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FIXED POINTS AND EXPONENTIAL STABILITY FOR UNCERTAIN STOCHASTIC NEURAL NETWORKS WITH MULTIPLE MIXED TIME-DELAYS

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ABSTRACT. In this paper we study the stability of a stochastic neural networks with parameter uncertainties and multiple time delays

$$\begin{aligned} dx &= [-(A+\triangle A(t))x(t)+(B+\triangle B(t))f(t,x(t),x(t-\tau_1(t)),\ldots,x(t-\tau_m(t))) \\ &\quad +\textstyle\sum_{p=1}^{k}(W_p+\triangle W_p(t))\int_{t-r_p(t)}^{t} g_p(x(s))ds]dt+\sum_{j=1}^{l} h_j(t,x(t),x(t-\sigma_j(t)))dw(t). \end{aligned}$$

Using fixed point theory and a linear matrix inequality(LMI), we obtain new criteria for exponential stability in mean square of the considered uncertain stochastic neural networks with multiple mixed time-delays.

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1. Introduction

Neural networks are important applications in various areas such as combinatorial optimization, signal processing, pattern recognition and solving nonlinear algebraic equations [6, 18, 19, 31]. Stability is one of the main properties of neural networks. It is a crucial feature in the design of neural networks and has received a lot of attention recently; see [1, 7, 11, 17, 22–24, 27, 30].

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Neural network can be stabilized or destabilized by certain stochastic inputs (see [5, 12, 13, 14, 26]). Stability for stochastic neural networks with parameter uncertainties and multiple time delays were discussed in [2, 15, 16, 21, 28, 32].

Let $(\Omega, \mathcal{F}, P)$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions, i.e. it is right continuous and $\mathcal{F}_0$ contains all P-null sets. Let $C^b_{\mathcal{F}_0}([-\tau, 0]; R^n)$ be the family of all bounded, $\mathcal{F}_0$-measurable functions. Let $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T$ and $B = [b_{ij}(t)]_{n\times n}$ with

$$|x(t)|_1 = \sum_{i=1}^{n} |x_i(t)|$$

and

$$\|B(t)\|_3 = \sum_{i,j=1}^{n} |b_{ij}(t)|.$$

We denote by $C([-\tau^*, 0]; R^n)$ the family of continuous functions $\varphi : [-\tau^*, 0] \to R^n$ with

$$\|\varphi\|_2 = \sup_{-\tau^*\leq\theta\leq 0} |\varphi(\theta)|_1,$$

where τ^* is a positive constant.

In this paper, using fixed point theory and a linear matrix inequality(LMI) [4, 8], we discuss the stability of a stochastic neural network with parameter uncertainties and multiple time-varying delays

$$dx = [-(A + \triangle A(t))x(t) + (B + \triangle B(t))f(t, x(t), x(t-\tau_1(t)), \ldots, x(t-\tau_m(t)))$$
$$+ \sum_{p=1}^{k}(W_p + \triangle W_p(t)) \int_{t-r_p(t)}^{t} g_p(x(s))ds]dt + \sum_{j=1}^{l} h_j(t, x(t), x(t-\sigma_j(t)))dw(t) \quad \text{(DW)}$$

with the initial condition

$$x(s) = \psi(s) \in C([-\tau^*, 0]; R^n), \quad -\tau^* \leq s \leq 0, \tag{1.1}$$

where $\tau^* \geq \max\{\tau_i(t), \sigma_j(t), r_p(t), i = 1, 2, \ldots, m, j = 1, 2, \ldots, l, p = 1, 2, \ldots, k\}$ is a positive constant, $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T$ is the state vector, $A = diag(a_1, a_2, \ldots, a_n) > 0$, B and W_p are the connection weight constant matrices with appropriate dimensions, $\triangle A(t)$, $\triangle B(t)$ and $\triangle W_p(t)$ represent the time-varying parameter uncertainties and bounded, $p = 1, 2 \ldots, k$. Here $w(t) = (w_1(t), w_2(t), \ldots, w_m (t))^T \in R^m$ is a m-dimensional Brownian motion defined on a complete probability space $(\Omega, \mathcal{F}, P)$ and

$$f(t, u_1, u_2, \ldots, u_n) \in C(R \times R^n \times R^n \times \cdots \times R^n)_{n+1}$$

is the neuron activation function and we assume $f(t, 0, 0, \dots, 0) = 0$. The stochastic disturbance term, $h_j(t, u_1, u_2) \in C(\mathbf{R} \times \mathbf{R}^n \times \mathbf{R}^n)$, can be viewed as stochastic perturbations on the neuron states and delayed neuron states.

Usually in literature the neuron activation function is assumed to be continuous, differentiable, monotonically increasing and bounded. However, in many real systems, such as electronic circuits, it may not be monotonically increasing or continuously differentiable. This paper was motivated by some ideas in [10, 20].

2. **Preliminaries**

For the sake of completeness, some definitions and lemmas will be stated here and they will be used in the proof of our main results.

Definition 2.1. The system (DW) with the initial condition is said to be exponentially stable in mean square for all admissible uncertainties if there exists a solution x of (DW) and there exists a pair of positive constants β and μ with

$$E|x(t)|_1^2 \le \mu E\|\psi\|_2^2 e^{-\beta t}, \quad t \ge 0. \tag{2.1}$$

Definition 2.2. The system (DW) with the initial condition is said to be globally exponentially stable in mean square for all admissible uncertainties if there exists a scalar $\varsigma > 0$, such that

$$\lim_{t\to\infty} \sup \frac{1}{t} \log(E\|x(t;\psi)\|_2^2) \le -\varsigma. \tag{2.2}$$

Let $C^{2,1}(R^+ \times R^n; R^+)$ denote the family of all nonnegative functions $V(t, x)$ on $R^+ \times R^n$ which are continuously twice differentiable in x and once differentiable in t. For each $V \in C^{2,1}([-\tau^*, \infty) \times R^+; R^+)$, define an operator LV, associated with the uncertain stochastic neural networks with multiple mixed time-delays (DW), from $(R^+ \times C[-\tau^*, \infty); R^n)$ to R by

$$\begin{aligned} LV(t,x) &= V_t(t,x) + V_x(t,x)[(B + \triangle B(t))f(t, x(t), x(t-\tau_1(t)), \dots, x(t-\tau_m(t))) \\ &+ \frac{1}{2}\text{trace}\left[\left(\sum_{i=1}^{l} h_i^T(t, x(t), x(t-\sigma_i(t)))\right) V_{xx}(t,x)(\sum_{i=1}^{l} h_i(t, x(t), x(t-\sigma_i(t))))\right] \\ &- (A + \triangle A(t))x(t) + \sum_{i=1}^{k}(W_i + \triangle W_i(t)) \int_{t-r_i(t)}^{t} g_i(x(s))ds], \end{aligned}$$

where

$$V_t(t,x) = \left(\frac{\partial V(t,x)}{\partial t}\right),$$

$$V_x(t,x) = \left(\frac{\partial V(t,x)}{\partial x_1}, \frac{\partial V(t,x)}{\partial x_2}, \dots, \frac{\partial V(t,x)}{\partial x_n}\right),$$

$$V_{xx}(t,x) = \left(\frac{\partial^2 V(t,x)}{\partial x_i \partial x_j}\right)_{n\times n}.$$

Lemma 2.1 ([29]). Let A, B, C and D be real matrices of appropriate dimension with D satisfying $D = D^T$. Then

$$ABC + C^T B^T A^T + D < 0, \quad \text{for all} \quad B^T B \le I$$

if and only if these exists a scale $\gamma > 0$ such that

$$\gamma^{-1} AA^T + \gamma C^T C + D < 0.$$

Lemma 2.2 ([25]). Let A, D, E, F and P be real matrices of appropriate dimension with $P > 0$ and F satisfying $F^T F \le I$. Then for any scalar $\varepsilon > 0$ satisfying $P^{-1} - \varepsilon^{-1} DD^T > 0$, we have

$$(A + DFE)^T P(A + DFE) \le A^T (P^{-1} - \varepsilon^{-1} DD^T)^{-1} A + \varepsilon E^T E.$$

Lemma 2.3 ([3]). The LMI

$$\begin{pmatrix} \Re_{11} & \Re_{12} \\ \Re_{12}^T & -\Re_{22} \end{pmatrix} < 0,$$

with $\Re_{11} = \Re_{11}^T$, $\Re_{22} = \Re_{22}^T$, is equivalent to

$$\Re_{22} > 0, \quad \Re_{11} + \Re_{12}\Re_{22}^{-1}\Re_{12}^T < 0.$$

Lemma 2.4 ([9]). For any positive definite matrix $M > 0$, scalar $\kappa > 0$, vector function $\varpi : [0, \kappa] \to R^n$ such that the integrations concerned are well defined, the following inequality holds:

$$\left(\int_0^\kappa \varpi(s)ds\right)^T M \left(\int_0^\kappa \varpi(s)ds\right) \le \kappa \left(\int_0^\kappa \varpi^T(s) M \varpi(s) ds\right).$$

3. Exponential stability (I)

In this section we prove that system (DW) is exponentially stable in mean square under the following conditions:

(H_1) $|g_p(x) - g_p(y)| \le |G_p(x - y)|$ and $g_p(0) = 0$, where $G_p \in R^{n\times n}$, $p = 1, 2, \dots, k$;

(H_2)

$$|f(t, x(t), x(t - \tau_1(t)), \dots, x(t - \tau_m(t))) - f(t, y(t), y(t - \tau_1(t)), \dots, y(t - \tau_m(t)))|$$

$$\le \sum_{i=1}^{m} |f_i(x(t - \tau_i(t))) - f_i(y(t - \tau_i(t)))| + |f_0(x(t)) - f_0(y(t))|;$$

(H_3) $|f_i(x) - f_i(y)| \le |K_i(x - y)|$ and $f_i(0) = 0$, where $K_i \in R^{n\times n}$, $i = 0, 1, 2, \dots, m$;

(H_4) There exist positive definite matrices $H_0, H_j (j = 1, 2, \dots, l)$ such that

$$\left\{\sum_{j=1}^{l} [h_j(t, x(t), x(t - \sigma_j(t))) - h_j(t, y(t), y(t - \sigma_j(t)))]\right\}^T$$

$$\times\left\{\sum_{j=1}^{l}[h_j(t,x(t),x(t-\sigma_j(t)))-h_j(t,y(t),y(t-\sigma_j(t)))]\right\}$$

$$\leq\sum_{j=1}^{l}[x(t-\sigma_j(t))-y(t-\sigma_j(t))]^T H_j[x(t-\sigma_j(t))-y(t-\sigma_j(t))]$$

$$+[x(t)-y(t)]^T H_0[x(t)-y(t)]$$

and $h_j(t,0,0)=0,\ j=1,2,\ldots,l$;

(H_5)

$$(m+k+l+1)\Big(\|\triangle A\|_3^2+\sum_{i=0}^{m}\|\overline{B}_i\|_3^2+\sum_{p=0}^{k}\|(W_p+\triangle W_p)G_p\|_3^2 + \|H_0\|_3+\sum_{j=1}^{l}\|H_j\|_3\Big)<1,$$

where $\overline{B}_0=(B+\triangle B(t))f_0, \overline{B}_i=(B+\triangle B(t))f_i, i=1,2,\ldots,m$;

(H_6) There exists a $\alpha>0$ such that

$$\min\{a_1,a_2,\ldots,a_n\}\geq 2\alpha.$$

Theorem 3.1. Suppose that conditions (H_1)–(H_6) are satisfied. Then the system (DW) is exponentially stable in mean square for all admissible uncertainties, that is, $e^{\alpha t}E|x(t)|_1^2\to 0$ as $t\to\infty$.

Proof of Theorem 3.1. From (DW), we have

$$x(t)=\exp(-At)\Big\{\psi(0)+\int_0^t\exp(As)[\sum_{p=1}^{k}(W_p+\triangle W_p(s))\int_{s-r_p(s)}^{s}g_p(x(v))dv$$
$$+(B+\triangle B(s))f(s,x(s),x(s-\tau_1(s)),\ldots,x(s-\tau_m(s)))-\triangle A(s)x(s)]ds$$
$$+\sum_{j=1}^{l}\int_0^t h_j(s,x(s),x(s-\sigma_j(s)))\exp(As)dw(s)\Big\}. \tag{3.1}$$

Let $(\mathcal{B},\|\cdot\|_{\mathcal{B}})$ be the Banach space of all bounded and continuous in mean square $\mathcal{F}_0$-adapted processes $\phi(t,\omega):[-\tau^*,\infty)\times\Omega\to R^n$ with the supremum norm

$$\|\phi\|_{\mathcal{B}}:=\sup\nolimits_{t\geq 0}E|\phi(t)|_1^2\quad\text{ for }\phi\in\mathcal{B}.$$

Denote by S the complete metric space with the supremum metric consisting of functions $\phi\in\mathcal{B}$ such that $\phi(s)=\psi(s)$ on $s\in[-\tau^*,0]$ and $e^{\alpha t}E|\phi(t,\omega)|_1^2\to 0$ as $t\to\infty$.

Define an operator Φ on S by $\Phi(x)(t) = \psi(t)$ for $t \in [-\tau^*, 0]$ and for $t \geq 0$,

$$\Phi(x)(t) := \sum_{i=1}^{3} \nu_i(t), \tag{3.2}$$

where

$$\nu_1(t) := \exp(-At)\psi(0), \tag{3.3}$$

$$\nu_2(t) := \sum_{j=1}^{l} \int_0^t \exp A(s-t) h_j(s, x(s), x(s-\sigma_j(s))) dw(s), \tag{3.4}$$

and

$$\begin{aligned} \nu_3(t) : = \int_0^t \exp A(s-t)\Big[- \triangle\, A(s)x(s) + \sum_{p=1}^{k} (W_p + \triangle W_p(s)) \int_{s-r_p(s)}^{s} g_p(x(v))dv \\ + (B + \triangle B(s)) f(s, x(s), x(s-\tau_1(s)), \dots, x(s-\tau_m(s)))\Big] ds. \end{aligned} \tag{3.5}$$

We first verify the mean square continuity of Φ.

Let $x \in S$, $t_1 \geq 0$, and $|r|$ be sufficiently small. Then

$$E|\Phi(x)(t_1 + r) - \Phi(x)(t_1)|_1^2 \leq 3 \sum_{i=1}^{3} E|\nu_i(t_1 + r) - \nu_i(t_1)|_1^2. \tag{3.6}$$

It is easy to see that

$$E|\nu_i(t_1 + r) - \nu_i(t_1)|_1^2 \to 0, \quad i = 1 \ \text{ or } \ i = 3 \tag{3.7}$$

as $r \to 0$. Further, by using the Burkhölder-Davis-Gundy inequality [20], we get

$$\begin{aligned} & E|\nu_2(t_1 + r) - \nu_2(t_1)|_1^2 \\ & = E\Big|\sum_{j=1}^{l} \int_0^{t_1+r} \exp A(s - t_1 - r) h_j(s, x(s), x(s-\sigma_j(s))) dw(s) \\ & \quad - \sum_{j=1}^{l} \int_0^{t_1} \exp A(s - t_1) h_j(s, x(s), x(s-\sigma_j(s))) dw(s)\Big|_1^2 \\ & = E\Big|\sum_{j=1}^{l} \int_0^{t_1} \exp A(s - t_1 - r) h_j(s, x(s), x(s-\sigma_j(s))) dw(s) \\ & \quad + \sum_{j=1}^{l} \int_{t_1}^{t_1+r} \exp A(s - t_1 - r) h_j(s, x(s), x(s-\sigma_j(s))) dw(s) \\ & \quad - \sum_{j=1}^{l} \int_0^{t_1} \exp A(s - t_1) h_j(s, x(s), x(s-\sigma_j(s))) dw(s)\Big|_1^2 \\ & = E\Big|\sum_{j=1}^{l} \int_0^{t_1} \exp A(s - t_1)[\exp A(-r) - I] h_j(s, x(s), x(s-\sigma_j(s))) dw(s) \end{aligned}$$

$$
\begin{aligned}
&+\sum_{j=1}^{l}\int_{t_1}^{t_1+r}\exp A(s-t_1-r)h_j(s,x(s),x(s-\sigma_j(s)))dw(s)|_1^2\\
&\le 2E\int_0^{t_1}\exp 2A(s-t_1)[\exp A(-r)-I]^2[\sum_{j=1}^{l}h_j^T(s,x(s),x(s-\sigma_j(s)))]\\
&\times[\sum_{j=1}^{l}h_j(s,x(s),x(s-\sigma_j(s)))]ds+2E\int_{t_1}^{t_1+r}\exp 2A(s-t_1-r)\\
&\times[\sum_{j=1}^{l}h_j^T(s,x(s),x(s-\sigma_j(s)))]\times[\sum_{j=1}^{l}h_j(s,x(s),x(s-\sigma_j(s)))]ds. \qquad (3.8)
\end{aligned}
$$

Note,

$$
\exp A(s-t)=\begin{pmatrix} e^{a_1(s-t)} & 0 & 0 & \cdots & 0\\ 0 & e^{a_2(s-t)} & 0 & \cdots & 0\\ \vdots & \vdots & \vdots & \vdots & \vdots\\ 0 & 0 & 0 & \cdots & e^{a_n(s-t)}\end{pmatrix}
$$

and

$$
\|\exp A(s-t)\|_3=\sum_{i=1}^{n}e^{a_i(s-t)}.
$$

From (H_4), (H_5) and (3.8), we have

$$
\begin{aligned}
&E|\nu_2(t_1+r)-\nu_2(t_1)|_1^2\\
&\le 2\left(\|H_0\|_3+\sum_{j=1}^{l}\|H_j\|_3\right)\left[E(\sup_{t_1-\tau^*\le s\le t_1+r}|x(s)|_1^2)\int_{t_1}^{t_1+r}\|\exp A(s-t_1-r)\|_3^2ds\right.\\
&\left.+E\left(\sup_{-\tau^*\le s\le t_1}|x(s)|_1^2\right)\int_0^{t_1}\|\exp A(s-t_1)\|_3^2\|\exp A(-r)-I\|_3^2ds\right]\to 0 \quad \text{as } r\to 0.
\end{aligned}
\tag{3.9}
$$

Thus, Φ is mean square continuous.

Next, we show that $\Phi(S)\subset S$. It is easy to see $e^{\alpha t}E|\nu_1(t)|_1^2\to 0$ as $t\to\infty$. It remains to prove $e^{\alpha t}E|\nu_2(t)|_1^2\to 0$ and $e^{\alpha t}E|\nu_3(t)|_1^2\to 0$ as $t\to\infty$. Note,

$$
\begin{aligned}
e^{\alpha t}E|\nu_3(t)|_1^2&=e^{\alpha t}E|\int_0^t\exp A(s-t)[\sum_{p=1}^{k}(W_p+\triangle W_p(s))\int_{s-r_p(s)}^{s}g_p(x(v))dv\\
&\quad+(B+\triangle B(s))f(s,x(s),x(s-\tau_1(s)),\ldots,x(s-\tau_m(s)))-\triangle A(s)x(s)]ds|_1^2\\
&\le e^{\alpha t}E\int_0^t\|\exp A(s-t)\|_3^2|[-\triangle A(s))x(s)\\
&\quad+(B+\triangle B(s))f(s,x(s),x(s-\tau_1(s)),\ldots,x(s-\tau_m(s)))\\
&\quad+\sum_{p=1}^{k}(W_p+\triangle W_p(t))\int_{s-r_p(s)}^{s}g_p(x(v))dv]|_1^2ds.
\end{aligned}
\tag{3.10}
$$

For any $\varepsilon > 0$, there exists $t^* > 0$ such that $s \geq t^* - \tau^*$ implies $e^{\alpha s}E|x(s)|^2 < \varepsilon$. Hence, we have from (H_6) and (3.10)

$$\begin{aligned}
e^{\alpha t}E|\nu_3(t)|_1^2 &\leq e^{\alpha t}E\int_0^{t^*} \|\exp A(s-t)\|_3^2|\Big[\sum_{p=1}^{k}(W_p+\triangle W_p(s))\int_{s-r_p(s)}^{s} g_p(x(v))dv \\
&\quad + (B+\triangle B(s))f(s,x(s),x(s-\tau_1(s)),\ldots,x(s-\tau_m(s))) - \triangle A(s)x(s)\Big]|_1^2 ds \\
&\quad + \int_{t^*}^{t} \|\exp A(s-t)\|_3^2|\Big[\sum_{p=1}^{k}(W_p+\triangle W_p(s))\int_{s-r_p(s)}^{s} g_p(x(v))dv \\
&\quad + (B+\triangle B(s))f(s,x(s),x(s-\tau_1(s)),\ldots,x(s-\tau_m(s))) - \triangle A(s)x(s)\Big]|_1^2 ds \\
&\leq e^{(\alpha-2\lambda_{\min}(A))t}C_1^*E\left(\sup_{-\tau^*\leq s\leq t^*}|x(s)|_1^2\right)\int_0^{t^*} e^{2\lambda_{\min}(A)s}ds \\
&\quad + e^{\alpha t}C_1^*E\int_{t^*}^{t} e^{\alpha s}e^{-\alpha s}x^T(s)x(s)e^{2\lambda_{\min}(A)(s-t)}ds \\
&\leq e^{(\alpha-2\lambda_{\min}(A))t}C_1^*E\left(\sup_{-\tau^*\leq s\leq t^*}|x(s)|_1^2\right)\int_0^{t^*} e^{2\lambda_{\min}(A)s}ds + \frac{\varepsilon}{\alpha},
\end{aligned} \tag{3.11}$$

where $\lambda_{\min}(A)$ represents the minimal eigenvalue of A, $C_1^* = n(\|\triangle A\|_3 + r^*\sum_{p=1}^{k}\|(W_p+\triangle W_p)G_p\|_3 + \sum_{i=0}^{m}\|\overline{B}_i\|)_3^2$, $\overline{B}_0 = (B+\triangle B(t))f_0$, $\overline{B}_i = (B+\triangle B(t))f_i$, $i = 1,2,\ldots,m$. Thus, we have $e^{\alpha t}E|\nu_2(t)|_1^2 \to 0$ as $t \to \infty$.

From (H_4) and (H_5), we have

$$\begin{aligned}
e^{\alpha t}E|\nu_2(t)|_1^2 &\leq e^{\alpha t}E|\sum_{j=1}^{l}\int_0^t \exp A(s-t)h_j(s,x(s),x(s-\sigma_j(s)))dw(s)|_1^2 \\
&\leq (l+1)e^{\alpha t}E\int_0^t \|\exp A(s-t)\|_3^2[\sum_{j=1}^{l} h_j^T(s,x(s),x(s-\sigma_j(s)))] \\
&\quad \times \left[\sum_{j=1}^{l} h_j(s,x(s),x(s-\sigma_j(s)))\right]ds \\
&= (l+1)e^{\alpha t}E\int_0^{t^*} \|\exp A(s-t)\|_3^2[\sum_{j=1}^{l} h_j^T(s,x(s),x(s-\sigma_j(s)))] \\
&\quad \times \left[\sum_{j=1}^{l} h_j(s,x(s),x(s-\sigma_j(s)))\right]ds \\
&\quad + (l+1)e^{\alpha t}E\int_{t^*}^{t} \|\exp A(s-t)\|_3^2\left[\sum_{j=1}^{l} h_j^T(s,x(s),x(s-\sigma_j(s)))\right] \\
&\quad \times \left[\sum_{j=1}^{l} h_j(s,x(s),x(s-\sigma_j(s)))\right]ds
\end{aligned}$$

$$\leq n(l+1)(\|H_0\|_3+\sum_{j=1}^{l}\|H_j\|_3)[e^{\alpha t}E\int_{t^*}^{t}x^T(s)x(s)e^{2\lambda_{\min}(A)(s-t)}ds$$
$$+e^{(\alpha-2\lambda_{\min}(A))t}E\left(\sup_{-\tau^*\leq s\leq t^*}|x(s)|_1^2\right)\int_0^{t^*}e^{2\lambda_{\min}(A)s}ds]\to 0 \qquad (3.12)$$

as $t\to\infty$. As a result $\Phi(S)\subset S$.

For $x,y\in S$, we have

$$E\sup_{s\in[0,t]}|\Phi(x)(s)-\Phi(y)(s)|_1^2$$
$$\leq E\sup_{s\in[0,t]}|\int_0^s\exp A(\eta-s)[\sum_{p=1}^{k}(W_p+\triangle W_p(s))\int_{\eta-r_p(\eta)}^{\eta}g_p(x(v))dv$$
$$+(B+\triangle B(\eta))f(\eta,x(\eta),x(\eta-\tau_1(\eta)),\ldots,x(\eta-\tau_m(\eta)))-\triangle A(\eta))x(\eta)]d\eta$$
$$+\sum_{j=1}^{l}\int_0^s\exp A(\eta-s)h_j(\eta,x(\eta),x(\eta-\sigma_j(\eta)))dw(\eta)$$
$$-\int_0^s\exp A(\eta-s)[-\triangle A(\eta))y(\eta)+\sum_{p=1}^{k}(W_p+\triangle W_p(s))\int_{\eta-r_p(\eta)}^{\eta}g_p(y(v))dv$$
$$+(B+\triangle B(\eta))f(\eta,y(\eta),y(\eta-\tau_1(\eta)),\ldots,y(\eta-\tau_m(\eta)))]d\eta$$
$$-\sum_{j=1}^{l}\int_0^s\exp A(\eta-s)h_j(\eta,y(\eta),y(\eta-\sigma_j(\eta)))dw(\eta)|_1^2$$
$$\leq C_2^*E\sup_{s\in[0,t]}|x(s)-y(s)|_1^2, \qquad (3.13)$$

where $C_2^*=(m+k+l+2)(\|\triangle A\|_3^2+\sum_{i=0}^{m}\|\overline{B}_i\|_3^2+\sum_{p=1}^{k}\|(W_p+\triangle W_p)G_p\|_3^2+\|H_0\|+\sum_{j=1}^{l}\|H_i\|_3)$. Thus Φ is a contraction since $0<C_2^*<1$.

Hence the Banach contraction principle guarantees that Φ has a fixed point x in S and note $x(s)=\psi(s)$ on $[-\tau^*,0]$ and $e^{\alpha t}E\|x(t)\|_1^2\to 0$ as $t\to\infty$. This completes the proof.

4. **Exponential stability (II)**

The proof in this section is based on the linear matrix inequality(LMI).

We now assume that following hypothesis is satisfied:

(V_1) The parameter uncertainties are of the $\triangle A$, $\triangle B$ and $\triangle W_i (i=1,2,\ldots,k)$ form:

$$(\triangle A(\cdot),\triangle W_1(\cdot),\triangle W_2(\cdot),\cdots,\triangle W_k(\cdot),\triangle B(\cdot)f_0(\cdot),\triangle B(\cdot)f_1(\cdot),\cdots,\triangle B(\cdot)f_m(\cdot))$$
$$=MF(N_A,N_{W_1},N_{W_2},\ldots,N_{W_k},N_0,N_1,\ldots,N_m)$$

in which M, $N_A, N_{W_1}, N_{W_2}, \ldots, N_{W_k}, N_0, N_1, \ldots, N_m$ are known constant matrices with appropriate dimensions. The uncertain matrix $F(t)$ satisfies

$$F^T(t)F(t) \leq I, \quad \forall t \in R.$$

(V_2) The time-varying delays $\tau_i(t), \sigma(t), r_p(t)$ satisfy

$$\tau_i'(t) \leq \tau < 1, \sigma_j'(t) \leq \sigma < 1, r_p(t) \leq r^*, i = 1, 2, \ldots, m, j = 1, 2, \ldots, l, p = 1, 2, \ldots, k,$$

where $t \in R$, τ, σ and r^* are constants.

For convenience, let $m \geq l$.

Theorem 4.1. Suppose (H_1)–(H_3) and (V_1)–(V_2) hold and assume that there exist matrices $P > 0$, $D_0 \geq 0$ and $D_i \geq 0 (j = 1, 2, \ldots, l)$ such that

$$\begin{aligned}&\text{trace}\left[\sum_{j=1}^{l} h_j^T(t, x(t), x(t-\sigma_j(t)))Ph_j(t, x(t), x(t-\sigma_j(t)))\right]\\ &\leq x^T(t)D_0x(t) + \sum_{j=1}^{l} x^T(t-\sigma_j(t))D_jx(t-\sigma_j(t)).\end{aligned} \tag{4.1}$$

Then the system (DW) is globally exponentially stable in mean square for all admissible uncertainties, if there exist positive scalar $\varrho > 0$, $\epsilon_p > 0$ $(p = 1, 2, \ldots, k)$ and positive definite matrices $Q_i > 0$ $(i = 1, 2, \ldots, m)$, $R_p > 0 (p = 1, 2, \ldots, k)$ such that the LMI holds:

$$\begin{pmatrix} \Xi & P(B_0f_0) & P(B_1f_1) & \cdots & P(B_mf_m) & \sqrt{m+2}PM \\ (B_0f_0)^TP & -\varrho N_0^TN_0 & 0 & \cdots & 0 & 0 \\ (B_1f_1)^TP & 0 & \Upsilon_1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ (B_mf_m)^TP & 0 & 0 & \cdots & \Upsilon_m & 0 \\ \sqrt{m+2}M^TP & 0 & 0 & \cdots & 0 & -\varrho I \end{pmatrix} < 0, \tag{4.2}$$

where

$$\Xi = (-PA - A^TP) + D_0 + \widetilde{W} + \sum_{p=1}^{k} \epsilon_p P(W_pW_p^T + N_{W_p}^TN_{W_p}^T)P + \varrho N_A^TN_A, \tag{4.3}$$

$$\widetilde{W} = \sum_{i=1}^{m} \frac{1}{1-\tau}Q_i + \sum_{j=1}^{l} \frac{1}{1-\sigma}D_j + \sum_{p=1}^{k} r^*R_p \tag{4.4}$$

and

$$\Upsilon_i = -\varrho N_i^TN_i, \quad i = 1, 2, \ldots, m. \tag{4.5}$$

Proof of Theorem 4.1. Let

$$V(t, x(t)) = \sum_{i=1}^{m} \frac{1}{1-\tau} \int_{t-\tau_i(t)}^{t} x^T(s)Q_ix(s)ds + \sum_{j=1}^{l} \frac{1}{1-\sigma} \int_{t-\sigma_j(t)}^{t} x^T(s)D_jx(s)ds$$

$$+\sum_{p=1}^{k}\int_{-r^*}^{0}\int_{t+s}^{t}x^T(\eta)R_p x(\eta)d\eta ds + x^T P x. \tag{4.6}$$

From Itô's differential formula (see, e.g.,[12]) we have along (DW)

$$\begin{aligned}LV(t,x(t)) &= \sum_{j=1}^{l}h_j^T(t,x(t),x(t-\sigma_j(t)))Ph_j(t,x(t),x(t-\sigma_j(t))) + x^T[-PA(t)\\ &- A^T(t)P]x + 2x^T P[B(t)f(t,x(t),x(t-\tau_1(t)),\dots,x(t-\tau_m(t))) \quad (4.1)\\ &+ \sum_{p=1}^{k}W_p(t)\int_{t-r_p(t)}^{t}g_p(x(s))ds] + \sum_{i=1}^{m}\frac{1}{1-\tau}x^T(t)Q_i x(t)\\ &+ \sum_{j=1}^{l}\frac{1}{1-\sigma}x^T(t)D_j x(t) + \sum_{p=1}^{k}r^* x^T(t)R_p x(t)\\ &- \Big[\sum_{i=1}^{m}\frac{1-\tau_i'(t)}{1-\tau}x^T(t-\tau_i(t))Q_i x(t-\tau_i(t)) + \sum_{p=1}^{k}\int_{t-r^*}^{t}x^T(s)R_p x(s)ds\\ &+ \sum_{j=1}^{l}\frac{1-\sigma_j'(t)}{1-\sigma}x^T(t-\sigma_j(t))D_j x(t-\sigma_j(t))\Big],\end{aligned} \tag{4.7}$$

where $A(t) = A + \triangle A(t)$, $B(t) = B + \triangle B(t)$ and $W_p(t) = W_p + \triangle W_p(t), p = 1, 2, \dots, k$.

From (V_2) and (4.7), we have

$$\begin{aligned}LV(t,x(t)) &\le \sum_{j=1}^{l}h_j^T(t,x(t),x(t-\sigma_j(t)))Ph_j(t,x(t),x(t-\sigma_j(t)))\\ &+ 2x^T P\Big[\sum_{p=1}^{k}W_p(t)\int_{t-r_p(t)}^{t}g_p(x(s))ds\\ &+ B(t)f(t,x(t),x(t-\tau_1(t)),\dots,x(t-\tau_m(t)))\Big]\\ &+ \sum_{i=1}^{m}\frac{1}{1-\tau}x^T(t)Q_i x(t) + \sum_{j=1}^{l}\frac{1}{1-\sigma}x^T(t)D_j x(t) + \sum_{p=1}^{k}r^* x^T(t)R_p x(t)\\ &- \Big[\sum_{i=1}^{m}x^T(t-\tau_i(t))Q_i x(t-\tau_i(t)) + \sum_{j=1}^{l}x^T(t-\sigma_j(t))D_j x(t-\sigma_j(t))\\ &+ \sum_{p=1}^{k}\int_{t-r_p(t)}^{t}x^T(s)R_p x(s)ds\Big] + x^T[-PA(t) - A^T(t)P]x.\end{aligned} \tag{4.8}$$

For the positive scalars $\epsilon_p > 0$ $(p = 1, 2, \dots, k)$, by using the relation (H_1), it follows from Lemma 2.4 that

$$2x^T P[\sum_{p=1}^{k}W_p(t)\int_{t-r_p(t)}^{t}g_p(x(s))ds$$

$$
\begin{aligned}
&\leq \sum_{p=1}^{k}[\epsilon_p x^T P W_p(t) W_p^T(t) P x + \epsilon_p^{-1}(\int_{t-r_p(t)}^{t} g_p(x(s))ds)^T(\int_{t-r_p(t)}^{t} g_p(x(s))ds)] \\
&\leq \sum_{p=1}^{k}[\epsilon_p x^T P W_p(t) W_p^T(t) P x + \epsilon_p^{-1} r_p(t) \int_{t-r_p(t)}^{t} g_p^T(x(s)) g_p(x(s))ds] \\
&\leq \sum_{p=1}^{k}[\epsilon_p x^T P W_p(t) W_p^T(t) P x + \epsilon_p^{-1} r_p(t) \int_{t-r_p(t)}^{t} x^T(s) G_p^T G_p x(s)ds] \\
&\leq \sum_{p=1}^{k}[\epsilon_p x^T P W_p(t) W_p^T(t) P x + (1-\eta_i) \int_{t-r_p(t)}^{t} x^T(s) R_p x(s)ds]ds \\
&= \sum_{p=1}^{k}[\epsilon_p x^T P (W_p + \triangle W_p(t))(W_p + \triangle W_p(t))^T P x \\
&\quad + (1-\eta_p) \int_{t-r_p(t)}^{t} x^T(s) R_p x(s)ds]ds \\
&\leq \sum_{p=1}^{k}[\epsilon_p x^T P (W_p W_p^T + W_p(\triangle W_p(t))^T + \triangle W_p(t) W_p^T + \triangle W_p(t)(\triangle W_p(t))^T) P x \\
&\quad + (1-\eta_p) \int_{t-r_p(t)}^{t} x^T(s) R_p x(s)ds]ds, \qquad (4.9)
\end{aligned}
$$

where

$$
\epsilon_p^{-1} r_p(t) G_p^T G_p \leq (1-\eta_p) R_p, \quad \eta_p \geq 0, \quad p = 1, 2, \ldots, k.
$$

For any scalar $\varepsilon > 0$, it is easy to get that

$$
W_p(\triangle W_p(t))^T + \triangle W_p(t) W_p^T \leq \varepsilon W_p W_p^T + \frac{1}{\varepsilon} \triangle W_p(t)(\triangle W_p(t))^T.
$$

From Lemma 2.2, there exists a scalar $\varepsilon' > 0$ that we have

$$
\triangle W_p(t)(\triangle W_p(t))^T = (MF(t)N_{W_p})(MF(t)N_{W_p})^T \leq \varepsilon' N_{W_p} N_{W_p}^T, \quad p = 1, 2, \ldots, k. \qquad (4.10)
$$

From (4.8)–(4.10), we have

$$
LV(t, x(t)) \leq \xi^T \Theta \xi - \sum_{p=1}^{k} \eta_p \int_{t-r_p(t)}^{t} x^T(s) R_p x(s)ds \leq \xi^T \Theta \xi, \qquad (4.11)
$$

where

$$
\xi = (x^T(t), f_0^T, f_1^T, \ldots, f_m^T, x(t-\tau_1(t)), \ldots, x(t-\tau_m(t))), \qquad (4.12)
$$

$$\Theta = \begin{pmatrix} \overline{\Xi} & P\overline{B}_0 & P\overline{B}_1 & \cdots & P\overline{B}_m & 0 & \cdots & 0 \\ \overline{B}_0^T P & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ \overline{B}_1^T P & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \overline{B}_m^T P & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 & -Q_1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & -Q_m \end{pmatrix}, \tag{4.13}$$

where $\overline{\Xi} = (-PA(t) - A^T(t)P) + D_0 + \widetilde{W} + \sum_{p=1}^{k} \epsilon_p P(W_p W_p^T + N_{W_p}^T N_{W_p}^T)P + \varrho N_A^T N_A$.

From Lemma 2.3, we have that $\Theta < 0$ is equivalent to $\widetilde{\Lambda} < 0$, where

$$\widetilde{\Lambda} = \begin{pmatrix} \overline{\Xi} & P\overline{B}_0 & P\overline{B}_1 & \cdots & P\overline{B}_m \\ \overline{B}_0^T P & 0 & 0 & \cdots & 0 \\ \overline{B}_1^T P & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & 0 \\ \overline{B}_m^T P & 0 & 0 & \cdots & 0 \end{pmatrix}. \tag{4.14}$$

From (V_1) and (4.14), the matrix $\widetilde{\Lambda}$ can be rewritten as

$$\widetilde{\Lambda} = \bigodot + X\widetilde{F}(t)Y + Y^T \widetilde{F}^T(t) X^T, \tag{4.15}$$

where

$$\bigodot = \begin{pmatrix} \Xi & P(B_0 f_0) & P(B_1 f_1) & \cdots & P(B_m f_m) \\ (B_0 f_0)^T P & \varrho N_0^T N_0 & 0 & \cdots & 0 \\ (B_1 f_1)^T P & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & 0 \\ (B_m f_m)^T P & 0 & 0 & \cdots & 0 \end{pmatrix}, \tag{4.16}$$

$$X = \begin{pmatrix} PM & PM & \cdots & PM \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix}_{(m+2)\times(m+2)}, \tag{4.17}$$

$$Y = diag\{-N_A, N_0, N_1, \ldots, N_m\}_{m+2} \tag{4.18}$$

and

$$\widetilde{F} = diag\{F(t), F(t), \ldots, F(t)\}_{m+2}. \tag{4.19}$$

From Lemma 2.1, there exists a positive scalar $\varrho > 0$, we have

$$\widetilde{\Lambda} \leq \bigodot + \varrho^{-1} X X^T + \varrho Y^T Y. \tag{4.20}$$

From (4.2), (4.13)–(4.15) and (4.20) with manipulations one can show that the LMI (4.2) is equivalent to $\widetilde{\Lambda} < 0$ (and so $\Theta < 0$).

Let $\widetilde{V}(t,x(t)) = e^{kt}V(t,x(t))$, where k is to be determined. It is easy to check that

$$V(t,x(t)) \le \lambda_{\max}(P)|x(t)|_1^2 + r^* \sum_{p=1}^{k} \int_{t-r^*}^{t} x^T(s)R_p x(s)ds + \sum_{i=1}^{m} \frac{1}{1-\tau} \int_{t-\tau_i(t)}^{t} x^T(s)Q_i x(s)ds + \sum_{j=1}^{l} \frac{1}{1-\sigma} \int_{t-\sigma_j(t)}^{t} x^T(s)D_j x(s)ds. \tag{4.21}$$

Thus

$$\begin{aligned} L\widetilde{V}(t,x(t)) &= e^{kt}[kV(t,x(t)) + LV(t,x(t)] \\ &\le e^{kt}\{\xi^T\Theta\xi + k[\lambda_{\max}(P)|x(t)|_1^2 + r^* \sum_{p=1}^{k} \int_{t-r^*}^{t} x^T(s)R_p x(s)ds \\ &\quad + \sum_{i=1}^{m} \frac{1}{1-\tau} \int_{t-\tau_i(t)}^{t} x^T(s)Q_i x(s)ds + \sum_{j=1}^{l} \frac{1}{1-\sigma} \int_{t-\sigma_j(t)}^{t} x^T(s)D_j x(s)ds]\}. \end{aligned} \tag{4.22}$$

Choose k sufficiently small so that

$$\xi^T\Theta\xi + k\Big[\lambda_{\max}(P)|x(t)|_1^2 + r^* \sum_{p=1}^{k} \int_{t-r^*}^{t} x^T(s)R_p x(s)ds + \sum_{i=1}^{m} \frac{1}{1-\tau} \int_{t-\tau_i(t)}^{t} x^T(s)Q_i x(s)ds + \sum_{j=1}^{l} \frac{1}{1-\sigma} \int_{t-\sigma_j(t)}^{t} x^T(s)D_j x(s)ds\Big] \le 0. \tag{4.23}$$

From (4.22) and (4.23), we have

$$L\widetilde{V}(t,x(t)) \le 0, \tag{4.24}$$

which implies that

$$E\widetilde{V}(t,x(t)) \le E\widetilde{V}(0,x(0)). \tag{4.25}$$

Therefore, we have

$$\begin{aligned} &e^{kt}EV(t,x(t) \le EV(0,x(0)) \\ &\le E\{\lambda_{\max}(P)|x(0)|_1^2 + r^* \sum_{p=1}^{k} \int_{-r^*}^{0} x^T(s)R_p x(s)ds \\ &\quad + \sum_{i=1}^{m} \frac{1}{1-\tau} \int_{-\tau_i(t)}^{0} x^T(s)Q_i x(s)ds + \sum_{j=1}^{l} \frac{1}{1-\sigma} \int_{-\sigma_j(t)}^{0} x^T(s)D_j x(s)ds\} \\ &\le [\lambda_{\max}(P) + k(r^*)^2\lambda_{\max}(R) + \frac{m\tau\lambda_{\max}(Q)}{1-\tau} + \frac{l\sigma\lambda_{\max}(D)}{1-\sigma}] \max_{-\tau^* \le s \le 0} E|x(s)|_1^2, \end{aligned} \tag{4.26}$$

where $\lambda_{\max}(R) = \max\{\lambda_{\max}(R_1), \lambda_{\max}(R_2), \ldots, \lambda_{\max}(R_k)\}$, $\lambda_{\max}(Q) = \max\{\lambda_{\max}(Q_1), \lambda_{\max}(Q_2), \ldots, \lambda_{\max}(Q_m)\}$, $\lambda_{\max}(D) = \max\{\lambda_{\max}(D_1), \lambda_{\max}(D_2), \ldots, \lambda_{\max}(D_l)\}$. Also, it is easy to see that

$$EV(t, x(t) \geq \lambda_{\min}(P)|x(t)|_1^2. \tag{4.27}$$

From (4.26) and (4.27), it follows that

$$\begin{aligned} E|x(t)|_1^2 \leq \lambda_{\min}^{-1}(P)[\lambda_{\max}(P) + k(r^*)^2\lambda_{\max}(R) \\ + \frac{m\tau\lambda_{\max}(Q)}{1-\tau} + \frac{l\sigma\lambda_{\max}(D)}{1-\sigma}]\, e^{-kt} \max_{-\tau^* \leq s \leq 0} E|x(s)|_1^2. \end{aligned} \tag{4.28}$$

Thus the system (DW) is globally exponentially stable in mean square.

Remark 4.1. Note the results in [14] and [27] are special cases of (DW).

5. **Some examples**

Now we provide some examples.

Example 5.1.

$$\begin{aligned} dx = [-(A + \triangle A(t))x(t) + (B + \triangle B(t))(x(t) + x(t-\tau)) \\ + (W + \triangle W(t)) \int_{t-\tau}^{t} x(s)ds]dt + [h_0(t)x(t) + h_1(t)x(t-\tau)]dw(t) \end{aligned} \tag{5.1}$$

and

$$x(t) = \varphi(t), \quad \forall t \in [-\tau, 0].$$

It is easy to see that $f_0 = f_1 = g_1 = I$ and (H_1)–(H_6) are satisfied when $(\| \triangle A\|_3 + \|\overline{B}_0\|_3 + \|\overline{B}_1\|_3 + \|W + \triangle W\|_3 + \|h_0\|_3 + \|h_1\|_3)$ are sufficiently small and $A = diag(a_1, a_2, \ldots, a_n) \geq 2diag(\alpha, \alpha, \ldots, \alpha)$. Theorem 3.1 guarantees that system (5.1) is exponentially stable in mean square for all admissible uncertainties.

Example 5.2.

$$\begin{aligned} dx = [-(A + \triangle A(t))x(t) + (B + \triangle B(t))x(t-\tau) + (W + \triangle W(t)) \int_{t-\tau}^{t} x(s)ds]dt \\ + [h_0(t)x(t) + h_1(t)x(t-\tau)]dw(t) \end{aligned} \tag{5.2}$$

and

$$x(t) = \varphi(t), \quad \forall t \in [-\tau, 0].$$

Let

$$A = \begin{pmatrix} -2 & 0 \\ 0 & -0.3 \end{pmatrix}, B = \begin{pmatrix} -0.3 & 0.08 \\ 0.11 & 0.36 \end{pmatrix}, W = \begin{pmatrix} 0.1 & 0 \\ 0.1 & 0.1 \end{pmatrix},$$

$$h_0 = \begin{pmatrix} 0.1 & 0 \\ 0 & 0.2 \end{pmatrix}, h_1 = \begin{pmatrix} 0.25 & 0 \\ 0 & 0.2 \end{pmatrix}, D_0 = \begin{pmatrix} 0.2 & 0 \\ 0 & 0.4 \end{pmatrix}, D_1 = \begin{pmatrix} 0.5 & 0 \\ 0 & 0.4 \end{pmatrix},$$

$$M = P = F = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, N_A = \begin{pmatrix} -0.04 & 0.01 \\ 0.03 & -0.008 \end{pmatrix}, N_1 = \begin{pmatrix} -0.03 & -0.02 \\ 0.04 & -0.06 \end{pmatrix}$$

and

$$N_W = \begin{pmatrix} -0.05 & 0 \\ 0.04 & 0.05 \end{pmatrix}.$$

Solving the LMI in Theorem 4.1, we get $\varrho = 8.3864$, $\epsilon = 8.8398$,

$$Q = \begin{pmatrix} 0.1124 & -0.0100 \\ -0.0100 & 0.2631 \end{pmatrix}, R = \begin{pmatrix} 0.4497 & -0,0396 \\ -0.0396 & 1.0523 \end{pmatrix}.$$

Theorem 4.1 guarantees that system (5.2) is globally exponentially stable in mean square for all admissible uncertainties.

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