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MODELING BOUNDARY LAYER FLOW AND HEAT TRANSFER OF A PARTICULATE SUSPENSION

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ABSTRACT. The flow of a steady viscous incompressible fluid with uniformly distributed suspended particles past a thin heated semi-infinite flat plate is considered. Finite difference technique with non-uniform grid is used to investigate the effect of various flow parameters on the flow and heat transfer characteristics of the particulate suspension. Heat is diffused away from the heated surface for smaller values of Prandtl number Pr more rapidly than that of higher values of Pr. The magnitude of particle velocity and particle phase density are increased whereas the particle temperature is reduced due to the presence of coarser particles with high material density. Higher values of Prandtl number Pr is to increase the magnitude of carrier fluid velocity, particle velocity, particle phase density and Nusselt number.

Key Words: Volume fraction, Suspended particulate matter, Particulate Suspension, Slip velocity, Heat Transfer.

AMS (MOS) Subject Classification. 35Q35, 35Q79.

1. INTRODUCTION

The flow of fluid with suspended particulate matter (SPM) has received the attention of many researchers due to its possible industrial applications like sedimentation, pipe flows, fluidized beds, gas-purification and transport process. Soo [1] and Chiu [2] have studied the boundary layer flow of fluid with suspended particles by neglecting the particle momentum equation in the normal direction. Both Marble [3] and Soo [7] have developed the conservation laws of mass, momentum and energy for two-phase flow and have obtained their closed form solutions. Marbel [3] has considered dynamics of a gas combining small solid particles and obtained the solutions valid for far downstream region of the plate assuming zero particulate velocity on the surface. Soo [7] has derived momentum integrals for the fluid and particle phases. Soo [4] has investigated the laminar mixing of a suspension with a clean fluid without considering the conservation of particulate phase momentum in the normal direction. Rudinger [5] has shown the effect of finite particle volume on the dynamics of gas-particle mixtures. Singleton [6] has considered compressible laminar boundary-layer

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flow of a dusty gas over a semi-infinite flat plate and obtained asymptotic solutions using the series expansion method for both small slip region and the large slip region close to the leading edge of the plate. Otterman [8] has studied the laminar mixing of a dusty fluid with clean fluid and has shown the effect of transverse force on the flow field. Tabakoff and Hammed [9] have studied boundary layer flow of particulate gas and pointed out that particle velocity decreases linearly and particle density increases continuously along the plate length. Jain and Ghosh [10] has studied the laminar boundary layer flow on a flat plate employing momentum integral method. Prabha and Jain [11] have employed the finite difference technique to study the laminar boundary layer flow over a flat plate, but have not considered the momentum equation in the normal direction. Dutta and Mishra [12] have studied Boundary Layer Flow of a Dusty Fluid over a Semi-Infinite Flat Plate.Wang and Glass [13] have studied compressible laminar boundary layer flows of a dusty gas over a semi-infinite flat plate considering a moderate slip region (a non-equilibrium transition region) in addition to the large and small slip regions. Chamakha [14] has considered a continuum mathematical model governing compressible boundary-layer fluid-particle flow and heat transfer over a semi-infinite flat plate. Panda et.al. [15] have studied the effect of diffusion of particles on the laminar two phase flow. Partha et. al. [16] have presented a similarity solution for mixed convection flow and heat transfer from an exponentially stretching surface by considering viscous dissipation effect in the medium. They showed that the buoyancy and viscous dissipation have significant influence on the non-dimensional skin friction and heat transfer coefficient. Panda et.al. [17] have studied the effect of volume- fraction and diffusion of SPM in in free convection flows in the vicinity of heated horizontal flat plate. Mishra and Tripathy [18, 20] have investigated the two-phase boundary layer flow over a flat plate to study the boundary layer flow characteristics by using momentum integral method, where effect of volume fraction on the flow has not been studied. Misra et.al.[19] have employed the Crank-Nicholson finite difference technique to show the effect of electrification of suspended particulate matter(SPM) in a two phase boundary layer flow and heat transfer over a semi-infinite flat plate. From most of the above literature, it is observed that studies relating to the boundary layer flow of dusty fluid with negligible volume fraction, omission of the momentum equation for particulate phase in normal direction and assumption of no slip condition for particle velocity are inadequate. The assumption of neglecting the volume fractions of the particles is not justified when the fluid density is high or particle mass fraction is large. Rudinger [5] has shown that the error in neglecting the volume fraction ranges from insignificant to large. Otterman [8] has shown that the boundary layer approximation of the momentum equation for the fluid phase is not necessary and that the particle momentum equation in the normal direction cannot be neglected. Further, the assumption of no slip condition for particle velocity in most of the above literature is not physically plausible since the particles do not flow in unison with fluid.In the boundary layer, the fluid decelerates from its free stream velocity to zero velocity at the solid surface, but since the density of the SPM is much greater than the fluid density, the SPM cannot accommodate this rapid deceleration but tends to slip through the fluid as they decelerate.

In the present problem, the influencing parameters like finite volume fraction,the momentum equation for particulate phase in the direction normal to the flow, heat due to conduction and viscous dissipation have been simultaneously considered to study their effects on the boundary layer flow and heat transfer of a steady viscous incompressible fluid with uniformly distributed suspended particles past a thin heated semi-infinite flat plate fulfilling the inadequacies of previous investigators. The diffusion equation is considered for calculating the particle concentration in the incompressible flow field. The heat due to conduction and viscous dissipation are included in the energy equations of both phases with a view to estimate the temperature profile as well as the rate of heat transfer on the surface of both phases.Further, the "no slip" condition is not satisfied by the particles, so the velocity, temperature and concentration of particle phase on the plate are considered as the functions of x only. The governing coupled, non-linear partial differential equations of the flow field are solved numerically using the finite difference technique with non-uniform-grid.

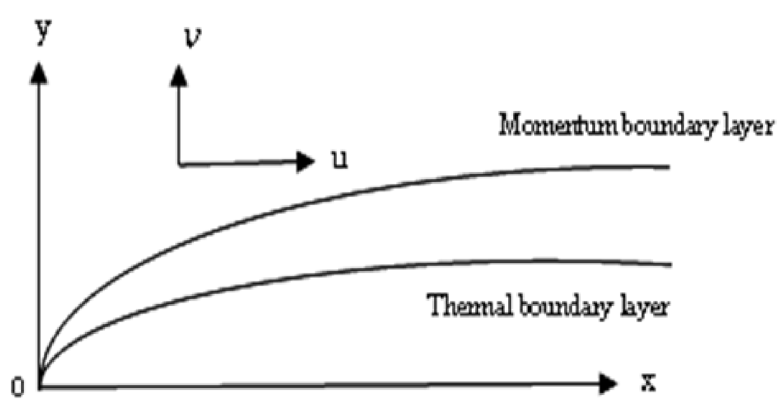

Figure 1. Schematic diagram of the flow field

NOMENCLATURE

$(u,\ v)$	Velocity components for the fluid phase in x- and y- directions respectively	μ	Coefficient of viscosity of fluid
(u_p, v_p)	Velocity components for the particle phase in x-and y- directions respectively	δ	Boundary layer thickness
(T, T_p)	Temperatures of fluid and particle phase	τ_T	Thermal equilibrium time
(T_w, T_∞)	Temperature at the wall and free-stream respectively	T_0	Temperature of the plate at η= 0
(ν,ν_p)	Kinematics coefficient of viscosity of fluid and particle phase respectively	a	Thermal diffusivity
(ρ,ρ_p)	Density of fluid and particle phase respectively	κ	Thermal conductivity
(ρ_s,ρ_m)	Material density of particle and mixture respectively	α	Concentration parameter
P_r	Prandtl number	ϵ	Diffusion parameter
E_c	Eckret number	J_{max}	Maximum number of Grid points along Y-axis
N_u	Nusselt number	L	Reference length
C_f	Skin friction coefficient	$W(x,y)$	Dummy variable
φ	Volume fraction of SPM	r_y	Grid growth ratio
D	Diameter of the particle	D_p	Binary diffusion coefficient
U	Free stream velocity		

2. MATHEMATICAL FORMULATION

A steady flow of a viscous incompressible fluid with uniformly distributed suspended particles past a heated thin semi-infinite flat plate at a constant temperature T_w is considered. The plate is placed along the direction of a uniform free stream of velocity U and temperature T_∞. Let the plate be placed in the direction of X-axis and Y-axis be perpendicular to it, as shown in Fig.1. Introducing the non- dimensional variables

$$x^* = x/L, \qquad n = y/LRe$$
$$u^* = u/U, \qquad v^* = v/URe$$
$$u_p^* = u_{\mathrm{p}}/U, \qquad v_p^* = v_{\mathrm{p}}/URe$$
$$T^* = \frac{T - T_\infty}{T_w - T_\infty}, \quad T_p^* = \frac{T_p - T_\infty}{T_w - T_\infty} \quad \rho_p^* = \frac{\rho_p}{\rho_{p_0}} \tag{2.1}$$

The governing boundary layer equations of the flow field following Misra et. al. [19], after dropping stars are given by

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial \eta}=0 \tag{2.2}$$

$$\frac{\partial \rho_p}{\partial x}+v_p\frac{\partial \rho_p}{\partial \eta}=\epsilon\frac{\partial^2 \rho_p}{\partial \eta^2} \tag{2.3}$$

$$u\frac{\partial u}{\partial x}+\nu\frac{\partial u}{\partial \eta}=\frac{\partial^2 u}{\partial \eta^2}-\alpha\frac{\psi}{1-\psi}\frac{FL}{U}\rho_p(u-u_p) \tag{2.4}$$

$$u_p\frac{\partial u_p}{\partial x}+v_p\frac{\partial u_p}{\partial \eta}=\epsilon\frac{\partial^2 u_p}{\partial \eta^2}+\frac{FL}{U}(u-u_p) \tag{2.5}$$

$$u_p\frac{\partial v_p}{\partial x}+v_p\frac{\partial v_p}{\partial \eta}=\epsilon\frac{\partial^2 v_p}{\partial \eta^2}+\frac{FL}{U}(v-v_p) \tag{2.6}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial \eta}=\frac{1}{Pr}\frac{\partial^2 T}{\partial \eta^2}+Ec\left(\frac{\partial u}{\partial \eta}\right)^2+\frac{2}{3}\frac{\alpha}{\mathrm{Pr}}\frac{\psi}{1-\psi}\frac{FL}{U}\rho_p(T_p-T) \tag{2.7}$$

$$u_p\frac{\partial T_p}{\partial x}+v+p\frac{\partial T_p}{\partial \eta}=\frac{FL}{U}(T-T_p)+\frac{\epsilon}{\mathrm{Pr}}\frac{\partial^2 T_p}{\partial \eta^2}+\epsilon\cdot Ec\left[\left(\frac{\partial u_p}{\partial \eta}\right)^2+u+p\frac{\partial^2 u_p}{\partial \eta^2}\right] \tag{2.8}$$

$$\eta=0: u=0, v=0, u+p=u_{\mathrm{pw}}(x), v_p=0, \rho_p=\rho_{\mathrm{pw}}(x), T=1, T_p=T_{\mathrm{pw}}(x) \tag{2.9}$$

$$\eta=\infty: u=u_p=\rho_p=1, v_p=0, T=0, T_p=0 \tag{2.10}$$

Introducing the finite difference expressions with non-uniform-grid for the various terms. The Equations (2.2) to (2.8) are reduced to

$$v_j^{n+1}=v_{j-1}^{n+1}-\frac{1}{2}\frac{\Delta y}{\Delta x}\left[(1.5u_j^{n+1}-2u_j^n+0.5u_j^{n-1})+(1.5u_{j-1}^{n+1}-2u_{j-1}^n+0.5u_{j-1}^{n-1})\right]$$

$$a_ju_{j-1}^{n+1}+b_ju_j^{n+1}+c_ju_{j+1}^{n+1}=d_j$$

$$a_j^*{u_p}_{j-1}^{n+1}+b_j^*+{u_p}_j^{n+1}+c_j^*{u_p}_{j+1}^{n+1}=d_j^*$$

$$a_j^{**}{v_p}_{j-1}^{n+1}+b_j^{**}{v_p}_j^{n+1}+c_j^{**}{v_p}_{j+1}^{n+1}=d_j^{**}$$

$$a_j^+T_{j-1}^{n+1}+b_j^+T_j^{n+1}+c_j^+T_{j+1}^{n+1}=d_j^+$$

$$a_j^+{T_p}_{j-1}^{n+1}+b_j^{++}{T_p}_j^{n+1}=d_j^{++}$$

$$a_j^{\#}{\rho_p}_{j-1}^{n+1}+b_j^{\#}{\rho_p}_j^{n+1}+c_j^{\#}{\rho_p}_{j+1}^{n+1}=d_j^{\#}$$

Where,

$$a_j = \frac{1}{\Delta x}\left[-pr_y - q\right]$$

$$b_j = \frac{1}{\Delta x}\left[1.5(2u_j^n - u_j^{n-1}) + p\left(r_y - \frac{1}{r_y}\right) + q\left(1 + \frac{1}{r_y}\right) + \frac{\psi}{1-\psi}\frac{FL}{U}\alpha\Delta x\left(2{\rho_p}_j^n - {\rho_p}_j^{n-1}\right)\right]$$

$$c_j = \frac{1}{\Delta x}\left[\frac{1}{r_y}(p - q)\right]$$

$$d_j = \frac{1}{\Delta x}\left[(2u_j^n - u_j^{n-1})(2u_j^n - 0.5u_j^{n-1}) - \frac{\psi}{1-\psi}\frac{FL}{U}\alpha\Delta x\left(2{\rho_p}_j^n - {\rho_p}_j^{n-1}\right)(-2{u_p}_j^n + {u_p}_j^{n-1})\right]$$

$$a_j^* = \frac{1}{\Delta x}[-pr_y - \epsilon q]$$

$$b_j^* = \frac{1}{\Delta x}\left[1.5(2{u_p}_j^n - {u_p}_j^{n-1}) + p\left(r_y - \frac{1}{r_y}\right) + \epsilon q\left(1 + \frac{1}{r_y}\right) + \frac{FL}{U}\Delta x\right]$$

$$c_j^* = \frac{1}{\Delta x}\left[\frac{1}{r_y}(p - \epsilon q)\right]$$

$$d^* = \frac{1}{\Delta x}\left[(2{u_p}_j^n - {u_p}_j^{n-1})(2{u_p}_j^n - 0.5{u_p}_j^{n-1}) + \frac{FL}{U}\Delta x u_j^{n+1}\right]$$

$$a_j^{**} = \frac{1}{\Delta x}[-pr_y - \epsilon q]$$

$$b_j^{**} = \frac{1}{\Delta x}\left[1.5{u_p}_j^{n+1} + p\left(r_y - \frac{1}{r_y}\right) + \epsilon q\left(1 + \frac{1}{r_y}\right) + \frac{FL}{U}\Delta x\right]$$

$$c_j^{**} = \frac{1}{\Delta x}\left[\frac{1}{r_y}(p - \epsilon q)\right]$$

$$d_j^{**} = \frac{1}{\Delta x}\left[{u_p}_j^{n+1}(2{v_p}_j^n - 0.5{v_p}_j^{n-1} + \frac{FL}{U}\Delta x v_j^{n+1}\right]$$

$$a_j^+ = \frac{1}{\Delta x}\left[-q\left(0.5r_y\Delta y v_j^{n+1} + \frac{1}{\mathrm{Pr}}\right)\right]$$

$$b_j^+ = \frac{1}{\Delta x}\left[1.5u_j^{n+1} + 0.5q\Delta y v_j^{n+1}\left(r_y - \frac{1}{r_y}\right) + \frac{1(1+r_y)}{\mathrm{Pr}\cdot r_y} + \frac{2\alpha}{2\mathrm{Pr}}\frac{\psi}{1-\psi}\frac{FL}{U}\Delta x{\rho_p}_j^{n+1}\right]$$

$$c_j^+ = \frac{1}{\Delta x}\left[\frac{q}{r_y}\left(0.5\Delta y v_j^{n+1} - \frac{1}{\mathrm{Pr}}\right)\right]$$

$$d_j^+ = \frac{1}{\Delta x}\left[\frac{2\alpha}{3\mathrm{Pr}}\frac{\psi}{1-\psi}\frac{FL}{U}{\rho_p}_j^{n+1}(2{T_p}_j^n - {T_p}_j^{n-1})\Delta x + \Delta\right] +$$

$$\frac{1}{\Delta x}\left[x \cdot Ec\left(\frac{u_{j+1}^{n+1} - u_j^{n+1}}{\Delta y}\right)^2 + u_j^{n+1}(2T_j^n - 0.5T_j^{n-1})\right]$$

$$a^{++} = \frac{1}{\Delta x}\left[-q\left(0.5r_y\Delta y v_j^{n+1} + \frac{\epsilon}{\mathrm{Pr}}\right)\right]$$

$$b_j^{++} = \frac{1}{\Delta x}\left[1.5u_{p_j}^{n+1} + 0.5q\Delta y v_{p_j}^{n+1}\left(r_y - \frac{1}{r_y}\right) + \frac{\epsilon q(1+r_y)}{\mathrm{Pr}\cdot r_y} + \frac{FL}{U}\Delta x\right]$$

$$c_j^{++} = \frac{1}{\Delta x}\left[\frac{q}{r_y}\left(0.5\Delta y\cdot v_{p_j}^{n+1} - \frac{\epsilon}{\mathrm{Pr}}\right)\right]$$

$$d^{++} = \frac{1}{\Delta x}\left[u_{p_j}^{n+1}(2T_{p_j}^{n} - 0.5T_{p_j}^{n-1}) + \frac{FL}{U}T_j^{n+1}\Delta x\right] +$$

$$\epsilon\cdot Ec\left[\left(\frac{u_{p_{j+1}}^{n+1} - u_{p_j}^{n+1}}{\Delta y}\right)^2 + u_{p_j}^{n+1}\left(\frac{u_{p_{j-1}}^{n+1} - \left(1+\frac{1}{r_y}\right)u_{p_j}^{n+1} + \frac{1}{r_y}u_{p_{j+1}}^{n+1}}{(1+r_y)\Delta y^2}\right)\right]$$

$$a^{\#} = -v_{p_j}^{n+1}r_{y_i}^2\Delta y - 2\epsilon r_y$$

$$b_j^{\#} = \frac{1.5p^{\#}u_{p_j}^{n+1}}{\Delta x} - v_{p_j}^{n+1}(1-r_y)^2\Delta y + 2\epsilon(1+r_y)$$

$$c_j^{\#} = v_{p_j}^{n+1}\Delta y - 2\epsilon$$

$$d_j^{\#} = p^{\#}u_{p_j}^{n+1}\frac{2\rho_{p_j}^{n} - 0.5\rho_{p_j}^{n-1}}{\Delta x}$$

$$p = (2v_j^n - v_j^{n-1})\frac{\Delta\Delta}{(1+r_y)\Delta y}$$

$$q = \frac{2\Delta x}{(1+r_y)\Delta y^2} \qquad p^{\#} = r_y(1+r_y)\Delta y$$

Using the boundary conditions (2.9) and (2.10) at $j = 1$ or $j = j_{\max}$, we get

$$a_2 = 0,\ u_1 = 0 \qquad \text{at } j = 2$$

$$d_j = d_j - c_j u_e \qquad \text{at } j = j_{\max} - 1$$

$$d_2^* = d_2^* - a_2^* v_{\mathrm{pw}} \qquad \text{at } j = 2$$

$$d_j^* = d_j^* - c_j u_e \qquad \text{at } j = j_{\max} - 1$$

$$a_2^{**} = 0 \qquad \text{at } j = 2$$

$$c_j^{**} = 0 \qquad \text{at } j = j_{\max} - 1$$

$$d_2^+ = d_2^+ - a_2^+ \qquad \text{at } j = 2$$

$$d_2^+ = d_2^+ - a_2^+ \qquad \text{at } j = j_{\max} - 1$$

$$c_j^+ = 0 \qquad \text{at } j = j_{\max} - 1$$

$$d_2^{++} = d_2^{++} - a_2^{++}T_{\mathrm{pw}} \qquad \text{at } j = 2$$

$$c_j^{++} = 0 \qquad \text{at } j = j_{\max} - 1$$

$$d_2^{\#} = d_2^{\#} - a_2^{\#}\rho_{\mathrm{pw}} \qquad j = 2$$

$$d_j^{\#} = d_j^{\#} - c_j^{\#} \qquad \text{at } j_{\max} - 1$$

As no slip condition is not satisfied by the particles, so $u_{\rm pw}$, $\rho_{\rm pw}$, and $T_{\rm pw}$ are calculated separately on the plate at $\eta = 0$. As $u_{\rm pw}$, $\rho_{\rm pw}$, and $T_{\rm pw}$ are functions of x only, so from Eqns. (2.5), (2.3) and (2.8) we obtain

$$\frac{\partial u_{pw}}{\partial x} = -\frac{FL}{U} \tag{2.11}$$

$$\frac{\partial}{\partial x}(\rho_{pw}u_{pw}) + \frac{\partial}{\partial y}(\rho_p v_p) = 0 \Rightarrow u_{pw}\frac{\partial \rho_{pw}}{\partial x} - \rho_{pw}\frac{FL}{U} = 0 \tag{2.12}$$

$$u_{pw}\frac{\partial T_{pw}}{\partial x} = \frac{FL}{U}(1 - T_{pw}) \tag{2.13}$$

Using finite differences, Eqns. (2.11), (2.12), and (2.13) are reduced to

$$u_1^{n+1} = -\frac{2}{3}\frac{FL}{U}\Delta x + \frac{4}{3}u_1^n - \frac{1}{3}u_1^{n-1} \tag{2.14}$$

$${\rho_{p}}_1^{n+1} = \frac{2{\rho_p}_1^n - 0.5{\rho_p}_1^{n-1}}{1.5 - \frac{\frac{FL}{U}\Delta x}{{u_p}_1^{n+1}}} \tag{2.15}$$

$${T_p}_1^{n+1} = \frac{2{T_p}_1^n - 0.5{T_p}_1^{n-1} + \frac{FL}{U}\frac{\Delta x}{{u_p}_1^{n+1}}}{1.5 + \frac{FL}{U}\frac{\Delta x}{{u_p}_1^{n+1}}} \tag{2.16}$$

3. Heat Transfer

The heat transfer characteristic is expressed in terms of Nusselt number, given by

$$Nu^{n+1} = -\sqrt{Re}\left[\frac{\partial T}{\partial \eta}\right]_{\eta=0}^{n+1} = -\sqrt{Re}\left[\frac{T_{j+1}^{n+1} - (1 - r_y^2)T_j^{n+1} - r_y^2 T_{j-1}^{n+1}}{r_y(1 + r_y)\Delta y}\right]_{j=2}$$

$$= -\sqrt{Re}\left[\frac{T_3^{n+1} - (1 - r_y^2)T_2^{n+1} - r_y^2 T_1^{n+1}}{r_y(1 + r_y)\Delta y}\right] \tag{3.1}$$

4. RESULTS AND DISCUSSION

The values of the various parameters involved are chosen as

$$\rho = 0.9752 \text{ kg/m}^3 \qquad \rho_p = 800, 2403, 8010 \text{ kg/m}^3 \quad \epsilon = 0.05, 0.1, 0.2$$

$$d = 50, 100 \quad \mu_m U = 60.96 msec \quad \mu = 1.5415 \times 10^{-5}\text{kg(m sec)}$$

$$\varphi = 0.001, 0.0003, 0.0001 \quad L = 0.3048m \quad \alpha = 0.1 Ec = 0.1 \quad Pr = 0.71, 1.0, 7.0$$

The accuracy of the numerical scheme is validated by comparing the results of Nusselt number and Displacement thickness obtained from the present investigation with the results obtained by Panda et.al.[17] and Mishra and Tripathy [20] for negligible volume fraction ψ and omission of the momentum equation v_p for particulate phase in normal direction. It is observed from the Tables 1 and 2 that the present result is in good agreement with the results obtained by Panda et.al.[17] and Mishra and Tripathy [20] for $\rho_p = 800, \epsilon = 0.05, Pr = 0.71, d = 50, \varphi = 0.0, \alpha = 0.1$ and $Ec = 0.1$.

Again the present results of Nusselt number and Displacement thickness for φ=0.0 without v_p are compared with that of obtained for φ=0.0001 with v_p. It is envisaged from the 3rd and 4th columns of Tables 1 and 2 that the rate of heat transfer in terms of Nusselt number as well as the Displacement thickness increase in case of φ=0.0001 with v_p as compared to that of φ=0.0 without v_p along the plate.

Table 1. Comparison of Nusselt number Nu for Particulate suspension .

X	Panda et.al.[17] Nu with φ = 0.0 and without v_p	Mishra [20] Nu with φ = 0.0 and without v_p μ	Present Study Nu with φ = 0.0 and without v_p	Present Study Nu with φ = 0.0001 and with v_p
1.20	4.54E+03	4.59E+03	4.57E+03	6.29E+02
2.00	4.37E+03	4.36E+03	4.38E+03	5.10E+03
2.80	3.37E+03	3.37E+03	3.37E+03	3.43E+03
3.60	3.36E+03	3.35E+03	3.37E+03	3.44E+03
4.40	3.36E+03	3.34E+03	3.36E+03	3.44E+03
5.00	3.37E+03	3.34E+03	3.37E+03	3.44E+03

Table 2. Comparison of Displacement thickness (DISP) for Particulate suspension.

X	Panda et.al.[17] DISP with φ = 0.0 and without v_p	Mishra [20] DISP with φ = 0.0 and without v_p μ	Present Study DISP with φ = 0.0 and without v_p	Present Study DISP with φ = 0.0001 and with v_p
1.20	5.37E-03	5.39E-03	5.34E-03	4.76E-03
2.00	4.66E-03	4.63E-03	4.64E-03	5.09E-03
2.80	4.35E-03	4.38E-03	4.36E-03	7.84E-03
3.60	4.27E-03	4.26E-03	4.28E-03	7.81E-03
4.40	4.24E-03	4.24E-03	4.25E-03	7.81E-03
5.00	4.24E-03	4.24E-03	4.25E-03	7.81E-03

To study the effect of various physical parameters on the velocity field, thermal boundary layer and coefficient of rate of heat transfer on the wall, the results obtained from numerical computation is depicted through Fig.2 to 10 and Tables 3 to 9. The values of Prandtl number Pr are taken as 0.71, 1.0 and 7.0 which physically correspond to air, electrolyte solution and water, respectively.

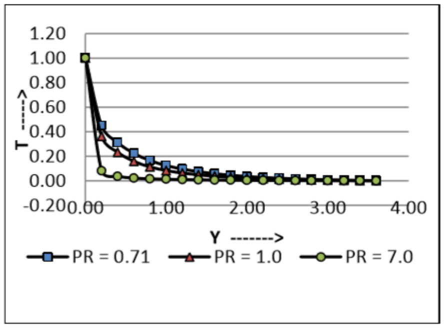

Figure 2. Variation of normalized temperature of carrier fluid T for different values of Pr

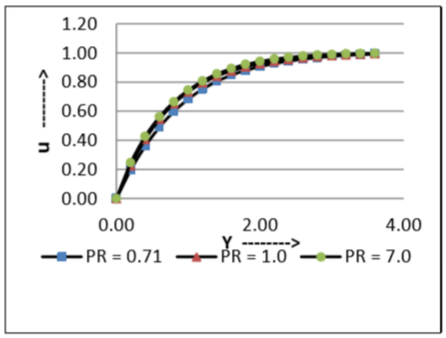

Figure 3. Variation of non-dimensional velocity profile of carrier fluid u for different values of Pr

Table 3. Variation of normalized Particle temperature T_p for different values of Prandtl number Pr.

Y	T_p for Pr= 0.71	T_p for Pr = 1.0	T_p for Pr= 7.0
0.00	2.63E-02	2.63E-02	2.63E-02
0.40	1.03E-03	7.03E-04	2.85E-05
0.80	6.23E-04	3.98E-04	7.15E-07
1.20	3.79E-04	2.33E-04	1.68E-06
1.60	2.31E-04	1.37E-04	1.74E-06
2.00	1.40E-04	8.08E-05	1.15E-06
2.40	8.23E-05	4.62E-05	6.34E-07
2.80	4.53E-05	2.48E-05	3.11E-07
3.20	2.14E-05	1.14E-05	1.29E-07
3.60	5.70E-06	2.97E-06	3.05E-08

From Fig.2, it is observed that the normalized temperature of carrier fluid T is higher for air (Pr = 0.71) as well as the temperature distribution is more uniform across the thermal boundary layer as compared to water (Pr=7.0) and electrolyte solution (Pr=1.0). Further it is concluded that heat is diffused away from the heated

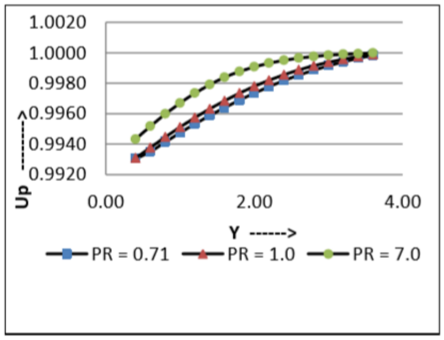

Figure 4. Variation of non-dimensional particle velocity up for different values of Pr

Table 4. Variation of non-dimensional particle density ρ_p for different values of material density of particles ρ_s

Y	$\rho_s = 800$	$\rho_s = 2403$	$\rho_s = 8010$
0.00	1.3462	1.1409	1.0834
0.40	0.2477	0.2490	0.2499
0.80	0.2974	0.2979	0.2985
1.20	0.3683	0.3688	0.3693
1.60	0.4644	0.4652	0.4657
2.00	0.5804	0.5813	0.5818
2.40	0.7030	0.7038	0.7041
2.80	0.8162	0.8168	0.8170
3.20	0.9084	0.9087	0.9088
3.60	0.9754	0.9755	0.9755

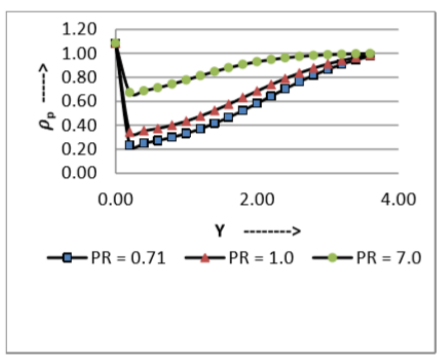

Figure 5. Variation of non-dimensional particle density ρ_p or different values of Pr

surface more rapidly for higher values of Prandtl number Pr. The normalized temperature is maximum near the plate and it asymptotically approaches to the free stream value towards the boundary layer. The same trend is noticed in case of normalized particle temperature T_p from Table 3.

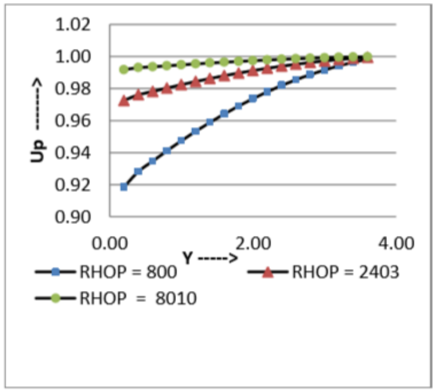

Figure 6. Variation of non-dimensional particle velocity up for different values of material density of particles ρ_s

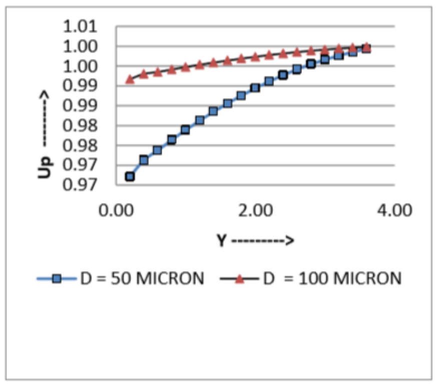

Figure 7. Variation of non-dimensional particle velocity up for different size of particles D

Table 5. Variation of non-dimensional Particle phase density ρ_p for different size of particles D.

Y	$\rho_P$for D= 50 micron	$\rho_P$for D= 100 micron
0.00	1.1585	1.0834
0.40	0.2488	0.2499
0.80	0.2978	0.2985
1.20	0.3687	0.3693
1.60	0.4651	0.4657
2.00	0.5812	0.5818
2.40	0.7037	0.7041
2.80	0.8167	0.8170
3.20	0.9087	0.9088
3.60	0.9755	0.9755

From Fig. 3, 4 and 5, it is observed that the magnitudes of non-dimensional carrier fluid velocity up, non-dimensional particle velocity up, non-dimensional particle

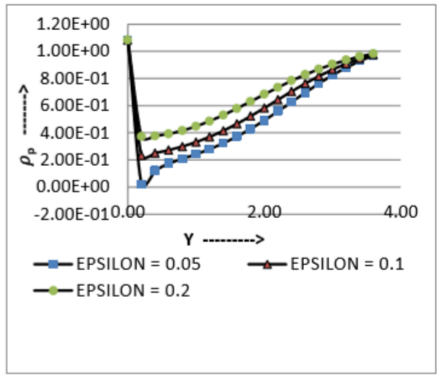

Figure 8. Variation of non-dimensional particle density ρ_p for different values of ϵ

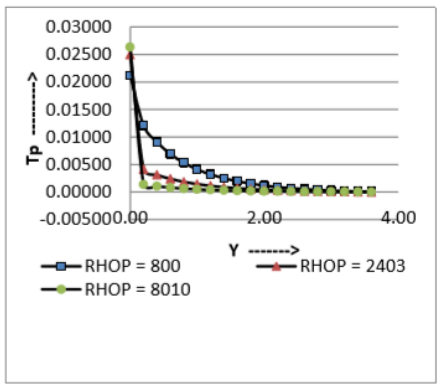

Figure 9. Variation of normalized particle temperature T_p for different values of material density of particles ρ_s

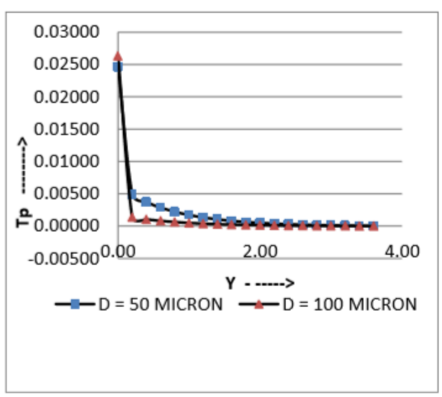

Figure 10. Variation of normalized particle temperature T_p for different size of particles D

phase density ρ_p increase with the increase of Prandtl number Pr for a fixed value of volume fraction ψ of the particle phase across the boundary layer.

The non-dimensional particle velocity up drastically increases near the plate attaining the free stream value towards the boundary layer due to the high material

Table 6. Variation of non-dimensional particle velocity up for various diffusion parameters ϵ.

Y	up for $\epsilon = 0.05$	up for $\epsilon = 0.1$	up for $\epsilon = 0.2$
0.00	0.9217	0.9217	0.9217
0.40	0.9922	0.9930	0.9927
0.80	0.9939	0.9941	0.9942
1.20	0.9951	0.9953	0.9955
1.60	0.9962	0.9964	0.9966
2.00	0.9971	0.9974	0.9976
2.40	0.9980	0.9982	0.9984
2.80	0.9987	0.9989	0.9990
3.20	0.9993	0.9994	0.9995
3.60	0.9998	0.9998	0.9999

Table 7. Variation of normalized Particle temperature T_p for various diffusion parameters ϵ

Y	T_p for $\epsilon = 0.05$	T_p for $\epsilon = 0.1$	T_p for $\epsilon = 0.2$
0.00	2.63E-02	2.63E-02	2.63E-02
0.40	7.31E-04	1.03E-03	1.24E-03
0.80	5.06E-04	6.23E-04	7.42E-04
1.20	3.07E-04	3.79E-04	4.56E-04
1.60	1.85E-04	2.31E-04	2.83E-04
2.00	1.10E-04	1.40E-04	1.73E-04
2.40	6.35E-05	8.23E-05	1.03E-04
2.80	3.44E-05	4.53E-05	5.73E-05
3.20	1.60E-05	2.14E-05	2.72E-05
3.60	4.22E-06	5.70E-06	7.31E-06

density of particles (Fig. 6), presence of coarser particles (Fig. 7) as well as high diffusion of particles through the carrier fluid (Table 6). The same trend is noticed in case of non-dimensional particle phase density ρ_p with respect to ρ_s, D and ϵ from Table 4, Table 5 and Fig. 8 respectively.

The normalized particle temperature T_p is reduced for higher material density of particles ρ_s (Fig. 9), where as it is enhanced for higher diffusion parameter ϵ across the boundary layer (Table 7). From Fig.10, it is concluded that the normalized particle phase temperature T_p decreases across the boundary layer with the increase of the size of particles. Corresponding to the carrier fluid with SPM, it is observed that the Nusselt number Nu increases (Table 8) and the displacement thickness decreases (Table 9) for the higher values of the Prandtl number Pr along the plate through the flow field.

Table 8. Variation of Nusselt number Nu for different values of Prandtl number Pr

Y	Nu for clear fluid	Nu for fluid with SPM where ψ = 0.0001	Nu for fluid with SPM with ψ=0.0001, Pr = 0.71	Nu for fluid with SPM with ψ=0.0001, Pr =1.0	Nu for fluid with SPM with ψ=0.0001, Pr = 7.0
1.20	2.63E+03	6.29E+02	8.52E+01	3.16E+02	5.12E+03
1.60	3.31E+03	4.72E+03	1.20E+04	2.08E+04	1.51E+05
2.00	3.31E+03	5.10E+03	1.09E+04	1.94E+04	1.34E+05
2.40	3.31E+03	2.06E+03	1.17E+04	2.01E+04	1.32E+05
2.80	3.31E+03	3.43E+03	1.11E+04	1.98E+04	1.33E+05
3.20	3.31E+03	3.44E+03	1.15E+04	1.98E+04	1.32E+05
3.60	3.31E+03	3.44E+03	1.13E+04	2.00E+04	1.32E+05
4.00	3.31E+03	3.44E+03	1.14E+04	1.97E+04	1.32E+05
4.40	3.31E+03	3.44E+03	1.13E+04	1.99E+04	1.32E+05
4.80	3.31E+03	3.44E+03	1.14E+04	1.98E+04	1.32E+05
5.00	3.31E+03	3.44E+03	1.13E+04	1.98E+04	1.32E+05

Table 9. Variation of Displacement thickness (DISP) for different values of Prandtl number Pr

Y	DISP for clear fluid	DISP for fluid with SPM where=0.0001	DISP for fluid with SPM with ψ=0.0001, Pr = 0.71	DISP for fluid with SPM with ψ=0.0001, Pr =1.0	DISP for fluid with SPM with ψ=0.0001, Pr = 7.0
1.20	5.37E-03	4.76E-03	4.24E-03	3.89E-03	2.87E-03
1.60	5.05E-03	3.82E-03	3.06E-03	2.77E-03	2.32E-03
2.00	4.66E-03	5.09E-03	3.68E-03	3.29E-03	2.11E-03
2.40	4.45E-03	7.75E-03	3.27E-03	3.04E-03	2.49E-03
2.80	4.35E-03	7.84E-03	3.55E-03	3.13E-03	2.31E-03
3.20	4.29E-03	7.82E-03	3.38E-03	3.14E-03	2.27E-03
3.60	4.27E-03	7.81E-03	3.49E-03	3.09E-03	2.41E-03
4.00	4.25E-03	7.81E-03	3.42E-03	3.16E-03	2.32E-03
4.40	4.24E-03	7.81E-03	3.46E-03	3.09E-03	2.32E-03
4.80	4.24E-03	7.81E-03	3.44E-03	3.15E-03	2.36E-03
5.00	4.24E-03	7.81E-03	3.46E-03	3.12E-03	2.33E-03

Further both the Nusselt number Nu and the displacement thickness corresponding to the carrier fluid with SPM of negligible volume fraction are more as compared to those respective values corresponding to the clear fluid throughout the flow field (Tables 8 and 9).

5. CONCLUSION

The main results of this investigation are briefly summarized as follows.

1. Higher Prandtl number is to increase the non-dimensional carrier fluid velocity, particle velocity and particle phase density across the boundary layer as well as to increase the Nusselt number along the plate throughout the flow field. The normalized temperatures of both carrier fluid and particle phase are reduced with the increase of Prandtl number envisaging that heat generated due to the flow of particulate suspension is diffused away from the heated surface more rapidly for higher values Prandtl number.
2. The non-dimensional particle velocity and particle phase density are increased with the increase of particle size, material density of particles and diffusion parameter across the boundary layer.

REFERENCES

[1] S.L. Soo, Boundary layer motion of a gas-solid suspension ,*Project Squid Report ILL-3P*,

[2] H.H. Chiu, Boundary layer flow with suspended particles, Princeton University Report 620, 1962.

[3] F.E. Marble, Dynamics of a gas containing small solid particles. Proceedings of thefifth AGARD combustion and propulsion colloquium, Pergamon Press, Oxford, 175-213, 1963.

[4] S.L. Soo, Laminar and separated flow of a particulate suspension, AstronauticaActa, 11:422-431, 1965.

[5] G. Rundienger, Some effects of finite particle volume on the dynamics of gas-particle mixtures, The American Institute of Aeronautics and Astronautics Journal, 3:1217-1222, 1965.

[6] R.E. Singleton, The compressible gas-solid particle flow over a semi-infinite flat plate, Journal of Applied Mathematics and Physics, 16:421- 449, 1965.

[7] S.L. Soo, Fluid dynamics of multiphase systems, Blaisdell Publishing Company, London,1967.

[8] B. Otterman, Particle migrations in laminar mixing of a suspension with a clean fluid, ZAMP 20:730-749, 1969.

[9] W. Tabakoff and A. Hamed, The boundary layer of particulate gas flow, ZFW, 20: 373 380, 1972.

[10] A.C. Jain and A.C. Ghosh, Gas particulate laminar boundary layer on a flat plate, ZFW, 3:29-37, 1979.

[11] S. Prabhaand and A.C. Jain, Laminar boundary layer of gas particulate flow on a flat plate, Proceedings of Indian Acad. Science 88A-III, 5:377-385, 1979.

[12] N. Datta and S.K. Mishra, Boundary layer flow of a dusty fluid over a semi-infinite flat plate, ActaMechanica, 42:71-83,1982.

[13] B.Y. Wang and I.I. Glass, Compressible laminar boundary-layer flows of a dusty gas over a semi-infinite flat plate, Journal of Fluid Mechanics, 186: 223- 241, 1988.

[14] A.J. Chamkha, Compressible dusty-gas boundary layer flow over a flat surface, J. of Fluids Engg., 118:179-185, 1996.

[15] T.C. Panda, S.K. Mishra and K. C.Panda, Laminar diffusion of suspended particulate matter sing a two phase flow model, Int. J. for Numerical Methods in Fluids, 40:841-853, 2002.

[16] M.K. Partha, P.V.S.N. Murthy and G.P. Rajasekhar, Effect of viscous dissipation on the mixed convection heat transfer from an exponentially stretching surface, Heat and Mass Transfer, 41: 360-366, 2005.

[17] T.C. Panda, S.K. Mishra and D. K.Dash, Modeling dispersion of SPM in free convection flows in the vicinity of heated horizontal flat plate, Impact J. Sc. Tech., 1:37-60, 2007.

[18] S.K. Mishra and P. K. Tripathy, Mathematical and numerical modeling of two phase flow and heat transfer using non-uniform grid, Far East J. of Applied Math., 54:107-126, 2011.

[19] A. Misra, S.K.Mishra and J. Prakash, Modelling of electrification of suspended particulate matter(SPM) in a two phase boundary layer flow and heat transfer over a semi-infinite flat plate, Int. Comm. in Heat and Mass Transfer, 38:1110-1118, 2011.

[20] S.K. Mishra and P. K. Tripathy, Approximate solution of two phase thermal boundary layer flow, Engg. Sc. and Tech.: An Int.J., 3:1-7,2013.