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EFFICIENT ALGORITHMS FOR CONTRAFLOW RECONFIGURATION IN EVACUATION PLANNING

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ABSTRACT. Contraflow reconfiguration allows the arc reversals that increases the outbound road capacities. During emergency, the maximum number of evacuees should be moved from the disastrous areas to the safe destinations. Contraflow technique is one of the widely accepted mathematical models for the efficient solution of evacuation planning problem. In literature, there are number of efficient algorithms to handle this issue however, the problem is NP-hard in general.

In this paper, we briefly overview the development of contraflow technique to solve the real life problems. The heuristic approaches with different applications will be highlighted. From the analytical point of view, the contraflow model increases the flow value up to double and decreases the time at most half to transship the given flow value. With contraflow reconfiguration, efficient algorithms for maximum dynamic, the earliest arrival (transshipment) and the lex-maximum dynamic contraflow problems are discussed in both discrete and continuous time settings. Moreover, the maximum dynamic contraflow and the earliest arrival contraflow problems are generalized including an additional constraint loss or gain for each arc of the evacuation network. These problems are illustrated on two terminal lossy network taking highest gain path from the source to he sink. The contraflow network is replaced by an abstract contraflow network with a system of linearly ordered sets, called paths satisfying the switching property and solved the maximum static contraflow problem and maximum dynamic abstract contraflow problem in continuous time setting.

Key words: Evacuation planning, contraflow, dynamic flow, transshipment problem, approximation algorithm, generalized contraflow.

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1. INTRODUCTION

The research in evacuation planning is being challenging due to the increasing number of noticed or unnoticed disasters, both natural and man-made. In spite of various discoveries and urbanization, terrible disasters e.g. earthquakes, tsunamis, landslides, volcanic eruptions, hurricanes, typhoons, floods, terrorist attacks, chemical explosions, etc. are creating threats to human life. The evacuation planning problem

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is the process of shifting the maximum evacuees from the disastrous areas to the safe destinations as quickly as possible with an optimal evacuation plan.

The evacuation network is defined as a network that corresponds to a region (or a building) to be evacuated in which the intersections of streets (or rooms in a building) represent the nodes and the connections between these parts (i.e., streets in region, or doors between rooms) denote the edges. The initial locations of evacuees are the source nodes and the locations at safety regions are the sink nodes. The nodes and edges are bounded by capacities. Each arc has transit time or cost function. The group of evacuees that passes through the network over time is modeled as a flow. The plan is dependent upon the number of sources, sinks, parameters on the arcs and nodes, like constant, time-dependent or flow dependent capacities or transit times as well as additional constraints. The time may be discrete or continuous. We refer to [4] for the detailed survey of evacuation network flow models.

Contraflow is a very useful model introduced in evacuation planning. It is a problem of congestion minimization. It increases the outbound roads capacities by reversing the direction of arcs towards the sinks from the sources. Through the network with increased capacity, contraflow problem shifts the maximum number of evacuees to the sinks and decreases the evacuation time as well. It seeks to remove traffic jams and makes the traffic systematic and smooth. It is emerging to react to different large scale natural and man-made disasters. However, it is a very challenging issue of finding a network reconfiguration with ideal lane directions satisfying the given constraints to optimize the given objective.

The concept of contraflow was developed to evacuate affected people in many terrible disasters, (Wolshon *et al*. [41]). However, it was used depending on past evacuation experiences without an appropriate guideline. Litman [19] criticized the unplanned contraflow concept and highlighted its importance after hurricanes Katrina and Rita. Hamza-Lup *et al*. [11] supported a smart traffic evacuation management system by developing contraflow algorithms for the first time. However, these algorithms are not effective for the fixed number of evacuees, road capacity and specific sinks, or for the spreading evacuees over many locations. First mathematical optimization model and a tube search heuristic for the contraflow concept have been investigated in [36, 37]. Kim and Shekhar [17] introduced the contraflow technique using graph and flow theory and presented two heuristics that not only find the ideal direction of arc reversals but also compute a local minimum of evacuation time. First integer programming formulation for contraflow was introduced in [18] with greedy and bottleneck heuristics that reduced the evacuation time by at least 40 percent with at most 30 percent of the total arc reversals.

There are different implementations of contraflow technique in real life. Authors in [42, 43] evacuated the Monticello, Minnesota region by solving the lane based contraflow and crossing elimination strategies simultaneously. Contraflow model and repair of damaged roads are simultaneously solved in [38] in which more than 50% time can be reduced for construction of one new road and 20% time can be reduced by re-planning the reseource. Wang *et al*. [39] presented contraflow evacaution model with the evacuation priorities and setup time for contraflow operation. If the root choice opportunity for evacuees in complete contraflow network model can be provided, then 30 to 60% evacuation time can be minimized, [20]. By reversing the shortest paths, Min & Lee [22] introduced a contraflow routing problem based on the maximum throughput flows. To evacuate a region with low mobility population that has little access to personal vehicles, unable to drive due to age, sickness, or any other reason, a multi-modal integrated contraflow mdoel has been presented in [14]. The transit-based models are initiated with vehicle routine problem whereas the integrated strategy contains non-contraflow to shorten the strategy setup time, full-lane contraflow to minimize the evacuation network capacity and bus contraflow to realize the transit cycle operation. Pyakurel *et al*. [32] study transit-based evacuation models and present a case study of Kathmandu metropolitan city for transit dependent evacuees as an application. Zhao *et al*. [44] presented bi-level model integrating lane based reversal design and routing with intersection crossing conflict elimination for an efficient evacuation by minimizing the total evacuation time to leave the evacuation zone.

In this study we mainly focus on analytical solutions of contraflow evacuation planning problems for dynamic, generalized dynamic and abstract networks. In dynamic network each arc has transit time and capacity, and the problem is to find the maximum flow from the sources to the sinks at each time period with arc reversal capability. The parameters we considered in the constraints are constant and the evacuation time means the evacuation egress time. Moreover, we assume that the arcs can be reversed without any processing cost. If the arc contains additional parameter, the gain factor, then the problem will be generalized contraflow in which we send the maximum flow to the sinks through highest gain paths with arc reversals [24]. If the network have linearly ordered sets, i.e., paths satisfying the switching property, the it is an abstract network. It has a set of elements and path system instead of node and arc.

In comparison with the development of heuristic and integrated approaches for contraflow problem, the analytical approach has not so long history. The first analytical approach for the contraflow configuration has been developed by Arulselvan [1] and Rebennack *et al*. [33] to solve the maximum dynamic contraflow and the quickest contraflow problems in discrete time on two terminal network. Recently, Pyakurel and Dhamala [28, 29] introduced the maximum dynamic contraflow problem in continuous time setting. They introduced continuous model for the problem and presented efficient algorithm to solve it. They used the natural transformation of Fleischer and Tardos [7] in the algorithm of Arulselvan [1] and Rebennack *et al*. [33] due to which it is converted into continuous time setting. The optimal solution for this problem can be computed in the same complexity as in discrete time setting. Moreover, the development of contraflow models and efficient algorithms in both discrete and continuous time settings can be found in [24, 5, 25, 26, 27]. The contraflow models and algorithms has been generalized in [30]. The concept of dynamic contraflow has been extended to the abstract network in [29]. We discussed these problems and algorithms in remaining sections.

This paper is organized as follows. We present the mathematical background that are necessary for this paper in Section 2. Section 3 gives a short summery of existing analytical solutions for the contraflow evacuation planning problems in both discrete and continuous time settings. Section 4 highlights the solution techniques for the generalized contraflow evacuation planning problem. The abstract contraflow approach is discussed in Section 5. Section 6 concludes the paper.

2. PROBLEM FORMULATIONS

An evacuation network $\mathcal{N}$ consists of a directed graph $G = (V, A)$ where V denotes a finite set of nodes and A denotes a finite set of arcs, i.e.,$|V| = n$ and $|A| = m$. Let $S \subset V$ be a set of source nodes and $D \subset V$ be a set of sink nodes. As we are considering the contraflow network, any two way network configuration is allowed. If a network has only one source and only one sink, then we represent them as s and d, respectively. Let $b_A : A \rightarrow \mathcal{Z}^+$, $b_V : V \rightarrow \mathcal{Z}^+$, $\tau : A \rightarrow \mathcal{Z}^+$ and $c_A : A \rightarrow \mathcal{Z}^+$ represent the arc capacities, node capacities, transit times and arc costs of the network, respectively. If supply and demand at each source and sink are fixed, they are represented by the vectors $\mu(s)$ and $\nu(d)$, respectively. The dynamic evacuation network is represented as $\mathcal{N} = (V, A, b_A, b_V, \tau, S, D, T)$ with predetermined time T. A finite time horizon T may be discrete or continuous within which evacuation process must be completed. We assume that a domain of time $\mathcal{T}$ is valid for both discrete and continuous time setting i.e., $\mathcal{T} = \{0, 1, 2, \ldots, T\}$ in a discrete model and $\mathcal{T} = [0, T]$ in a continuous model. The group of evacuees is modeled as a flow which passes through the network over time. Let $A_v^{out} = \{(v, w) \in A\}$ and $A_v^{in} = \{(w, v) \in A\}$ be the sets of outgoing arcs and incoming arcs, respectively, for the node $v \in V$. Unless otherwise stated, we assume that there isn't any incoming arc to source node s and outgoing arc from sink node d, i.e., $A_d^{out} = A_s^{in} = \emptyset$.

Let the reversal of an arc $e = (v, w)$ be denoted by $e^{-1} = (w, v)$. For a contraflow configuration of a given network $\mathcal{N}$ with symmetric transit times, the auxiliary network $\overline{\mathcal{N}} = (V, E, b_E, \tau, S, D, T)$ consists of the modified arc capacities and constant transit times as follows

$$b_E(\overline{e}) = b_A(e) + b_A(e^{-1}), \text{ and } \tau(\overline{e}) = \begin{cases} \tau(e) & \text{if } e \in A \\ \tau(e^{-1}) & \text{otherwise} \end{cases}$$

where, an edge $\overline{e} \in E$ in the auxiliary $\overline{\mathcal{N}}$ if $e \vee e^{-1} \in A$ in $\mathcal{N}$. The remaining graph structure and data are unaltered.

Let the non-negative function $x_{dyna} : A \times \mathbf{T} \to \mathcal{R}^+$ represents the dynamic flow. A dynamic $s-d$ flow x_{dyna} for given time T satisfies the flow capacity and conservation constraints (2.1 -2.3). The inequality flow conservation constraint (2.1) allows the flow to wait at intermediate nodes, however, the equality constraint (2.2) forces the flow entering an intermediate node to leave it again immediately. Ford and Fulkerson [6] studied the maximum dynamic flow models.

$$\sum_{\sigma=\tau(e)}^{t} \sum_{e \in A_v^{in}} x_{dyna}(e, \sigma - \tau(e)) - \sum_{\sigma=0}^{t} \sum_{e \in A_v^{out}} x_{dyna}(e, \sigma) \geq 0, \forall v \notin \{s, d\}, t \in \mathbf{T} \tag{2.1}$$

$$\sum_{\sigma=\tau(e)}^{T} \sum_{e \in A_v^{in}} x_{dyna}(e, \sigma - \tau(e)) - \sum_{\sigma=0}^{T} \sum_{e \in A_v^{out}} x_{dyna}(e, \sigma) = 0, \ \forall\, v \notin \{s, d\} \tag{2.2}$$

$$0 \leq x_{dyna}(e, t) \leq b_A(e, t), \qquad \forall\, e \in A,\ t \in \mathbf{T} \tag{2.3}$$

The maximum dynamic flow and the earliest arrival flow maximizes $val(x_{dyna}, T)$ and $val(x_{dyna}, t)$ in (2.4) and (2.5) satisfying the constraints (2.1-2.3).

$$val(x_{dyna}, T) = \sum_{\sigma=0}^{T} \sum_{e \in A_s^{out}} x_{dyna}(e, \sigma) = \sum_{\sigma=\tau(e)}^{T} \sum_{e \in A_d^{in}} x_{dyna}(e, \sigma - \tau(e)) \tag{2.4}$$

$$val(x_{dyna}, t) = \sum_{\sigma=0}^{t} \sum_{e \in A_s^{out}} x_{dyna}(e, \sigma) = \sum_{\sigma=\tau(e)}^{t} \sum_{e \in A_d^{in}} x_{dyna}(e, \sigma - \tau(e)) \tag{2.5}$$

If the flow is static, then we represent it as $x_{stat} : A \to \mathcal{R}^+$.

The continuous flow models are similar to the discrete flow models, with the sum over time replaced by an integral. The continuous time models would give more accurate results with higher computational complexity. Thus, discretization of the models are better options for good approximations to real-life solutions. The distinction between the discrete and continuous time approaches depends on whether the flow that entering an arc e at time $t - \tau(e)$ has already arrived at the head node by time t or is still on the arc at that moment. In the former we assume that such a flow is already at the head node at time t. However, in the latter it will be reached to head node at time $[t + 1)$. Then this continuous flow is feasible and the amount of flow that can be sent from the source to the sink at any integer time interval

$[t, t+k)$, for $t = 0, 1, \ldots, T, k \in N$, will be the same for both flows. This is the natural transformation introduced in [7].

For a network $\mathcal{N}$, the time expanded network $\mathcal{N}(T) = (V_T, A_M \cup A_H)$ is defined as an expansion of the dynamic network where each node v of the static graph is copied T times to obtain a node $v(t)$ for each $v \in V$ and each $t \in \{0, ..., T\}$. For each arc $e = (v, w) \in A$, the arc from $v(t)$ to $w(t + \tau(v, w))$ has capacity $b_A(v, w)$, called movement arc. For each arc $e = (v, w) \in A$, the arc from $v(t)$ to $v(t+1)$ has capacity $b_V(v)$, called holdover arc which allows storage at the node.

3. CONTRAFLOW ALGORITHMS

3.1. **Maximum dynamic contraflow.** The maximum dynamic contraflow (MDCF) problem maximizes the flow value that can be sent from the sources to the sinks in given time horizon by reversing the direction of arcs towards the sinks. In general, this problem is not solved yet. But in particular case of a single source and a single sink network with arc reversal capability at time zero, Arulselvan [1] and Rebennack *et al.* [33] presented a polynomial time algorithm to solve it by extending the temporally repeated flow algorithm of Ford and Fulkerson [6]. Arc reversal at time zero means when an arc is reversed, it remains reversed from time 0 to T. Their solution is in discrete time setting and the processing cost of contraflow configuration is neglected. The network is allowed to be asymmetric with respect to the arc capacities. However, transit time of two way arcs are same between any two nodes. Their algorithm for *s-d* maximum dynamic flow solution determines temporally repeated dynamic flow in auxiliary network $\overline{\mathcal{N}}$ which optimizes the overall dynamic flows in it. An arc $(w, v) \in A$ is reversed if and only if the flow on (v, w) is greater than $b_A(v, w)$, or if there is a nonnegative flow along $(v, w) \notin A$. An optimal solution in $\overline{\mathcal{N}}$ is at most equal to the optimal solution in $\mathcal{N}$ and an optimal MDCF solution $\mathcal{N}$ is not greater than an optimal MSCF solution in the corresponding time expanded network $\mathcal{N}(T)$. An *s-d* maximum static contraflow problem has been introduced as a foundation of dynamic contraflow problem.

Theorem 3.1. [33] *The s-d maximum dynamic contraflow problem can be solved in time $O(h_2(n, m)+h_3(n, m))$, where $h_2(n, m) = O(n.m)$ and $h_3(n, m) = O(n^2.m^3. \log n)$ are the time required for the flow decomposition and the maximum static flow computation, respectively.*

For the multi terminal networks with arc reversal capability at time zero, Kim *et al.* [18] and Rebennack *et al.* [33] showed that the maximum dynamic contraflow problem is $\mathcal{NP}$-hard in the strong sense even with two sources and one sink or vice versa. When we choose arcs, we have to know if an arc has been reversed or not in every time. This memory and decision of reversing the arc now or at a later time makes the problem $\mathcal{NP}$-complete. The proofs follow by reductions from the problems 3-SAT and PARTITION.

3.2. **Lex-maximum dynamic contraflow.** Depending upon the given priority order, the lex-maximum dynamic contraflow (LMDCF) problem finds the maximum dynamic flow with the contraflow approach. That is, we maximize the flow leaving the sources and entering the sinks in that given priority order with contraflow reconfiguration of evacuation networks for given time horizon T. After disasters, evacuees are of different categories, for example, injured, frustrated, disabled, etc. We may not consider the evacuees as a single group. The advantage of the LMDCF problem is that we can shift the evacuees according to the priority ordering. For example, we can shift the injured evacuees before the others. Moreover, some regions may be more dangerous in comparison to another so that we have to rescue the most dangerous regions first. Pyakurel and Dhamala [26] introduced the LMDCF problem in discrete time setting by extending the lex-maximum dynamic flow solution technique of Hoppe and Tardos [13] and presented polynomial time algorithm to solve it.

First of all, the dynamic network $\mathcal{N}$ is converted into auxiliary network $\overline{\mathcal{N}}$ by reversing the direction of arcs towards the sinks using contraflow configuration in which the capacities of the arcs are added to obtain new capacities and the transit times remain the same. Then algorithm of Hoppe and Tardos [13] solves the LMDF problem polynomially on the auxiliary network $\overline{\mathcal{N}}$. It starts with zero flow and calculates successive layers of minimum cost static flows in the residual network of previous layers. Each layer adds standard chain decomposition to the previous chain. The dynamic flow constructed by the polynomial time algorithm computed via δ minimum cost flow (MCF) computations is feasible and is lexicographically maximal for a given time horizon T, where δ is the number of iterations.

Theorem 3.2. [26] *Lexicographically maximum dynamic contraflow solution can be computed in $O(\delta \times MCF(m,n))$ time, where $MCF(m,n)$ represents the time complexity $O(m \log n)(m + n \log n))$ of the minimum cost flow problem in the residual network.*

Recently, Pyakurel and Dhamala [29] introduced the lex-maximum dynamic contraflow problem in continuous time setting. They used the natural transformation to transform the discrete solution into continuous. Recall that in continuous time also, their algorithm solves the problem with same complexity as in discrete time setting.

3.3. **Earliest arrival contraflow.** In fact, the estimation of evacuation time for shifting evacuees from the sources to the sinks is difficult because an efficient method to estimate it is not known yet. So, the problem of shifting maximum number of evacuees from the beginning of time point is important in evacuation planning. In the dynamic contraflow evacuation networks, such kind of problem is considered as the earliest arrival contraflow. The main advantage of the problem is that it does not need estimated time period in advance. In general, the efficient solution for this problem is not developed yet. However in some particular cases, this problem has been solved with efficient algorithms in both discrete and continuous time setting.

Dhamala and Pyakurel [5, 25] introduced the earliest arrival contraflow problem on two terminal series parallel network for discrete time. They extended the earliest arrival flow algorithm of Ruzika *et al*. [34] to the contraflow framework and presented a polynomial time algorithm. Recall that a minimum cost circulation flow (MCCF) solution has minimum cost if and only if the corresponding residual network does not contain a cycle with negative cost. The main advantage on series parallel network is that every cycle in the residual network has non-negative cycle length. This solves the MCCF problem introduced in (Ford and Fulkerson [6]) for the MDF problem in the auxiliary network $\overline{\mathcal{N}}$. The temporally repeated flow thus obtained is an optimal solution to the s-d EACF problem as well. However, it is not true that every earliest arrival contraflow on series-parallel network satisfies the temporarily repeated property.

Theorem 3.3. [5, 25] *Earliest arrival contraflow problem can be solved in time* $O(nm + mlogm)$ *on two terminal series-parallel network.*

With natural transformation, Pyakurel and Dhamala [28, 29] extended the earliest arrival contraflow solution in continuous time setting. They presented an algorithm to solve the problem with the same time complexity as in discrete time.

In fact the earliest arrival flow problem continues the already obtained flows in earlier steps to forthcoming flows in forward steps, the final solution may change the direction of arcs and obeys the backward flow laws in its processing. With this knowledge, Pyakurel and Dhamala [26] extended the earliest arrival flow model of (Wilkinson [40] and Minieka [23]) in two terminal contraflow network with the relaxation of the arc reversal capability at a number of times when an earliest arrival flow solution demands this property. They presented an algorithm to solve it however, its time complexity is pseudo-polynomial because it works on time expanded network. Using natural transformation of Fleischer and Tardos [7], the earliest arrival contraflow solution on two terminal general network has been solve with same complexity in continuous time, (Pyakurel and Dhamala [29]).

Theorem 3.4. *Earliest arrival contraflow problem can be solved in pseudo-polynomial time complexity on two terminal general network.*

Recall that there isn't any polynomial algorithm to solve even the earliest arrival flow problem on two-terminal general network. Thus, an approximate earliest arrival contraflow problem has been introduced in [29] that computes an earliest arrival contraflow from the source s to the sink d within a factor of $(1+\epsilon)$, for $\epsilon > 0$ in time T if the direction of the arcs can be reversed without any processing cost. Their algorithm works as follows. First, the auxiliary network is obtained by reversing the direction of arcs towards sink. Then, with natural transformation, algorithm of Hoppe [12] is applied that computes the minimum cost flow using shortest augmenting paths in a repeatedly rounded auxiliary network. The obtained flow is decomposed into chains as the sequence of augmentation that gives a dynamic flow on auxiliary network $\overline{\mathcal{N}}$. The number of augmentations can be bounded by a polynomial in n, log$U$, and ϵ^{-1}, where $U = max_{e \in E} b_E(e)$ is the maximum capacity on $\overline{\mathcal{N}}$. The algorithm starts to compute flow by augmenting along exact shortest paths. Then for given ϵ, it periodically rounds down the edge capacities according to an increasing scaling factor Δ, where Δ is increased with $\Delta = 2^i$ during the inner loop of phase i if there are $i = 0, 1, \ldots, k$ scaling phases. That implies all residual capacities are integer multiples of Δ, so that subsequent augmentations obtain at least Δ units of flow in the static network.

Theorem 3.5. *An $(1+\epsilon)$-approximate earliest arrival contraflow solution can be computed in $O(m\epsilon^{-1}(m + nlogn)logU)$ time on two terminal general network for both discrete and continuous time settings.*

3.4. **Earliest Arrival Transshipment contraflow.** If the supplies and demands are known in advance, and we have to shift all the supplies within given time horizon T then the earliest arrival contraflow problem will be the earliest arrival transshipment contraflow. In general, its solution is more demanding which is not found yet. Pyakurel and Dhamala [27] introduced the problem for evacuation planning in discrete time. The problem is extended into continuous time in (Pyakurel and Dhamala [29]). However, it is solved only in particular networks. In multi-source and single sink network, they extended the algorithm of (Baumann and Skutella [2]) and presented a polynomial time algorithm to solve the problem. Their algorithm works as follows.

Firstly an auxiliary network is constructed according to the contraflow configuration. Then, the multi-source auxiliary network is converted into a single source network, called extended auxiliary network. It is obtained by adding a super-terminal node s_0 that is connected to each source $s \in S$ by uncapacitated arcs (s_0, s) with zero transit time, and that can be reached from the sink d by an uncapacitated dummy arc (d, s_0). The supplies of nodes in $s \in S$ are shifted to s_0. Then, a feasible dynamic flow from the source s_0 to the sink d can be obtained by computing the minimum cost circulation flow as in [2] on extended auxiliary network. However, the individual supplies at the source nodes might be violated by the induced dynamic flow on auxiliary network. To overcome this difficulty, an earliest arrival flow pattern $p(t)$ is defined on extended auxiliary network as the maximum flow $val_S(x_{dyna}, t)$ where $p(t) \leq val_S(x_{dyna}, t)$ holds for every $t \geq 0$. If $p(t) = val_S(x_{dyna}, t)$, for all $t \geq 0$, we are done. Otherwise, it is obtained using the algorithm of Baumann and Skutella [2] on extended auxiliary network. Thus, the continuous time solution for earliest arrival transshipment is computed on extended auxiliary network. The obtained earliest arrival pattern in continuous time model is now turned into discrete time model using the natural transformation of Fleischer and Tardos [7] that finds the earliest arrival transshipment on extended auxiliary network which is equal to the earliest arrival transshipment on auxiliary network. The obtained solution is equivalent to the earliest arrival transshipment contraflow in both discrete and continuous time settings.

Theorem 3.6. *In both discrete and continuous time setting, the multi-source earliest arrival transshipment contraflow problem can be solved in polynomial time in the input plus output size of the problem.*

For a single source and multi-sink networks with a special case that each arc has zero transit time, they solved the problem with efficient algorithms. In zero transit time, arc capacities of networks restrict the quantity of flow that can be sent at any one time with arc reversal capability. With the help of (Schmidt and Skutella [35]), they categorized the number of networks that allow the earliest arrival transshipment contraflow in both discrete and continuous time, [27, 29].

Theorem 3.7. *For a multi-sink network, an earliest arrival transshipment contraflow solution with zero transit time can be obtained in polynomial time complexity on each of the following networks construction.*

- **Type 1.:** *Network with single length path*
- **Type 2.:** *Network with path of length two*
- **Type 3.:** *Network with two paths starting in the same node but continuing disjointly*
- **Type 4.:** *Network with two paths of length two starting in different nodes but end in the same node*
- **Type 5.:** *Network with containing at least one path of length two*

3.5. **Approximate earliest arrival transshipment contraflow.** Efficient solution technique for the earliest arrival transshipment contraflow problem with discrete and continuous time model on multi-terminal network has not been found yet, [27, 24]. When it is impossible to find efficient algorithms for $\mathcal{NP}$-hard problems, approximation algorithms have been developed. The aim is to find a polynomial algorithm that probably does not find the optimal solution but finds a solution that is as good as necessary. Authors in [27, 24] presented first discrete time solution algorithms for the approximate earliest arrival transshipment contraflow problem. For the network with arbitrary transit time, their solution is obtained in pseudo-polynomial time complexity. By assuming transit time zero on each arc, the solution is computed in polynomial time complexity. Based on the value approximate algorithms of [10, 15], they proved that 2 is the best approximation factor for the problem, i.e., an optimal solution is computed within 2 times the approximation solution. Recently, Pyakurel *et al.* [31] presented efficient approximate algorithm for the earliest arrival transshipment contraflow problem with continuous time setting. With natural transformation, they solved the problem with the same complexity as in discrete time setting.

3.6. **Quickest contraflow.** The quickest contraflow problem transships the given amount of integer flow value from the sources to the sinks in minimum time period. In general, this problem is unsolved. For particular network of a single source and a single sink, Arulselvan [1] and Rebennack *et al.* [33]) realized a strongly polynomial time algorithm to solve the quickest contraflow problem. Their algorithm based on the parametric search algorithms of (Megiddo [21], Burkard *et al.* [3]). Computing $s-d$ paths, they first obtained an upper bound on the quickest time in polynomial time and applied a binary search repeatedly to compute maximum dynamic contraflow along the path until all supplies at the source are sent to the sink. They also proved that the multi-terminal quickest contraflow problem is harder than 3-SAT and PARTITION. For the network with given supply-demand vector and the arc reversal are allowed back and forth at inter time points, the multi-terminal quickest transshipment contraflow problem is polynomially solvable as it is equivalent to the quickest transshipment problem of Hoppe and Tardos [13]. The solution procedure is similar to the maximum dynamic contraflow solution algorithm.

In continuous time setting, Pyakurel and Dhamala [28] introduced the quickest contraflow model and presented a polynomial time algorithm on two-terminal networks for given integral flow value. Moreover, Pyakurel and Dhamala [29] presented a polynomial time algorithm to solve the quickest transshipment contraflow problem. In fact, the quickest transshipment contraflow in dynamic network is a feasible dynamic transshipment contraflow by transshipping all given flow value in minimum time with arc reversals whenever it necessary.

4. GENERALIZED CONTRAFLOW ALGORITHMS

The generalized evacuation contraflow problem has been investigated to evacuate the maximum number of evacuees from the sources to the sinks through highest gain paths by reversing the direction of arcs towards the sinks, [30, 24]. Recall that the flow is physically transformed due to unfortunate death or hold-over in arcs so that flow conservation may not be satisfied in generalized evacuation contraflow problem. As the flow may not be conserved, we cannot solve the problem in arbitrary networks. Arbitrary generalized networks allow each gain factor to be any positive number. We refer to a network with no gain factor exceeding one as a lossy network. Lossy network captures many natural generalized networks where flow values are only loss or conserve when these are sent from the sources to the sinks through the networks. On generalized evacuation planning, as there is no hope of increasing evacuees on the arcs during rescue process, lossy network is acceptable.

4.1. **Generalized Maximum Dynamic Contraflow.** The generalized maximum dynamic contraflow problem is a maximum dynamic contraflow problem in which each arc contains capacity, transit time as well as gain factor. We consider a two terminal lossy network and we assume each arc has the proportional gain factor i.e., $\lambda \equiv 2^{c.\tau}$ for some constant $c < 0$. We also assumed that, the gain per arc remains same in either direction, i.e., the gain factor is symmetric on the generalized contraflow network. Then the problem is to find the maximum amount of flow with highest gain that can be sent from the source s to the sink d through the highest gain $s - d$ paths within the given integer time T if the direction of the arcs can be reversed at time zero.

Authors in [30, 24] presented an efficient algorithm to solve the generalized maximum dynamic contraflow problem. By developing the generalized contraflow reconfiguration framework, they solved the generalized maximum dynamic flow problem using algorithm of [9, 8] within the framework. It starts with the zero flow, computes a maximum flow in the highest gain path, i.e., in the shortest path of the static residual network, augments this flow and repeats this process until no $s - d$ path exists in the static residual network. Then, it uses the augmented maximum flows to construct an optimal solution by sending each flow as long as possible through the network similar to the temporally repeated flow technique for standard maximum dynamic flow on the auxiliary network. The obtained flow is equal to the generalized maximum dynamic contraflow on original network. As it works on time-expanded network, its time complicity is pseudo-polynomial.

Theorem 4.1. *The generalized maximum dynamic contraflow problem is solved in pseudo-polynomial time complexity on two-terminal lossy network.*

4.2. **Generalized Earliest Arrival Contraflow.** For a two terminal lossy network with integer capacity, integer transit time and gain factor on each arc, the generalized earliest arrival contraflow problem is to find the maximum amount of flow with highest gain that can be sent in every time period $0 \leq t \leq T$, from the source s to the sink d if the arcs can be reversed at time zero. Authors in [30, 24] introduced the problem and solved it with efficient algorithm. They proved that the algorithm developed for the generalized maximum dynamic contraflow problem also solves the generalized earliest arrival contraflow problem because the solution satisfies the earliest arrival property, i.e., a cumulative amount of flows reaching the sink in every considered time period and all preceding time periods of the considered one have to be maximal. Analogously, the flows leaving the source have to be maximal. Thus, the generalized earliest arrival contraflow problem is also solved in pseudo-polynomial time complexity. Its polynomial solution is demanding.

5. ABSTRACT CONTRAFLOW ALGORITHMS

An abstract network contains the element set $\mathcal{A}$ and path sets $\mathcal{P}$. Each element a in $\mathcal{A}$ has capacity $b_{\mathcal{A}}(a) : \mathcal{A} \to \mathcal{R}^+$ and travel time $\tau_{\mathcal{A}}(a) : \mathcal{A} \to \mathcal{Z}^+$ or cost (weight) $c_{\mathcal{A}}(a) : \mathcal{A} \to \mathcal{R}^+$. The capacity limits flow amount moving along each element, $\tau_{\mathcal{A}}$ measures the time needed to travel an element and $c_{\mathcal{A}}$ represents the cost needed to move one unit of flow along each element. There are paths from s-d as well as d-s. However, the movement of flow along the paths d-s are forbidden. Pyakurel *et al*. [31] introduced the abstract contraflow model, where the contraflow has been extended to more general setting of abstract flows by replacing the underline contraflow network with an abstract system of linearly ordered sets, called paths satisfying the switching property. The paths P and Q cross at element a if $a \in P \cap Q$ and satisfy the switching property if there exists a path R that only uses elements at the beginning of P and at the end of Q (and vice versa). Mathematically, both sets $\{R \subseteq \mathcal{P} \mid R \subseteq (P, a) \cup (a, Q)\}$ and $\{R \subseteq \mathcal{P} \mid R \subseteq (Q, a) \cup (a, P)\}$ are non-empty. Let the switching paths of P and Q be

$$\begin{aligned} P \times_a Q &\in \{R \subseteq \mathcal{P} \mid R \subseteq (P, a) \cup (a, Q)\} \text{ and} \\ Q \times_a P &\in \{R \subseteq \mathcal{P} \mid R \subseteq (Q, a) \cup (a, P)\} \end{aligned}$$

where, $(P, a) = \{p \in P : p \leq_P a\}$ and $(a, Q) = \{q \in Q : a \leq_Q q\}$, [16, 15].

Contraflow configuration with path reversal capability has been introduced in [31]. It is to reverse the direction of empty paths to increase the flow value. Notice that the reversal of a path is equivalent to the reversal of each element contained in the path. However, the transit time remains same. Moreover each path with increased capacity satisfies the switching property. In the abstract contraflow framework, authors introduced the abstract maximum static and abstract maximum dynamic contraflow problems on two terminal abstract network and presented polynomial time algorithms to solve them. Using maximum flow and minimum cut relation in abstract static network, they proved that double flow can be sent from s to d if the minimum cut capacities are symmetric. Moreover, they solved the abstract maximum dynamic contraflow in continuous time setting with the same complexity as in without contraflow. This solution also exists for discrete time setting on two terminal abstract network.

Theorem 5.1. *The abstract maximum dynamic contraflow problem can be solved in polynomial time complexity and the flow value can be increased up to double with contraflow reconfiguration if the dynamic minimum cut capacities are symmetric.*

6. CONCLUDING REMARKS

The most crucial evacuation planning problems, that are being frequently studied by most of the emergency planning researchers, are considered in this paper. Compact surveys on the contraflow models and algorithms of the maximum dynamic, lexicographic maximum dynamic, earliest arrival, earliest arrival transshipment and the quickest problem are presented in depth. The contraflow approach that increases the flow value significantly with reasonably less evacuation time are extended with their analytical optimal solutions. These techniques are mostly accepted as heuristics solutions earlier. Majority of the dynamic flow solution with or without contraflow are proved to be computationally hard except for two-terminal evacuation circumstances or special structured networks.

This work highlights the approximation algorithms on earliest arrival transshipment contraflow that yield near optimal solutions with efficient algorithms. The generalized contraflow approaches are the extensions of the results on solution of dynamic flow and lossy networks. The generalized maximum and earliest arrival solutions are studied briefly in this paper. The abstract contraflow approach has been highlighted. The summery of results in this paper highlights many insights on the efficient solutions and opens further interesting research problems in dynamic evacuation planning problems.

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