A NOTE ON THE LEE–CHANG–PHAM–SONG
SOFTWARE RELIABILITY MODEL

OLGA RAHNEVA[1], HRISTO KISKINOV[2],
ANNA MALINOVA[3], AND GEORGI SPASOV[4]

[1]Faculty of Economy and Social Sciences
University of Plovdiv "Paisii Hilendarski"
24, Tzar Asen Str., 4000 Plovdiv, BULGARIA

[2,3,4]Faculty of Mathematics and Informatics
University of Plovdiv Paisii Hilendarski
24, Tzar Asen Str., 4000 Plovdiv, BULGARIA

ABSTRACT: In this paper we study the Hausdorff approximation of the shifted Heaviside step function $h_{t_0}(t)$ by sigmoidal function based on the Lee–Chang–Pham–Song cumulative function and find an expression for the error of the best approximation. We give real examples with small on–line data provided by IBM entry software package using the model. The potentiality of the software reliability models is analyzed. Lee–Chang–Pham–Song's idea of including the characteristic t^* (the time when debugging starts after modifying the code causing syntax errors) in the study of models in debugging theory can be successfully expanded. For instance, for the Goel (1980) software reliability model.

1. INTRODUCTION

Detailed description of all elements in the area of debugging theory may be found in the following books [5]–[6] and [4].

In the books [7]–[8], we pay particular attention to both deterministic approaches and probability models for debugging theories. Some of the existing cumulative distributions (Gompertz–Makeham, Yamada-exponential, Yamada–Rayleigh, Yamada–Wei–bull, transmuted inverse exponential, transmuted Log-Logistic, Ku–maraswamy–Dagum and Kumaraswamy Quasi Lindley) are considered in the light of modern debugging and test theories.

Some software reliability models, can be found in [9]–[39].

In this note we study the Hausdorff approximation of the shifted Heaviside step function $h_{t_0}(t)$ by sigmoidal function based on the Lee–Chang–Pham–Song cumulative function [1] and find an expression for the error of the best approximation.

We propose a software modules (intellectual properties) within the programming environment CAS Mathematica for the analysis.

The models have been tested with real-world data.

2. PRELIMINARIES

Definition 1. [1] The Lee–Chang–Pham–Song software reliability model considering the syntax error in uncertainty environments is given as follows

$$m(t) = N \left(1 - \frac{\beta}{\beta + a(t - t^*)^b} \right)^\alpha \tag{1}$$

where t^* is the time when debugging starts after modifying the code causing syntax errors, a is a scale parameter, b is the shape parameter, α, $\beta > 0$

$$0 \xrightarrow{Syntax\ error} t^* \xrightarrow{Testing\ time} t$$

Definition 2. [2] The Hausdorff distance (the H–distance) $\rho(f,g)$ between two interval functions f, g on $\Omega \subseteq \mathbb{R}$, is the distance between their completed graphs $F(f)$ and $F(g)$ considered as closed subsets of $\Omega \times \mathbb{R}$. More precisely,

$$\rho(f,g) = \max\{ \sup_{A \in F(f)} \inf_{B \in F(g)} ||A - B||, \sup_{B \in F(g)} \inf_{A \in F(f)} ||A - B||\},$$

wherein $||.||$ is any norm in $\mathbb{R}^2$, e. g. the maximum norm $||(t,x)|| = \max\{|t|, |x|\}$; hence the distance between the points $A = (t_A, x_A)$, $B = (t_B, x_B)$ in $\mathbb{R}^2$ is $||A - B|| = \max(|t_A - t_B|, |x_A - x_B|)$.

Definition 3. The shifted Heaviside function is defined by

$$h_{t_0}(t) = \begin{cases} 0, & \text{if} \quad t < t_0, \\ [0,1], & \text{if} \quad t = t_0 \\ 1, & \text{if} \quad t > t_0 \end{cases} \tag{2}$$

3. MAIN RESULTS

3.1. A NOTE ON THE LEE–CHANG–PHAM–SONG SOFTWARE RELIABILITY MODEL

Without loosing of generality, for $N = 1$ and $t^* = 0$ we consider the following family:

$$M^*(t) = \left(1 - \frac{\beta}{\beta + at^b}\right)^\alpha, \tag{3}$$

with

$$t_0 = \left(\frac{\beta}{a} \frac{\left(\frac{1}{2}\right)^{\frac{1}{\alpha}}}{1 - \left(\frac{1}{2}\right)^{\frac{1}{\alpha}}}\right)^{\frac{1}{b}}; \quad M^*(t_0) = \frac{1}{2}. \tag{4}$$

The one–sided Hausdorff distance d between the Heaviside step function $h_{t_0}(t)$ and the sigmoid ((3)–(4)) satisfies the relation

$$M^*(t_0 + d) = 1 - d. \tag{5}$$

The following theorem gives upper and lower bounds for d.

Theorem. *Let*

$$p = -\frac{1}{2}$$

$$q = 1 + \frac{ab\alpha}{\beta} \left(\frac{\beta}{a}\right)^{\frac{b-1}{b}} \left(\frac{1}{2}\right)^{\frac{\alpha-1}{\alpha}} \left(1 - \left(\frac{1}{2}\right)^{\frac{1}{\alpha}}\right)^2 \left(\frac{\left(\frac{1}{2}\right)^{\frac{1}{\alpha}}}{1 - \left(\frac{1}{2}\right)^{\frac{1}{\alpha}}}\right)^{\frac{b-1}{b}}.$$

For the one–sided Hausdorff distance d between h_{t_0} and the sigmoid ((3)–(4)) the following inequalities hold for:

$$2.1q > e^{1.05}$$

$$d_l = \frac{1}{2.1q} < d < \frac{\ln(2.1q)}{2.1q} = d_r. \tag{6}$$

Proof. Let us examine the functions:

$$F(d) = M^*(t_0 + d) - 1 + d. \tag{7}$$

$$G(d) = p + qd. \tag{8}$$

From Taylor expansion we obtain $G(d) - F(d) = O(d^2)$.

Hence $G(d)$ approximates $F(d)$ with $d \to 0$ as $O(d^2)$ (see Figure 1).

In addition $G'(d) > 0$.

Further, for $2.1q > e^{1.05}$ we have $G(d_l) < 0$ and $G(d_r) > 0$.
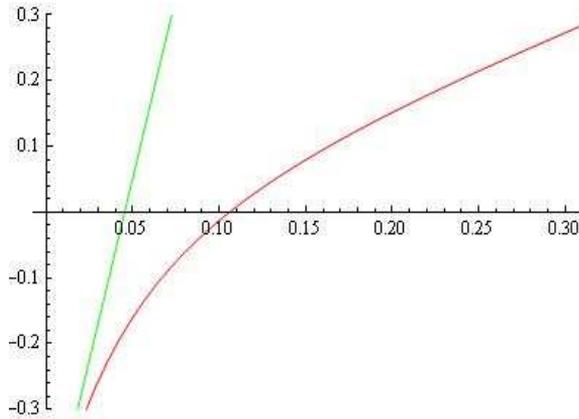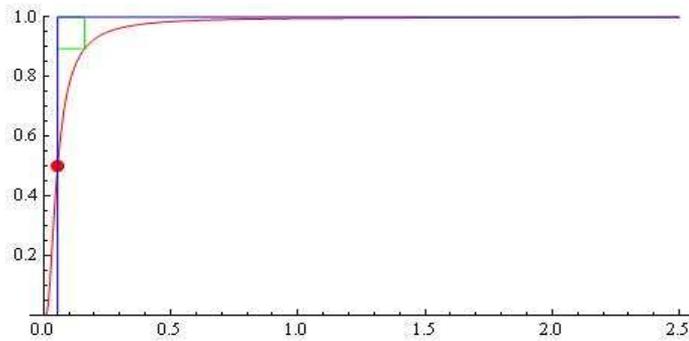
Figure 1: The functions $F(d)$ and $G(d)$.

Figure 2: The model ((3)–(4)) for $\beta = 0.01$, $\alpha = 2.95$, $a = 6.9$, $b = 1.8$ $t_0 = 0.0553865$; H–distance $d = 0.105906$, $d_l = 0.0431416$, $d_r = 0.135606$.

This completes the proof of the theorem.

The model ((3)–(4)) for $\beta = 0.01$, $\alpha = 2.95$, $a = 6.9$, $b = 1.8$ $t_0 = 0.0553865$ is visualized on Figure 2.

From nonlinear equation (5) and inequalities (6) we find $d = 0.105906$, $d_l = 0.0431416$ and $d_r = 0.135606$.

The model ((3)–(4)) for $\beta = 0.005$, $\alpha = 35$, $a = 7$, $b = 1.5$ $t_0 = 0.0196195$ is visualized on Figure 3.

From (5) and (6) we have $d = 0.0726818$, $d_l = 0.0193112$ and $d_r = 0.0762226$.
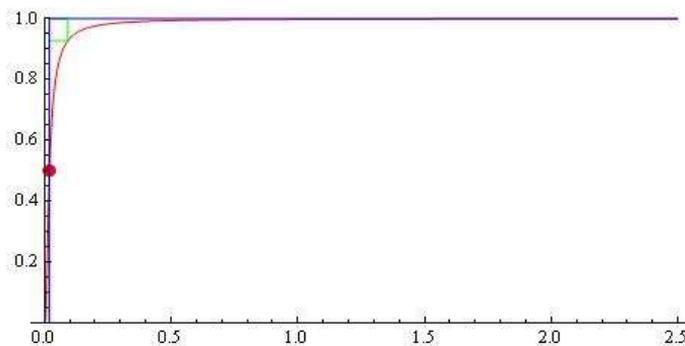
Figure 3: The model ((3)–(4)) for $\beta = 0.005$, $\alpha = 35$, $a = 7$, $b = 1.5$ $t_0 = 0.0196195$; H–distance $d = 0.0726818$, $d_l = 0.0193112$, $d_r = 0.0762226$.

3.2. NUMERICAL EXAMPLE

We examine the following data. (The small on–line data entry software package test data, available since 1980 in Japan [3], is shown in Table 1. For more details, see [4]).

For $t^* = 0.05$ the fitted model (1)

$$m(t) = N \left(1 - \frac{\beta}{\beta + a(t - 0.05)^b} \right)^\alpha$$

based on the data of Table 1 for the estimated parameters:

$$N = 71; \;\; \beta = 247.4; \;\; \alpha = 0.413126; \;\; a = 0.00413126; \;\; b = 3.42921$$

is plotted on Figure 4.

Remarks. Specifically, we will note that the reliability of the software model functions is checked by an additional six criteria, the consideration of which go beyond this article.

For example, for the predictive power (PP) criterion [4]

$$PP = \sum_{i=1}^{n} \left(\frac{m(t_i) - y_i}{y_i} \right)^2$$

measures the distance of model actual data from the estimates against the actual data, we find $PP = 0.633296$.

Based on the methodology proposed in the present note, for given $t^* > 0$, the reader may formulate the corresponding approximation problems for the model $m(t)$ (1) on his/her own.

In conclusion, we will note that the determination of compulsory in area of the Software Reliability Theory components, such as confidence intervals and confidence

$Testing\,time\,(day)$	$Failures$	$Cumulative\,failures$
1	2	2
2	1	3
3	1	4
4	1	5
5	2	7
6	2	9
7	2	11
8	1	12
9	7	19
10	2	21
11	1	22
12	2	24
13	2	26
14	4	30
15	1	31
16	6	37
17	1	38
18	3	41
19	1	42
20	3	45
21	1	46

Table 1: On–line IBM entry software package [3]

bounds, should also be accompanied by a serious analysis of the value of the best Hausdorff approximation - the subject of study in the present paper.

We propose a software modules (intellectual properties) within programming environment CAS Mathematica for the analysis.

An example for the usage of dynamical and graphical representation is plotted on Figure 5.

We hope that the results will be useful for specialists in this scientific area.

The results have independent significance in the study of issues related to lifetime analysis, population dynamics and impulse technics [40]– [50].
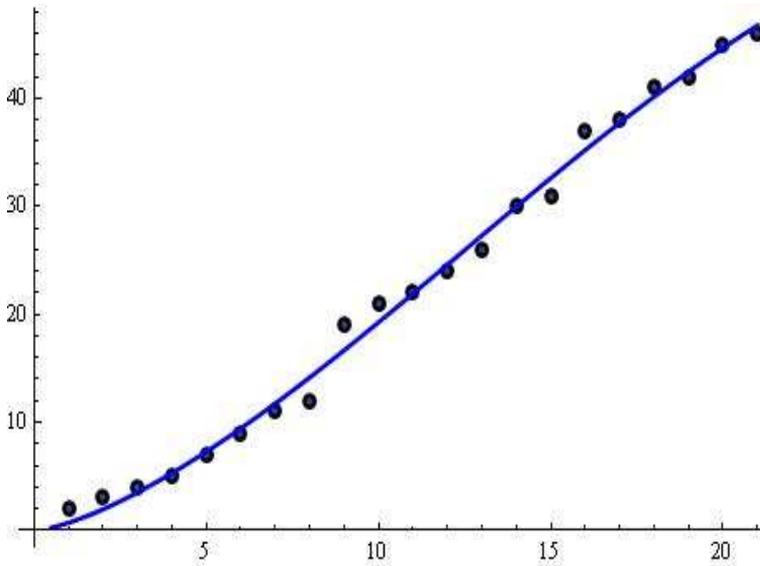
Figure 4: The model $m(t)$

4. APPENDIX

Lee–Chang–Pham–Song's idea of including the characteristic t^* in the study of models in debugging theory can be successfully expanded.

For instance, the Goel (1980) software reliability model considering the syntax error in uncertainty environments is given as follows

$$M_1(t) = N \left(1 - e^{-b(t-t^*)^c}\right) \tag{9}$$

where t^* is the time when debugging starts after modifying the code causing syntax errors

$$0 \xrightarrow{Syntax\,error} t^* \xrightarrow{Testing\,time} t$$

For $t^* = 0.5$ the fitted model (9)

$$M_1(t) = N \left(1 - e^{-b(t-0.5)^c}\right)$$

based on the data (see, Figure 6) for the estimated parameters:

$$N = 136; \ \ b = 0.234025; \ \ c = 0.789945$$

is plotted on Figure 7.

For the predictive power criterion we have $PP = 0.19843$.

Behavior of the software reliability factor R [7] is plotted on Figure 8.

```
Manipulate[Dynamic@Show[Plot[f[t], {t, 1, 21}, LabelStyle → Directive[Green, Bold],
    PlotLabel → N0 * (1 - β / (β + a * (t - tSyntax)^b))^α], ListPlot[{{1, 2}, {2, 3}, {3, 4}, {4, 5},
    {5, 7}, {6, 9}, {7, 11}, {8, 12}, {9, 19}, {10, 21}, {11, 22}, {12, 24}, {13, 26}, {14, 30},
    {15, 31}, {16, 37}, {17, 38}, {18, 41}, {19, 42}, {20, 45}, {21, 46}},
    PlotStyle → Red, PlotMarkers → {Automatic, Small}], PlotRange → {Automatic, {0, 46}},
    AxesOrigin → {0, 0}], {{N0, 0}, 0.001, 100, Appearance → "Open"}, {{tSyntax, 0}, 0.001,
    100, Appearance → "Open"}, {{β, 0}, 0, 300, Appearance → "Open"}, {{α, 0}, 0.001, 10,
    Appearance → "Open"}, {{a, 0}, 0.001, 10, Appearance → "Open"}, {{b, 0}, 0.001, 10,
    Appearance → "Open"}, Initialization :→ (f[t_] := N0 * (1 - β / (β + a * (t - tSyntax)^b))^α)]
```
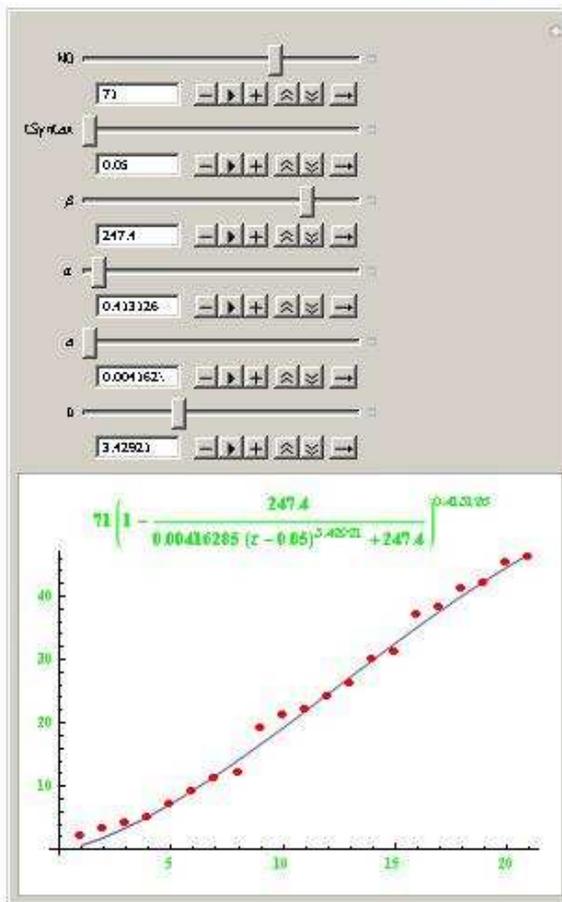

$$PP = \sum_{i=1}^{n} \left(\frac{m(t_i) - Y_i}{Y_i} \right)^2 = 0.633296$$

Figure 5: An example for the usage of dynamical and graphical representation for the model $m(t)$

Hour	Number of failures	Cum. Failures
1	27	27
2	16	43
3	11	54
4	10	64
5	11	75
6	7	83
7	2	84
8	5	89
9	3	92
10	1	93
11	4	97
12	7	104
13	2	106
14	5	111
15	5	116
16	6	122
17	0	122
18	5	127
19	1	128
20	1	129
21	2	131
22	1	132
23	2	134
24	1	135
25	1	136

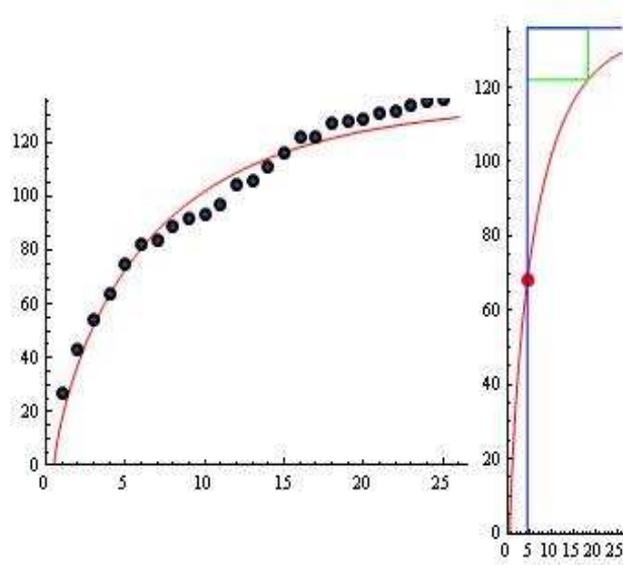Figure 6: Data Set: Real–Time Command and Control Data (see, for example [4])

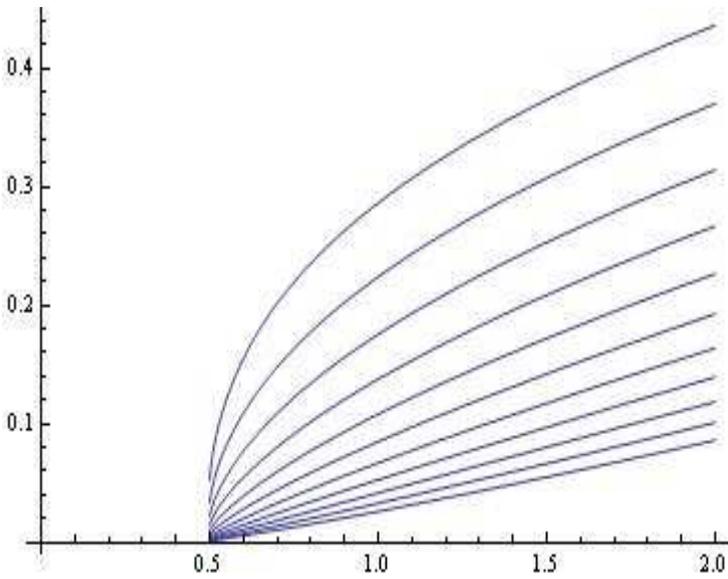Figure 7: a) The model $M_1(t)$; b) The "saturation" in Hausdorff since

Figure 8: The software reliability factor R

5. ACKNOWLEDGMENTS

REFERENCES

[1] D. H. Lee, I. H. Chang, H. Pham, K. Y. Song, A software reliability model considering the syntax error in uncertainty environments, optimal release time, and sensitivity analysis, *Appl. Sci.*, **8**, (2018), 1483.

[2] F. Hausdorff, *Set Theory* (2 ed.) (Chelsea Publ., New York, (1962 [1957]) Republished by AMS-Chelsea, (2005), ISBN: 978–0–821–83835–8.

[3] M. Ohba, Software reliability analysis models, *IBM J. Research and Development*, **21**, No. 4 (1984).

[4] H. Pham, *System Software Reliability*, In: Springer Series in Reliability Engineering, Springer–Verlag London Limited (2006).

[5] S. Yamada, *Software Reliability Modeling: Fundamentals and Applications*, Springer (2014).

[6] S. Yamada, Y. Tamura, *OSS Reliability Measurement and Assessment*, In: Springer Series in Reliability Engineering (H. Pham, Ed.), Springer International Publishing Switzerland (2016).

[7] N. Pavlov, A. Iliev, A. Rahnev, N. Kyurkchiev, *Some software reliability models: Approximation and modeling aspects*, LAP LAMBERT Academic Publishing (2018), ISBN: 978-613-9-82805-0.

[8] N. Pavlov, A. Iliev, A. Rahnev, N. Kyurkchiev, *Nontrivial Models in Debugging Theory (Part 2)*, LAP LAMBERT Academic Publishing (2018), ISBN: 978-613-9-87794-2.

[9] K. Ohishi, H. Okamura, T. Dohi, Gompertz software reliability model: Estimation algorithm and empirical validation, *J. of Systems and Software*, **82**, No. 3 (2009), 535–543.

[10] D. Satoh, A discrete Gompertz equation and a software reliability growth model, *IEICE Trans. Inform. Syst.*, **E83-D**, No. 7 (2000), 1508–1513.

[11] D. Satoh, S. Yamada, Discrete equations and software reliability growth models, In: *Proc. 12th Int. Symp. on Software Reliab. and Eng.*, (2001), 176–184.

[12] S. Yamada, A stochastic software reliability growth model with Gompertz curve, *Trans. IPSJ*, **33**, (1992), 964–969. (in Japanese)

[13] P. Oguntunde, A. Adejumo, E. Owoloko, On the flexibility of the transmuted inverse exponential distribution, *Proc. of the World Congress on Engineering*, July 5–7, 2017, London, U.K., **1**, (2017).

[14] W. Shaw, I. Buckley, The alchemy of probability distributions: Beyond Gram–Charlier expansions and a skew–kurtotic–normal distribution from a rank transmutation map, *Research report*, (2009).

[15] M. Khan, Transmuted generalized inverted exponential distribution with application to reliability data, *Thailand Statistician*, **16**, No. 1 (2018), 14–25.

[16] A. Abouammd, A. Alshingiti, Reliability estimation of generalized inverted exponential distribution, *J. Stat. Comput. Simul.*, **79**, No. 11 (2009), 1301–1315.

[17] I. Ellatal, Transmuted generalized inverted exponential distribution, *Econom. Qual. Control*, **28**, No. 2 (2014), 125–133.

[18] E. P. Virene, Reliability growth and its upper limit, in: *Proc. 1968, Annual Symp. on Realib.*, (1968), 265–270.

[19] S. Rafi, S. Akthar, Software Reliability Growth Model with Gompertz TEF and Optimal Release Time Determination by Improving the Test Efficiency, *Int. J. of Comput. Applications*, **7**, No. 11 (2010), 34–43.

[20] F. Serdio, E. Lughofer, K. Pichler, T. Buchegger, H. Efendic, Residua–based fault detection using soft computing techniques for condition monitoring at rolling mills, *Information Sciences*, **259**, (2014), 304–320.

[21] S. Yamada, M. Ohba, S. Osaki, S–shaped reliability growth modeling for software error detection, *IEEE Trans, Reliab.*, **R–32**, (1983), 475–478.

[22] S. Yamada, S. Osaki, Software reliability growth modeling: Models and Applications, *IEEE Transaction on Software Engineering*, **SE–11**, (1985), 1431–1437.

[23] N. Pavlov, G. Spasov, A. Rahnev, N. Kyurkchiev, A new class of Gompertz–type software reliability models, *International Electronic Journal of Pure and Applied Mathematics*, **12**, No. 1 (2018), 43–57.

[24] N. Pavlov, G. Spasov, A. Rahnev, N. Kyurkchiev, Some deterministic reliability growth curves for software error detection: Approximation and modeling aspects, *International Journal of Pure and Applied Mathematics*, **118**, No. 3 (2018), 599–611.

[25] N. Pavlov, A. Golev, A. Rahnev, N. Kyurkchiev, A note on the Yamada–exponential software reliability model, *International Journal of Pure and Applied Mathematics*, **118**, No. 4 (2018), 871–882.

[26] N. Pavlov, A. Iliev, A. Rahnev, N. Kyurkchiev, A Note on The "Mean Value" Software Reliability Model, *International Journal of Pure and Applied Mathematics*, **118**, No. 4 (2018), 949–956.

[27] N. Pavlov, A. Golev, A. Rahnev, N. Kyurkchiev, A note on the generalized inverted exponential software reliability model, *International Journal of Advanced Research in Computer and Communication Engineering*, **7**, No. 3 (2018), 484–487.

[28] A. L. Goel, Software reliability models: Assumptions, limitations and applicability, *IEEE Trans. Software Eng.*, **SE–11**, (1985), 1411–1423.

[29] J. D. Musa, *Software Reliability Data, DACS, RADC*, New York (1980).

[30] N. Pavlov, A. Iliev, A. Rahnev, N. Kyurkchiev, Transmuted inverse exponential software reliability model, *Int. J. of Latest Research in Engineering and Technology*, **4**, No. 5 (2018), 1–6.

[31] N. Pavlov, A. Iliev, A. Rahnev, N. Kyurkchiev, On the extended Chen's and Pham's software software reliability models. Some applications, *Int. J. of Pure and Appl. Math.*, **118**, No. 4 (2018), 1053–1067.

[32] N. Pavlov, A. Iliev, A. Rahnev, N. Kyurkchiev, Some deterministic growth curves with applications to software reliability analysis, *Int. J. of Pure and Appl. Math.*, **119**, No. 2 (2018), 357–368.

[33] N. Pavlov, A. Iliev, A. Rahnev, N. Kyurkchiev, Investigations of the K-stage Erlangian software reliability growth model, *Int. J. of Pure and Appl. Math.*, **119**, No. 3 (2018), 441–449.

[34] N. Pavlov, A. Iliev, A. Rahnev, N. Kyurkchiev, A Note on Ohbas Inflexion S–shaped Software Reliability Growth Model, *Collection of scientific works from conference "Mathematics. Informatics. Information Technologies. Application in Education"*, Pamporovo, Bulgaria, October 10-12, (2018). (to appear)

[35] N. Pavlov, A. Iliev, A. Rahnev, N. Kyurkchiev, Analysis of the Chen's and Pham's Software Reliability Models, *Cybernetics and Information Technologies*, **18**, No. 3 (2018), 37–47.

[36] N. Pavlov, A. Iliev, A. Rahnev, N. Kyurkchiev, On Some Nonstandard Software Reliability Models, *Dynamic Systems and Applications*, **27**, No. 4 (2018),757–771.

[37] N. Pavlov, A. Iliev, A. Rahnev, N. Kyurkchiev, Application of a new class cumulative lifetime distribution to software reliability analysis, *Comm. in Appl. Analysis*, **22**, No. 4 (2018), 555–565.

[38] V. Kyurkchiev, A. Malinova, O. Rahneva, P. Kyurkchiev, Some Notes on the Extended Burr XII Software Reliability Model, *Int. J. of Pure and Appl. Math.*, **120**, No. 1 (2018), 127–136.

[39] V. Kyurkchiev, H. Kiskinov, O. Rahneva, G. Spasov, A Note on the Exponentiated Exponential Poisson Software Reliability Model, *Neural, Parallel, and Scientific Computations*, **26**, No. 3 (2018), 257–267.

[40] N. Kyurkchiev, S. Markov, *Sigmoid functions: Some Approximation and Modelling Aspects*, LAP LAMBERT Academic Publishing, Saarbrucken (2015), ISBN: 978-3-659-76045-7.

[41] N. Kyurkchiev, A. Iliev, S. Markov, *Some techniques for recurrence generating of activation functions*, LAP LAMBERT Academic Publishing (2017), ISBN: 978-3-330-33143-3.

[42] N. Pavlov, A. Golev, A. Iliev, A. Rahnev, N. Kyurkchiev, On the Kumaraswamy-Dagum log-logistic sigmoid function with applications to population dynamics, *Biomath Communications*, **5**, No. 1 (2018).

[43] R. Anguelov, N. Kyurkchiev, S. Markov, Some properties of the Blumberg's hyper-log-logistic curve, *BIOMATH*, **7**, No. 1 (2018).

[44] S. Markov, N. Kyurkchiev, A. Iliev, A. Rahnev, On the approximation of the generalized cut functions of degree $p + 1$ by smooth hyper-log-logistic function, *Dynamic Systems and Applications*, **27**, No. 4 (2018), 715–728.

[45] S. Markov, A. Iliev, A. Rahnev, N. Kyurkchiev, A note on the Log-logistic and transmuted Log-logistic models. Some applications, *Dynamic Systems and Applications*, **27**, No. 3 (2018), 593–607.

[46] A. Malinova, V. Kyurkchiev, A. Iliev, N. Kyurkchiev, Some new approaches to Kumaraswamy-Lindley cumulative distribution function, *Int. J. of Innovative Sci. and Techn.*, **5** No. 3 (2018), 233–236.

[47] A. Malinova, V. Kyurkchiev, A. Iliev, N. Kyurkchiev, A note on the transmuted Kumaraswamy Quasi Lindley cumulative distribution function, *Int. J. for Sci., Res. and Developments*, **6** No. 2 (2018), 561–564.

[48] N. Kyurkchiev, A new class activation functions with application in the theory of impulse technics, *Journal of Mathematical Sciences and Modelling*, **1**, No. 1 (2018), 15–20.

[49] S. Markov, N. Kyurkchiev, A. Iliev, A. Rahnev, On the approximation of the cut functions by hyper-log-logistic function, *Neural, Parallel and Scientific Computations*, **26**, No. 2 (2018), 169–182.

[50] N. Kyurkchiev, A. Iliev, *Extension of Gompertz-type Equation in Modern Science: 240 Anniversary of the birth of B. Gompertz*, LAP LAMBERT Academic Publishing (2018), ISBN: 978-613-9-90569-0.