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PERFORMANCE ANALYSIS OF PULPING AND SCREENING SYSTEM OF A PAPER PRODUCTION PLANT

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Abstract
The performance evaluation of a paper plant is an important factor in improving its production. This paper aims to present a practical framework to measure the reliability characteristics of pulping and screening systems of a paper plant. There is an increasing demand of paper that must be reconciled with a growing environment concern. This requires focusing research on the development of more effective production of paper. There are a number of processes involved in paper making. Out of these pulping and screening system are the foremost concern for the effectively paper production. The aim of pulping system is to collapse bulk of fiber structures and then the outcome goes for screening and then for further processes of paper making. A mathematical model has been developed in this work, for the evaluation of the reliability characteristics of the pulping and screening units of a paper plant.

Key Words: Production system, Sensitivity Analysis, Cost-effectiveness.

1. Introduction
In the present hi-tech scenario, industrial system and its component availability have a high rank of status. System availability is a specific combination of dependability and maintainability. It is anticipated standard of the performance of the system under the specified conditions. In most of complicated industries, it is observed that these consist of structures and substructures associated in series or parallel or a union of these two. For a paper plant point of view, paper making is the process of making paper from the paper machine. In today's era, paper is used universally for printing, writing, packaging and many other purposes. Firstly the mixture of pulp and water goes into the pulping system and then water is removed from this mixture by pressing and drying. A comprehensive literature review reflects that several approaches have been used to analyze the steady state behaviour of a paper plant.
In earlier research work related to the reliability field, a lot of work has been done by a number of the authors [Suhail (1983); Pan et al. (1986); Goel and Mumtaz (1994); Ram et al. (2013a,b), Khatab et al. (2013), Ram and Manglik (2014), Ram and Kumar (2015), Manglik and Ram] on industrial based complex systems and it has become an even greater concern in recent years, because high-tech industry processes with increasing levels of sophistication comprise most engineering systems today [El-Neweihi and Proschan (1984); Verma et al. (2010), Ram (2013)]. Dhillion (1992) presented the reliability measures analysis of a two unit parallel system with warm stand by and common cause failure and failed system repair time is assumed to be distributed arbitrary. Castro and Cavalca (2003) presented an availability optimization problem of an engineering system assembled in series configuration by using genetic algorithm. The objective of this paper is to reach the maximum value of availability

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considering the maintenance cost, weight, volume and available maintenance terms as constraints. For this, the authors have used an algorithm which is based on biological concepts of species evolution. Sachdeva et al. (2008) dealt with the reliability analysis of the pulping system in paper industry using petri nets technique and had drawn some important results and concluded that the digester is a critical part of the paper plant. Khanduja et al. (2010) have discussed the performance evaluation for washing unit of a paper plant and of the digesting system of paper plant using genetic algorithm. Besides, the effect of genetic algorithm parameters such as number of generations, population size and crossover probability on the system performance i.e. availability has also been analyzed. Rani et al. (2011) also discussed about the washing unit of paper mill using artificial bee colony technique. This paper presents an artificial bee colony technique to search the optimal solution for availability redundancy allocation problem with non-linear resource constraints of a parallel-series system. The decision variable corresponding to the washing unit is identified, which may be targeted for optimal performance of washing unit of paper plant. Garg et al. (2012) presented a cost minimization of a washing unit of a paper mill using artificial bee colony technique. The objective of this paper is to improve the design efficiency and to find the most optimal policies in mean time between failure and mean time to repair. Results are shown by the mean of the pooled *t* - test with other evolutionary algorithm.

Thus, we see that a lot of research has been done in reliability theory for pulping and screening system of the paper plant, but none of them have considered both of the systems simultaneously i.e. the pulping and screening system of paper plant. So from here, one has got an idea to develop a mathematical model which consists both of these two systems.

2. System Description

In the present paper, the authors have developed a mathematical model which deals with the pulping and screening system of a paper plant. These are one of the most essential parts of a paper plant. There are four segments in pulping system, namely digester, knotter, washing system and opener. Knotter and opener, both contain two units in parallel configuration. Also the washing unit is of 2-out-of-3: F type configuration. The pulping system is connected with the screening system in series configuration as shown in Fig. 1(a).

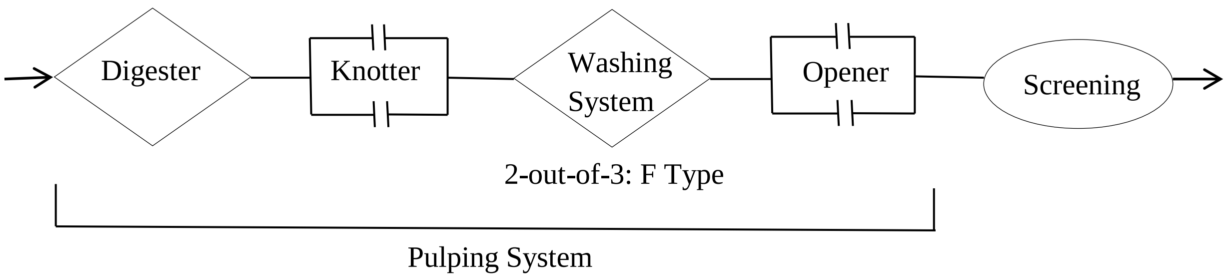

Fig. 1(a): System Configuration

The considered system has three states, namely good, degraded and failed state. The failure rates are considered to be constant and repairs follow the general time distribution. The state transition diagram of the paper plant has been shown in Fig. 1(b).

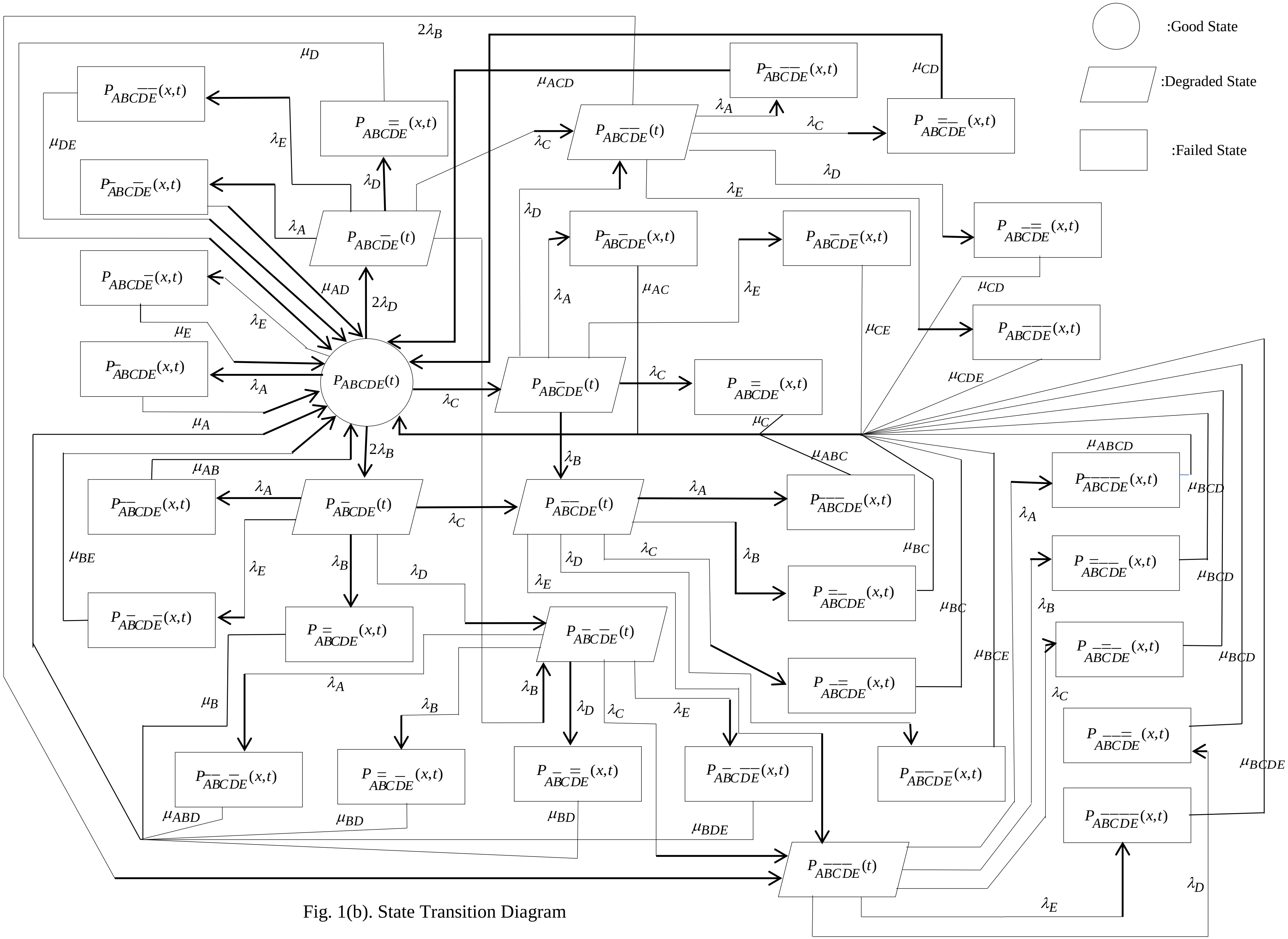

Fig. 1(b). State Transition Diagram

3. Assumptions and Notations

The following assumptions have been taken to study the proposed model:
(i) Initially the system is free from all failures.
(ii) Each failure is either present or absent.
(iii) A repaired unit is as good as new one.
(iv) There is no waiting time for repair of a failed unit.
(v) The system may work with reduced capacity.
(vi) Failure rates of the system are taken to be constant.

The following notations are used in the design model:

t/s	Time scale/ Laplace transform variable
$P_{AB\bar{C}DE}(t)$	The probability that at time t the system is working with one failed washing system.
$P_{A\bar{B}CDE}(t)$	The probability that at time t the system is working with one failed unit of knotter.
$P_{A\bar{B}\bar{C}DE}(t)$	The probability that at time t the system is working with one failed unit of knotter and one failed washing system.
$P_{A\bar{B}C\bar{D}E}(t)$	The probability that at time t the system is working with one failed unit of knotter and one failed unit of opener.
$P_{A\bar{B}\bar{C}\bar{D}E}(t)$	The probability that at time t the system is working with one failed unit of knotter, one failed washing system and one failed unit of opener.
$P_{ABC\bar{D}E}(t)$	The probability that at time t the system is working with one failed unit of opener.
$P_{AB\bar{C}\bar{D}E}(t)$	The probability that at time t the system is working with one failed washing system and one failed unit of opener.
$P_{AB\bar{\bar{C}}DE}(x,t)$	The probability that at time t the system is failed due to failure of washing unit.
$P_{\bar{A}BCDE}(x,t)$	The probability that at time t the system is failed due to failure of digester.
$P_{ABCD\bar{E}}(x,t)$	The probability that at time t the system is failed due to failure of the screening system.
$P_{\bar{A}BC\bar{D}E}(x,t)$	The probability that at time t the system is failed due to failure of digester and one unit of opener.
$P_{ABC\bar{D}\bar{E}}(x,t)$	The probability that at time t the system is failed due to failure of the screening system and one unit of opener.
$P_{ABC\bar{\bar{D}}E}(x,t)$	The probability that at time t the system is failed due to complete failure of opener.
$P_{\bar{A}B\bar{C}DE}(x,t)$	The probability that at time t the system is failed due to failure of digester and one failed washing system.
$P_{AB\bar{C}D\bar{E}}(x,t)$	The probability that at time t the system is failed due to one failed washing system and failure of screening system.
$P_{\bar{A}B\bar{C}\bar{D}E}(x,t)$	The probability that at time t the system is failed due to failure of digester, one failed washing system and one unit failure of opener.
$P_{AB\bar{C}\bar{\bar{D}}E}(x,t)$	The probability that at time t the system is failed due to one failed washing system and complete failure of opener.
$P_{AB\bar{\bar{C}}\bar{D}E}(x,t)$	The probability that at time t the system is failed due to one failed unit of opener and complete failure of washing unit.
$P_{AB\bar{C}\bar{D}\bar{E}}(x,t)$	The probability that at time t the system is failed due to one failed washing system, one failed unit of opener and complete failure of screening system.
$P_{\bar{A}\bar{B}CDE}(x,t)$	The probability that at time t the system is failed due to failure of digester and failure of one unit of knotter.

$P_{A\bar{B}CD\bar{E}}(x,t)$	The probability that at time *t* the system is failed due to one failed unit of knotter and complete failure of screening system.
$P_{A\bar{\bar{B}}CDE}(x,t)$	The probability that at time *t* the system is failed due to complete failure of knotter.
$P_{\bar{A}\bar{B}C\bar{D}E}(x,t)$	The probability that at time *t* the system is failed due to failure of one unit of knoter, one unit of opener and complete failure of digester.
$P_{A\bar{B}C\bar{\bar{D}}E}(x,t)$	The probability that at time *t* the system is failed due to failure of one unit of knotter and complete failure of opener.
$P_{A\bar{\bar{B}}C\bar{D}E}(x,t)$	The probability that at time *t* the system is failed due to failure of one unit of opener and complete failure of knotter.
$P_{A\bar{B}C\bar{D}\bar{E}}(x,t)$	The probability that at time *t* the system is failed due to failure of one unit of knotter, one unit of opener and complete failure of screening system.
$P_{A\bar{B}\bar{C}D\bar{E}}(x,t)$	The probability that at time *t* the system is failed due to failure of one unit of knotter, one washing system and complete failure of screening system.
$P_{A\bar{B}\bar{\bar{C}}DE}(x,t)$	The probability that at time *t* the system is failed due to failure of one unit of knotter and complete failure of washing unit.
$P_{A\bar{\bar{B}}\bar{C}DE}(x,t)$	The probability that at time *t* the system is failed due to failure of one washing system and complete failure knotter.
$P_{\bar{A}\bar{B}\bar{C}DE}(x,t)$	The probability that at time *t* system is failed due to failure of one unit of knotter, one washing system and complete failure of digester.
$P_{A\bar{B}\bar{C}\bar{D}\bar{E}}(x,t)$	The probability that at time *t* the system is failed due to failure of one unit of knotter, one washing system, one unit of opener and complete failure of screening system.
$P_{A\bar{B}\bar{C}\bar{\bar{D}}E}(x,t)$	The probability that at time *t* the system is failed due to failure of one unit of knotter, one washing system and complete failure of opener.
$P_{A\bar{B}\bar{\bar{C}}\bar{D}E}(x,t)$	The probability that at time *t* the system is failed due to failure of one unit of knotter, one unit of opener and complete failure of washing unit.
$P_{A\bar{\bar{B}}\bar{C}\bar{D}E}(x,t)$	The probability that at time *t* the system is failed due to failure of one unit of opener, one washing system and complete failure of knotter.
$P_{\bar{A}\bar{B}\bar{C}\bar{D}E}(x,t)$	The probability that at time *t* the system is failed due to failure of one unit of knotter, one washing system, one unit of opener and complete failure of digester.
$\lambda_A / \lambda_B / \lambda_C / \lambda_D / \lambda_E$	Failure rate of digester/ knotter/ washing system/screening system.
$\mu_A / \mu_B / \mu_C / \mu_D / \mu_E$	Repair rate of digester/ knotter/ washing system/screening system.
$\mu_{BD} / \mu_{BC} / \mu_{BE} / \mu_{AB}$ $\mu_{AD} / \mu_{DE} / \mu_{CE} /$ μ_{AC} / μ_{CD}	Simultaneous repair rate of knotter and opener/knotter and washing system/knotter and screening system/digester and knotter/ digester and opener/opener and screening system/washing system and screening system/digester and washing system/washing system and opener.
$\mu_{ABD} / \mu_{BDE} / \mu_{BCE} /$ $\mu_{ABC} / \mu_{BCD} / \mu_{ACD} /$ μ_{CDE}	Simultaneous repair rate of digester, knotter and opener/knotter, opener and screening system/knotter, washing system and screening system/ digester, knotter and washing system/ knotter washing system and opener/ digester, washing system and opener/ washing system, opener and screening system.

μ_{ABCD}/μ_{BCDE}	Simultaneous repair rate of digester, knotter washing system and opener /knotter, washing system, opener and screening system.
K_1/K_2	Revenue/ service cost per unit time.

4. Mathematical Formulation and Solution of the Model

By the probability considerations and Markov birth-death process, we can obtain the following set of differential equations from the state transition diagram.

$$\left(\frac{\partial}{\partial t}+\lambda_A+2\lambda_B+\lambda_C+2\lambda_D+\lambda_E\right)P_{ABCDE}(t)=\sum_{i,j}\int_0^\infty \mu_i P_j(x,t)dx$$

where $i = DE, AD, E, A, D, AB, BE, B, ABD, BD, BD, BDE, BCE, BC, BC, ABC, ABCD, BCD, BCD, BCD, BCDE, C, AC, CE, ACD, CD, CD, CDE$

$j = ABC\bar{D}\bar{E}, \bar{A}BC\bar{D}E, ABCD\bar{E}, \bar{A}BCDE, ABC\bar{\bar{D}}E, \bar{A}\bar{B}CDE, A\bar{B}CD\bar{E}, A\bar{\bar{B}}CDE, \bar{A}\bar{B}C\bar{D}E, A\bar{\bar{B}}C\bar{D}E, A\bar{B}C\bar{\bar{D}}E, A\bar{B}C\bar{D}\bar{E}, \bar{A}\bar{B}CD\bar{E}, A\bar{B}\bar{\bar{C}}DE, A\bar{\bar{B}}\bar{C}DE, \bar{A}\bar{B}\bar{C}DE, \bar{A}\bar{B}\bar{C}\bar{D}E, A\bar{\bar{B}}\bar{C}\bar{D}E, A\bar{B}\bar{\bar{C}}\bar{D}E, A\bar{B}\bar{C}\bar{\bar{D}}E, A\bar{B}\bar{C}\bar{D}\bar{E}, AB\bar{\bar{C}}DE, \bar{A}B\bar{C}DE, AB\bar{C}D\bar{E}, \bar{A}B\bar{C}D\bar{E}, AB\bar{\bar{C}}\bar{D}E, AB\bar{C}\bar{\bar{D}}E, AB\bar{C}\bar{D}\bar{E}$

(1)

$$\left(\frac{\partial}{\partial t}+\lambda_A+2\lambda_B+\lambda_C+2\lambda_D+\lambda_E\right)P_{AB\bar{C}DE}(t)=\lambda_C P_{ABCDE}(t) \tag{2}$$

$$\left(\frac{\partial}{\partial t}+\lambda_A+\lambda_B+\lambda_C+2\lambda_D+\lambda_E\right)P_{A\bar{B}CDE}(t)=2\lambda_B P_{ABCDE}(t) \tag{3}$$

$$\left(\frac{\partial}{\partial t}+\lambda_A+\lambda_B+\lambda_C+2\lambda_D+\lambda_E\right)P_{A\bar{B}\bar{C}DE}(t)=\lambda_C P_{A\bar{B}CDE}(t)+\lambda_B P_{AB\bar{C}DE}(t) \tag{4}$$

$$\left(\frac{\partial}{\partial t}+\lambda_A+\lambda_B+\lambda_C+\lambda_D+\lambda_E\right)P_{A\bar{B}C\bar{D}E}(t)=2\lambda_D P_{A\bar{B}CDE}(t)+2\lambda_B P_{ABC\bar{D}E}(t) \tag{5}$$

$$\left(\frac{\partial}{\partial t}+\lambda_A+\lambda_B+\lambda_C+\lambda_D+\lambda_E\right)P_{A\bar{B}\bar{C}\bar{D}E}(t)=2\lambda_B P_{AB\bar{C}\bar{D}E}(t)+\lambda_C P_{A\bar{B}C\bar{D}E}(t)+2\lambda_D P_{A\bar{B}\bar{C}DE}(t) \tag{6}$$

$$\left(\frac{\partial}{\partial t}+\lambda_A+2\lambda_B+\lambda_C+\lambda_D+\lambda_E\right)P_{ABC\bar{D}E}(t)=2\lambda_D P_{ABCDE}(t) \tag{7}$$

$$\left(\frac{\partial}{\partial t}+\lambda_A+2\lambda_B+\lambda_C+\lambda_D+\lambda_E\right)P_{AB\bar{C}\bar{D}E}(t)=\lambda_C P_{ABC\bar{D}E}(t)+2\lambda_D P_{AB\bar{C}DE}(t) \tag{8}$$

$$\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial t}+\mu_k\right)P_l(x,t)=0$$

where $k = DE, AD, E, A, D, AB, BE, B, ABD, BD, BD, BDE, BCE, BC, BC, ABC, BCDE, BCD, BCD, BCD, ABCD, CDE, CD, CD, CE, ACD, AC, C$

$l = ABC\bar{D}\bar{E}, \bar{A}BC\bar{D}E, ABCD\bar{E}, \bar{A}BCDE, ABC\bar{\bar{D}}E, \bar{A}\bar{B}CDE, A\bar{B}CD\bar{E}, A\bar{\bar{B}}CDE, \bar{A}\bar{B}C\bar{D}E, A\bar{\bar{B}}C\bar{D}E, A\bar{B}C\bar{\bar{D}}E, A\bar{B}C\bar{D}\bar{E}, \bar{A}\bar{B}CD\bar{E}, A\bar{B}\bar{\bar{C}}DE, A\bar{\bar{B}}\bar{C}DE, \bar{A}\bar{B}\bar{C}DE, \bar{A}\bar{B}\bar{C}\bar{D}E, A\bar{B}\bar{C}\bar{\bar{D}}E, A\bar{B}\bar{\bar{C}}\bar{D}E, A\bar{\bar{B}}\bar{C}\bar{D}E, \bar{A}\bar{B}\bar{C}\bar{D}E, A\bar{B}\bar{C}\bar{D}E, AB\bar{C}\bar{\bar{D}}E, AB\bar{\bar{C}}\bar{D}E, AB\bar{C}D\bar{E}, \bar{A}B\bar{C}\bar{D}E, \bar{A}B\bar{C}DE, AB\bar{\bar{C}}DE$

(9)

Boundary conditions

$$P_i(0,t)=\lambda_j P_k(t)$$

where $i = ABC\bar{\bar{D}}E, ABC\bar{D}\bar{E}, \bar{A}BC\bar{D}E, ABCD\bar{E}, \bar{A}BCDE, \bar{A}\bar{B}CDE, A\bar{B}CD\bar{E}, A\bar{\bar{B}}CDE,$
$\bar{A}\bar{B}C\bar{D}E, A\bar{\bar{B}}C\bar{D}E, A\bar{B}C\bar{\bar{D}}E, A\bar{B}C\bar{D}\bar{E}, A\bar{B}\bar{C}D\bar{E}, A\bar{B}\bar{\bar{C}}DE, A\bar{\bar{B}}\bar{C}DE, \bar{A}\bar{B}\bar{C}DE,$
$A\bar{B}\bar{C}\bar{D}\bar{E}, A\bar{B}\bar{C}\bar{\bar{D}}E, A\bar{B}\bar{\bar{C}}\bar{D}E, A\bar{\bar{B}}\bar{C}\bar{D}E, \bar{A}\bar{B}\bar{C}\bar{D}E, AB\bar{C}\bar{D}\bar{E}, AB\bar{C}\bar{\bar{D}}E, AB\bar{\bar{C}}\bar{D}E,$
$\bar{A}B\bar{C}\bar{D}E, AB\bar{\bar{C}}DE, AB\bar{C}D\bar{E}, \bar{A}B\bar{C}DE$

$j = D, E, A, E, A, A, E, B, A, B, D, E, E, C, B, A, E, D, C, B, A, E, D, C, A, C, E, A$

$k = ABC\bar{D}E, ABC\bar{D}E, ABC\bar{D}E, ABCDE, ABCDE, A\bar{B}CDE, A\bar{B}CDE, A\bar{B}CDE,$
$A\bar{B}C\bar{D}E, A\bar{B}C\bar{D}E, A\bar{B}C\bar{D}E, A\bar{B}C\bar{D}E, A\bar{B}\bar{C}DE, A\bar{B}\bar{C}DE, A\bar{B}\bar{C}DE, A\bar{B}\bar{C}DE,$
$A\bar{B}\bar{C}\bar{D}E, A\bar{B}\bar{C}\bar{D}E, A\bar{B}\bar{C}\bar{D}E, A\bar{B}\bar{C}\bar{D}E, A\bar{B}\bar{C}\bar{D}E, AB\bar{C}\bar{D}E, AB\bar{C}\bar{D}E, AB\bar{C}\bar{D}E,$
$AB\bar{C}\bar{D}E, AB\bar{C}DE, AB\bar{C}DE, AB\bar{C}DE$

(10)

Initial condition

$P_{ABCDE}(0)=1$ and all other state probabilities are zero at t = 0 (11)

Taking the Laplace transformation from Equations (1) to (10)

$$(s+\lambda_A+2\lambda_B+\lambda_C+2\lambda_D+\lambda_E)\bar{P}_{ABCDE}(s)=1+\sum_{i,j}\int_0^\infty \mu_i \bar{P}_j(x,s)dx \tag{12}$$

$$(s+\lambda_A+2\lambda_B+\lambda_C+2\lambda_D+\lambda_E)\bar{P}_{AB\bar{C}DE}(s)=\lambda_C\bar{P}_{ABCDE}(s) \tag{13}$$

$$(s+\lambda_A+\lambda_B+\lambda_C+2\lambda_D+\lambda_E)\bar{P}_{A\bar{B}CDE}(s)=2\lambda_B\bar{P}_{ABCDE}(s) \tag{14}$$

$$(s+\lambda_A+\lambda_B+\lambda_C+2\lambda_D+\lambda_E)\bar{P}_{A\bar{B}\bar{C}DE}(s)=\lambda_C\bar{P}_{A\bar{B}CDE}(s)+\lambda_B\bar{P}_{AB\bar{C}DE}(s) \tag{15}$$

$$(s+\lambda_A+\lambda_B+\lambda_C+\lambda_D+\lambda_E)\bar{P}_{A\bar{B}C\bar{D}E}(s)=2\lambda_D\bar{P}_{A\bar{B}CDE}(s)+2\lambda_B\bar{P}_{ABC\bar{D}E}(s) \tag{16}$$

$$(s+\lambda_A+\lambda_B+\lambda_C+\lambda_D+\lambda_E)\bar{P}_{A\bar{B}\bar{C}\bar{D}E}(s)=2\lambda_B\bar{P}_{AB\bar{C}\bar{D}E}(s)+\lambda_C\bar{P}_{A\bar{B}C\bar{D}E}(s)+2\lambda_D\bar{P}_{A\bar{B}\bar{C}DE}(s) \tag{17}$$

$$(s+\lambda_A+2\lambda_B+\lambda_C+\lambda_D+\lambda_E)\bar{P}_{ABC\bar{D}E}(s)=2\lambda_D\bar{P}_{ABCDE}(s) \tag{18}$$

$$(s+\lambda_A+2\lambda_B+\lambda_C+\lambda_D+\lambda_E)\bar{P}_{AB\bar{C}\bar{D}E}(s)=\lambda_C\bar{P}_{ABC\bar{D}E}(s)+2\lambda_D\bar{P}_{AB\bar{C}DE}(s) \tag{19}$$

$$\bar{P}_i(0,s)=\lambda_j\bar{P}_k(s) \tag{20}$$

Solving the equations from (12) to (19) with the help of boundary conditions and initial condition, we get the state probabilities of the system as given below

$$\bar{P}_{ABCDE}(s)=\frac{1}{[H_3-H_5-H_6-H_7-H_8-H_9-H_{10}-H_{11}]} \tag{21}$$

$$\bar{P}_{A\bar{B}C\bar{D}E}(s)=\bar{P}_{ABCDE}(s)\left(\frac{4\lambda_B\lambda_D}{H_1H_4}+\frac{4\lambda_B\lambda_D}{H_1H_2}\right) \tag{22}$$

$$\bar{P}_{AB\bar{C}DE}(s)=\bar{P}_{ABCDE}(s)\left(\frac{\lambda_C}{H_3}\right) \tag{23}$$

$$\bar{P}_{A\bar{B}CDE}(s)=\bar{P}_{ABCDE}(s)\left(\frac{2\lambda_B}{H_4}\right) \tag{24}$$

$$\bar{P}_{ABC\bar{D}E}(s)=\bar{P}_{ABCDE}(s)\left(\frac{2\lambda_D}{H_2}\right) \tag{25}$$

$$\bar{P}_{AB\bar{C}\bar{D}E}(s)=\bar{P}_{ABCDE}(s)\left(\frac{2\lambda_C\lambda_D}{H_2{}^2}+\frac{2\lambda_C\lambda_D}{H_2H_3}\right) \tag{26}$$

$$\overline{P}_{A\bar{B}\bar{C}DE}(s)=\overline{P}_{ABCDE}(s)\left(\frac{2\lambda_B\lambda_C}{H_4{}^2}+\frac{\lambda_B\lambda_C}{H_3H_4}\right) \tag{27}$$

$$\overline{P}_{A\bar{B}\bar{C}\bar{D}E}(s)=\overline{P}_{ABCDE}(s)\left(\frac{4\lambda_B\lambda_C\lambda_D}{H_1H_2{}^2}+\frac{4\lambda_B\lambda_C\lambda_D}{H_1H_2H_3}+\frac{4\lambda_B\lambda_C\lambda_D}{H_1{}^2H_4}\right.$$
$$\left.+\frac{4\lambda_B\lambda_C\lambda_D}{H_1{}^2H_2}+\frac{4\lambda_B\lambda_C\lambda_D}{H_1H_4{}^2}+\frac{4\lambda_B\lambda_C\lambda_D}{H_1H_3H_4}\right) \tag{28}$$

$$\overline{P}_{ABC\bar{D}\bar{E}}(s)=\frac{\lambda_E\overline{P}_{ABCDE}(s)}{(s+\mu_{DE})}\left(\frac{2\lambda_D}{H_2}\right) \tag{29}$$

$$\overline{P}_{\bar{A}BC\bar{D}E}(s)=\frac{\lambda_A\overline{P}_{ABCDE}(s)}{(s+\mu_{AD})}\left(\frac{2\lambda_D}{H_2}\right) \tag{30}$$

$$\overline{P}_{ABCD\bar{E}}(s)=\frac{\overline{P}_{ABCDE}(s)\lambda_E}{(s+\mu_E)} \tag{31}$$

$$\overline{P}_{\bar{A}BCDE}(s)=\frac{\overline{P}_{ABCDE}(s)\lambda_A}{(s+\mu_A)} \tag{32}$$

$$\overline{P}_{ABC\bar{\bar{D}}E}(s)=\frac{\lambda_D\overline{P}_{ABCDE}(s)}{(s+\mu_D)}\left(\frac{2\lambda_D}{H_2}\right) \tag{33}$$

$$\overline{P}_{\bar{A}\bar{B}CDE}(s)=\frac{\lambda_A\overline{P}_{ABCDE}(s)}{(s+\mu_{DE})}\left(\frac{2\lambda_B}{H_4}\right) \tag{34}$$

$$\overline{P}_{A\bar{B}CD\bar{E}}(s)=\frac{\lambda_E\overline{P}_{ABCDE}(s)}{(s+\mu_E)}\left(\frac{2\lambda_B}{H_4}\right) \tag{35}$$

$$\overline{P}_{A\bar{\bar{B}}CDE}(s)=\frac{\lambda_B\overline{P}_{ABCDE}(s)}{(s+\mu_B)}\left(\frac{2\lambda_B}{H_4}\right) \tag{36}$$

$$\overline{P}_{\bar{A}\bar{B}C\bar{D}E}(s)=\frac{\lambda_A\overline{P}_{ABCDE}(s)}{(s+\mu_{ABD})}\left(\frac{4\lambda_B\lambda_D}{H_1H_4}+\frac{4\lambda_B\lambda_D}{H_1H_2}\right) \tag{37}$$

$$\overline{P}_{A\bar{\bar{B}}C\bar{D}E}(s)=\frac{\lambda_B\overline{P}_{ABCDE}(s)}{(s+\mu_{BD})}\left(\frac{4\lambda_B\lambda_D}{H_1H_4}+\frac{4\lambda_B\lambda_D}{H_1H_2}\right) \tag{38}$$

$$\overline{P}_{A\bar{B}C\bar{\bar{D}}E}(s)=\frac{\lambda_D\overline{P}_{ABCDE}(s)}{(s+\mu_{BD})}\left(\frac{4\lambda_B\lambda_D}{H_1H_4}+\frac{4\lambda_B\lambda_D}{H_1H_2}\right) \tag{39}$$

$$\overline{P}_{A\bar{B}C\bar{D}\bar{E}}(s)=\frac{\lambda_E\overline{P}_{ABCDE}(s)}{(s+\mu_{BDE})}\left(\frac{4\lambda_B\lambda_D}{H_1H_4}+\frac{4\lambda_B\lambda_D}{H_1H_2}\right) \tag{40}$$

$$\overline{P}_{A\bar{B}\bar{C}D\bar{E}}(s)=\frac{\lambda_E\overline{P}_{ABCDE}(s)}{(s+\mu_{BCE})}\left(\frac{2\lambda_B\lambda_C}{H_4{}^2}+\frac{\lambda_B\lambda_C}{H_3H_4}\right) \tag{41}$$

$$\overline{P}_{A\bar{B}\bar{\bar{C}}DE}(s)=\frac{\lambda_E\overline{P}_{ABCDE}(s)}{(s+\mu_{BC})}\left(\frac{2\lambda_B\lambda_C}{H_4{}^2}+\frac{\lambda_B\lambda_C}{H_3H_4}\right) \tag{42}$$

$$\overline{P}_{A\bar{\bar{B}}\bar{C}DE}(s)=\frac{\lambda_BP_{ABCDE}(s)}{(s+\mu_{BC})}\left(\frac{2\lambda_B\lambda_C}{H_4{}^2}+\frac{\lambda_B\lambda_C}{H_3H_4}\right) \tag{43}$$

$$\overline{P}_{\bar{A}\bar{B}\bar{C}DE}(s)=\frac{\lambda_AP_{ABCDE}(s)}{(s+\mu_{ABC})}\left(\frac{2\lambda_B\lambda_C}{H_4{}^2}+\frac{\lambda_B\lambda_C}{H_3H_4}\right) \tag{44}$$

$$\bar{P}_{A\bar{B}\bar{C}\bar{D}\bar{E}}(s)=\frac{\lambda_E\bar{P}_{ABCDE}(s)}{(s+\mu_{BCDE})}\left(\frac{4\lambda_B\lambda_C\lambda_D}{H_1H_2^{\,2}}+\frac{4\lambda_B\lambda_C\lambda_D}{H_1H_2H_3}+\frac{4\lambda_B\lambda_C\lambda_D}{H_1^{\,2}H_4}+\frac{4\lambda_B\lambda_C\lambda_D}{H_1^{\,2}H_2}+\frac{4\lambda_B\lambda_C\lambda_D}{H_1H_4^{\,2}}+\frac{4\lambda_B\lambda_C\lambda_D}{H_1H_3H_4}\right) \tag{45}$$

$$\bar{P}_{A\bar{B}\bar{C}\bar{\bar{D}}E}(s)=\frac{\lambda_D\bar{P}_{ABCDE}(s)}{(s+\mu_{BCD})}\left(\frac{4\lambda_B\lambda_C\lambda_D}{H_1H_2^{\,2}}+\frac{4\lambda_B\lambda_C\lambda_D}{H_1H_2H_3}+\frac{4\lambda_B\lambda_C\lambda_D}{H_1^{\,2}H_4}+\frac{4\lambda_B\lambda_C\lambda_D}{H_1^{\,2}H_2}+\frac{4\lambda_B\lambda_C\lambda_D}{H_1H_4^{\,2}}+\frac{4\lambda_B\lambda_C\lambda_D}{H_1H_3H_4}\right) \tag{46}$$

$$\bar{P}_{A\bar{B}\bar{\bar{C}}\bar{D}E}(s)=\frac{\bar{P}_{ABCDE}(s)\lambda_C}{(s+\mu_{BCD})}\left(\frac{4\lambda_B\lambda_C\lambda_D}{H_1H_2^{\,2}}+\frac{4\lambda_B\lambda_C\lambda_D}{H_1H_2H_3}+\frac{4\lambda_B\lambda_C\lambda_D}{H_1^{\,2}H_4}+\frac{4\lambda_B\lambda_C\lambda_D}{H_1^{\,2}H_2}+\frac{4\lambda_B\lambda_C\lambda_D}{H_1H_4^{\,2}}+\frac{4\lambda_B\lambda_C\lambda_D}{H_1H_3H_4}\right) \tag{47}$$

$$\bar{P}_{A\bar{\bar{B}}\bar{C}\bar{D}E}(s)=\frac{\bar{P}_{ABCDE}(s)\lambda_B}{(s+\mu_{BCD})}\left(\frac{4\lambda_B\lambda_C\lambda_D}{H_1H_2^{\,2}}+\frac{4\lambda_B\lambda_C\lambda_D}{H_1H_2H_3}+\frac{4\lambda_B\lambda_C\lambda_D}{H_1^{\,2}H_4}+\frac{4\lambda_B\lambda_C\lambda_D}{H_1^{\,2}H_2}+\frac{4\lambda_B\lambda_C\lambda_D}{H_1H_4^{\,2}}+\frac{4\lambda_B\lambda_C\lambda_D}{H_1H_3H_4}\right) \tag{48}$$

$$\bar{P}_{\bar{A}\bar{B}\bar{C}\bar{D}E}(s)=\frac{\bar{P}_{ABCDE}(s)\lambda_A}{(s+\mu_{ABCD})}\left(\frac{4\lambda_B\lambda_C\lambda_D}{H_1H_2^{\,2}}+\frac{4\lambda_B\lambda_C\lambda_D}{H_1H_2H_3}+\frac{4\lambda_B\lambda_C\lambda_D}{H_1^{\,2}H_4}+\frac{4\lambda_B\lambda_C\lambda_D}{H_1^{\,2}H_2}+\frac{4\lambda_B\lambda_C\lambda_D}{H_1H_4^{\,2}}+\frac{4\lambda_B\lambda_C\lambda_D}{H_1H_3H_4}\right) \tag{49}$$

$$\bar{P}_{AB\bar{C}\bar{D}\bar{E}}(s)=\frac{\bar{P}_{ABCDE}(s)\lambda_E}{(s+\mu_{CDE})}\left(\frac{2\lambda_C\lambda_D}{H_2^{\,2}}+\frac{2\lambda_C\lambda_D}{H_2H_3}\right) \tag{50}$$

$$\bar{P}_{AB\bar{\bar{C}}\bar{D}E}(s)=\frac{\bar{P}_{ABCDE}(s)\lambda_C}{(s+\mu_{CD})}\left(\frac{2\lambda_C\lambda_D}{H_2^{\,2}}+\frac{2\lambda_C\lambda_D}{H_2H_3}\right) \tag{51}$$

$$\bar{P}_{AB\bar{C}\bar{\bar{D}}E}(s)=\frac{\bar{P}_{ABCDE}(s)\lambda_D}{(s+\mu_{CD})}\left(\frac{2\lambda_C\lambda_D}{H_2^{\,2}}+\frac{2\lambda_C\lambda_D}{H_2H_3}\right) \tag{52}$$

$$\bar{P}_{\bar{A}B\bar{C}\bar{D}E}(s)=\frac{\bar{P}_{ABCDE}(s)\lambda_A}{(s+\mu_{ACD})}\left(\frac{2\lambda_C\lambda_D}{H_2^{\,2}}+\frac{2\lambda_C\lambda_D}{H_2H_3}\right) \tag{53}$$

$$\bar{P}_{AB\bar{C}D\bar{E}}(s)=\frac{\bar{P}_{ABCDE}(s)\lambda_E}{(s+\mu_{CE})}\left(\frac{\lambda_C}{H_3}\right) \tag{54}$$

$$\bar{P}_{\bar{A}B\bar{C}DE}(s)=\frac{\bar{P}_{ABCDE}(s)\lambda_A}{(s+\mu_{AC})}\left(\frac{\lambda_C}{H_3}\right) \tag{55}$$

$$\bar{P}_{AB\bar{\bar{C}}DE}(s)=\frac{\bar{P}_{ABCDE}(s)\lambda_C}{(s+\mu_C)}\left(\frac{\lambda_C}{H_3}\right) \tag{56}$$

where $H_1=(s+\lambda_A+\lambda_B+\lambda_C+\lambda_D+\lambda_E)$, $H_2=(s+\lambda_A+2\lambda_B+\lambda_C+\lambda_D+\lambda_E)$,
$H_3=(s+\lambda_A+2\lambda_B+\lambda_C+2\lambda_D+\lambda_E)$, $H_4=(s+\lambda_A+\lambda_B+\lambda_C+2\lambda_D+\lambda_E)$

$$H_5=\lambda_E\frac{\mu_E}{(s+\mu_E)}+\lambda_A\frac{\mu_A}{(s+\mu_A)}-2\frac{\lambda_D}{H_2}\left[\lambda_E\frac{\mu_{DE}}{(s+\mu_{DE})}+\lambda_A\frac{\mu_{AD}}{(s+\mu_{AD})}+\lambda_D\frac{\mu_D}{(s+\mu_D)}\right]$$

$$H_6=\frac{2\lambda_B}{H_4}\left[\lambda_A\frac{\mu_{AB}}{(s+\mu_{AB})}+\lambda_E\frac{\mu_{BE}}{(s+\mu_{BE})}+\lambda_B\frac{\mu_B}{(s+\mu_B)}\right]$$

$$H_7=\left[\frac{4\lambda_B\lambda_D}{H_1H_4}+\frac{4\lambda_B\lambda_D}{H_1H_2}\right]\left[\lambda_A\frac{\mu_{ABD}}{(s+\mu_{ABD})}+\lambda_B\frac{\mu_{BD}}{(s+\mu_{BD})}+\lambda_D\frac{\mu_{BD}}{(s+\mu_{BD})}+\lambda_E\frac{\mu_{BDE}}{(s+\mu_{BDE})}\right]$$

$$H_8=\left[\frac{2\lambda_B\lambda_C}{H_4^{\ 2}}+\frac{\lambda_B\lambda_C}{H_3H_4}\right]\left[\lambda_E\frac{\mu_{BCE}}{(s+\mu_{BCE})}+\lambda_C\frac{\mu_{BC}}{(s+\mu_{BC})}+\lambda_B\frac{\mu_{BC}}{(s+\mu_{BC})}+\lambda_A\frac{\mu_{ABC}}{(s+\mu_{ABC})}\right]$$

$$H_9=\frac{\lambda_C}{H_3}\left[\lambda_C\frac{\mu_C}{(s+\mu_C)}+\lambda_A\frac{\mu_{AC}}{(s+\mu_{AC})}+\lambda_E\frac{\mu_{CE}}{(s+\mu_{CE})}\right]$$

$$H_{10}=\left[\frac{2\lambda_C\lambda_D}{H_2^{\ 2}}+\frac{2\lambda_C\lambda_D}{H_2H_3}\right]\left[\lambda_A\frac{\mu_{ACD}}{(s+\mu_{ACD})}+\lambda_C\frac{\mu_{CD}}{(s+\mu_{CD})}+\lambda_D\frac{\mu_{CD}}{(s+\mu_{CD})}+\lambda_E\frac{\mu_{CDE}}{(s+\mu_{CDE})}\right]$$

$$H_{11}=\left[\frac{4\lambda_B\lambda_C\lambda_D}{H_1H_2^{\ 2}}+\frac{4\lambda_B\lambda_C\lambda_D}{H_1H_2H_3}+\frac{4\lambda_B\lambda_C\lambda_D}{H_1^{\ 2}H_4}\frac{4\lambda_B\lambda_C\lambda_D}{H^2{}_1H_2}+\frac{4\lambda_B\lambda_C\lambda_D}{H_1H_4^{\ 2}}+\frac{2\lambda_B\lambda_C\lambda_D}{H_2H_3H_4}\right]$$
$$\times\left[\lambda_A\frac{\mu_{ABCD}}{(s+\mu_{ABCD})}+\lambda_B\frac{\mu_{BCD}}{(s+\mu_{BCD})}+\lambda_C\frac{\mu_{BCD}}{(s+\mu_{BCD})}+\lambda_D\frac{\mu_{BCD}}{(s+\mu_{BCD})}\right]$$

The Laplace transformation of the probabilities that the system is in up (i.e.in good or degraded state) and down (failed state) state at any time is as follows:

$$\bar{P}_{up}(s)=\bar{P}_{ABCDE}(s)+\bar{P}_{ABC\bar{D}E}(s)+\bar{P}_{AB\bar{C}\bar{D}E}(s)+\bar{P}_{A\bar{B}\bar{C}DE}(s)+\bar{P}_{A\bar{B}\bar{C}\bar{D}E}(s)+\bar{P}_{A\bar{B}C\bar{D}E}(s)+\bar{P}_{AB\bar{C}DE}(s)$$
$$+\bar{P}_{A\bar{B}CDE}(s) \tag{57}$$

$$\bar{P}_{down}(s)=\bar{P}_{ABC\bar{D}\bar{E}}(s)+\bar{P}_{\bar{A}BC\bar{D}E}(s)+\bar{P}_{ABCD\bar{E}}(s)+\bar{P}_{\bar{A}BCDE}(s)+\bar{P}_{ABC\bar{\bar{D}}E}(s)+\bar{P}_{\bar{A}\bar{B}CDE}(s)+\bar{P}_{AB\bar{C}\bar{D}\bar{E}}(s)$$
$$+\bar{P}_{A\bar{\bar{B}}\bar{C}DE}(s)+\bar{P}_{\bar{A}\bar{B}\bar{C}DE}(s)+\bar{P}_{\bar{A}\bar{B}C\bar{D}E}(s)+\bar{P}_{\bar{A}\bar{B}\bar{C}\bar{D}E}(s)+\bar{P}_{A\bar{\bar{B}}\bar{C}\bar{D}E}(s)+\bar{P}_{A\bar{B}\bar{\bar{C}}\bar{D}E}(s)+\bar{P}_{A\bar{B}\bar{C}\bar{\bar{D}}E}(s)$$
$$+\bar{P}_{A\bar{B}\bar{C}\bar{D}\bar{E}}(s)+\bar{P}_{AB\bar{\bar{C}}DE}(s)+\bar{P}_{\bar{A}B\bar{C}DE}(s)+\bar{P}_{AB\bar{C}D\bar{E}}(s)+\bar{P}_{\bar{A}B\bar{C}\bar{D}E}(s)+\bar{P}_{AB\bar{\bar{C}}\bar{D}E}(s)+\bar{P}_{AB\bar{C}\bar{\bar{D}}E}(s)$$
$$\tag{58}$$

6. Numerical Computations

6.1. Availability Analysis

Setting the values of different failure rates as $\lambda_A=0.048$, $\lambda_B=0.24$, $\lambda_C=0.07$, $\lambda_D=0.36$, $\lambda_E=0.09$ per day and repair facility is always available in equation (57) and then taking the inverse Laplace transform, we get the availability of the system as

$$P_{up}(t)=0.6979131693\ e^{(-0.00583211177t)}+0.0032190094\ 16e^{(-1.212486741t)}-0.0004647700\ 253e^{(-0.8609226602t)}$$
$$+0.0845749556\ 0e^{(-1.636266537t)}+0.0000623932\ e^{(-1.509815349t)}+0.0011084207\ 79e^{(-1.074813361t)}$$
$$\cos(0.0427145522\ 6t)+0.0005610698\ 560e^{(-1.074813361t)}\sin(0.0427145522\ 6t)+0.3706406445$$
$$e^{(-1.244373390t)}\sin(0.7420428270\ t)+0.2135868162\ ^{(-1.244373390t)}\cos(0.7420428270\ t) \tag{59}$$

Now varying time unit t from 0 to 10 unit of time in Equation (59), we get the following Table 1 and Fig. 2 for availability.

Time (t)	Availability
0	1.000
1	0.829
2	0.725
3	0.690
4	0.680
5	0.677
6	0.673
7	0.669
8	0.666
9	0.662
10	0.658

Table1. Availability as function of time

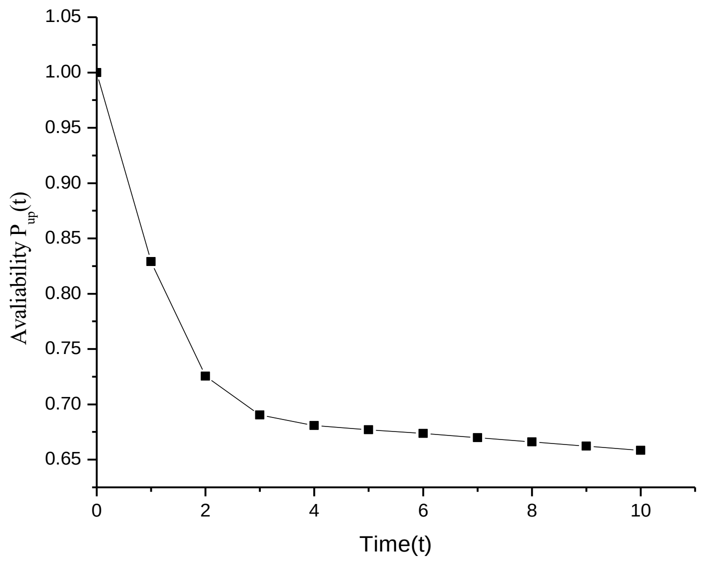

Fig.2: Availability as function of time

6.2 Reliability Analysis

Setting all repairs zero and the various failure rates as $\lambda_A = 0.048$, $\lambda_B = 0.24$, $\lambda_C = 0.07$, $\lambda_D = 0.36, \lambda_E = 0.09$ in equation (57) and taking the inverse Laplace transform, the reliability of the system is given as

$$R(t) = (3.906666667 + 0.28t)e^{(-0.808t)} + (-0.14t - 3.708333333)e^{(-1.168t)} - (4 + 0.28t)e^{(-1.048t)} + 4e^{(-1.288t)}\sinh(0.12t)$$

$$+ (0.28t + 4)e^{(-1.228\,t)}\sinh(0.18t) + (4.801666667 + 0.196t)e^{(-1.408t)} \quad (60)$$

Now varying time unit *t* from 0 to 10 in equation (60), one get the Table 2 and Fig. 3.

Time (*t*)	Reliability
0	1.000
1	0.750
2	0.469
3	0.266
4	0.142
5	0.073
6	0.036
7	0.017
8	0.008
9	0.004
10	0.001

Table 2. Reliability as function of time

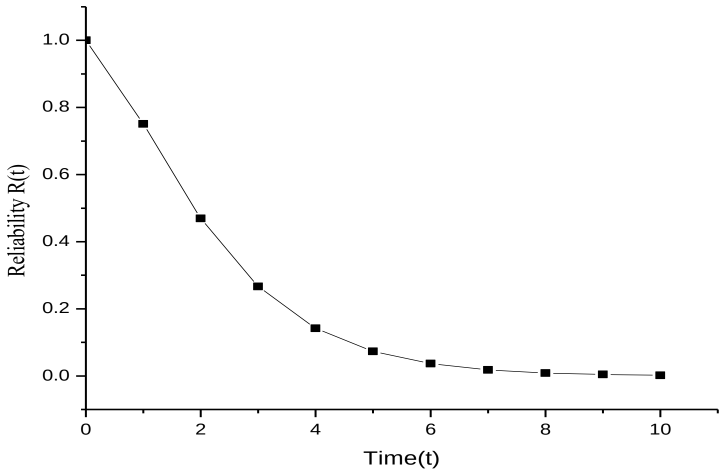

Fig. 3. Reliability as function of time

6.3 Mean Time to Failure (MTTF) Analysis
Taking all repairs equal to zero in (57) and taking as *s* tends to zero, one can obtain the MTTF of the system. Further, setting $\lambda_A = 0.048, \lambda_B = 0.24,$ $\lambda_C = 0.07,$ $\lambda_E = 0.09$ $\lambda_E = 0.09$ and varying various failure rates from 0.01 to 0.09 one by one, we get the Table 3 and Fig. 4 for MTTF.

Variations in λ_A , λ_B, λ_C, λ_D, λ_E	MTTF with respect to various failure rates				
	λ_A	λ_B	λ_C	λ_D	λ_E
0.01	2.421	2.791	2.297	3.480	2.626
0.02	2.377	2.783	2.293	3.470	2.574
0.03	2.334	2.770	2.288	3.452	2.525
0.04	2.292	2.755	2.282	3.428	2.477
0.05	2.252	2.738	2.276	3.400	2.431
0.06	2.213	2.718	2.268	3.368	2.386
0.07	2.175	2.697	2.260	3.333	2.342
0.08	2.138	2.674	2.251	3.296	2.300
0.09	2.102	2.650	2.242	3.258	2.260

Table3. MTTF as function of failure rates

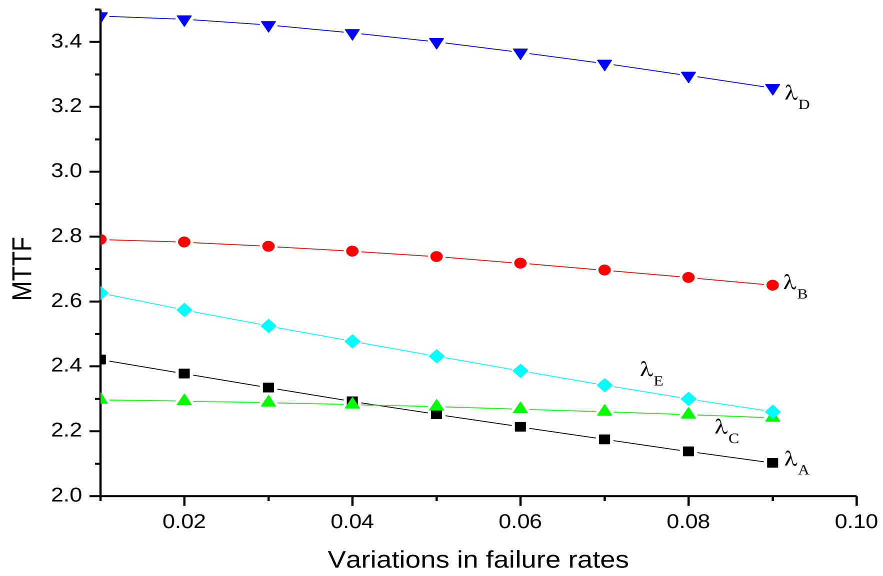

Fig. 4. MTTF as function of failure rates

6.4. Sensitivity Analysis

6.4.1 Sensitivity of Reliability

For sensitivity analysis of reliability, differentiating the reliability expression with respect to failure rates, then putting $\lambda_A = 0.048, \lambda_B = 0.24, \lambda_C = 0.07$, $\lambda_D = 0.36$, $\lambda_E = 0.09$ we get the values of $\frac{\partial R(t)}{\partial \lambda_A}, \frac{\partial R(t)}{\partial \lambda_B}, \frac{\partial R(t)}{\partial \lambda_C}, \frac{\partial R(t)}{\partial \lambda_D}, \frac{\partial R(t)}{\partial \lambda_E}$. Now, setting time unit t from 0 to 10, in the partial derivatives of reliability with respect to different failure rates, one can obtain the Table 4 and Fig. 5 respectively.

Time(t)	$\frac{\partial R(t)}{\partial \lambda_A}$	$\frac{\partial R(t)}{\partial \lambda_B}$	$\frac{\partial R(t)}{\partial \lambda_C}$	$\frac{\partial R(t)}{\partial \lambda_D}$	$\frac{\partial R(t)}{\partial \lambda_E}$
0	0	0	0	0	0
1	-0.750	-0.272	-0.087	-0.346	-0.750
2	-0.939	-0.526	-0.163	-0.631	-0.939
3	-0.798	-0.546	-0.174	-0.630	-0.798
4	-0.568	-0.436	-0.145	-0.488	-0.568
5	-0.365	-0.301	-0.105	-0.329	-0.365
6	-0.218	-0.189	-0.070	-0.204	-0.218
7	-0.125	-0.112	-0.043	-0.119	-0.155
8	-0.069	-0.063	-0.025	-0.066	-0.069
9	-0.037	-0.034	-0.014	-0.036	-0.037
10	-0.019	-0.018	-0.008	-0.019	-0.019

Table 4. Sensitivity of reliability as function of time

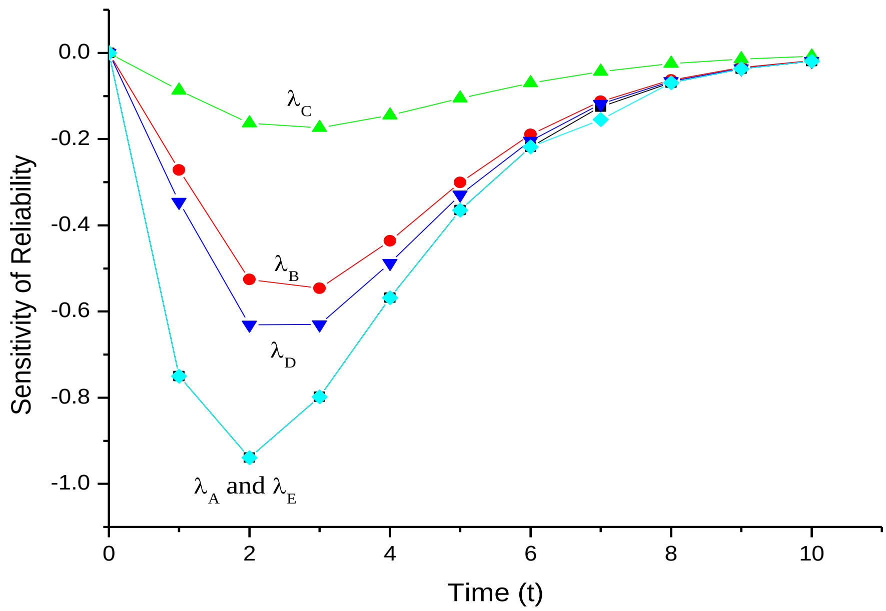

Fig. 5. Sensitivity of reliability as function of time

6.4.2 Sensitivity of MTTF

By differentiating MTTF expression with respect to failure rates and then putting the values of different failure rates as $\lambda_A = 0.048, \lambda_B = 0.24, \lambda_C = 0.07,\ \lambda_D = 0.36\ , \lambda_E = 0.09$ .We get the values of $\frac{\partial(MTTF)}{\partial\lambda_A}, \frac{\partial(MTTF)}{\partial\lambda_B}, \frac{\partial(MTTF)}{\partial\lambda_C}, \frac{\partial(MTTF)}{\partial\lambda_D}, \frac{\partial(MTTF)}{\partial\lambda_E}$. Varying the failure rates one by one respectively as 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09 in the partial derivatives of MTTF with respect to different failure rates, one can obtain the Table 5 and Fig. 6 respectively.

Variations in $\lambda_A, \lambda_B, \lambda_C, \lambda_D, \lambda_E$	$\frac{\partial(MTTF)}{\partial\lambda_A}$	$\frac{\partial(MTTF)}{\partial\lambda_B}$	$\frac{\partial(MTTF)}{\partial\lambda_C}$	$\frac{\partial(MTTF)}{\partial\lambda_D}$	$\frac{\partial(MTTF)}{\partial\lambda_E}$
0.01	-4.522	-0.665	-0.340	-4.830	-9.012
0.02	-4.375	-1.054	-0.571	-8.949	-8.590
0.03	-4.234	-1.377	-0.774	-11.493	-8.196
0.04	-4.100	-1.645	-0.953	-13.005	-7.827
0.05	-3.972	-1.867	-1.110	-13.832	-7.481
0.06	-3.849	-2.049	-1.248	-14.200	-7.157
0.07	-3.732	-2.198	-1.369	-14.261	-6.852
0.08	-3.620	-2.319	-1.474	-14.118	-6.566
0.09	-3.512	-2.417	-1.566	-13.839	-6.296

Table5. Sensitivity of MTTF as function of failure rates

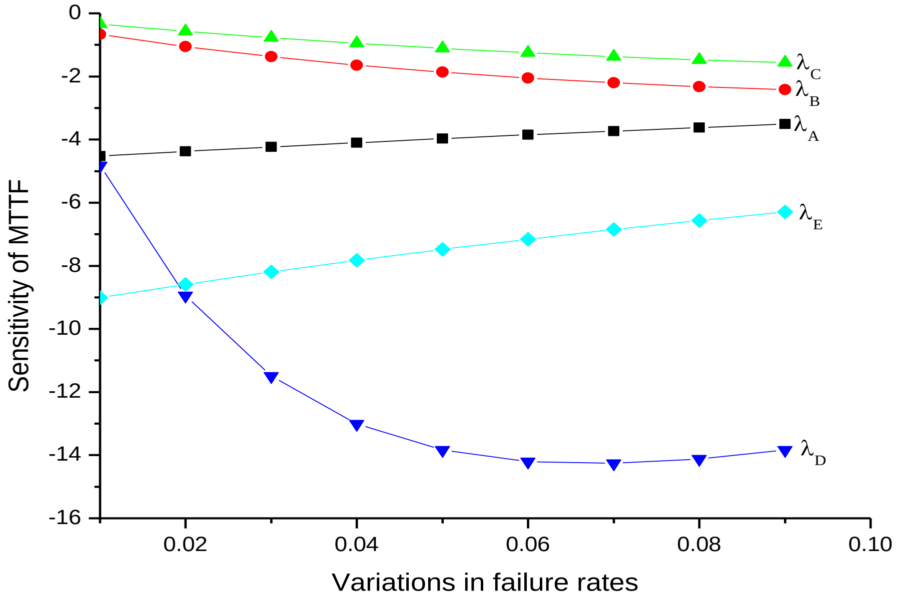

Fig.6. Sensitivity of MTTF as function of failure rates

6.5 Expected Profit

The expected profit during the interval [0, t) is given as

$$E_p(t) = K_1 \int_0^t P_{up}(t)dt - tK_2 \tag{61}$$

Using equation (59), the expected profit for the same set of parameters, we have

$$\begin{aligned} E_p(t) = K_1[&-119.6037552\ e^{(-0.005835211177t)} - 0.0026548821\ 58e^{(-1.212486741\ t)} + 0.0005398510\ 77e^{(-0.8609226602t)} \\ &- 0.0516782872\ 5e^{(-1.636566537t)} 0.0000413250\ 5345e^{(-1.509815349t)} - 0.0010503548\ 70e^{(-1.074813361t)} \cos(0.0427145522\ 6t) \\ &- 0.0004802735\ 403e^{(-1.074813361t)} \sin(0.0427145522\ 6t) - 0.257641306\ e^{(-1.244373390t)} \cos(0.742042827\ t) \\ &- 0.1442169713\ e^{(-1.24437339t)} \sin(0.742042827\ ) + 119.9162815\ ] - K_2 t \end{aligned} \tag{62}$$

Setting K_1= 1 and K_2= 0.1, 0.2, 0.0.3, 0.4, 0.5 respectively and varying t from 0 to 10 in (171) we get the Table 6 and correspondingly Fig. 7.

Time(t)	Expected Profits				
	$K_2 = 0.1$	$K_2 = 0.2$	$K_2 = 0.3$	$K_2 = 0.4$	$K_2 = 0.5$
0	0	0	0	0	0
1	0.814	0.714	0.614	0.514	0.414
2	1.484	1.284	1.084	0.884	0.684
3	2.088	1.788	1.488	1.188	0.888
4	2.673	2.273	1.873	1.473	1.073
5	3.252	2.752	2.252	1.752	1.252
6	3.827	3.227	2.627	2.027	1.427
7	4.399	3.699	2.999	2.299	1.599
8	4.967	4.167	3.367	2.567	1.767
9	5.531	4.631	3.731	2.831	1.931
10	6.091	5.091	4.091	3.091	2.091

Table 6. Expected profit as function of failure rates

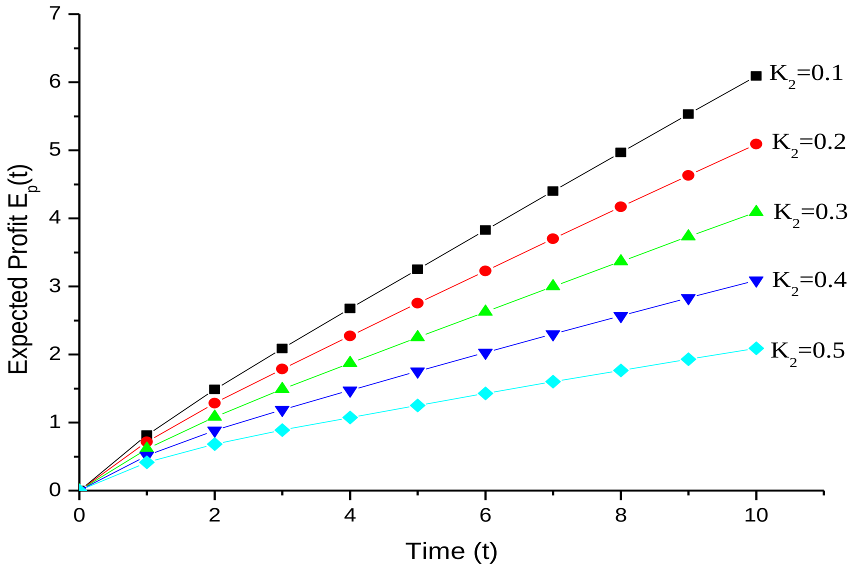

Fig.7. Expected profit as function of failure rates

7. Results Analysis

In conclusion, it is observed that

- The graph of availability vs. Time (Fig. 2) yields that the availability of the system decreases continuously with increment in time, but after a specific time, it becomes approximate constant.
- The graph of reliability vs. Time (Fig. 3) yields that the reliability of the system initially decreases fastly and then smoothly decreases with increment in time.
- The graph of MTTF (Fig. 4) shows that MTTF of the system decreases with respect to all types of failure except the failure rate of washing system. Further, MTTF is lowest with respect to the failure rate of digester and highest with respect to the opener failure rate.
- The sensitivities of the system reliability with respect to different failure rates are shown in Fig. 5. It is clear from the graph that the system reliability is at the lowest sensitive with respect to the digester and the screening system and highest with respect to the washing system.
- Fig. 6 shows the sensitivity of MTTF with respect to different failure rates of the system. Critical observation of the graph point out that MTTF of the system is more sensitive with respect to the opener.
- Keeping the revenue cost per unit time fixed at 1 and varying service cost at 0.1, 0.2, 0.3, 0.4, and 0.5 we obtain Fig.7, which reveal that the profit decreases as the service cost increases.

8. Conclusion

The mathematical modelling and performance evaluation of pulping and screening system of a paper plant is discussed in this work. With the help of Markov death-birth process, the authors have found the reliability measures of the pulping and screening unit of paper plant. One can accurately identify the performance of each individual unit. We can see that system reliability is more sensitive with respect to digester and screening system, which indicate that to make the system more reliable, one have to focus more on these two units. It asserts that the result of this research will be useful to the management of the paper plant.

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