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SUFFICIENT CONDITIONS FOR MULTI-DIMENSIONAL BLOW-UP PROBLEM

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ABSTRACT. This article studies a multi-dimensional parabolic problem with a concentrated nonlinear source having local and nonlocal features. It is shown that if its solution exists, then it blows up everywhere on the boundary of a ball where the concentrated source is situated. A criterion for the solution to blow up in a finite time is also given.

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1. INTRODUCTION

Let $\varsigma = (\varsigma_1, \varsigma_2, \varsigma_3, \ldots, \varsigma_n)$ be a point in the n-dimensional Euclidean space in $\mathbb{R}^n$, $\hat{T}(> 0)$ and $m(> 1/2)$ be real numbers, $\hat{D}$ be a bounded domain in $\mathbb{R}^n, \partial\hat{D}$ be the boundary of $\hat{D}$, $\hat{B}$ be an n-dimensional ball, $\{\varsigma \in \mathbb{R}^n : |\varsigma - \hat{b}| < \hat{R}\}$, centered at a given point $\hat{b}$ with radius $\hat{R}, \bar{\hat{B}} \subset \hat{D}, \partial\hat{B}$ be the boundary of $\hat{B}, v(\varsigma)$ denote the unit inward normal vector at $\varsigma \in \partial\hat{B}$, $\chi_{\hat{B}}(\varsigma) = 1$ if $\varsigma \in \hat{B}$ and $\chi_{\hat{B}}(\varsigma) = 0$ if $\varsigma \in \hat{D}\backslash\hat{B}$ be the characteristic function. Without loss of generality, let $\hat{b}$ be the origin. We would like to study the following multi-dimensional nonlinear parabolic problem:

$$(1.1)\qquad \begin{cases} u_\gamma - \Delta_\varsigma u = \dfrac{\partial \chi_{\hat{B}}(\varsigma)}{\partial v} F(u(\varsigma,\gamma)) Z^m(\gamma) \text{ in } \hat{D} \times (0, \hat{T}], \\ u(\varsigma, 0) = \psi(\varsigma) \text{ on } \hat{D} \text{ and } u(\varsigma,\gamma) = 0 \text{ for } \varsigma \in \partial\hat{D}, 0 < \gamma \leq \hat{T}. \end{cases}$$

We note that F and S are given functions, $\Delta_\varsigma = \sum_{i=1}^n \partial^2/\partial\varsigma_i^2$, and $Z(\gamma) = \int_{\hat{D}} S(u(\varsigma,\gamma))d\varsigma$. The nonlinear source in the problem (1.1) is the product of a local contribution $(\partial\chi_B(\varsigma)/\partial v)F(u(\varsigma,\gamma))$ and a global contribution $Z^m(\gamma)$. In order to study the behavior of the solution over a unit domain, we consider a domain having the same shape as $\hat{D}$ [1]. If the shape of the domain is given, then the domain can be uniquely determined by its size. Let D be a bounded n-dimensional domain having the same

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shape as $\hat{D}$. Then, there is $x_0 \in \hat{D} \cap D$ and a positive constant A such that

$$\hat{D} = \{\varsigma : \varsigma = x_0 + A(x - x_0) \text{ for } x \in D\}.$$

Let the size of D be one, that is $|D| = \int_D dx = 1$. Without lost of generality, we can let x_0 be the origin. We note that

$$A = \left(\frac{\text{size of } \hat{D}}{\text{size of } D} \right)^{\frac{1}{n}} = \left(\frac{\int_{\hat{D}} d\varsigma}{\int_D dx} \right)^{\frac{1}{n}} = \left(\frac{|\hat{D}|}{|D|} \right)^{\frac{1}{n}} = |\hat{D}|^{\frac{1}{n}}.$$

Consider the change of variables $\gamma = A^2 t$ and $\varsigma = Ax$. Let $B = \{x \in \mathbb{R}^n : |x| < R\}$, where ∂B is its boundary and $R = \hat{R}/A$, ν denote the inward normal at $x \in \partial B$, $\varphi(x)$ be an infinitely differentiable function with compact support, and $\delta(x)$ denote the usual Dirac delta function. By using the spherical coordinates [7], $\partial \chi_{\hat{B}}(\varsigma)/\partial v$ can be rewritten as following:

$$\begin{aligned}
&\int_{\mathbb{R}^n} \frac{\partial \chi_{\hat{B}}(Ax)}{\partial v} \varphi(x) dx \\
&= \int_0^\infty \int_{-\pi}^{\pi} \int_0^{\pi} \cdots \int_0^{\pi} \delta(Ar - \hat{R}) \varphi(r, \omega_1, \ldots, \omega_{n-1}) r^{n-1} \\
&\times \prod_{i=1}^{n-2} \sin^{n-1-i} \omega_i d\omega_1 \cdots d\omega_{n-1} dr \\
&= \int_0^\infty \int_{-\pi}^{\pi} \int_0^{\pi} \cdots \int_0^{\pi} \delta(\sigma - \hat{R}) \varphi\left(\frac{\sigma}{A}, \omega_1, \ldots, \omega_{n-1}\right) \left(\frac{\sigma}{A}\right)^{n-1} \\
&\times \prod_{i=1}^{n-2} \sin^{n-1-i} \omega_i d\omega_1 \cdots d\omega_{n-1} \frac{1}{A} d\sigma \\
&= \frac{1}{A} \int_{-\pi}^{\pi} \int_0^{\pi} \cdots \int_0^{\pi} \varphi\left(\frac{\hat{R}}{A}, \omega_1, \ldots, \omega_{n-1}\right) \left(\frac{\hat{R}}{A}\right)^{n-1} \\
&\times \prod_{i=1}^{n-2} \sin^{n-1-i} \omega_i d\omega_1 \cdots d\omega_{n-1} \\
&= \frac{1}{A} \int_0^\infty \int_{-\pi}^{\pi} \int_0^{\pi} \cdots \int_0^{\pi} \delta\left(r - \frac{\hat{R}}{A}\right) \varphi\left(\frac{\hat{R}}{A}, \omega_1, \ldots, \omega_{n-1}\right) \left(\frac{\hat{R}}{A}\right)^{n-1} \\
&\times \prod_{i=1}^{n-2} \sin^{n-1-i} \omega_i d\omega_1 \cdots d\omega_{n-1} dr \\
&= \frac{1}{A} \int_{\mathbb{R}^n} \frac{\partial \chi_B(x)}{\partial \nu} \varphi(x) dx.
\end{aligned}$$

Hence,

$$\frac{\partial \chi_B(Ax)}{\partial v} = \frac{1}{A} \frac{\partial \chi_B(x)}{\partial \nu},$$

and

$$Z(\gamma) = \int_{\hat{D}} S(u(\varsigma,\gamma))d\varsigma = A^n \int_{\hat{D}} s(u(x,t))dx,$$

where $S(u(\varsigma,\gamma)) = s(u(x,t))$. Let $\Delta = \sum_{i=1}^{n} \partial^2/\partial x_i^2, H = \partial/\partial t - \Delta$, $F(u(\varsigma,\gamma)) = f(u(x,t)), U(t) = \int_D s(u(x,t))dx, T = \hat{T}/A^2$, ∂D be the boundary of D, $\bar{D}$ be the closure of D, and $\Omega = D \times (0,T]$. Then, the problem (1.1) becomes

$$\begin{cases} Hu = \left|\hat{D}\right|^{m+\frac{1}{n}} \dfrac{\partial \chi_B(\varsigma)}{\partial v} f(u(x,t))U^m(t) \text{ in } \Omega, \\ u(x,0) = \psi(x) \text{ on } \bar{D} \text{ and } u(x,t) = 0 \text{ for } x \in \partial D, 0 < t \leq T. \end{cases} \tag{1.2}$$

We assume that $f(u), s(u), f'(u)$ and $f''(u)$ are positive, $s'(u)$ and $s''(u)$ are nonnegative, and $\psi(x)$ is nontrivial on ∂B, nonnegative and continuous function such that

$$\Delta\psi(x) + |\hat{D}|^{m+\frac{1}{n}} \frac{\partial \chi_B(x)}{\partial \nu} f(\psi(x)) \left(\int_D s(\psi(x))dx \right)^m \geq 0. \tag{1.3}$$

This intuitively means that at the beginning, the temperature will rise up.

The problem (1.2) describes a temperature u due to a nonlinear source having local and nonlocal features subject to the initial condition $\psi(x)$ and zero temperature on the lateral boundary.

A solution of (1.2) is a continuous function on $\bar{\Omega}$ satisfying (1.2). A solution u of the problem (1.2) is said to be *blow up* at a point (x, t_b) if there exists a sequence $\{u(x_n, t_n)\} \to \infty$ as $(x_n, t_n) \to (x, t_b)$.

Instead of studying $u(b,t)$ for any point $b \in B$, we would like to investigate a solution $u(x,t)$ of (1.2).

We also assume Ω has the property that for any point $P \in \partial D \times (0,T]$, there exists an $(n+1)-$ dimensional neighborhood Σ such that $\Sigma \cap \partial D \times (0,T]$ can be represented, for some $i \in \{1,2,\dots,n\}$ in the form

$$x_i = \beta(x_1, x_2, \dots, x_{i-1}, x_{i+1}, \dots, x_n, t),$$

where $\beta, D_x\beta, D_1^2\beta$ are Hölder continuous of exponent $\alpha \in (0,1)$ while $D_xD_t\beta$ and $D_t^2\beta$ are continuous.

Let $G(x,t;\xi,\tau)$ denote Green's function corresponding to the problem (1.2). With the above assumptions on Ω, $G(x,t;\xi,\tau)$ has the following properties [6]:

(a) There exists a unique $G(x,t;\xi,\tau)$ that is continuous in $\bar{\Omega} \times (D \times [0,T))$, $t > \tau$. Furthermore, $\partial G/\partial x$, $\partial^2 G/\partial x^2$ and $\partial G/\partial t$ are continuous functions of $(x,t;\xi,\tau)$ in $\Omega \times (D \times [0,T))$, $t > \tau$.

(b) For each $(\xi,\tau) \in D \times [0,T)$ on $\partial D \times (\tau,T]$, $G(x,t;\xi,\tau) > 0$ in $D \times (\tau,T]$.

(c) For any fixed $(\xi,\tau) \in D \times [0,T)$ and any $\epsilon > 0$, $\partial G/\partial x$, $\partial^2 G/\partial x^2$ and $\partial G/\partial t$ are uniformly continuous functions of $(x,t) \in \Omega$ with $t - \tau > \epsilon$.

The problem with the case $n = 1$ was studied by Chan and Tian [4]. They proved that there was a unique continuous solution before the blow-up has occurred and they also gave a blow-up criterion. For an n-dimensional problem, Chan and Tian [5] studied a blow-up with nonlinear source of the form

$$\frac{\partial \chi_B(x)}{\partial \nu} f(u(x,t)).$$

They showed that the nonlinear source represented a correct formulation of a concentrated source in multidimensional problems. They also showed that the problem had a unique solution before the blow-up occurred on the boundary of a ball.

We extend the problem of Chan and Tian [4] into a multidimensional version base on the formulation of Chan and Tian [5]. In section 2, we quote Boonklurb and Siriroop's results [1] that the integral equation corresponding to (1.2) has a unique continuous solution u, which is nondecreasing function of t. Then, it leads to the conclusion that u is a unique solution of problem (1.2). Finally, we give sufficient conditions for a finite time blow-up. The blow-up set of the problem (1.2) is also given.

2. BLOW-UP SET AND SUFFICIENT CONDITIONS FOR BLOW-UP IN A FINITE TIME

Boonklurb and Siriroop [1] transformed the problem (1.2) into an integral representation. To construct the representation, they used the adjoint operator of the heat operator, $H^* = -\partial/\partial t - \Delta$, and the Green's second identity to obtain

$$u(x,t) = \int_D G(x,t;\xi,0)\psi(\xi)d\xi + |\hat{D}|^{m+\frac{1}{n}} \int_0^t \int_D G(x,t;\xi,\tau)\frac{\partial \chi_B(\xi)}{\partial \nu} f(u(\xi,\tau))U^m(\tau)d\xi d\tau. \tag{2.1}$$

Using the divergent theorem and integration by parts, (2.1) becomes

$$u(x,t) = \int_D G(x,t;\xi,0)\psi(\xi)d\xi + \int_0^t \int_{\partial B} G(x,t;\xi,\tau) f(u(\xi,\tau))U^m(\tau)d\xi d\tau. \tag{2.2}$$

For ease of reference, we will quote theorems 2.3 and 3.1 of Boonklurb and Siriroop [1] as theorems 2.1 and 2.2, respectively.

Theorem 2.1. *There exists t_b such that for all $0 \le t < t_b$, the integral equation* (2.2) *has a unique continuous solution $u \ge \psi(x)$, and u is nondecreasing function of t. If t_b is finite, then u is unbounded in $[0, t_b)$.*

Theorem 2.2. *The problem* (1.2) *has a unique continuous solution u for $0 \le t < t_b$.*

By modifying the technique of Chan [2], we can show that if the solution u of the problem (1.2) blows up, then it blows up everywhere on the boundary of the ball B.

Theorem 2.3. *If t_b is finite and ψ attains its maximum on ∂B, then u blows everywhere on ∂B.*

Proof. By Theorems 2.1 and 2.2, the problem (1.2) has a unique continuous solution u for $0 \leq t < t_b$. We will first show that, for any $t > 0$, the solution u of the problem (1.2) satisfies

$$u\,(x,t) > u\,(y,t) \text{ for all } x \in \partial B \text{ and } y \notin \partial B.$$

Since $u\,(x,t)$ is known on $\partial B \times (0,t_b)$, let it be denoted by $g\,(x,t)$, and rewrite the problem (1.2) as the following two initial-boundary value problems:

$$\begin{cases} Hu = 0 \text{ in } B \times (0,t_b)\,, \\ u(x,0) = \psi(x) \text{ on } \bar{B} \text{ and } u(x,t) = g\,(x,t) \text{ on } \partial B \times (0,t_b)\,, \end{cases} \tag{2.3}$$

$$\begin{cases} Hu = 0 \text{ in } D\backslash\bar{B} \times (0,t_b)\,, \\ u(x,0) = \psi(x) \text{ on } D\backslash\bar{B} \text{ and } u(x,t) = g\,(x,t) \text{ on } \partial B \times (0,t_b)\,0. \end{cases} \tag{2.4}$$

Consider the problem (2.3). It follows from the strong maximum principle that (cf. Friedman [6]) that u attains its maximum on $\partial B \times (0,t_b)$. Since u is a nondecreasing function of t, we have, for each given $\eta \in (0,t_b)$, u attains its maximum for $0 \leq t \leq \eta$ somewhere on $\partial B \times \{\eta\}$. Suppose that there exists a smallest positive number, say t_0, and some $y_0 \notin \partial B$ such that $u\,(y_0,t_0) = \min_{x\in\partial B} u\,(x,t_0)$. We claim that for $x \in \partial B$, $u\,(x,t_0) = u\,(y_0,t_0)$. If this is not true, then there exists some $x_0 \in \partial B$ such that $u\,(x_0,t_0) > \min_{x\in\partial B} u\,(x,t_0)$. Since u is continuous, there is a point (y',t_0) in a neighborhood of (x_0,t_0) such that $y' \notin \partial B$ and $u\,(y',t_0) > \min_{x\in\partial B} u\,(x,t_0)$. This contradicts to t_0 being the smallest number such that $u\,(y_0,t_0) = \min_{x\in\partial B} u\,(x,t_0)$. Thus, the claim is proved. We now have that u attains its maximum at (y_0,t_0) for $0 \leq t \leq t_0$. If $y_0 \in B$, then $u \equiv u\,(y_0,t_0)$ in $B \times (0,t_0]$ by the strong maximum principle. Since u is continuous, we have $u \equiv u\,(y_0,t_0)$ in $\bar{B} \times [0,t_0]$. Then, u is constant in $\bar{B} \times [0,t_0]$. This gives a contradiction since ψ is not constant on $\bar{B}$. If $y_0 \in \left(D\backslash\bar{B}\right)$, then $u \equiv u\,(y_0,t_0)$ in $\left(D\backslash\bar{B}\right) \times (0,t_0]$. By using the continuity of u, we have that $u \equiv u\,(y_0,t_0)$ in $\left(D\backslash\bar{B}\right) \times [0,t_0]$. We again have a contradiction since ψ is not constant on $D\backslash\bar{B}$. Therefore, for any $t > 0$,

$$u\,(x,t) > u\,(y,t) \text{ for all } x \in \partial B \text{ and } y \notin \partial B. \tag{2.5}$$

We claim that for each $t > 0$, u attains the same value for $x \in \partial B$. Suppose there is a point $x_0 \in \partial B$ such that $u\,(x_0,t) > \min_{x\in\partial B} u\,(x,t)$ for some $t > 0$. Since u is continuous, there exists a point $(\tilde{y},t)$ in a neighborhood of (x_0,t) such that $\tilde{y} \notin \partial B$ and $u\,(\tilde{y},t) > \min_{x\in\partial B} u\,(x,t)$. This contradicts (2.5). Hence, for any $t > 0$,

$$u\,(x,t) = \max_{x\in\bar{D}} u\,(x,t) \text{ for } x \in \partial B \text{ and } \max_{x\in\bar{D}} u\,(x,t) > u\,(y,t) \text{ for any } y \notin \partial B. \tag{2.6}$$

From (2.6), u blows up everywhere on ∂B as $t \to t_b$. $\square$

Let

$$\mu(t) = \int_D \phi(x)u(x,t)dx,$$

where ϕ is the normalized fundamental eigenfunction of the problem,

$$\Delta\phi + \lambda\phi = 0 \text{ in } D \text{ and } \phi = 0 \text{ on } \partial D,$$

with λ denoting its corresponding eigenvalue.

We will investigate the sufficient conditions for finite time blow-up by developing the method of Chan and Tian [4].

Theorem 2.4. *Let ω be the n-dimensional solid angle, $M(t) = \sup_{x\in\bar{D}} u(x,t)$, and ψ attains its maximum on ∂B. If*

$$\mu(0) > \left(\frac{\lambda}{|\hat{D}|^{m+\frac{1}{n}}R^{n-1}\omega}\right)^{\frac{1}{2m-1}}, \tag{2.7}$$

$$\phi(x)f(u(x,t)) \geq u^m(x,t) \text{ for all } x \in \partial B \text{ and for all } t > 0, \tag{2.8}$$

$$s(u(x,t)) \geq u(x,t) \text{ for all } x \in D \text{ and for all } t > 0, \tag{2.9}$$

then the solution u of (1.2) *blows up everywhere on ∂B in a finite time.*

Proof. Multiplying the normalized eigenfunction ϕ to (1.2) and integrating over D, we obtain

$$\begin{aligned}
\mu'(t) + \lambda\mu(t) &= \int_D \phi(x)\left|\hat{D}\right|^{m+\frac{1}{n}} \frac{\partial\chi_B}{\partial\nu} f(u(x,t))\,U^m(t)\,dx \\
&= \int_D \left|\hat{D}\right|^{m+\frac{1}{n}} \phi(x)\,f(u(x,t))\,U^m(t)\,(\nu(x)\cdot\nabla\chi_B(x))\,dx \\
&= -\int_B \sum_{i=1}^{n} \left|\hat{D}\right|^{m+\frac{1}{n}} \frac{\partial}{\partial\nu_i}\phi(x)\,f(u(x,t))\,U^m(t)\,\nu_i dx \\
&= \int_{\partial B} \left|\hat{D}\right|^{m+\frac{1}{n}} \phi(x)\,f(u(x,t))\,U^m(t)\,dx.
\end{aligned}$$

By using (2.9) and the supremum property, we have $s(u(x,t)) \geq M(t)$ for all $t > 0$, and

$$\int_D s(u(x,t))\,dx \geq \int_D M(t)\,dx = M(t) \text{ for all } t > 0.$$

By (2.8) and the above argument, we obtain

$$\begin{aligned}\int_{\partial B}\left|\hat{D}\right|^{m+\frac{1}{n}}\phi(x)\,f(u(x,t))\,U^m(t)\,dx &\geq \int_{\partial B}\left|\hat{D}\right|^{m+\frac{1}{n}}u^m(x,t)\,U^m(t)\,dx\\ &= \left|\hat{D}\right|^{m+\frac{1}{n}}M^m(t)\,U^m(t)\int_{\partial B}dx\\ &= \left|\hat{D}\right|^{m+\frac{1}{n}}M^m(t)\,U^m(t)\,R^{n-1}\omega\\ &\geq \left|\hat{D}\right|^{m+\frac{1}{n}}M^{2m}(t)\,R^{n-1}\omega,\end{aligned}$$

where $R^{n-1}\omega$ is an n-dimensional surface area of a sphere. Thus,

$$\mu'(t)+\lambda\mu(t)\geq\left|\hat{D}\right|^{m+\frac{1}{n}}M^{2m}(t)\,R^{n-1}\omega \text{ for all } t>0. \tag{2.10}$$

By the Schwarz's inequality,

$$\mu(t)=\int_D\phi(x)\,u(x,t)\,dx\leq M(t)\int_D\phi(x)\,dx\leq M(t)\,|D|^{\frac{1}{2}}\left(\int_D\phi^2(x)\,dx\right)^{\frac{1}{2}}.$$

Since $\int_D\phi^2(x)\,dx=1$ and $|D|=1$, we have

$$\mu(t)\leq M(t) \text{ for all } t>0.$$

Thus, (2.10) becomes

$$\mu'(t)+\lambda\mu(t)\geq\left|\hat{D}\right|^{m+\frac{1}{n}}R^{n-1}\omega\mu^{2m}(t).$$

Solving this Bernoulli's inequality, we obtain

$$\mu^{2m-1}(t)\geq\frac{\lambda e^{(1-2m)t}}{\left|\hat{D}\right|^{m+\frac{1}{n}}R^{n-1}\omega e^{(2m-1)\lambda t}+\left(\lambda\mu^{1-2m}(0)-\left|\hat{D}\right|^{m+\frac{1}{n}}R^{n-1}\omega\right)}.$$

Hence, $\mu^{2m-1}(t)$ tends to infinity whenever

$$\left|\hat{D}\right|^{m+\frac{1}{n}}R^{n-1}\omega e^{(2m-1)\lambda t}+\left(\lambda\mu^{1-2m}(0)-\left|\hat{D}\right|^{m+\frac{1}{n}}R^{n-1}\omega\right)\to 0.$$

Thus, we obtain that, if

$$t\to\frac{1}{(1-2m)\,\lambda}\ln\left(\frac{-\lambda\mu^{1-2m}(0)-\left|\hat{D}\right|^{m+\frac{1}{n}}R^{n-1}\omega}{\left|\hat{D}\right|^{m+\frac{1}{n}}R^{n-1}\omega}\right),$$

then $\mu^{2m-1}(t)$ tends to infinity. From (2.7), we have $\mu^{2m-1}(0)<\left|\hat{D}\right|^{m+\frac{1}{n}}R^{n-1}\omega/\lambda$. Hence, μ tends to infinity for some finite time $t=t_b$, where

$$t_b\leq\frac{1}{(1-2m)\,\lambda}\ln\left(\frac{-\lambda\mu^{1-2m}(0)-\left|\hat{D}\right|^{m+\frac{1}{n}}R^{n-1}\omega}{\left|\hat{D}\right|^{m+\frac{1}{n}}R^{n-1}\omega}\right).$$

It follows from Theorem 2.3 and the above argument that u blows up for some finite time t_b on the boundary of the concentrated ball B. □

From the above theorem, we obtain the upper bound for the blow-up time of our problem. One can try to use the numerical technique to find the numerical blow-up time for some specific domains. Especially, one may see the relation between the size of the domain and the blow-up time. Moreover, one may notice that if the radius of the concentrated ball is too big, the effect of the boundary condition may prevent the blow-up to occur. If this is the case, we may extend the idea of Chan and Boonklurb [3] to find a critical radius of the concentrated ball in order to guarantee the blow-up.

REFERENCES

[1] R. Boonklurb and A. Siriroop, Bow-up set for multi-dimensional blow-up problem due to concentrated source having local and nonlocal features, *Proc. 30th National Grad. Res. Conf.*, 31–38, 2003.

[2] C. Y. Chan, Multi-dimensional quenching due to a concentrated nonlinear source, *DCDIS Proceedings*, 2:273–278, 2006.

[3] C. Y. Chan and R. Boonklurb, A blow-up criterion for a degenerate parabolic problem due to a concentrated nonlinear source, *Quart. Appl. Math.*, 65:781–787, 2007.

[4] C. Y. Chan and H. Y. Tain, Single point blow-up for a degenerate parabolic problem with a nonlinear source of local and nonlocal features, *Appl. Math. Comput.*, 145:371–390, 2003.

[5] C. Y. Chan and H. Y. Tain, A multi-dimensional explosion due to a concentrated nonlinear source, *J. Math. Anal. Appl.*, 295:174–190, 2004.

[6] A. Friedmann, *Partial differential equations of parabolic type*, Prentince Hall Inc., Englewood Cliffs, New Jersey, 1964.

[7] K. R. Stromberg, *An Introduction to Classical Real Analysis*, Chapman & Hall Wadsworth, Belmont, 1981.