*Neural, Parallel, and Scientific Computations*, **26**, No. 3 (2018), 257-267 ISSN: 1061-5369

# A NOTE ON THE EXPONENTIATED EXPONENTIAL–POISSON SOFTWARE RELIABILITY MODEL

VESSELIN KYURKCHIEV[1], HRISTO KISKINOV[2],
OLGA RAHNEVA[3], AND GEORGI SPASOV[4]

[1,2,4]Faculty of Mathematics and Informatics
University of Plovdiv Paisii Hilendarski
24, Tzar Asen Str., 4000 Plovdiv, BULGARIA

[3]Faculty of Economy and Social Sciences
University of Plovdiv Paisii Hilendarski
24, Tzar Asen Str., 4000 Plovdiv, BULGARIA

**ABSTRACT:** In this paper we consider an application to the debugging theory of a class of cumulative exponentiated exponential–Poisson distribution functions introduced by Ramos, Dey, Louzada and Lachos.

By this family of cumulative distribution functions we study the Hausdorff approximation of the shifted Heaviside step function.

Numerical examples, illustrating our results using the programming environment Mathematica are presented.

As application in the field of debugging and test theory are given examples with real data including compatibility modifications, operating system upgrade and signaling message processing from year 2000 using the new software reliability model.

**AMS Subject Classification:** 68N30, 41A46
**Key Words:** exponentiated exponential–Poisson cumulative distribution function (EEPcdf), Heaviside function, Hausdorff approximation, upper and lower bounds

**Received:** July 4, 2018; **Accepted:** September 26, 2018;
**Published:** October 3, 2018 **doi:** 10.12732/npsc.v26i3.3
Dynamic Publishers, Inc., Acad. Publishers, Ltd. https://acadsol.eu/npsc

## 1. INTRODUCTION

Some extensions of the well-known Poisson, Poisson–exponential, Chen, Exponentiated Chen, modified Weibull and Burr distributions can be found in [1]–[8].

A software reliability model that uses the "Gompertz–type correction" is the Exponentiated Exponential–Poisson cumulative distribution function (EEPcdf) introduced by Ramos, Dey, Louzada and Lachos in [9]:

$$M(t;\lambda;\theta) = \left( \frac{e^{\lambda \frac{1-e^{-\theta t}}{1-e^{-\theta}}} - 1}{e^{\lambda} - 1} \right)^{\alpha} \tag{1}$$

where $\theta > 0$; $\lambda > 0$; $\alpha > 0$.

For other software reliability models see [10]– [29].

Without loss of generality we consider the following class of the family (1) with application to the debugging theory:

$$M_1(t) = \left( \frac{e^{\lambda \frac{1-e^{-\theta t}}{1-e^{-\theta}}} - 1}{e^{\lambda} - 1} \right)^{\alpha} \tag{2}$$

with

$$t_0 = -\frac{1}{\theta} \ln \left( 1 - \frac{1-e^{-\theta}}{\lambda} \ln \left( 1 + \frac{e^{\lambda} - 1}{2^{\frac{1}{\alpha}}} \right) \right); \qquad M_1(t_0) = \frac{1}{2}. \tag{3}$$

In this note we study the Hausdorff approximation of the *shifted Heaviside step function*

$$h_{t_0}(t) = \begin{cases} 0, & \text{if} \quad t < t_0, \\ [0,1], & \text{if} \quad t = t_0, \\ 1, & \text{if} \quad t > t_0, \end{cases}$$

by the family (2)–(3).

Hausdorff approximation of some modeling functions can be found in [30]–[35].

Furthermore, we propose a software module (intellectual property) within the programming environment CAS Mathematica for the analysis. Numerical examples, illustrating our results are presented.

As application in the field of debugging and test theory we give also real examples with data provided in [36] using the new software reliability model. The dataset includes [37] Year 2000 compatibility modifications, operating system upgrade and signaling message processing.

## 2. HAUSDORFF APPROXIMATION OF THE SHIFTED HEAVISIDE STEP FUNCTION

**Definition 1.** [38] The Hausdorff distance (the H–distance) $\rho(f,g)$ between two interval functions $f, g$ on $\Omega \subseteq \mathbb{R}$, is the distance between their completed graphs $F(f)$

and $F(g)$ considered as closed subsets of $\Omega \times \mathbb{R}$. More precisely,

$$\rho(f,g) = \max\{\sup_{A\in F(f)} \inf_{B\in F(g)} ||A-B||, \sup_{B\in F(g)} \inf_{A\in F(f)} ||A-B||\},$$

wherein $||.||$ is any norm in $\mathbb{R}^2$, e. g. the maximum norm $||(t,x)|| = \max\{|t|,|x|\}$; hence the distance between the points $A = (t_A, x_A)$, $B = (t_B, x_B)$ in $\mathbb{R}^2$ is $||A-B|| = \max\{|t_A - t_B|, |x_A - x_B|\}$.

The one–sided Hausdorff distance $d$ between the function $h_{t_0}(t)$ and the function (2)–(3) satisfies the relation

$$M_1(t_0 + d) = 1 - d. \tag{4}$$

The following theorem gives upper and lower bounds for $d$.

**Theorem 2.** *Let*

$$p = -\frac{1}{2},$$

$$q = 1 + \frac{\alpha\theta\lambda\left(1 + \frac{e^{\lambda}-1}{2^{\frac{1}{\alpha}}}\right)}{2^{\frac{\alpha-1}{\alpha}}(1-e^{-\theta})(e^{\lambda}-1)}\left(1 - \frac{1-e^{-\theta}}{\lambda}\ln\left(1 + \frac{e^{\lambda}-1}{2^{\frac{1}{\alpha}}}\right)\right),$$

$$r = 2.1q$$

*For the one–sided Hausdorff distance $d$ between $h_{t_0}(t)$ and the function* (2)–(3) *for*

$$q > \frac{e^{1.05}}{2.1}$$

*the following inequalities hold:*

$$d_l = \frac{1}{r} < d < \frac{\ln r}{r} = d_r. \tag{5}$$

**Proof.** Let us examine the function:

$$F(d) = M_1(t_0 + d) - 1 + d. \tag{6}$$

From $F'(d) > 0$ we conclude that the function $F$ is increasing.

Consider the function

$$G(d) = p + qd. \tag{7}$$

From the Taylor expansion we obtain $G(d) - F(d) = O(d^2)$.

Hence $G(d)$ approximates $F(d)$ with $d \to 0$ as $O(d^2)$ (see Fig. 1).

In addition $G'(d) > 0$.

Further, for $q > \frac{e^{1.05}}{2.1}$ we have $G(d_l) < 0$ and $G(d_r) > 0$.

This completes the proof of the theorem. □

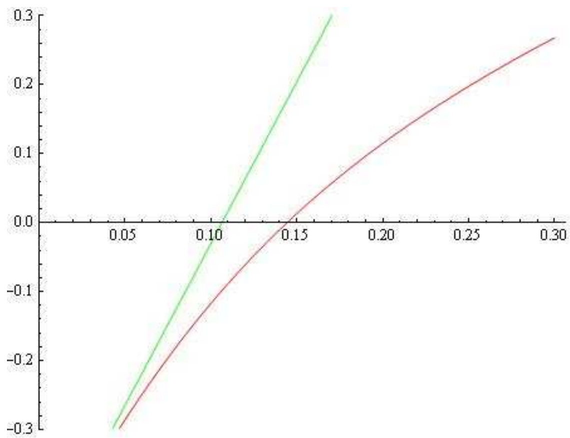


Figure 1: The functions $F(d)$ and $G(d)$ for $\theta = 10; \lambda = 0.5; \alpha = 3.5$.

The model (2)–(3) for $\theta = 10$, $\lambda = 0.5$, $\alpha = 3.5$, $t_0 = 0.191957$ is visualized on Fig. 2. From the nonlinear equation (4) and inequalities (5) we have: $d = 0.14436$, $d_l = 0.101423$, $d_r = 0.232102$.

The model (2)–(3) for $\theta = 10$, $\lambda = 0.05$, $\alpha = 0.5$, $t_0 = 0.0293988$ is visualized on Fig. 3. From the nonlinear equation (4) and inequalities (5) we have: $d = 0.120798$, $d_l = 0.0569498$, $d_r = 0.163195$.

From the above examples, it can be seen that the proven estimates in Theorem 2 for the value of the Hausdorff approximation are reliable when assessing the important characteristic - "saturation". This characteristic has its equal participation together with the other two characteristics - "confidence intervals" and "confidence bounds" in the area of the software reliability theory.

We propose a software module (intellectual property) within the programming environment *CAS Mathematica* for the analysis of the considered family $M_1(t)$.

The module offers the following possibilities:

- generation of the function under user defined values of the parameters $\lambda$, $\alpha$ and $\theta$;

- calculation of the H-distance $d$ between the function $h_{t_0}(t)$ and the function $M_1(t)$;

- software tools for animation and visualization.

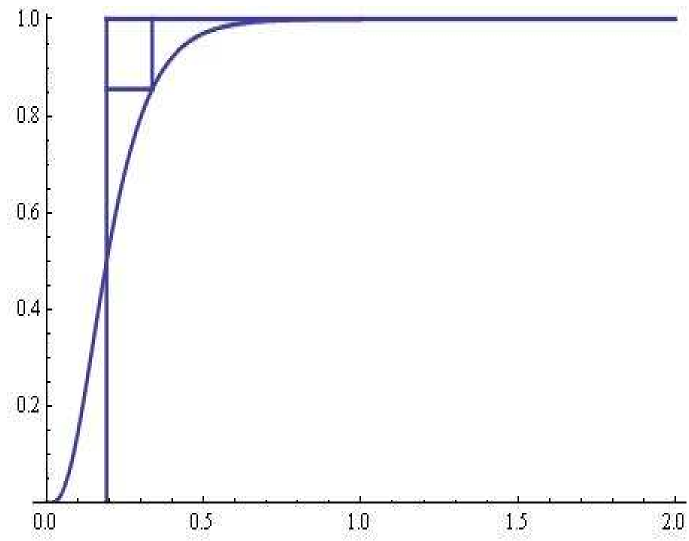


Figure 2: The model (2)–(3) for $\theta = 8$, $\lambda = 0.5$, $t_0 = 0.103098$; H–distance $d = 0.155378$, $d_l = 0.103958$, $d_r = 0.235337$.

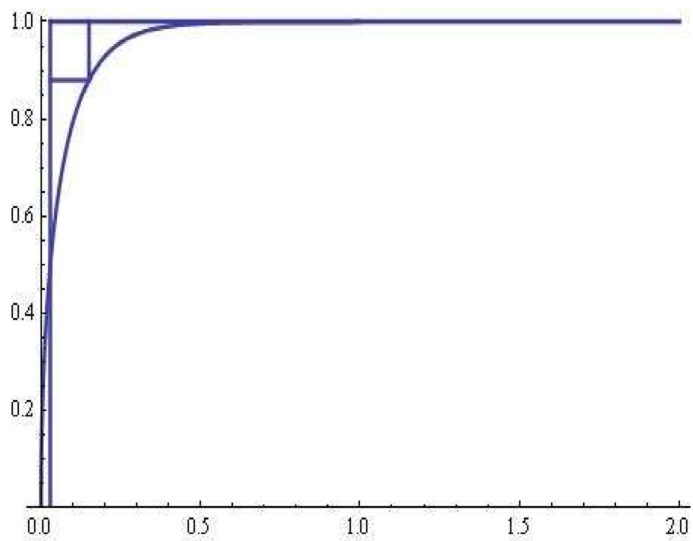


Figure 3: The model (2)–(3) for $\beta = 15$, $\lambda = 0.01$, $t_0 = 0.0463767$; H–distance $d = 0.104496$, $d_l = 0.0561458$, $d_r = 0.161689$.

## 3. APPLICATION IN THE FIELD OF DEBUGGING AND TEST THEORY

We give real examples with data provided in [36].

The operating time of the software is 167,900 days. 115 failures are detected for these days which contain 71 unique failures.

Table 1 shows the failures data which are united for each of the 13 months.

The dataset includes [37] Year 2000 compatibility modifications, operating system upgrade and signaling message processing.

| Month Index | System Days (Days) | System Days (Cumulative) | Failures | Cumulative Failures |
|---|---|---|---|---|
| 1 | 961 | 961 | 7 | 7 |
| 2 | 4170 | 5131 | 3 | 10 |
| 3 | 8789 | 13,920 | 14 | 24 |
| 4 | 11,858 | 25,778 | 8 | 32 |
| 5 | 13,110 | 38,888 | 11 | 43 |
| 6 | 14,198 | 53,086 | 8 | 51 |
| 7 | 14,265 | 67,351 | 7 | 58 |
| 8 | 15,175 | 82,526 | 19 | 77 |
| 9 | 15,376 | 97,902 | 17 | 94 |
| 10 | 15,704 | 113,606 | 6 | 100 |
| 11 | 18,182 | 131,788 | 11 | 111 |
| 12 | 17,760 | 149,548 | 4 | 115 |
| 13 | 18,352 | 167,900 | 0 | 115 |

Table 1. Field failure data [36].

The fitted model

$$M_1(t) = \omega \left( \frac{e^{\lambda \frac{1-e^{-\theta t}}{1-e^{-\theta}}} - 1}{e^{\lambda} - 1} \right)^{\alpha}$$

based on the data of Table 1 for the estimated parameters:

$$\omega = 402; \;\; \theta = 0.23822; \;\; \lambda = 0.830748; \;\; \alpha = 19.0736$$

is plotted on Fig. 4.

We hope that the results will be useful for a lot of specialists in this scientific area.

**Remark 3.** In [29] the authors developed the following new software reliability model

$$M(t) = \omega \left( \frac{e^{\lambda \frac{1-e^{-\theta t}}{1-e^{-\theta}}} - 1}{e^{\lambda} - 1} \right)$$

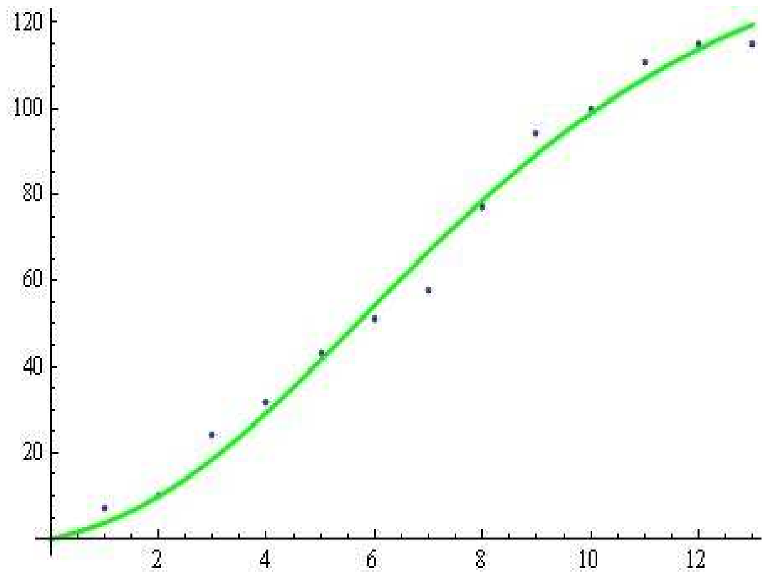


Figure 4: The fitted model $M_1(t)$.

The model for

$$\omega = 7;\ \ \theta = 0.142302;\ \ \lambda = 0.317432$$

is plotted on Fig. 5.

We will explicitly note that in some cases the software reliability model $M_1(t)$ provides better results than other much more sophistical models.

## ACKNOWLEDGMENTS

This work has been supported by project FP17-FMI-008 of Department for Scientific Research, Paisii Hilendarski University of Plovdiv.

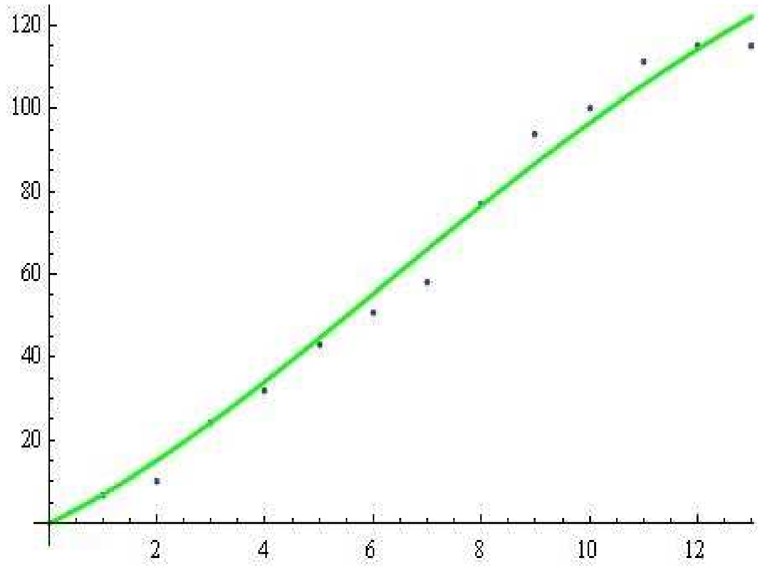


Figure 5: The fitted model $M(t)$.

## REFERENCES

[1] Z. Chen, A new two-parameter lifetime distribution with bathtub shape or increasing failure rate function, *Stat. and Prob. Letters*, **49** (2000), 155-161.

[2] M. Xie, Y. Tang, T. Goh, A modified Weibull extension with bathtub - shaped failure rate function, *Reliability Eng. and System Safety*, **76** (2002), 279-285.

[3] M. Khan, A. Sharma, Generalized order statistics from Chen distribution and its characterization, *J. of Stat. Appl. and Prob.*, **5** (2016), 123-128.

[4] S. Dey, D. Kumar, P. Ramos, F. Louzada, Exponentiated Chen distribution: Properties and Estimations, *Comm. in Stat. - Simulation and Computation*, (2017), 1-22.

[5] Y. Chaubey, R. Zhang, An extension of Chen's family of survival distributions with bathtub shape or increasing hazard rate function, *Comm. in Stat. - Theory and Methods*, **44** (2015), 4049-4069.

[6] V. Cancho, F. Louzada, G. Barriga, The Poisson-exponential lifetime distribution, *Comp. Stat. Data Anal.*, **55** (2011), 677-686.

[7] G. Rodrigues, F. Louzada, P. Ramos, Poisson-exponential distribution: different methods of estimation, J. of Appl. Stat., **45** (2018), no. 1, 128-144.

[8] F. Louzada, P. Ramos, P. Ferreira, Exponential-Poisson distribution: estimation

and applications to rainfall and aircraft data with zero occurrence, *Communication in Statistics-Simulation and Computation*, (2018).

[9] P. Ramos, D. Dey, F. Louzada, V. Lachos, An extended Poisson family of lifetime distribution: A unified approach in competitive and complementary risk, *arXiv*: submit/2267507 [stat.AP] 19 May 2018.

[10] N. Pavlov, A. Iliev, A. Rahnev, N. Kyurkchiev, *Some software reliability models: Approximation and modeling aspects*, LAP LAMBERT Academic Publishing (2018), ISBN: 978-613-9-82805-0.

[11] N. Pavlov, A. Iliev, A. Rahnev, N. Kyurkchiev, *Nontrivial Models in Debugging Theory (Part 2)*, LAP LAMBERT Academic Publishing (2018), ISBN: 978-613-9-87794-2.

[12] N. Pavlov, A. Iliev, A. Rahnev, N. Kyurkchiev, On the extended Chen's and Pham's software reliability models. Some applications, *Int. J. of Pure and Appl. Math.*, **118** (2018) no. 4, 1053-1067.

[13] N. Pavlov, G. Spasov, A. Rahnev, N. Kyurkchiev, A new class of Gompertz–type software reliability models, *International Electronic Journal of Pure and Applied Mathematics*, **12** (2018), no. 1, 43–57.

[14] N. Pavlov, G. Spasov, A. Rahnev, N. Kyurkchiev, Some deterministic reliability growth curves for software error detection: Approximation and modeling aspects, *Int. J. of Pure and Appl. Math.*, **118** (2018), no. 3, 599-611.

[15] N. Pavlov, A. Golev, A. Rahnev, N. Kyurkchiev, A note on the Yamada–exponential software reliability model, *Int. J. of Pure and Appl. Math.*, **118** (2018), no. 4, 871–882.

[16] N. Pavlov, A. Iliev, A. Rahnev, N. Kyurkchiev, A note on the "Mean Value" software reliability model, *Int. J. of Pure and Appl. Math.*, **118** (2018), no. 4, 949–956.

[17] N. Pavlov, A. Golev, A. Rahnev, N. Kyurkchiev, A note on the generalized inverted exponential software reliability model, *International Journal of Advanced Research in Computer and Communication Engineering*, **7** (2018), no. 3, 484–487.

[18] N. Pavlov, A. Iliev, A. Rahnev, N. Kyurkchiev, Transmuted inverse exponential software reliability model, *Int. J. of Latest Research in Engineering and Technology*, **4** (2018), no. 5, 1–6.

[19] N. Pavlov, A. Iliev, A. Rahnev, N. Kyurkchiev, Analysis of the Chen's and Pham's software reliability models, *Cybernetics and Information Technologies*, **18** (2018), no. 3, 37–47.

[20] N. Pavlov, A. Iliev, A. Rahnev, N. Kyurkchiev, On some nonstandard software reliability models, *Dynamic Systems and Applications*, **27** (2018), no. 4, 757–771.

[21] N. Pavlov, A. Iliev, A. Rahnev, N. Kyurkchiev, Some deterministic growth curves with applications to software reliability analysis, *Int. J. of Pure and Appl. Math.*, **119** (2018), no. 2, 357–368.

[22] N. Pavlov, A. Iliev, A. Rahnev, N. Kyurkchiev, Investigations of the K-stage Erlangian software reliability growth model, *Int. J. of Pure and Appl. Math.*, **119** (2018), no. 3, 441–449.

[23] N. Pavlov, A. Iliev, A. Rahnev, N. Kyurkchiev, Some transmuted software reliability models, *Journal of Mathematical Sciences and Modeling*, **1** (2018), no. 2, (to appear).

[24] N. Pavlov, A. Iliev, A. Rahnev, N. Kyurkchiev, A note on Ohbas inflexion S–shaped software reliability growth model, *Collection of scientific works from conference "Mathematics. Informatics. Information Technologies. Application in Education"*, Pamporovo, Bulgaria, October 10-12, (2018), (to appear).

[25] V. Kyurkchiev, A. Malinova, O. Rahneva, P. Kyurkchiev, On the Burr XII-Weibull software reliability model, *Int. J. of Pure and Appl. Math.*, **119** (2018), no. 4, 639–650.

[26] V. Kyurkchiev, A. Malinova, O. Rahneva, P. Kyurkchiev, Some notes on the extended Burr XII software reliability model, *Int. J. of Pure and Appl. Math.*, **120** (2018), no. 1, 127–136.

[27] S. Markov, N. Kyurkchiev, A. Iliev, A. Rahnev, A note on the Log–logistic and transmuted Log–logistic Models. Some applications, *Dynamic Systems and Applications*, **27** (2018), no. 3, 593–607.

[28] N. Kyurkchiev, A. Iliev, *Extension of Gompertz-type Equation in Modern Science: 240 Anniversary of the birth of B. Gompertz*, LAP LAMBERT Academic Publishing (2018), ISBN: 978-613-9-90569-0.

[29] N. Pavlov, A. Iliev, A. Rahnev, N. Kyurkchiev, Application of a new class cumulative lifetime distribution to software reliability analysis, *Communications in Applied Analysis*, **22** (2018), no. 4, 555–565.

[30] O. Rahneva, H. Kiskinov, H. Melemov, M. Stieger, Some approximation Aspects for a new class cumulative distribution functions, *Neural, Parallel, and Scientific Computations*, **26** (2018), no. 2, 225-235, doi: 10.12732/npsc.v26i2.6

[31] O. Rahneva, H. Kiskinov, H. Melemov, M. Stieger, On the approximation of one class of impulse functions, *Neural, Parallel, and Scientific Computations*, **26** (2018), no. 3, 237-246, doi: 10.12732/npsc.v26i3.1

[32] N. Kyurkchiev, A. Andreev, *Approximation and Antenna and Filter Synthesis: Some Moduli in Programming Environment Mathematica*, LAP LAMBERT Academic Publishing, Saarbrucken, (2014), ISBN 978-3-659-53322-8.

[33] N. Kyurkchiev, S. Markov, On the Hausdorff distance between the Heaviside step function and Verhulst logistic function, *J. Math. Chem.*, **54** (2016), no. 1, 109-119.

[34] A. Andreev, N. Kyurkchiev, Approximation of some impulse functions - implementation in programming environment MATHEMATICA, *Proceedings of the 43 Spring Conference of the Union of Bulgarian Mathematicians, Borovetz, April 2-6*, (2014), 111-117.

[35] S. Markov, N. Kyurkchiev, A. Iliev, A. Rahnev, On the approximation of the cut functions by hyper-log-logistic function, *Neural, Parallel, and Scientific Computations*, **26** (2018), no. 2, 169-182, doi: 10.12732/npsc.v26i2.3

[36] D. R. Jeske, X. Zhang, Some successful approaches to software reliability modeling in industry, *J. Syst. Softw.*, **74** (2005), 85–99.

[37] K. Song, H. Pham, A software reliability model with a Weibull fault detection rate function subject to operating environments, *Appl. Sci.*, **7** (2017), doi:10.3390/app7100983, 16 pp.

[38] F. Hausdorff, *Set Theory*, (2 ed.) (Chelsea Publ., New York, (1962 [1957]) (Republished by AMS-Chelsea 2005), ISBN: 978-0-821-83835-8.