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RELIABILITY AND AVAILABILITY OF MACHINES WITH TWO TYPES OF FAILURES OPERATED UNDER PERIODIC SURVEILLANCE TEST

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ABSTRACT. This paper deals with study of reliability and availability of machines operated under periodic surveillance test. Machines fail randomly in Poisson probability distribution and are repaired exponentially. The main objective of the paper is to find probability distribution function for the system with the help of which reliability, availability, unavailability of the system are obtained explicitly. The mathematical model is constructed in terms of system of equations with the assumption of the defects of machines (standby, common cause and independent) with two types of failure mode (minor and major) and their repair (minor and major). To solve these equations, matrix pseudo-inverse technique has been used. Numerical results are obtained by using computational software to show the applicability of the model.

Keywords: Reliability, Availability, Periodic surveillance, Standby system, Matrix pseudo-inverse
AMS (MOS) Subject Classification. 62B05.

1. INTRODUCTION

Reliability of a system plays an important role in machining system, manufacturing system, quality control, communication system, internet and intranet system, robot and robotic system. So it is much more worth while to mention some of the works done on the line. Carr and Savage [1] studied the reliability model of a system with redundancy which depends on two factors: first the component failures which is maintained by functional system topology and second the probability of adequate system performance in each functional configuration. Lu and Lewis [8] analyzed the reliability model of k-out-of-n configuration for partially redundant safety system which ensure the high level of reliability and safety with limited financial and space resources. Lee and Wang [7] proposed the approximately optimal testing policy for two non identical units parallel standby system. Singh [18] studied K-out-of-M+N warm standby redundant system with common cause failures under the assumption that

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failure rates for both operating and standby units are constant with heterogeneous repair rate. Dhillon and Yang [2] analyzed the reliability model of warm standby system with common cause failure and human error failure and they obtained the expressions for system availability, reliability, mean time to failure (MTTF), and time to failure variance. Huang et al. [5] developed the reliability model and obtained its analytical solution for a warm standby redundant system with two identical non-repairable sets having N functional units. Permila et al. [15] analyzed the stochastic modeling of a 2-out-of-2: G system of identical units with one more identical unit as spare in cold standby. Wu et al. [19] studied a k-out-of-n:G repairable system with one replaceable repair equipment, where the lifetimes and repair times of components follow exponential distributions and arbitrary distributions, respectively. They also made the provision that when one component breaks down, it is repaired by the repair equipment. If the repair equipment fails during the repair period, then it is replaced by a new one.

Some times machining system fails not only by erroson, wearout, breaking but it also fails due to vibration, sound, temperature, environmental change, humidity, moisture which are not certain. Such failures are termed as common cause failure. Some of the authors threw the light on some of reliability models with the imposition of common cause failure. Jacob et al. [6] analyzed the reliability of deteriorating standby system with repairs for common cause failure and critical human error and obtained point-wise availability. Mosleh [12] developed the reliability model to prevent the common cause failures of the nuclear power plant. Pan and Nonaka [14] constructed the reliability model with the provision of common cause failures and evaluated the structure importance, probability importance and β-importance of the systems which helped reliability analysis to limit the common cause failure so that the system structure may work for long time.

Availability is another part of concept in the study of system reliability. Availability is the ability of an item to perform its required function at a stated instant of time or over a stated period of time [17]. Several authors contributed to the study of the availability of the system. Raje et al. [16] developed the Markov model for availability assessment of a two unit standby pumping system. Nourelfath and Dutuit [13] solved the redundancy optimization problem for multi-state systems using universal moment generating function (UGF) technique to evaluate the system availability. Heising [3] investigated the time dependent unavailability models which are combined with Markov models, including common cause failures using β-factor.

Surveillance requirements involve periodic tests such as monthly or weekly. The main purpose of testing is to assure that equipment of the safety system normally in standby will be operable when the equipment in the operation fails. Martorell et al. [11] gave the insight into allowed outage time (AOT), surveillance test interval

(STI) and their intersections using probabilistic methods. Martorell et al. [10] worked on the surveillance and maintenance tasks to prevent the dominant failure cause of critical components by using reliability centered maintenance (RCM) method to establish maintenance task for critical component. Marton et al. [9] suggested the new model of ageing probabilistic safety assessment (APSA) which is used to support risk-informed decision and technical specification requirements of nuclear power plant (NPP). Hellmich and Berg [4] investigated a new model of probabilistic safety assessment (PSA) introducing the Markov model to discuss various strategies for organizing repair and periodic surveillance test of two-train standby safety system.

2. MODEL FORMULATION

This paper deals with periodic surveillance test, taken periodically to improve the reliability and availability of the system. Our model is 1-out-of-2 standby system operated under periodic surveillance test where one machine is in operation and the other is under surveillance test. When the surveillance test of second machine is completed, the second machine comes into the operation where the first machine will go in surveillance test. During the surveillance test if failure in the machine is detected, it is repaired immediately. At the same time no additional test is done on the other machine. When the repair is completed, the periodic surveillance test on that machine is resumed. Two types of failures (minor and major) and their repairs (minor and major) are taken into account. Reliability and availability of the system we have studied with repair, surveillance test and standby system provision can rarely be found in the area. Our work differs from the earlier works conducted by several authors in a way that we have provisioned minor and major failures and their minor and major repairs which not only enhance the reliability and availability of the system but also lead our problem more realistic.

2.1. **Notations Used in the Model:** We have used the following notations in the model:

A = The machine is available and is able to provide the service.

S = The machine is under surveillance test state.

F_f = The machine is in the state of minor failure.

F_m = The machine is in the state of major failure.

R_f = The machine is in the state of minor repair.

R_m = The machine is in the state of major repair.

β = Beta factor, $\beta \in [0, 1]$

$\bar{\beta}$ = 1-β

λ_1 = Standby minor failure rate.

λ_2 = Standby major failure rate.

$\beta\lambda_1$ = Common cause minor failure rate of the both machine.
$\beta\lambda_2$ = Common cause major failure rate of the both machine.
$\beta\lambda_3$ = Common cause minor in one and major in other failure rate.
$\bar{\beta}\lambda_1$ = Independent minor failure rate.
$\bar{\beta}\lambda_2$ = Independent major failure rate.
μ_r = Minor repair rate.
μ_R = Major repair rate.
η = Transition rate emanating from a surveillance test state.
$\frac{1}{\eta}$ = Mean surveillance test duration (mean STD).
ξ = Transition rate from a standby to a surveillance test state.
$\frac{1}{\xi}$ = the mean surveillance time interval (mean STI).
r = The probability that a machine under surveillance test can be brought successfully into operation when a system demand occur.
E = Set of states which are included in the mathematical model of the system.
Q_1 = Set of states corresponding to system availability.
Q_2 = Set of states corresponding to system availability with probability r.
P_i = Probability distribution of i^{th} state.

2.2. **Description of the Model and Methodology.** In the transition diagram (Figure 1), the states are numbered from 1 to 36. Combination of two letters describe the state of the first and second machine respectively. The Markov model is introduced in the transition diagram.

Machines of the system under consideration have constant standby minor failure rate λ_1 and the major failure rate λ_2. Only one standby redundant system has been provisioned. The beta factor model is applied for common cause failures. Here $\beta \in [0,1]$ denotes beta factor, $\beta\lambda_1$ is the common cause minor failure rate whereas $\beta\lambda_2$ is the common cause major failure rate. Again $\bar{\beta} = 1 - \beta$, then $\bar{\beta}\lambda_1$ is the independent minor failure rate and $\bar{\beta}\lambda_2$ is the independent major failure rate.
In the transition diagram (Figure 1), both machines are available in the green states 1,14,23,36. In the orange color states 2,3,8,9,17,18,19,20,28,29,34,35 one machine is available whereas other is either failure or under repair. The blue states 15,16,21,22 stand for the states in which one machine is failure where other is under surveillance test. We assume the probability that a machine which is under surveillance test can be brought successfully into operation when a system demand be r. Then the probability of the system availability which is in the state where one machine is failed and other is under surveillance test is r. The red states 4,5,6,7,10,11,12,13,24,25,26,27,30,31,32,33 represent the system failure states either failure both machines or one machine is failure and other is under repair.

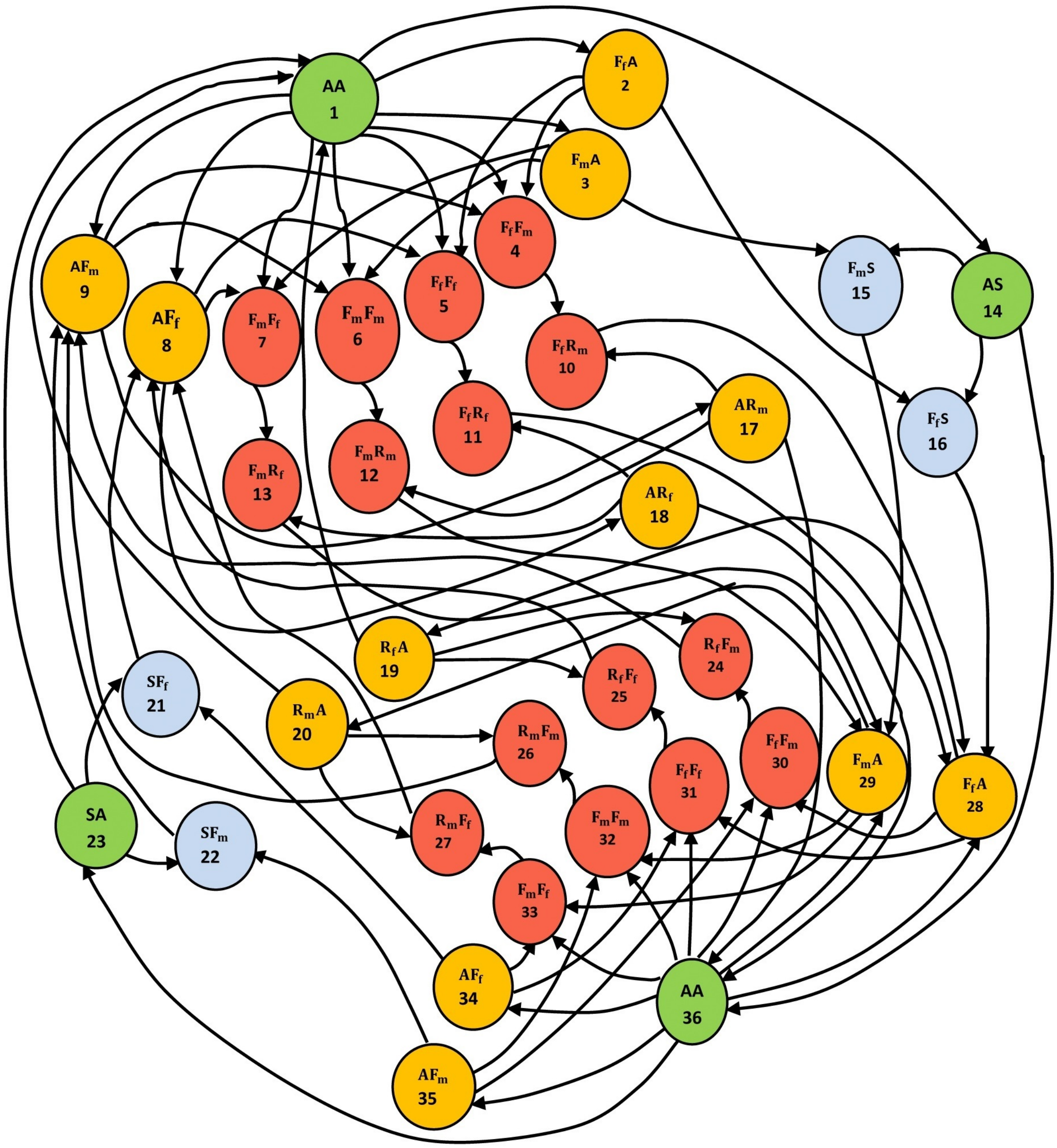

FIGURE 1. Transition diagram for two machines.
(Source: Basu Dev Ghimire and Ram Prasad Ghimire)

2.3. **Balance Equations.** From the transition diagram (Figure 1) with the transition rates listed above, for each state, using sum of the probabilities incoming to the state = sum of the probabilities outgoing from this state we obtained the following balance equations:

$$-(2\bar{\beta}\lambda_1 + 2\bar{\beta}\lambda_2 + 2\beta\lambda_3 + \beta\lambda_1 + \beta\lambda_2 + \xi)P_1 + \mu_r P_{19} + \mu_R P_{20} + \eta P_{23} = 0 \tag{2.1}$$

$$\bar{\beta}\lambda_1 P_1 - (\lambda_1 + \lambda_2 + \xi)P_2 = 0 \tag{2.2}$$

$$\bar{\beta}\lambda_2 P_1 - (\lambda_1 + \lambda_2 + \xi)P_3 = 0 \tag{2.3}$$

$$\beta\lambda_3 P_1 + \lambda_2 P_2 - \xi P_4 + \lambda_1 P_9 = 0 \tag{2.4}$$

$$\beta\lambda_1 P_1 + \lambda_1 P_2 - \xi P_5 + \lambda_1 P_8 = 0 \tag{2.5}$$

$$\beta\lambda_2 P_1 + \lambda_2 P_3 - \xi P_6 + \lambda_2 P_9 = 0 \tag{2.6}$$

$$\beta\lambda_3 P_1 + \lambda_1 P_3 - \xi P_7 + \lambda_2 P_8 = 0 \tag{2.7}$$

$$\bar{\beta}\lambda_1 P_1 - (\lambda_1 + \lambda_2 + \xi)P_8 + \eta P_{21} + \mu_r P_{25} + \mu_R P_{27} = 0 \tag{2.8}$$

$$\bar{\beta}\lambda_2 P_1 - (\lambda_1 + \lambda_2 + \xi)P_9 + \eta P_{22} + \mu_r P_{24} + \mu_R P_{26} = 0 \tag{2.9}$$

$$\xi P_4 - \mu_R P_{10} + \lambda_1 P_{17} = 0 \tag{2.10}$$

$$\xi P_5 - \mu_r P_{11} + \lambda_1 P_{18} = 0 \tag{2.11}$$

$$\xi P_6 - \mu_R P_{12} + \lambda_2 P_{17} = 0 \tag{2.12}$$

$$\xi P_7 - \mu_r P_{13} + \lambda_2 P_{18} = 0 \tag{2.13}$$

$$\xi P_1 - (\lambda_1 + \lambda_2 + \eta)P_{14} \tag{2.14}$$

$$\xi P_3 + \lambda_2 P_{14} - \eta P_{15} = 0 \tag{2.15}$$

$$\xi P_2 + \lambda_1 P_{14} - \eta P_{16} = 0 \tag{2.16}$$

$$\xi P_9 - (\lambda_1 + \lambda_2 + \mu_R)P_{17} = 0 \tag{2.17}$$

$$\xi P_8 - (\lambda_1 + \lambda_2 + \mu_r)P_{18} = 0 \tag{2.18}$$

$$-(\lambda_1 + \lambda_2 + \mu_r)P_{19} + \xi P_{28} = 0 \tag{2.19}$$

$$-(\lambda_1 + \lambda_2 + \mu_R)P_{20} + \xi P_{29} = 0 \tag{2.20}$$

$$-\eta P_{21} + \lambda_1 P_{23} + \xi P_{34} = 0 \tag{2.21}$$

$$-\eta P_{22} + \lambda_2 P_{23} + \xi P_{35} = 0 \tag{2.22}$$

$$-(\lambda_1 + \lambda_2 + \eta)P_{23} + \xi P_{36} = 0 \tag{2.23}$$

$$\lambda_2 P_{19} - \mu_r P_{24} + \xi P_{30} = 0 \tag{2.24}$$

$$\lambda_1 P_{19} - \mu_r P_{25} + \xi P_{31} = 0 \tag{2.25}$$

$$\lambda_2 P_{20} - \mu_R P_{26} + \xi P_{32} = 0 \tag{2.26}$$

$$\lambda_1 P_{20} - \mu_R P_{27} + \xi P_{33} = 0 \tag{2.27}$$

$$\mu_R P_{10} + \mu_r P_{11} + \eta P_{16} - (\lambda_1 + \lambda_2 + \xi) P_{28} + \bar{\beta} \lambda_1 P_{36} = 0 \tag{2.28}$$

$$\mu_R P_{12} + \mu_r P_{13} + \eta P_{15} - (\lambda_1 + \lambda_2 + \xi) P_{29} + \bar{\beta} \lambda_2 P_{36} = 0 \tag{2.29}$$

$$\lambda_2 P_{28} - \xi P_{30} + \lambda_1 P_{35} + \beta \lambda_3 P_{36} = 0 \tag{2.30}$$

$$\lambda_1 P_{28} - \xi P_{31} + \lambda_1 P_{34} + \beta \lambda_1 P_{36} = 0 \tag{2.31}$$

$$\lambda_2 P_{29} - \xi P_{32} + \lambda_2 P_{35} + \beta \lambda_2 P_{36} = 0 \tag{2.32}$$

$$\Rightarrow \lambda_1 P_{29} - \xi P_{33} + \lambda_2 P_{34} + \beta \lambda_3 P_{36} = 0 \tag{2.33}$$

$$-(\lambda_1 + \lambda_2 + \xi) P_{34} + \bar{\beta} \lambda_1 P_{36} = 0 \tag{2.34}$$

$$-(\lambda_1 + \lambda_2 + \xi) P_{35} + \bar{\beta} \lambda_2 P_{36} = 0 \tag{2.35}$$

$$\eta P_{14} + \mu_R P_{17} + \mu_r P_{18} - (2\bar{\beta}\lambda_1 + 2\bar{\beta}\lambda_2 + 2\beta\lambda_3 + \beta\lambda_1 + \beta\lambda_2 + \xi) P_{36} = 0 \tag{2.36}$$

Again,

$$P_1 + P_2 + P_3 + P_4 + P_5 + P_6 + P_7 + P_8 + P_9 + P_{10} + P_{11} + P_{12} + P_{13} + P_{14} \tag{2.37}$$
$$+ P_{15} + P_{16} + P_{17} + P_{18} + P_{19} + P_{20} + P_{21} + P_{22} + P_{23} + P_{24} + P_{25} + P_{26} + P_{27} + P_{28} + P_{29} + P_{30} + P_{31} + P_{32} + P_{33} + P_{34} + P_{35} + P_{36} = 1$$

Setting
$2\bar{\beta}\lambda_1 + 2\bar{\beta}\lambda_2 + 2\beta\lambda_3 + \beta\lambda_1 + \beta\lambda_2 + \xi = \delta_1$,
$\lambda_1 + \lambda_2 + \xi = \delta_2$,
$\lambda_1 + \lambda_2 + \eta = \delta_3$,
$\lambda_1 + \lambda_2 + \mu_R = \delta_4$,
$\lambda_1 + \lambda_2 + \mu_r = \delta_5$,
$\bar{\beta}\lambda_1 = \delta_6$,
$\bar{\beta}\lambda_2 = \delta_7$,
$\beta\lambda_1 = \delta_8$,
$\beta\lambda_2 = \delta_9$,
$\beta\lambda_3 = \delta_0$,
the system of equations (2.1)-(2.37) can be written in the matrix form as,

$$BP = C \tag{2.38}$$

where B $= \begin{bmatrix} B_1 & B_2 \\ B_3 & B_4 \end{bmatrix}$, and

$B_1 =$

$$\begin{bmatrix}
-\delta_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\delta_6 & -\delta_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\delta_7 & 0 & -\delta_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\delta_0 & \lambda_2 & 0 & -\xi & 0 & 0 & 0 & 0 & \lambda_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\delta_8 & \lambda_1 & 0 & 0 & -\xi & 0 & 0 & \lambda_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\delta_9 & 0 & \lambda_2 & 0 & 0 & -\xi & 0 & 0 & \lambda_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\delta_0 & 0 & \lambda_1 & 0 & 0 & 0 & -\xi & \lambda_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\delta_6 & 0 & 0 & 0 & 0 & 0 & 0 & -\delta_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\delta_7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\delta_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \xi & 0 & 0 & 0 & 0 & 0 & -\mu_R & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_1 & 0 \\
0 & 0 & 0 & 0 & \xi & 0 & 0 & 0 & 0 & 0 & -\mu_r & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_1 \\
0 & 0 & 0 & 0 & 0 & \xi & 0 & 0 & 0 & 0 & 0 & -\mu_R & 0 & 0 & 0 & 0 & \lambda_2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \xi & 0 & 0 & 0 & 0 & 0 & -\mu_r & 0 & 0 & 0 & 0 & \lambda_2 \\
\xi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\delta_3 & 0 & 0 & 0 & 0 \\
0 & 0 & \xi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_2 & -\eta & 0 & 0 & 0 \\
0 & \xi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_1 & 0 & -\eta & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \xi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\delta_4 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \xi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\delta_5 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix},$$

$$B_2 = \begin{bmatrix}
\mu_r & \mu_R & 0 & 0 & \eta & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \eta & 0 & 0 & 0 & \mu_r & 0 & \mu_R & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \eta & 0 & \mu_r & 0 & \mu_R & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\delta_5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \xi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix},$$

$$B_3 = \begin{bmatrix} 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&\mu_R&\mu_r&0&0&0&0&\eta&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&\mu_R&\mu_r&0&\eta&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0&\eta&0&0&\mu_R&\mu_r\\ 1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1 \end{bmatrix},$$

$B_4 =$

$$\begin{bmatrix} 0&-\delta_4&0&0&0&0&0&0&0&0&\xi&0&0&0&0&0&0&0\\ 0&0&-\eta&0&\lambda_1&0&0&0&0&0&0&0&0&0&0&\xi&0&0\\ 0&0&0&-\eta&\lambda_2&0&0&0&0&0&0&0&0&0&0&0&\xi&0\\ 0&0&0&0&-\delta_3&0&0&0&0&0&0&0&0&0&0&0&0&\xi\\ \lambda_2&0&0&0&0&-\mu_r&0&0&0&0&0&\xi&0&0&0&0&0&0\\ \lambda_1&0&0&0&0&0&-\mu_r&0&0&0&0&0&\xi&0&0&0&0&0\\ 0&\lambda_2&0&0&0&0&0&-\mu_R&0&0&0&0&0&\xi&0&0&0&0\\ 0&\lambda_1&0&0&0&0&0&0&-\mu_R&0&0&0&0&0&\xi&0&0&0\\ 0&0&0&0&0&0&0&0&0&-\delta_2&0&0&0&0&0&0&0&\delta_6\\ 0&0&0&0&0&0&0&0&0&0&-\delta_2&0&0&0&0&0&0&\delta_7\\ 0&0&0&0&0&0&0&0&0&\lambda_2&0&-\xi&0&0&0&0&\lambda_1&\delta_0\\ 0&0&0&0&0&0&0&0&0&\lambda_1&0&0&-\xi&0&0&\lambda_1&0&\delta_8\\ 0&0&0&0&0&0&0&0&0&0&\lambda_2&0&0&-\xi&0&0&\lambda_2&\delta_9\\ 0&0&0&0&0&0&0&0&0&0&\lambda_1&0&0&0&-\xi&\lambda_2&0&\delta_0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&-\delta_2&0&\delta_6\\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&-\delta_2&\delta_7\\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&-\delta_1\\ 1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1&1 \end{bmatrix},$$

$$\mathrm{P} = \begin{bmatrix} P_m & P_n \end{bmatrix}',$$

$$P_m = \begin{bmatrix} P_1 & P_2 & P_3 & P_4 & P_5 & P_6 & P_7 & P_8 & P_9 & P_{10} & P_{11} & P_{12} & P_{13} & P_{14} & P_{15} & P_{16} & P_{17} & P_{18} \end{bmatrix},$$

$$P_n = \begin{bmatrix} P_{19} & P_{20} & P_{21} & P_{22} & P_{23} & P_{24} & P_{25} & P_{26} & P_{27} & P_{28} & P_{29} & P_{30} & P_{31} & P_{32} & P_{33} & P_{34} & P_{35} & P_{36} \end{bmatrix},$$

$$\mathrm{C} = \begin{bmatrix} C_1 & C_2 \end{bmatrix}',$$

$$C_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$C_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The order of matrix B is 37x36, that of B_1 is 19x18, B_2 is 19x18, B_3 is 18x18, B_4 is 18x18, P is 36x1, P_m is 1x18, P_n is 1x18, C is 37x1, C_1 is 1x18 and C_2 is 1x19.
For numerical computations, we have applied pseudo-inverse technique to solve the system of equations (2.1) - (2.37).
From equation (2.38)

$$P = (B^T B)^{-1}(B^T C) \tag{2.39}$$

2.4. **Availability and Unavailability.** The system availability A and unavailability $\bar{A}$ at steady state are given by

$$A = \sum_{i\in Q_1} P_i + r\sum_{i\in Q_2} P_i \tag{2.40}$$

where $\sum_{i\in Q_1} P_i = \sum_{i=1}^{3} P_i + \sum_{i=8}^{9} P_i + P_{14} + \sum_{i=17}^{20} P_i + P_{23} + \sum_{i=28}^{29} P_i + \sum_{i=34}^{36} P_i$

and $\sum_{i\in Q_2} P_i = \sum_{i=15}^{16} P_i + \sum_{i=21}^{22} P_i$

$$\bar{A} = \sum_{i\in E-(Q_1\cup Q_2)} P_i + (1-r)\sum_{i\in Q_2} P_i \tag{2.41}$$

Clearly,

$$A + \bar{A} = 1 \tag{2.42}$$

2.5. **Numerical Results and Interpretation.** Numerical results have been obtained by using computing software by varying one parameter while fixing remaining parameters in equation (2.40) which we describe in figure 2 through figure 16 one by one. To observe how the availability depends on the different parameters, we have taken the following values of effective parameters: r = 0.9; $\xi = 0.01$; $\eta = 0.2$; $\beta = 0.02$; $\bar{\beta} = 1 - \beta$; $\mu_r = 0.2$; $\mu_R = 0.1$; $\lambda_1 = 0.002$; $\lambda_2 = 0.0001$; $\lambda_3 = 0.001$

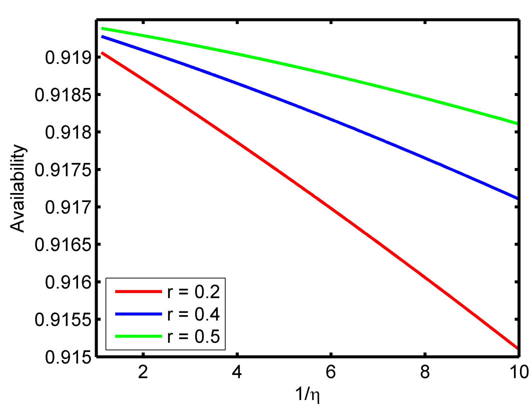

FIGURE 2. Availability vs. mean surveillance test duration (hours).

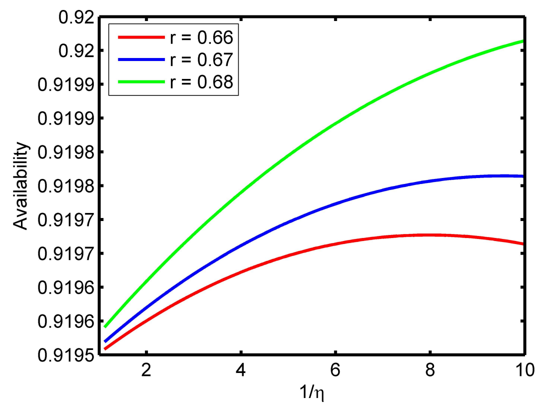

FIGURE 3. Availability vs. mean surveillance test duration (hours).

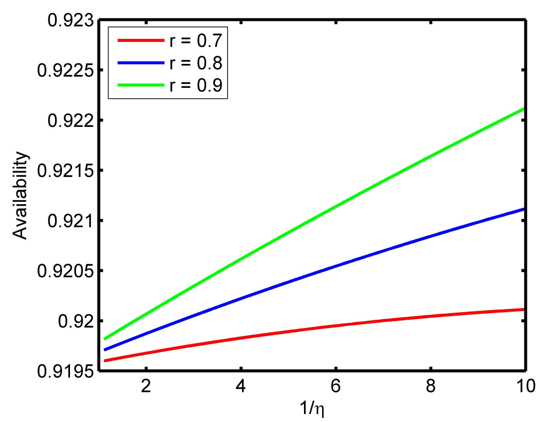

FIGURE 4. Availability vs. mean surveillance test duration (hours).

The dependency of system availability on mean STD at different values of r are shown in figures 2, 3 and 4. Figure 2 shows availability decreases almost linearly with the increase of mean STD when the value of r is less than 0.5. Figure 3 reveals in one hand that, initially availability of machine at r = 0.66 increases and decreases gradually with the increase of mean STD. On the other hand, the availability of the machine at r = 0.67 and 0.68 increases with the increase of mean STD. Figure 4 explores that when r is greater than 0.7 the availability increases almost linearly with the increase of mean STD.

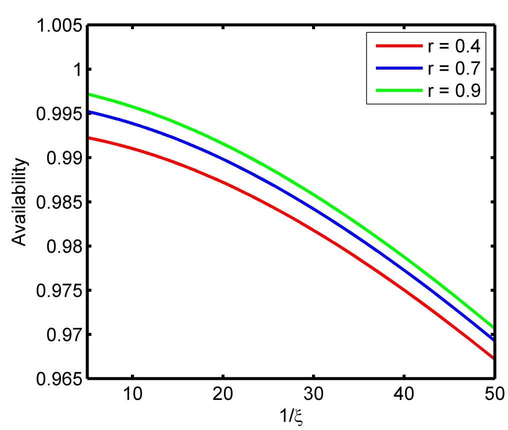

FIGURE 5. Availability vs. mean surveillance test interval (days).

Figure 5 deals with availability as the function of mean surveillance test interval (mean STI) for three different values of r. The availability decreases gradually with the increase of mean STI for different values of r = 0.4, r = 0.7 and r = 0.9.

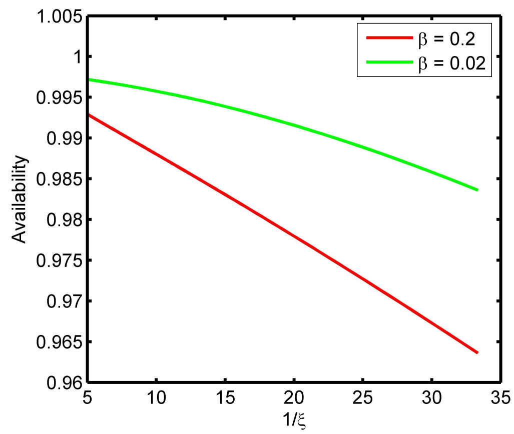

FIGURE 6. Availability vs. mean surveillance test interval (days).

Similarly figure 6 demonstrates the availability as the function of mean STI for two different values of beta factors. The availability decreases with the increase of mean STI. When β changes from 0.02 to 0.2, the trade-off between two availability significantly wide with the increase of mean STI.

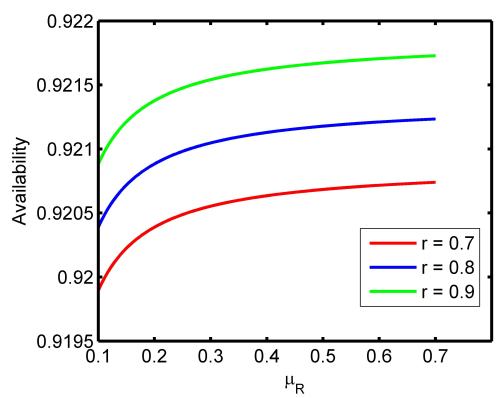

FIGURE 7. Availability vs. major repair rate.

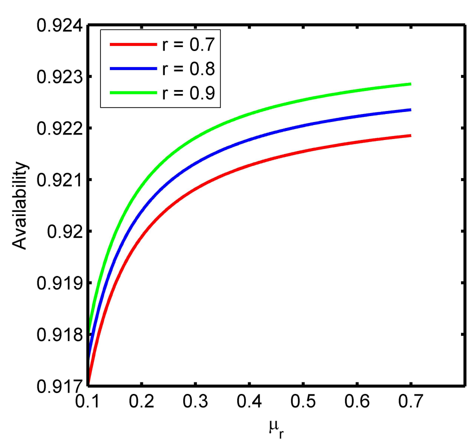

FIGURE 8. Availability vs. minor repair rate.

The dependency of availability in the system on major repair rate and minor repair rate can be observe in figure 7 and figure 8 respectively for different values of r. The availability increases gradually with the increase of major repair rate in figure 7 and minor repair rate in figure 8 at r = 0.7, r = 0.8 and r = 0.9.

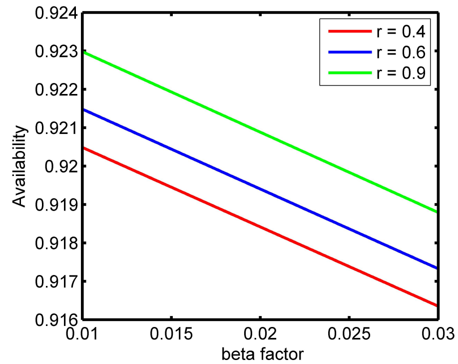

FIGURE 9. Availability vs. beta factor.

Linearly decrements of availability with the increment of beta factors for r = 0.4, r = 0.6 and r = 0.9 have been viewed in figure 9.

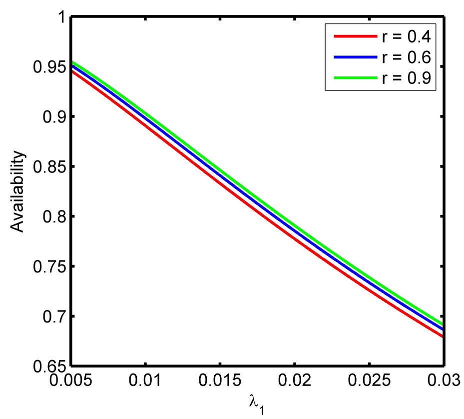

FIGURE 10. Availability vs. standby minor failure rate.

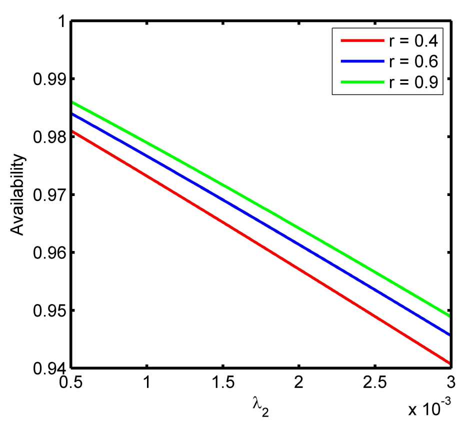

FIGURE 11. Availability vs. standby major failure rate.

Figure 10 and figure 11 predicts that availability decreases steeply with the increase of standby minor failure rates and standby major failure rates respectively.

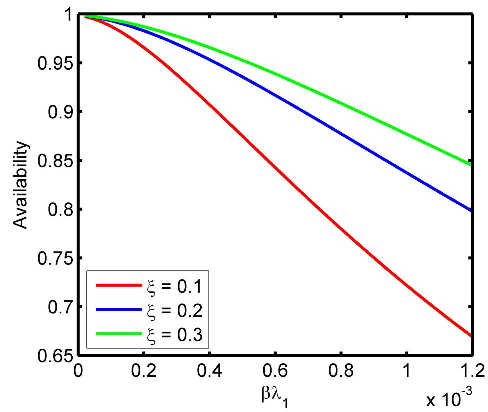

FIGURE 12. Availability vs. common cause minor failure rate of both machines.

Figure 12 and figure 13 reveal the impact of common cause minor failure rate of both machines and common cause major failure rate of both machines respectively on the availability for three different values of $\xi = 0.1,\ \xi = 0.2,\ \xi = 0.3$. But the decreasing rate of availability is more for the lesser value of ξ when $\beta\lambda_1$ and $\beta\lambda_2$ increasing.

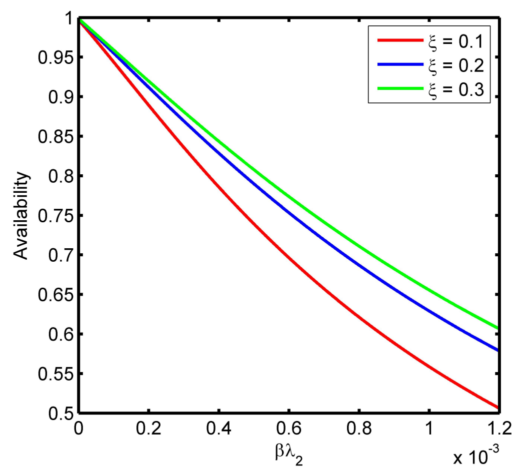

FIGURE 13. Availability vs. common cause major failure rate of both machines.

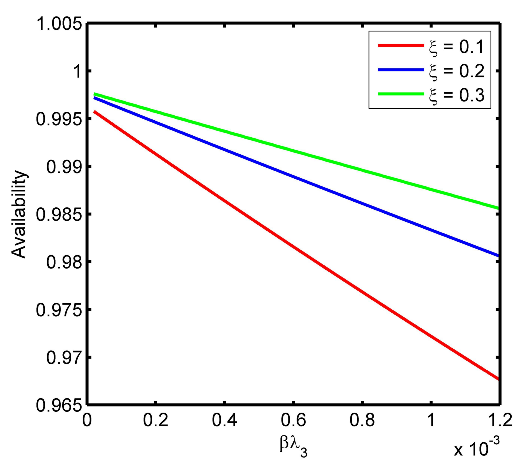

FIGURE 14. Availability vs. common cause minor failure rate in one and major failure rate in other machines.

Figure 14 explores the application of common cause minor failure rate in one and major failure rate in other machine on availability. As $\beta\lambda_3$ increases, availability decreases for three different values of $\xi = 0.1,\ \xi = 0.2,\ \xi = 0.3$. This shows the better agreement with real life situation.

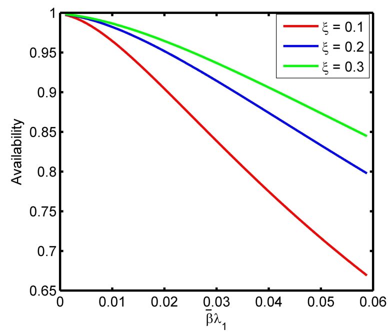

FIGURE 15. Availability vs. independent minor failure rate.

Figure 15 and figure 16 analyze the availability decreases significantly when the independent minor failure rate and independent major failure rate respectively increases at $\xi = 0.1,\ \xi = 0.2,\ \xi = 0.3$. But the decreasing rate of availability is less in figure 15 with compare to figure 16, which is practically true because less time is required to repair independent minor failure than independent major failure.

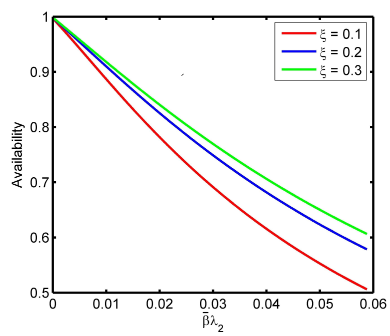

FIGURE 16. Availability vs. independent major failure rate.

3. **Conclusion**

Mathematical model under study has been constructed with major and minor states from which set of system of equations has been established. For the solution purpose matrix-pseudo-inverse-method has been deployed. Various measures of performances have numerically been obtained by varying some of the influenced parameters into consideration. Numerical results obtained get better agreement with real life situations. Model under study may be applicable to machining system, manufacturing system, communication system, internet and intranet system, robot and robotic system.

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