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SENSITIVE ANALYSIS ON SOME TRANSFORMATIONS TO CONSTRUCT A GENERALIZED 2–COMPONENT WEIBULL DISTRIBUTION

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ABSTRACT: In literature, several transformations exists to obtain a new cumulative distribution function (cdf) using other(s) well-known cdf(s).

In this paper we study the important "saturation" characteristic for the generalized 2–component Weibull c.d.f. in the Hausdorff sense.

We will conduct a sensitive analysis of this family of cumulative functions.

The results have independent significance in the study of issues related to life time analysis, insurance mathematics, biochemical kinetics, population dynamics and debugging theory.

Numerical examples, illustrating our results are presented using programming environment Mathematica.

AMS Subject Classification: 41A46, 68N30

Key Words: modified 2–component Weibull c.d.f., G–transformation, saturation in Hausdorff sense

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1. INTRODUCTION

The Weibull distribution [1]–[3] and Bur XII–Weibull c.d.f are of interest from the theoretical and applied aspects.

They find applications in insurance mathematics, population dynamics and debugging theory [4]–[5], [17].

For some approximation and modeling aspects, see [6].

In literature, several transformations exists to obtain a new cumulative distribution function (cdf) using other(s) well-known cdf(s) [7]– [13], [18]–[31].

Many researches have used the quadratic rank transmuted map (QRTM) to develop new life time distribution.

Another popular transformations by using a (cdf) $F(t)$ are [11] (with applications in data analysis):

$$G_1(t) = \frac{1}{e-1}\left(e^{F(t)} - 1\right) \tag{1}$$

and [13]:

$$G_2(t) = e^{1-\frac{1}{F(t)}}. \tag{2}$$

Kumar et al. [14] proposed the cdf distribution by the use of any two cdf $F_1(t)$ and $F_2(t)$ of baseline continuous distribution(s) with common spectrum, by the transformation:

$$G_3(t) = \frac{F_1(t) + F_2(t)}{1 + F_1(t)}. \tag{3}$$

The transformation (3) has great applications in lifetime analysis.

A new c.d.f. based on m existing ones is the following [15]:

$$G_4(t) = \frac{\sum_{k=1}^{m} F_k(t)}{m - 1 + \prod_{k=1}^{m} \left(F_k(t)\right)^{\delta_k}}. \tag{4}$$

The GM–transformation yields the following cdf

$$G_5(t) = \frac{mF(t)}{m - 1 + F^q(t)}, \quad q \in \{0, 1, \ldots, m\}. \tag{5}$$

In this paper we study the important ”saturation” characteristic for the modified Weibull cumulative distribution function (based on transformations (1), (2) and (5) in the Hausdorff sense (see definition 1)).

The results have independent significance in the study of issues related to lifetime analysis, population dynamics and debugging theory.

Definition 1. [16] The one–sided Hausdorff distance $\overrightarrow{\rho}(f, g)$ between two interval functions f, g on $\Omega \subseteq \mathbb{R}$, is the one–sided Hausdorff distance between their completed graphs $\mathcal{F}(f)$ and $\mathcal{F}(g)$ considered as closed subsets of $\Omega \times \mathbb{R}$. More precisely,

$$\overrightarrow{\rho}(f, g) = \sup_{B \in \mathcal{F}(g)} \inf_{A \in \mathcal{F}(f)} ||A - B||,$$

where $|| \cdot ||$ is a norm in $\mathbb{R}^2$.

We recall that completed graph of f is the closure of the graph of f as a subset of $\Omega \times \mathbb{R}$. If the graph of an interval function f equals $\mathcal{F}(f)$, then the f is called S-continuous. The Hausdorff distance $\rho(f,g) = \max\{\overrightarrow{\rho}(f,g), \overrightarrow{\rho}(g,f)\}$ defines a metric in the set of the S-continuous interval functions.

2. THE TRANSFORMATION (2) WITH CORRECTION OF WEIBULL CDF – TYPE

Let

$$F(t) = 1 - e^{-\left(\frac{t}{\lambda}\right)^k}$$

is the Weibull c.d.f. with parameters (k, λ).

Based on the transformation $G_2(t)$ (2) we have:

$$G_2(t) = e^{1 - \frac{1}{1 - e^{-\left(\frac{t}{\lambda}\right)^k}}}. \tag{6}$$

Let

$$t_0 = \lambda \left(\ln \frac{1 + \ln 2}{\ln 2}\right)^{\frac{1}{k}}; \quad G_2(t_0) = \frac{1}{2}.$$

The one–sided Hausdorff distance d between the shifted Heaviside function and the c.d.f. function - (6) satisfies the relation

$$G_2(t_0 + d) = 1 - d. \tag{7}$$

The following theorem gives upper and lower bounds for d

Theorem. Let

$$p = -\frac{1}{2},$$

$$q = -1 + \frac{k \ln 2(1 + \ln 2)}{2\lambda} \left(\ln \frac{1 + \ln 2}{\ln 2}\right)^{\frac{k-1}{k}},$$

$$r = 2.1q.$$

For the one–sided Hausdorff distance d the following inequalities hold for:

$$r > e^{1.05}$$

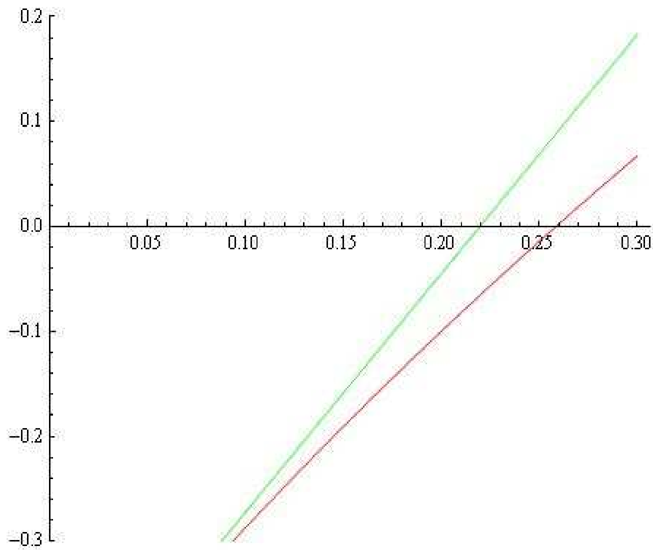

Figure 1: The functions $F(d)$ and $G(d)$ for $k = 1.1; \lambda = 0.5$.

$$d_l = \frac{1}{r} < d < \frac{\ln r}{r} = d_r. \tag{8}$$

Proof. Let us examine the function:

$$F(d) = G_2(t_0 + d) - 1 + d. \tag{9}$$

The function F is increasing.

Consider the function

$$G(d) = p + qd. \tag{10}$$

From Taylor expansion we obtain $G(d) - F(d) = O(d^2)$.

Hence $G(d)$ approximates $F(d)$ with $d \to 0$ as $O(d^2)$ (see Fig. 1).

In addition $G'(d) > 0$.

Further, for $r > e^{1.05}$ we have $G(d_l) < 0$ and $G(d_r) > 0$.

This completes the proof of the theorem.

The model (6) for $\lambda = 0.5$; $k = 1.1$ and $\lambda = 0.5$, $k = 2.2$, respectively is visualized on Fig. 2.

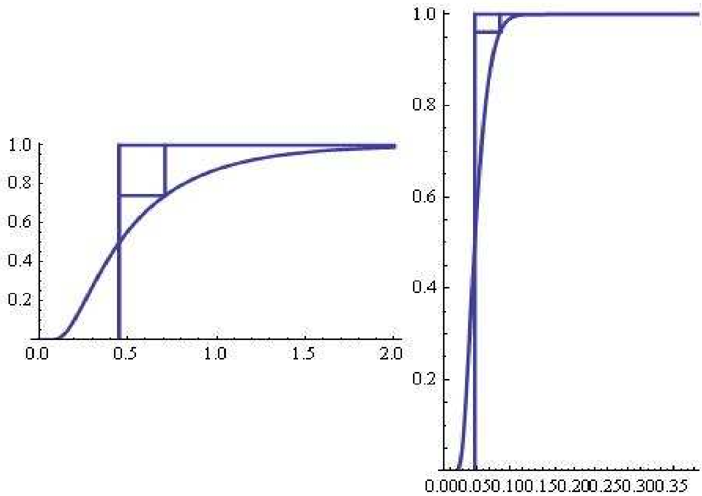

Figure 2: a) The model (6) for $\lambda = 0.5$; $k = 1.1$, $t_0 = 0.451164$ The H–distance $d = 0.258289$, $d_l = 0.209061$, $d_r = 0.327207$; b) The model (6) for $\lambda = 0.5$; $k = 2.2$, $t_0 = 0.0474955$ The H–distance $d = 0.0383095$, $d_l = 0.0188403$, $d_r = 0.074829$.

3. THE TRANSFORMATION (1) WITH CORRECTION OF WEIBULL CDF – TYPE

Based on the transformation $G_1(t)$ (1) with correction of Weibull cdf – type we have:

$$G_1(t) = \frac{1}{e-1}\left(e^{1-e^{-\left(\frac{t}{\lambda}\right)^k}} - 1\right). \tag{11}$$

The one–sided Hausdorff distance d between the Heaviside function and the c.d.f. function - (11) satisfies the relation

$$G_1(d) = 1 - d. \tag{12}$$

The model (11) for $\lambda = 0.05$; $k = 0.9$ and $\lambda = 0.01$, $k = 0.9$, respectively is visualized on Fig. 3.

Some computational examples using nonlinear equation (12) are presented in Table 1.

k	λ	d *computed by* (12)
0.9	0.1	0.209935
0.9	0.05	0.133845
0.9	0.01	0.0417732
0.7	0.05	0.158851
1.1	0.03	0.0803209
1.5	0.03	0.0647972
2.1	0.01	0.0201538
4	0.01	0.0147033

Table 1: Bounds for d computed by nonlinear equation (12) for various k and λ.

4. THE TRANSFORMATION (5) WITH CORRECTION OF WEIBULL CDF – TYPE

Based on the transformation $G_5(t)$ (5) for $m = 2$ with correction of Weibull cdf – type we have:

$$G_5(t) = \frac{2\left(1 - e^{-\left(\frac{t}{\lambda}\right)^k}\right)}{1 + \left(1 - e^{-\left(\frac{t}{\lambda}\right)^k}\right)^2}. \tag{13}$$

The one–sided Hausdorff distance d between the Heaviside function and the c.d.f. function - (13) satisfies the relation

$$G_5(d) = 1 - d. \tag{14}$$

The model (13) for $\lambda = 0.05$; $k = 0.9$; $m = 2$ and $q = 2$ is visualized on Fig. 4.

A comparison between models (13) and (11) is also shown in Fig. 4.

The applied comparisons show that ”saturation” using the model (13) is faster.

We hope that the results will be useful for specialists in this scientific area.

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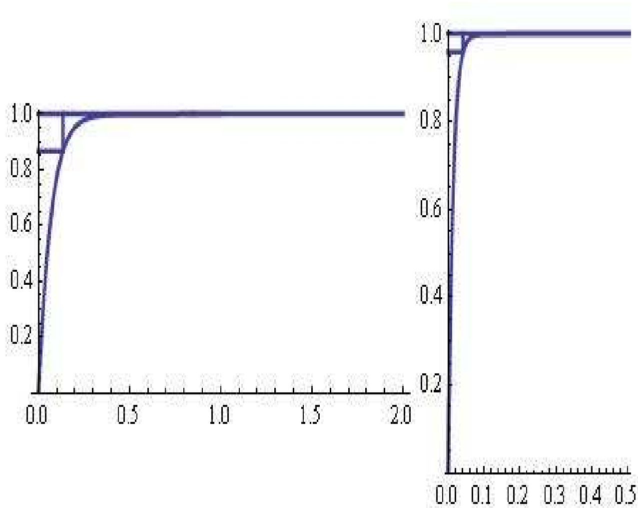

Figure 3: a) The model (11) for $\lambda = 0.05$; $k = 0.9$, The H–distance $d = 0.133845$; b) The model (11) for $\lambda = 0.01$, $k = 0.9$, The H–distance $d = 0.0417732$.

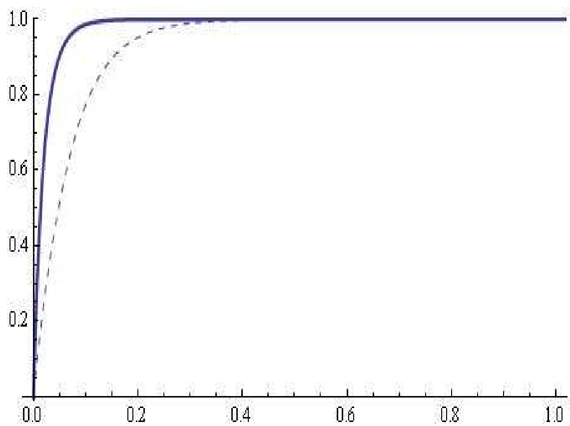

Figure 4: The model (13)- (thick) for $\lambda = 0.05$; $k = 0.9$; $m = 2$ and $q = 2$; The H–distance $d = 0.0611484$; b) The model (11) - (dashed) for $\lambda = 0.05$, $k = 0.9$; The H–distance $d = 0.133845$.

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