. The problem in the multiple sources and multiple sinks are NP-hard. We formulated the earliest arrival contraflow problem, where as many evacuees as possible should be sent from the sources to the sinks in every time period by reversing the road directions at time zero. A polynomial time algorithm for this problem on a two-terminal series-parallel graph having capacities and transit times on the arcs has been provided.

Key words: Evacuation planning, maximum dynamic contraflow, earliest arrival contraflow.

AMS (MOS) Subject Classification. 46N10, 90B35, 90C25.

1. INTRODUCTION

Due to the different disasters both natural and man-made, the challenges of emergency management have been increased day to day. Evacuation transportations, logistics supports and facility locations are the major problems for the evacuation planning after different disasters. After any kind of disasters like hurricane, earthquakes, tsunami, flooding, industrial and nuclear accidents, fire and terrorist attacks, etc, there are different real life questions where we have limited time to take the evacuees from the dangerous states to safe places. We want to evacuate as many evacuees as possible within the limited time because no one wants to die. Chichi, Bam and Kashmir earthquakes in Taiwan, Iran and Pakistan, the tsunami in the

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Indian Ocean and Japan, Asian flooding including China, the September 11 attacks, two major hurricanes Katrina and Rita and recent hurricane Sandy in the USA are the most cited examples of horrible disasters. The transportation network has an important role during the evacuation process. The mathematical models developed on the transportation network not only controls the traffic congestion, makes the evacuation plane systematic and effective but also have applications in transportation planning like rush office hours, sporting events, concert and mass-meetings.

Dynamic network flow model is an important tool to deal evacuation problems. An array of dynamic network flow problems are considered for evacuation planning for example: the maximum dynamic flow (MDF) problem, where we reallocate the maximal amount of evacuees in safe place in a given time, the earliest arrival flow (EAF) (also called universal maximum flow) problem, where we postpone the maximum number of evacuees to the safe destination in every time periods from the beginning of the plan, the quickest flow problem, where we reallocate the evacuees to a safe zone in minimum time. The EAF obtains the maximum amount of flow for every time steps from the sources to the sinks. In general, no polynomial algorithm has been found to solve the EAF. A polynomial algorithm for the EAF problem has been presented on two terminal series-parallel (TTSP) graph [6].

The transportation network is interpreted by a directed graph where nodes represent the intersection of roads, and edges represent the road segments between them. In the evacuation problem, the accidental areas where evacuees start to move are considered as source nodes and the safe places where they are supposed to arrive are destination (sink) nodes. Each node and edge has non-negative integer capacity where node capacity gives the maximum number of evacuees at this node and the edge capacity gives the maximum flow amount. This network also has total number of evacuees, non-negative integer travel times on edges. The problem contraflow is a way of increasing outbound capacity of a transportation network by reversing the direction of inbound roads during evacuations. It is considered a potential remedy to solve traffic jam during evacuations due to different natural disasters and homeland security. It is a challenging NP-complete computational problem.

In the literature, there are only few mathematical models developed for static network and dynamic network contraflow (maximum contraflow, maximum dynamic contraflow, quickest contraflow and earliest arrival contraflow) problems for simple graph with single source and single sink by reversing the direction of way towards the safe destination. Rebennack et al. [5] investigated the complexity of network flow problems with the possibility of arc reversal. They presented two strongly polynomial time algorithms to solve maximum contraflow (MCF) problems in general graph with constant capacities on every arcs and maximum dynamic contraflow (MDCF) problems having a single source and a single sink network with constant capacities and transit times on every arcs. They also proved that the MDCF problem with only an additional source or sink turns out to NP-complete. In their contribution, it is shown that the quickest contraflow problem can be solved in a strongly polynomial time complexity for a single source and a single sink but the problems in the multiple sources and multiple sinks are NP-hard. Dhamala and Pyakurel [2] presented a polynomial time algorithm for the earliest arrival contraflow (EACF) problem on two terminal series parallel graph. Integer programming formulation and some heuristics for the contraflow problems have been presented by Kim et al.[4].

In this work, we formulated the EACF problem, where as many evacuees as possible should be sent from the sources to the sinks in every time period by reversing the road directions at time zero. A polynomial time algorithm for this problem on a TTSP-graph having capacities and transit times on the arcs has been presented.

The organization of the paper is as follows. In Section 2, we provide the problem formulations for evacuation planning. We present the development on contraflow and EACF problem for evacuation planning in Section 3. Concluding remarks and future works are mensioned in Section 4.

2. PROBLEM FORMULATIONS

Let $G = (V, E)$ be a directed graph with set of nodes V, set of arcs E, capacities $u_e \in \mathcal{Z}^+$. The capacity $u_e \in \mathcal{Z}^+$ denotes the maximum amount of flow which may enter the arc $e \in E$ per time period. Let $S^+ \in V$ be the source node that denotes the dangerous place where evacuees are situated and $S^- \in V$ be the sink node that denotes the safe destination having enough capacities for the evacuees from the source. The directed path from source to sink is called a chain. Let $x_1, ..., x_n$, for all $x_i \in V$ be chain if $(x_i, x_{i+1}) \in E$, for all $i = 1, ..., n$. The set of all chains from source to sink is represented by P. The function $f : E \to \Re_0^+$ defines the feasible static flow from source S^+ to sink S^- if it satifies the following constraints.

$$f_e \leq u_e \tag{2.1}$$

$$\sum_{(i,j)\in E} f_{i,j} - \sum_{(j,k)\in E} f_{j,k} = \begin{cases} v, & j = S^+ \\ 0, & \forall j \in V\backslash\{S^+, S^-\} \\ -v, & j = S^- \end{cases} \tag{2.2}$$

Equation (2.1) gives the capacity constraint. In a graph, the upper bound of flow is the capacity of the arc and the lower bound is assumed to be zero. The total flow at source S^+ and at sink S^- are v and $-v$, respectively, and the flow conservation is satisfied in other nodes according to equation (2.2).

In dynamic network $G = (V, E)$ each arc is associated with a travel time $\tau : E \to \mathcal{Z}^+$ with capacity function. The travel time gives the time required to travel an arc $e \in E$ from tail node $t(e)$ to head node $h(e)$. Let T be the given integer time horizon.

A dynamic network flow funtion is defined as $v : E \times \{0, ..., T\} \rightarrow \Re_0^+$ and $v_e(\theta)$ be the amount of flow that enters arc e from the node $t(e)$ at time θ and reaches node $h(e)$ at time $\theta + \tau_e$. Then, the maximum dynamic flow (MDF) requires the maximum amount of flow that can be sent from source S^+ to sink S^- in the given integer time horizon T. The linear programming formulation of MDF problem is as defined in ([3], [6]), as follows:

$$\max \mathcal{F} \tag{2.3}$$

$$s.t. \sum_{e \in E,\, t(e)=S^+} \sum_{\theta=0}^{T} v_e(\theta) = \mathcal{F} \tag{2.4}$$

$$\sum_{e \in E,\, h(e)=S^-} \sum_{\theta=0}^{T} v_e(\theta - \tau_e) = \mathcal{F} \tag{2.5}$$

$$\sum_{e \in E,\, h(e)=i} v_e(\theta + \tau_e) = \sum_{e \in E,\, t(e)=i} v_e(\theta) \quad \text{for all } i \in V \backslash \{S^+, S^-\},\ \theta \in \{0, ..., T\}. \tag{2.6}$$

$$0 \leq v_e(\theta) \leq u_e \text{ for all } e \in E,\ \theta \in \{0, ..., T\}. \tag{2.7}$$

Equation (2.3) gives the objective of the MDF problem that maximizes the total amount of flow value $\mathcal{F}$ that can be sent from the source S^+ to the sink S^- within the time horizon T. The flow conservation at nodes is described by the constraints (2.4), (2.5) and (2.6) and the constraint (2.7) gives the boundary of capacity.

The dynamic flow for a graph $G = (V, E, T)$ is equivalent to the static flow in time expanded graph $G_T = (V_T, E_T)$ and vice versa [3]. Time expanded networks are defined as the expansion of the dynamic network where each node i of the static graph is copied T times to obtain a node $i(\theta)$ for each $i \in V$ and each $\theta \in \{0, ..., T\}$. For each arc $e = (i, j) \in E$, let $i(\theta)$ be the tail and $j(\theta + \tau_e)$ be the head of arc e having capacity u_e, called movement arcs. For each arc $e = (i, j) \in E$, let $i(\theta)$ be the tail and $i(\theta + \tau_e)$ be the head of arc e having capacity ∞, called holdover arcs which allows storage at the nodes. We use only movement arcs of the time expanded network because holdover arcs need not be considered for the MDF [3]

Ruzika et al. [6] considered the earliest arrival flow problem on TTSP-graphs. A single arc $e = (S^+, S^-)$ is series-parallel with starting terminal S^+ and end terminal S^-. Let G_1 and G_2 be two series-parallel graphs with starting terminals S_1^+ and S_2^+ and the end terminals S_1^- and S_2^-, respectively. Then, the graph $S(G_1, G_2)$ obtained by identifying S_1^- as S_2^+ in the series combination is a series-parallel graph, with S_1^+ and S_2^- as its terminals. The graph $P(G_1, G_2)$ obtained by identifying S_1^+ as S_2^+ and also S_1^- as S_2^- in the parallel combination is a series-parallel graph with $S_1^+(= S_2^+)$ and $S_1^-(= S_2^-)$ as its terminals. Then, they find temporally repeated flow by standard chain decomposition for the MDF problem and the particular EAF problem in this class of graphs.

3. CONTRAFLOW

In this section, we discuss about the development of optimization techniques to deal contraflow for evacuation planning. In Subsection 3.1, we provide the MCF problem for static network and its solution procedure. In the case of dynamic network, we discuss the MDCF problem and its optimal solution techniques in Subsection 3.2. In Subsection 3.3, we provide the polynomial algorithm to solve earliest arrival contraflow problem on TTSP-graphs.

3.1. **Maximum (Static) Contraflow.** The MCF problem is also known as maximum flow problem with arc reversal. For a directed graph $G = (V, E)$, with source $S^+ \in V$ and sink $S^- \in V$, the MCF problem finds the maximum flow from source S^+ to sink S^- with boundary of capacity $u_e \in \mathcal{Z}^+$ on each arc $e \in E$, if the direction of arcs can be reversed.

In order to find the contraflow configuration of a graph, the original graph is transformed by summing the capacity of arcs (i, j) and (j, i) such that the MCF problem will reduce to the maximum flow problem (MFP). Then, the MFP is solved on the transformed graph to find the optimal solution of MCF problem. Rebennack et al.[5] presented an algorithm to find the MCF that solves the MCF problem in strongly polynomial time complexity for single source and single sink garph. For multiple sources and multiple sinks graph, they proved that the MCF problem is polynomially solvable.

Rebennack et al.[5] proved that the MCF problem is equivalent to the MFP on an undirected (transformed)graph. The running time of their algorithm is $O((S_1(|V|, |E|)+ S_2(|V|, |E|))$ where $S_1(|V|, |E|) = O(|V|^2.\sqrt{E})$ solves the maximum flow problem and $S_2(|V|, |E|) = O(|V|.|E|)$ solves the flow decomposition problem.

3.2. **Maximum Dynamic Contraflow.** In the MDCF problem, we maximize the flow sent from the source to the sink within the given time horizon by reversing the direction of arc. For a directed graph $G = (V, E, T)$ with a single source $\mathrm{S}^+ \in V$ and a single sink $S^- \in V$ having travel time $\tau_{ij} \in \mathcal{Z}^+$ with $\tau_{ij} = \tau_{ji}$ for $(i, j), (j, i) \in E$ and capacity $u_{ij} \in \mathcal{Z}^+$ for each $(i, j) \in E$, the MDCF problem requires to find the maximum amount of flow that can be sent within the given integer time T units from the source S^+ to the sink S^- if the direction of the arcs can be reversed at time 0.

First polynomial algorithm for MDCF problem has been given by Arulselvan [1] and Rebennack et al. [5]. According to the constrction of MDCF problem, the graph is allowed to be asymmetric with respect to the arc capacities. Although, both directions of an arc are included in the graph, the travelling time of these two arcs must be the same. This assumption implies that the switching of an arc only changes the capacities of the arcs but the travelling time remains same. After the contraflow configuration of the original graph, a temporally repeated chain flow is obtained to find the MDF in the transformed graph using algorithm of Ford and Fulkerson [3]. Then, the resulting flow gives the MDCF for the original graph. The algorithm presented by Rebennack et al. [5] solves the MDCF problem in strongly polynomial time for sinlge source single sink transportation network. They also solved the quickest flow problem where the time to send a given flow from source to sink is minimized, is also polynomially solvable for single source and single sink network.

The running time of the MDCF algorithm is $O(S_2(|V|,|E|)+S_3(|V|,|E|))$ where $S_2(|V|,|E|) = O(|V|.|E|)$ solves the flow decomposition problem and $S_3(|V|,|E|) = O(|V|^2.|E|^3.log|V|)$ solves the minimum cost flow problem. By reductions from the problems 3-SAT and from PARTITION, respectively, Kim et al. [4] and Rebennack et al. [5] proved that the MDCF problem remains NP-hard in the strong sense even with two sources and one sink.

3.3. **Earliest Arrival Contraflow.** Given a TTSP-graph $G = (V, E, T)$ with a single source $\mathrm{S}^+ \in V$ and a single sink $S^- \in V$, travel time $\tau_{ij} \in \mathcal{Z}^+$ with $\tau_{ij} = \tau_{ji}$ for $(i, j), (j, i) \in E$ and capacity $u_{ij} \in \mathcal{Z}^+$ for each $(i, j) \in E$, the EACF problem on TTSP-graph requires the maximum amount of flow that can be sent in every time period $\theta, 0 \leq \theta \leq T$ from the source S^+ to the sink S^- if the direction of the arcs can be reversed at time 0.

The model of EACF has been developed by Dhamala and Pyakurel [2]. They developed an algorithm (c.f. Algorithm 3.1) on TTSP-graphs that solves the EACF problem in strongly polynomial time complexity. The development of their algorithm has been based on the MDCF algorithm on general graphs of Rebennack et al. [5] and the minimum cost circulation flows (MCCF) algorithm on TTSP-graphs of Ruzika et al. [6].

Dhamala and Pyakurel [2] varified with an example that the MDCF algorithm on general graph [5] can not solve the EACF problem. Then, they modified the MDCF algorithm using MCCF algorithm and obtained the MDCF algorithm on TTSP-graphs that solve the EACF problem in strongly polynomial time complexity.

Algorithm 3.1. Maximum Dynamic Contraflow on TTSP-Graphs

1. Let the transformed graph be $G' = (V, E', T)$ with arc set defined as $(i, j) \in E'$ if $(i, j) \in E$ or $(j, i) \in E$.
   For all arcs $(i, j) \in E'$ the capacity function u' is $u'_{ij} = u_{ij} + u_{ji}$
   and the transit time is

$$\tau'_{ij}(= \tau'_{ji}) = \begin{cases} \tau_{ij} & \mathrm{i}f(i,j) \in E \\ \tau_{ji} & otherwise \end{cases}$$

2. In the transformed graph $G^{'}$, we generate a dynamic temporally repeated flow with capacity $u^{'}_{ij}$ and travel time $\tau^{'}_{ij}$ using the MCCF algorithm [6].
3. Obtain flow decomposition into path and cycle flows of the resulting network from Step 2. Remove the cycle flows.
4. Arc $(j, i) \in E$ is reversed, if and only if the flow along arc (i, j) is greater than u_{ij} or if there is a non-negative flow along arc $(i, j) \notin E$ with highest gain and the resulting flow is MDF with the arc reversal for the network $G = (V, E, T)$.

Theorem 3.1. *The Algorithm 3.1 solves the MDCF problem for TTSP-graph $G = (V, E, T)$ optimally.*

The Theorem 3.1 has been proved in two steps [2]. First, it has been verified whether it is feasible or not. Obiously, it is feasible. Then, the optimality has been proved with the help of time expanded network and Lemma 3.2 on the TTSP-graph.

Lemma 3.2. *For TTSP-graph $G = (V, E, T)$, the maximum amount of flow in the MDCF problem is less than or equal to the optimal flow in the MCF for the corresponding time expanded graph $G_T = (V_T, E_T)$.*

The complexity of the Algorithm 3.1 in case of TTSP-graph has been proved in comparision to the general graph.

Theorem 3.3. [2] *Algorithm 3.1 solves the MDCF problem in strongly polynomial time for TTSP-graphs.*

Proof: Step 1 and Step 4 of the Algorithm 3.1 are solved in linear time. The flow decomposition in Step 3 can be done in O($|V||E^{'}|$) time. The running time of the MCCF algorithm is $O(|V||E^{'}| + |E^{'}|log|E^{'}|)$ [6]. Hence, total complexity of the Algorithm 3.1 is $O(|V||E^{'}| + |E^{'}|log|E^{'}|)$.

The MDCF Algorithm 3.1 gives the temporally repeated chain flow which is the main requirement for EACF problem. Thus, Dhamala and Pyakurel [2] proved the existance of EACF on TTSP-graphs giving optimal solution in strongly polynomial time with the Theorem 3.4 and Theorem 3.5.

Theorem 3.4. *Any MDCF solution induced by Algorithm 3.1 has earliest arrival property for TTSP-graph.*

Theorem 3.5. *Algorithm 3.1 solves the EACF problem optimally in strongly polynomial time for TTSP-graphs.*

Example 3.6. We use the contraflow model to the network given in fig 3.1. By Step 1 of Algorithm 3.1, reverse all backward arcs towards the sink S^{-} as shown in fig

3.2. The contraflow configuration of this series-parallel graph will give the MDCF solution using the MCCF algorithm [6] of the original graph fig 3.1. The flow along the backward arc (S^-, S^+) is ignored and the flows from S^+ to S^- are decomposed into the directed cycles. Here the MDF value is 30.

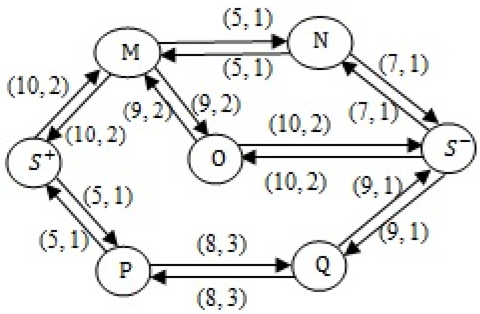

Figure 3.1. Evacuation scenario of TTSP-graph

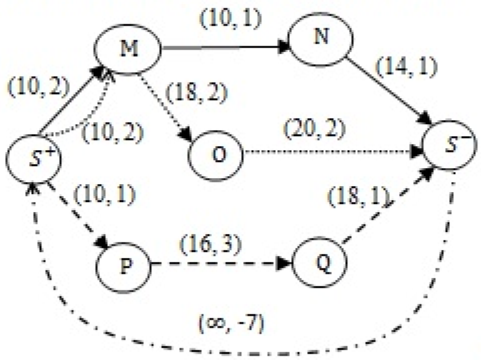

Figure 3.2. Contraflow configuration of TTSP-graph

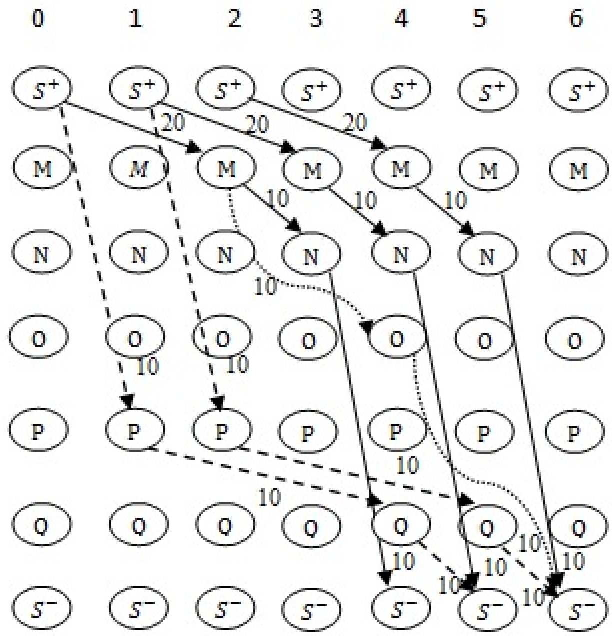

Figure 3.3. Earliest arrival contraflow for TTSP- graph of fig. 3.2

The time expanded network of fig 3.2 is represented by fig 3.3 that gives an EAF solution. At time 4, the first 10 flow units reach to the sink through the path $S^+ - M - N - S^-$. At time 5, another 20 flow units is added through the paths $S^+ - M - N - S^-$ and $S^+ - P - Q - S^-$. Similarly, another 30 more flow units reach to the sink at time 6 through three paths $S^+ - M - N - S^-$, $S^+ - P - Q - S^-$ and $S^+ - M - O - S^-$. Hence, up to time six, 60 flow units has reached at the sink in total. Note that this flow is temporally repeated because all paths are used continuously to flow from the source to sink and it will be continuous with the increment of the time horizon T. There is a temporally repeated flow obtained by Algorithm 3.1 and has the earliest arrival property in TTSP-graphs.

4. CONCLUDING REMARKS

In this paper, we discussed the earliest arrival contraflow model for evacuation planning developed on TTSP-graphs [2] that solved the EACF problem in strongly polynomial time complexity. The earlist arrival contraflow problem has not solved in general graph. In our future work, we are going to generalize the maximum dynamic contraflow problem and the earliest arrival contraflow problem in more general transportation network with additional constraints.

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