EXISTENCE OF POSITIVE SOLUTIONS FOR INTEGRAL EQUATIONS WITH VANISHING KERNELS

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Dedicated to Professor Jeffrey R. Webb on the occasion of his retirement

ABSTRACT. We study the existence of positive solutions of integral equations in $C[0,1]$ where the kernel is supposed to be non-negative on $[0,1] \times [0,1]$ but may vanish at the interior points which prevent us of some standard cones. We prove existence of one or two positive solutions under some sharp conditions, and we do not need any convexity assumptions on the nonlinearities. The proof of the main results is based upon bifurcation techniques.

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1. INTRODUCTION

In recent years, there has been an extensive study of the existence of positive solutions of boundary value problems (BVPs) for differential equations involving both local and nonlocal boundary conditions (BCs). A standard method used to show the existence of positive solutions is to find fixed points of the integral equation

$$u(t) = \int_0^1 G(t,s)g(s)f(s,u(s))ds =: Tu(t) \qquad (1.1)$$

in the space $C[0,1]$ of continuous functions, where G is the Green's function of the differential equation with the given BCs. To show existence of a positive solution (when $g, f \geq 0$), it is required that $G(t,s) \geq 0$ and one seeks fixed points of T in the cone

$$P := \{u \in C[0,1] : u(t) \geq 0\}.$$

For Sturm-Liouville BCs, see for example [1, 2]; for periodic BCs, see for example [3, 4]; for nonlocal BCs, see for example [5, 6] and references therein. It is worth remarking that the Green's functions in [1–6] satisfy the following Lan-Webb condition:

(LW) *There exist a subinterval $[a, b] \subset [0, 1]$, a measurable function ϕ, and a constant $c = c(a, b) \in (0, 1]$ such that*

$$G(t, s) \leq \Phi(s) \quad for \ t \in [0, 1] \ and \ s \in [0, 1],$$
$$G(t, s) \geq c\Phi(s) \quad for \ t \in [a, b] \ and \ s \in [0, 1]. \tag{1.2}$$

Recently, Graef, Kong and Wang [7] studied the nonlinear periodic BVP

$$u''(t) + a(t)u(t) = g(t)f(u(t)), \quad t \in (0, 1),$$
$$u(0) = u(1), \quad u'(0) = u'(1), \tag{1.3}$$

where the Green's function is assumed to be non-negative on the square $[0, 1] \times [0, 1]$, but can be zero at some interior points of the square. For example, in the case $a(t) \equiv \pi^2$, the Green's function can be explicitly given by

$$G(t, s) = \begin{cases} \dfrac{\cos(\pi(1/2 - t + s))}{2\pi}, & s \leq t, \\ \dfrac{\cos(\pi(1/2 - s + t))}{2\pi}, & s > t. \end{cases}$$

Obviously $G(s, s) = 0$ for $s \in [0, 1]$. (In fact the authors of [7] worked on the interval $[0, 2\pi]$ but, since nothing essential is changed, we consider $[0, 1]$.) In such a case the Green's function does not satisfy the (LW) condition. In fact, in [7], a new cone of the form

$$K = \left\{ u \in P : \int_0^1 u(t)dt \geq c||u|| \right\}, \tag{1.4}$$

(where $c > 0$ is a constant independent of u, and $||u|| := \max_{t \in [0,1]} |u(t)|$), was used and the authors proved the existence of one positive solution under a sub-linear condition on f and also under a super-linear condition on f provided that f was convex. A key assumption made by the authors of [7] is that

$$\min_{0 \leq s \leq 1} \int_0^1 G(t, s)dt > 0. \tag{1.5}$$

It was also assumed that the functions f, g are continuous and non-negative and that

$$\min_{t \in [0,1]} g(t) > 0. \tag{1.6}$$

Motivated by [7], Webb [8] used fixed point index theory to study the integral equation

$$u(t) = \int_0^1 G(t, s)g(s)f(u(s))ds \tag{1.7}$$

under the assumptions

(H1) The kernel G is non-negative and is continuous on $[0, 1] \times [0, 1]$, with

$$G(t, s) \leq G_0, \quad (t, s) \in [0, 1] \times [0, 1];$$

(H2) The function g is non-negative almost everywhere, $g \in L^1[0,1]$, and satisfies

$$g_1 := \int_0^1 g(t)dt > 0;$$

(H3) There is a constant $\alpha > 0$ such that

$$\int_0^1 G(t,s)g(t)dt \geq \alpha, \qquad s \in [0,1];$$

(H4) The nonlinearity $f : [0,\infty) \to [0,\infty)$ is continuous.

Denote

$$\left(\bar{f}\right)_0 = \limsup_{u \to 0^+} \frac{f(u)}{u}, \qquad \left(\underline{f}\right)_0 = \liminf_{u \to 0^+} \frac{f(u)}{u},$$

$$\left(\bar{f}\right)_\infty = \limsup_{u \to \infty} \frac{f(u)}{u}, \qquad \left(\underline{f}\right)_\infty = \liminf_{u \to \infty} \frac{f(u)}{u}.$$

Let

$$\tilde{K} := \{u \in P : \int_0^1 u(t)g(t)dt \geq \frac{\alpha}{G_0}||u||\}. \tag{1.8}$$

Define an operator $L : C[0,1] \to C[0,1]$ by

$$Lu(t) := \int_0^1 G(t,s)g(s)u(s)ds.$$

Then by [8, Lemma 2.1], we have

$$L(P) \subset \tilde{K}. \tag{1.9}$$

Assume that

(H5) $r(L) > 0$.

Remark 1.1 By the Krein-Rutman theorem, L has an eigenfunction $\varphi \in P \setminus \{0\}$ corresponding to the principal eigenvalue $r(L)$.

We suppose that $||\varphi|| = 1$.

Since $L(P) \subset \tilde{K}$, $\varphi \in \tilde{K}$. We set

$$\lambda_1 := 1/r(L), \tag{1.10}$$

and call it the principal characteristic value of L.

Applying fixed point index theory and an open set in $\tilde{K}$, Webb [8] proved the following

Theorem A ([8, Theorem 2.2]) Assume that (H1)–(H5) hold. Then the integral equation (1.7) has at least one positive solution, that is, a nonzero solution in the cone $\tilde{K}$ if either of the following conditions (S1), (S2) hold.

(S1) $\left(\underline{f}\right)_0 > \lambda_1$ and $\left(\bar{f}\right)_\infty < \lambda_1$.

(S2) $\left(\bar{f}\right)_0 < \lambda_1$ and there exists $R > 0$ such that $\frac{f(R)}{R} > \frac{1}{\alpha}$ and f is convex on $\left[0, \frac{G_0 g_1 R}{\alpha}\right]$.

The equation (1.7) has at least two positive solutions if

(D) $(\bar{f})_0 < \lambda_1$, there exists $R > 0$ such that $\frac{f(R)}{R} > \frac{1}{\alpha}$ and f is convex on $\left[0, \frac{G_0 g_1 R}{\alpha}\right]$, and $(\bar{f})_\infty < \lambda_1$.

Notice that a key assumption made by the authors of [7,8] is that f is convex on $[0, \infty)$ or on some subinterval of $[0, \infty)$. It is the purpose of our paper to use bifurcation techniques to prove the existence and multiplicity of positive solutions without any convexity restriction of f. More precisely, we will prove the following

Theorem 1.1 Assume that (H1)–(H3) and (H5) hold. Moreover, assume that

(A1) $\int_0^1 G(t,s)\varphi(t)dt \geq \beta$ for some constant $\beta > 0$;

(A2) $f : \mathbb{R} \to \mathbb{R}$ is continuous and

$$sf(s) > 0, \qquad s \in (0, \infty),$$

and there exist $f_0, f_\infty \in (0, \infty)$ such that

$$f_0 = \lim_{|u| \to 0^+} \frac{f(u)}{u}, \quad f_\infty = \lim_{|u| \to \infty} \frac{f(u)}{u}.$$

Then (1.7) has one positive solution $u^+ \in \tilde{K}$ and one negative solution u^- with $-u^- \in \tilde{K}$ if either of the following conditions (A3), (A4) hold.

(A3) $f_0 > \lambda_1$ and $f_\infty < \lambda_1$;

(A4) $f_0 < \lambda_1$ and $f_\infty > \lambda_1$.

Theorem 1.2 Assume that (H1)–(H3),(H5) and (A1)-(A2) hold. Then (1.7) has at least two positive solutions if the following condition (A5) holds.

(A5) $f_0 > \lambda_1$, $f_\infty > \lambda_1$, and there exists $R > 0$ and $\gamma \in \left(0, \frac{\alpha}{g_1^2 G_0^2}\right)$, such that

$$\frac{\hat{f}(R)}{R} < \gamma$$

with

$$\hat{f}(R) := \max\{f(s) : s \in [0, R]\}.$$

Remark 1.2. In Theorem 1.1 and 1.2, we do not need any convexity restriction on f.

2. PROOF OF THE MAIN RESULTS

Let us denote by $|| \cdot ||$ the sup norm in $C[0,1]$. Recall that the operator $L : C[0,1] \to C[0,1]$ defined by

$$Lu(t) := \int_0^1 G(t,s)g(s)u(s)ds.$$

It is easy to verify that L is a compact operator in $C[0,1]$.

Let ζ, $\xi \in C(\mathbb{R}, \mathbb{R})$ be such that

$$f(u) = f_0 u + \zeta(u), \qquad f(u) = f_\infty u + \xi(u).$$

Clearly,

$$\lim_{|u| \to 0} \frac{\zeta(u)}{u} = 0, \qquad \lim_{|u| \to \infty} \frac{\xi(u)}{u} = 0.$$

Let

$$\xi^*(u) = \max_{0 \le |s| \le u} |\xi(s)|,$$

then $\bar{\xi}$ is nondecreasing and

$$\lim_{u \to \infty} \frac{\xi^*(u)}{u} = 0. \tag{2.1}$$

Proof of Theorem 1.1. Let us consider

$$u(t) := \mu \int_0^1 G(t,s)g(s)f(u(s))ds, \qquad \mu \in [0, \infty) \tag{2.2}$$

as a bifurcation problem from the trivial solution $u \equiv 0$.

From (2.2), we obtain

$$u(t) = \mu \int_0^1 G(t,s)g(s)[f_0 u(s) + \zeta(u(s))]ds$$

$$=: (\mu L[f_0 u(\cdot)] + \mu L[\zeta(u(\cdot))])(t). \tag{2.3}$$

Further, we note that $\|L[\zeta(u(\cdot))]\| = o(\|u\|)$ for u near 0 in $C[0,1]$, since

$$\|L[\zeta(u(\cdot))]\| = \max_{t \in [0,1]} \left| \int_0^1 G(t,s)g(s)\zeta(u(s))ds \right|$$

$$\le G_0 \cdot g_1 \cdot \|\zeta(u(\cdot))\|.$$

In what follows, we use the terminology of Rabinowitz [9]. Let S_1^+ denote the set of functions in $C[0,1]$ which is positive on $[0,1]$, and set $S_1^- = -S_1^+$, and $S_1 = S_1^- \cup S_1^+$. They are disjoint and open in $C[0,1]$. Finally, let $\Phi_1^\pm = \mathbb{R} \times S_1^\pm$. The result of Rabinowitz [9] for problem (2.2) can be stated as follows: For $\nu = \{+, -\}$, there exists a continuum $\mathcal{C}_1^\nu$ of solutions of (2.2) joining $(\frac{\lambda_1}{f_0}, 0)$ to infinity. Moreover, $\mathcal{C}_1^\nu \setminus \{(\frac{\lambda_1}{f_0}, 0)\} \subset \Phi_1^\nu$.

Proof of Theorem 1.1. It is clear that any solution of (2.2) of the form $(1, u)$ yields a solution u of integral equation (1.7). We will show that $\mathcal{C}_1^\nu$ crosses the hyperplane $\{1\} \times C[0,1]$ in $\mathbb{R} \times C[0,1]$. To do this, it is enough to show that $\mathcal{C}_1^\nu$ joins $(\frac{\lambda_1}{f_0}, 0)$ to $(\frac{\lambda_1}{f_\infty}, \infty)$. Let $(\mu_n, y_n) \in \mathcal{C}_1^\nu$ satisfy

$$\mu_n + \|y_n\| \to \infty.$$

We note that $\mu_n > 0$ for all $n \in \mathbb{N}$ since $(0,0)$ is the only solution of (2.2) for $\mu = 0$ and $\mathcal{C}_1^\nu \cap (\{0\} \times C[0,1]) = \emptyset$.

Case 1. $\frac{\lambda_1}{f_\infty} < 1 < \frac{\lambda_1}{f_0}$.

In this case, we show that the interval

$$(\frac{\lambda_1}{f_\infty}, \frac{\lambda_1}{f_0}) \subseteq \{\lambda \in \mathbb{R} \mid \exists(\lambda, u) \in \mathcal{C}_1^\nu\}.$$

We divide the proof into two steps.

Step 1. We show that if there exists a constant $M_1 > 0$ such that $\mu_n \subset (0, M_1]$, then $\mathcal{C}_1^\nu$ joins $(\frac{\lambda_1}{f_0}, 0)$ to $(\frac{\lambda_1}{f_\infty}, \infty)$.

In this case, it follows that $\|y_n\| \to \infty$. We divide the equation

$$y_n(t) := \mu_n \int_0^1 G(t, s) g(s) f(y_n(s)) ds. \tag{2.4}$$

by $\|y_n\|$ and set $\bar{y}_n = \frac{y_n}{\|y_n\|}$. Since $\bar{y}_n$ is bounded in $C[0, 1]$, it follows from (2.4) and the compactness of L (after taking a subsequence if necessary) that $\bar{y}_n \to \bar{y}$ for some $\bar{y} \in C[0, 1]$ with $\|\bar{y}\| = 1$. From ξ^* is nondecreasing and $\lim_{u \to \infty} \frac{\xi^*(u)}{u} = 0$, we know that

$$\lim_{n \to \infty} \frac{|\xi(y_n(t))|}{\|y_n\|} = 0,$$

since

$$\frac{|\xi(y_n(t))|}{\|y_n\|} \le \frac{\xi^*(|y_n(t)|)}{\|y_n\|} \le \frac{\xi^*(\|y_n\|)}{\|y_n\|}.$$

Thus,

$$\bar{y}(t) = \int_0^1 G(t, s) \bar{\mu} g(s) f_\infty \bar{y}(s) ds,$$

where $\bar{\mu} := \lim_{n \to \infty} \mu_n$, again choosing a subsequence and relabeling, if necessary. Thus

$$\bar{y} = \bar{\mu} \int_0^1 G(t, s) g(s) f_\infty \bar{y}(s) ds, \tag{2.5}$$

and accordingly

$$\bar{\mu} = \frac{\lambda_1}{f_\infty}, \qquad \bar{y} = \varphi. \tag{2.6}$$

Therefore $\mathcal{C}_1^\nu$ joins $(\frac{\lambda_1}{f_0}, 0)$ to $(\frac{\lambda_1}{f_\infty}, \infty)$.

Step 2. We show that there exists a constant $M_1 > 0$ such that $\mu_n \in (0, M_1]$ for all $n \in \mathbb{N}$.

Suppose there is no such M_1. Choosing a subsequence and relabeling, if necessary, it follows that

$$\lim_{n \to \infty} \mu_n = \infty.$$

$(\mu_n, y_n) \in \mathcal{C}_1^\nu$ implies that

$$y_n(t) := \mu_n \int_0^1 G(t, s) g(s) f(y_n(s)) ds. \tag{2.7}$$

By (A2), there exists a constant $\delta > 0$ such that

$$f(s) > \delta s, \qquad s \in [0, \infty). \tag{2.8}$$

Combining this with (2.7) and using (A1), it concludes that

$$
\begin{aligned}
||y_n|| &\geq \int_0^1 \varphi(t) y_n(t) dt \\
&= \mu_n \int_0^1 \Big[\int_0^1 G(t,s) g(s) f(y_n(s)) ds \Big] \varphi(t) dt \\
&\geq \mu_n \int_0^1 \Big[\int_0^1 G(t,s) g(s) \delta y_n(s) ds \Big] \varphi(t) dt \\
&= \delta \mu_n \int_0^1 \Big[\int_0^1 G(t,s) \varphi(t) dt \Big] g(s) y_n(s) ds \\
&\geq \delta \mu_n \beta \int_0^1 g(s) y_n(s) ds \\
&\geq \delta \mu_n \beta \cdot \frac{\alpha}{G_0} ||y_n||.
\end{aligned}
\tag{2.9}
$$

This is impossible as $n \to \infty$. Therefore,

$$
|\mu_n| \leq M_1
$$

for some constant $M_1 > 0$, independent of $n \in \mathbb{N}$.

Case 2. $\frac{\lambda_1}{f_0} < 1 < \frac{\lambda_1}{f_\infty}$.

In this case, if $(\mu_n, y_n) \in \mathcal{C}_1^\nu$ is such that

$$
\lim_{n \to \infty} (\mu_n + ||y_n||) = \infty,
$$

and

$$
\lim_{n \to \infty} \mu_n = \infty,
$$

then

$$
(\frac{\lambda_1}{f_0}, \frac{\lambda_1}{f_\infty}) \subset \{\lambda \in (0, \infty) : (\lambda, u) \in \mathcal{C}_1^\nu\},
$$

and subsequently,

$$
(\{1\} \times C[0,1]) \cap \mathcal{C}_1^\nu \neq \emptyset.
$$

Assume that $\{\mu_n\}$ is bounded, applying a similar argument to that used in Step 1 of Case 1, after taking a subsequence and relabeling, if necessary, it follows that

$$
(\mu_n, y_n) \to (\frac{\lambda_1}{f_\infty}, \infty), \quad n \to \infty.
$$

Again $\mathcal{C}_1^\nu$ joins $\left(\frac{\lambda_1}{f_0}, 0\right)$ to $\left(\frac{\lambda_1}{f_\infty}, \infty\right)$ and the result follows.

$\square$

Proof of Theorem 1.2. As in the proof of Theorem 1.1, there exists a continuum $\mathcal{C}_1^+$ of solutions of (2.2) joining $(\frac{\lambda_1}{f_0}, 0)$ to $(\frac{\lambda_1}{f_\infty}, \infty)$. Moreover, $\mathcal{C}_1^+ \setminus \{(\frac{\lambda_1}{f_0}, 0)\} \subset \Phi_1^+$.

It follows from (A5) that

$$
\frac{\lambda_1}{f_0} < 1, \qquad \frac{\lambda_1}{f_\infty} < 1.
$$

To complete the proof of the Theorem, it is enough to show that

$$\mathcal{C}_1^+ \cap \{(\mu, y) : 0 < \mu \leq 1, \ ||y|| = R\} = \emptyset. \tag{2.10}$$

Suppose on the contrary that there exists $(\mu, y) \in \mathcal{C}_1^+ \cap \{(\mu, y) : 0 < \mu \leq 1, \ ||y|| = R\}$, then $y \in \tilde{K}$ and

$$y(t) = \mu \int_0^1 G(t, s) g(s) f(y(s)) ds.$$

This implies that

$$\frac{\alpha}{G_0} ||y|| \leq \int_0^1 g(t) y(t) dt$$

$$= \int_0^1 \left[\mu \int_0^1 G(t, s) g(s) f(y(s)) ds \right] g(t) dt$$

$$\leq \int_0^1 \left[\int_0^1 G(t, s) g(s) f(y(s)) ds \right] g(t) dt$$

$$= \int_0^1 \left[\int_0^1 G(t, s) g(t) dt \right] g(s) f(y(s)) ds$$

$$\leq G_0 g_1 \int_0^1 g(s) f(y(s)) ds$$

$$\leq G_0 g_1 \int_0^1 g(s) \hat{f}(||y||) ds$$

$$\leq G_0 g_1 \int_0^1 g(s) \gamma ||y|| ds$$

This contradicts the assumption $\gamma < \frac{\alpha}{g_1^2 G_0^2}$. Therefore, (2.10) is valid.

$$\square$$

3. FURTHER RESULTS

In Theorem 1.1 and 1.2, we made the assumption:

$$(\bar{f})_0 = (\underline{f})_0 =: f_0, \quad (\bar{f})_\infty = (\underline{f})_\infty =: f_\infty.$$

In fact, this assumption is too strong. We may apply the techniques of *bifurcating from interval* (see [10]) and the same method to prove Theorem 1.1 and 1.2, with obvious changes, to establish the following

Theorem 3.1 Let (H1)–(H3), (H5) and (A1) hold. Moreover, assume that

$(A2_w)$ $f : \mathbb{R} \to \mathbb{R}$ is continuous,

$$s f(s) > 0, \qquad s \in (0, \infty),$$

and $(\bar{f})_0, (\underline{f})_0, (\bar{f})_\infty, (\underline{f})_\infty \in (0, \infty)$.

Then (1.7) has one positive solution $u^+ \in \tilde{K}$ and one negative solution u^- with $-u^- \in \tilde{K}$ if either of the following conditions (A6), (A7) hold.

(A6) $\left(\underline{f}\right)_0 > \lambda_1$ and $\left(\bar{f}\right)_\infty < \lambda_1$;

(A7) $\left(\bar{f}\right)_0 < \lambda_1$ and $\left(\underline{f}\right)_\infty > \lambda_1$.

Theorem 3.2 Assume that (H1)–(H3), (H5), (A1) and ($A2_w$) hold. Then (1.7) has at least two positive solutions if the following condition (A8) holds.

(A8) $\left(\underline{f}\right)_0 > \lambda_1$, $\left(\underline{f}\right)_\infty > \lambda_1$, and there exists $R > 0$ and $\gamma \in \left(0, \frac{\alpha}{g_1^2 G_0^2}\right)$, such that

$$\frac{\hat{f}(R)}{R} < \gamma.$$

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