ASYMPTOTIC STABILITY OF LINEAR STATE-DELAYED NEUTRAL SYSTEMS WITH POLYTOPE TYPE UNCERTAINTIES

P. T. NAM, H. M. HIEN, AND V. N. PHAT

Department of Mathematics, QuiNhon University, BinhDinh, Vietnam
Department of Mathematics, QuiNhon University, BinhDinh, Vietnam
Institute of Mathematics, 18 Hoang Quoc Viet, Hanoi, Vietnam
Corresponding author: `vnphat@math.ac.vn`

ABSTRACT. In this paper, a class of linear state-delayed neutral systems with polytope type uncertainties is studied. Using an improved Lyapunov Krasovskii parameter-dependent functional and linear matrix inequality (LMI) technology, new delay-dependent sufficient conditions for the asymptotic stability of the system are first established in terms of Mondie-Kharitonov type's LMI conditions.

AMS (MOS) Subject Classification. 34D20, 37C75, 93D20.

1. INTRODUCTION

Stability analysis of linear neutral systems has received much attention in the past decades, e.g. see [1, 2, 3, 11] and the references therein. Theoretically, the linear neutral system with time delays is much more complicated, especially for the case where the system matrices belong to some convex polytope [9]. By using parameter-dependent Lyapunov functionals, some less conservative results for asymptotic stability of uncertain polytopic delay systems have been proposed in [5, 10] via LMIs. Although these results improve the estimate of asymptotic stability domain, some conservatism still remain since common matrix variable required to satisfy the whole sets of LMIs. An LMI-based method is proposed in [8, 12] for asymptotic stability of neutral systems with time delays, but the polytope uncertainties are not taken into account therein. To the best of our knowledge, so far, no result on the stability for linear neutral state-delayed systems with polytope uncertainties is available in the literature, which is still open and remains unsolved. This motivates our present investigation.

In this paper, we study asymptotic stability of linear neutral time-delay systems with polytope uncertainties. The novel feature of the results obtained in this paper in comparison with [5, 8, 10, 12] is twofold. First, the neutral system considered in this

paper is convex polytopic uncertain subjected to state delay. Second, by employing an improved parameter-dependent Lyapunov Krasovskii functional and LMI technology, delay-dependent sufficient conditions for the asymptotic stability of the system are first obtained in terms of Mondie-Kharitonov type's LMIs [7], which can be effectively solved by various convex optimization algorithms and LMI Toolbox of Matlab.

The paper is organized as follows: Section 2 presents notations, definitions and some well-known technical propositions needed for the proof of the main result. Delay-dependent stability conditions of the system is presented in Section 3 with numerical example. The paper ends with conclusions and cited references.

2. PRELIMINARIES

The following notations and definitions will be employed throughout this paper. R^+ denotes the set of all real non-negative numbers; R^n denotes the n-dimensional Euclidean space with the scalar product $\langle \cdot, \cdot \rangle$; $R^{n \times r}$ denotes the space of all matrices of $(n \times r)$-dimensions; A^T denotes the transpose of A; $\lambda(A)$ denotes the set of all eigenvalues of A, $\lambda_{\max}(A) = \max\{\operatorname{Re} \lambda : \lambda \in \lambda(A)\}$, $\lambda_{\min}(A) = \min\{\operatorname{Re} \lambda : \lambda \in \lambda(A)\}$; matrix $Q \geq 0$ ($Q > 0$, resp.) means Q is semi-positive definite matrix i.e. $\langle Qx, x \rangle \geq 0$, $\forall x \in R^n$ (positive definite, resp. i.e. $\langle Qx, x \rangle > 0, \forall x \in R^n, x \neq 0$), $A \geq B$ means $A - B \geq 0$; $C([a, b], R^n)$ denotes the set of all R^n-valued continuous functions on $[a, b]$; the segment of the trajectory $x(t)$ is denoted by $x_t = \{x(t+s) : s \in t \in [-h, 0]\}$ with its norm $\|x_t\| = \sup_{s \in [-h,0]} \|x)(+s)\|$.

Consider the following linear polytopic neutral system with state delay:

$$(2.1) \quad \begin{cases} \dot{x}(t) - C(\xi)\dot{x}(t - \tau_2) = A_0(\xi)x(t) + A_1(\xi)x(t - \tau_1) & t \geq 0 \\ x(\theta) = \varphi(\theta), \quad \forall \theta \in [-h, 0] \end{cases}$$

where $x(t) \in R^n$ is the state, τ_1, τ_2 are time delays, $h = \max\{\tau_1, \tau_2\}$. The state-space data are subject to uncertainties and belong to the polytope Ω given by

$$\Omega = \left\{ [C, A_0, A_1](\xi) := \sum_{i=1}^{p} \xi_i [C_i, A_{0i}, A_{1i}], \quad \sum_{i=1}^{p} \xi_i = 1, \xi_i \geq 0 \right\},$$

where $C_i, A_{0i}, A_{1i}, (i = 1, \ldots, p)$ are constant matrices with appropriate dimensions and $\xi_i, (i = 1, \ldots, p)$ are time-invariant uncertainties.

Let

$$\|x_t\|_w = \sqrt{\|x(t)\|^2 + \int_{-h}^{0} \|\dot{x}(t + s)\|^2 ds}.$$

Proposition 2.1. [6] *Assume* $\|C_i\| < 1, i = 1, 2, \ldots, p$. *If there is a Lyapunov-Krasovskii functional* $V(t, x_t)$ *satisfying*

i) $\exists c_1, c_2 > 0 \quad c_1 \|x(t)\|^2 \leq V(t, x_t) \leq c_2 \|x_t\|_w^2,$

ii) $\exists c_3 > 0 \quad \dot{V}(t, x_t) < -c_3 \|x(t)\|^2,$

for all solution $x(t)$ of the system, then the zero solution of system (2.1) is asymptotically stable.

Proposition 2.2. [4] *For any symmetric positive definite $M \in R^{n \times n}$, and number $\sigma > 0$, vector function $w : [0, \sigma] \to \mathbb{R}^n$ such that the integrations concerned are well defined, we have*

$$\left(\int_0^\sigma w(s)ds \right)^T M \left(\int_0^\sigma w(s)ds \right) \leq \sigma \int_0^\sigma w^T(s)Mw(s)ds.$$

3. MAIN RESULT

Let $Q_{1i}, Q_{2i}, Q_{3i}, Q_{4i}, Q_{5i}, P_i, T_i, H_i, S_i, M$ be $n \times n$ matrices. We denote

$$\Xi_i(P_j, T_j, H_j, S_j, K_j, \mathcal{Q}_j) =: \begin{bmatrix} \Xi_{11} & A_{0i}^T Q_{2j}^T & \Xi_{13} & -Q_{1j} + A_{0i}^T Q_{4j}^T & \Xi_{15} \\ \star & -S_j & Q_{2j}A_{1i} & -Q_{2j} & Q_{2j}C_i \\ \star & \star & \Xi_{33} & A_{1i}^T Q_{4j} - Q_{3j} & \Xi_{35} \\ \star & \star & \star & \Xi_{44} & -Q_{5j}^T + Q_{4j}C_i \\ \star & \star & \star & \star & \Xi_{55} \end{bmatrix},$$

where

$\Xi_{11} = (P_j + Q_{1j})A_{0i} + A_{0i}^T(P_j + Q_{1j}^T) + S_j - H_j + K_j,$

$\Xi_{13} = H_j + (P_j + Q_{1j})A_{1i} + A_{0i}^T Q_{3j},$

$\Xi_{15} = (P_j + Q_{1j})C_i + A_{0i}^T Q_{5j}^T,$

$\Xi_{33} = -H_j + Q_{3j}A_{1i} + A_{1i}^T Q_{3j}^T - K_j,$

$\Xi_{35} = A_{1i}^T Q_{5j}^T + Q_{3j}C_i,$

$\Xi_{44} = T_j + \tau_1^2 H_j - Q_{4j} - Q_{4j}^T,$

$\Xi_{55} = -T_j + Q_{5j}C_i + C_i^T Q_{5j}^T,$

$$\mathbb{M} = \begin{pmatrix} M & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$\lambda_1 = \min_{i=1,2,\ldots,p} \lambda_{\min}(P_i), \quad \alpha_P = \max_{i=1,2,\ldots,p} \lambda_{\max}(P_i), \quad \alpha_T = \max_i \lambda_{\max}(T_i),$

$\alpha_S = \max_i \lambda_{\max}(S_i), \quad \alpha_K = \max_i \lambda_{\max}(K_i), \quad \alpha_H = \max_i \lambda_{\max}(H_i),$

$\lambda_2 = \alpha_P + \alpha_T + 2h(1+h)\alpha_S + 2h(1+h)\alpha_K + h^2\alpha_H.$

Theorem 3.1. *Assume that $||C_i|| < 1, (i = 1, \ldots, p)$. The zero solution of system (2.1) is asymptotically stable if there exist matrices $Q_{1i}, Q_{2i}, Q_{3i}, Q_{4i}, Q_{5i}(i = 1, \ldots, p)$, a symmetric semi-positive definite matrix M and symmetric positive definite matrices P_i, T_i, H_i, S_i, K_i $(i = 1, \ldots, p)$, such that the following linear matrix inequalities hold:*

(3.1) $\qquad\qquad \Xi_i(P_i, T_i, H_i, S_i, K_i, \mathcal{Q}_i) < -\mathbb{M}, \quad i = 1, \ldots, p$

(3.2) $\Xi_i(P_j, T_j, H_j, S_j, K_j, \mathcal{Q}_j) + \Xi_j(P_i, T_i, H_i, S_i, K_i, \mathcal{Q}_i) < \dfrac{2}{p-1}\mathbb{M},$

$i = 1, \ldots, p-1, j = i+1, \ldots, p.$

Proof. We denote $[P, T, S, M, H, K, Q_k](\xi) := \sum_{i=1}^{p} \xi_i[P_i, T_i, S_i, M_i, H_i, K_i, Q_{ki}]$, and consider the following Lyapunov functional:

$$V(t, x_t) = V_1(\cdot) + V_2(\cdot) + V_3(\cdot) + V_4(\cdot) + V_5(\cdot),$$

where

$V_1(\cdot) = x^T(t)P(\xi)x(t),$
$V_2(\cdot) = \int_{t-\tau_2}^{t} \dot{x}^T(s)T(\xi)\dot{x}(s)ds,$
$V_3(\cdot) = \int_{t-\tau_2}^{t} x^T(s)S(\xi)x(s)ds,$
$V_4(\cdot) = \int_{t-\tau_1}^{t} x^T(s)K(\xi)x(s)ds,$
$V_5(\cdot) = \tau_1 \int_{t-\tau_1}^{t} (s - (t-\tau_1))\dot{x}^T(s)H(\xi)\dot{x}(s)ds.$ It is easy to verify that

(3.3) $\lambda_1||x(t)||^2 \le V(t, x_t) \le \lambda_2||x_t||_w^2.$

The derivatives of $V_i(.), i = 1, 2, \ldots, 5$ along the trajectory of system (2.1) are given by

$\dot{V}_1(t) = 2x(t)^T P(\xi)\dot{x}(t)$

$\qquad = 2x(t)^T P(\xi)[A_0(\xi)x(t) + A_1(\xi)x(t-\tau_1) + C(\xi)\dot{x}(t-\tau_2)]$

$\qquad = \sum_{i=1}^{p} \xi_i \Big[\sum_{j=1}^{p} \xi_j \Big(x^T(t)(P_i A_{0j} + A_{0j}^T P_i)x(t) + 2x^T(t)P_i A_{1j}x(t-\tau_1)$

(3.4) $+ 2x^T(t)P_i C_j x(t-\tau_2) \Big) \Big],$

$\qquad \dot{V}_2(t) = \dot{x}^T(t)T(\xi)\dot{x}(t) - \dot{x}^T(t-\tau_2)T(\xi)\dot{x}(t-\tau_2)$

(3.5) $\qquad = \sum_{i=1}^{p} \xi_i \Big[\sum_{j=1}^{p} \xi_j \Big(\dot{x}^T(t)T_i\dot{x}(t) - \dot{x}^T(t-\tau_2)T_i\dot{x}(t-\tau_2) \Big) \Big],$

$\qquad \dot{V}_3(t) = x^T(t)S(\xi)x(t) - x^T(t-\tau_2)S(\xi)x(t-\tau_2)$

(3.6) $\qquad = \sum_{i=1}^{p} \xi_i \Big[\sum_{j=1}^{p} \xi_j \Big(x^T(t)S_i x(t) - x^T(t-\tau_2)S_i x(t-\tau_2) \Big) \Big],$

$\qquad \dot{V}_4(t) = x^T(t)K(\xi)x(t) - x^T(t-\tau_1)K(\xi)x(t-\tau_1)$

(3.7) $\qquad = \sum_{i=1}^{p} \xi_i \Big[\sum_{j=1}^{p} \xi_j \Big(x^T(t)K_i x(t) - x^T(t-\tau_1)K_i x(t-\tau_1) \Big) \Big].$

Using Proposition 2.2, we have

$$\dot{V}_5(t) = \tau_1{}^2 \dot{x}^T(t)H(\xi)\dot{x}(t) - \tau_1 \int_{t-\tau_1}^{t} \dot{x}^T(s)H(\xi)\dot{x}(s)ds$$

$$\leq \tau_1{}^2 \dot{x}^T(t) H(\xi) \dot{x}(t) - \left(\int_{t-\tau_1}^t \dot{x}^T(s) ds \right)^T H(\xi) \left(\int_{t-\tau_1}^t \dot{x}^T(s) ds \right)$$

$$= \tau_1{}^2 \dot{x}^T(t) H(\xi) \dot{x}(t) - x^T(t) H(\xi) x(t) + 2 x^T(t) H(\xi) x(t - \tau_1)$$

$$\qquad - x^T(t - \tau_1) H(\xi) x(t - \tau_1)$$

$$= \sum_{i=1}^p \xi_i \Big[\sum_{j=1}^p \xi_j \Big(\tau_1{}^2 \dot{x}^T(t) H_i \dot{x}(t) - x^T(t) H(\xi) x(t) + 2 x^T(t) H_i x(t - \tau_1)$$

(3.8) $$\qquad - x^T(t - \tau_1) H_i x(t - \tau_1) \Big) \Big].$$

Since $-\dot{x}(t) + C(\xi)\dot{x}(t - \tau_2) + A_0(\xi)x(t) + A_1(\xi)x(t - \tau_1) = 0$, we have

$$2[x^T(t) Q_1(\xi) + x^T(t - \tau_2) Q_2(\xi) + x^T(t - \tau_1) Q_3(\xi) + \dot{x}^T(t) Q_4(\xi) + \dot{x}^T(t - \tau_2) Q_5(\xi)]$$

$$\times [-\dot{x}(t) + C(\xi)\dot{x}(t - \tau_2) + A_0(\xi)x(t) + A_1(\xi)x(t - \tau_1)] = 0.$$

From (3.4)–(3.8) it follows that

$$\dot{V}(t, x_t) \leqslant \sum_{i=1}^p \xi_i \Big[\sum_{j=1}^p \xi_j \Big(x^T(t)[P_i A_{0j} + A_{0j}^T P_i + S_i - H_i + K_i + Q_{1i} A_{0j} + A_{0j}^T Q_{1i}^T] x(t)$$

$$- 2 x^T(t) A_{0j}^T Q_{2i}^T x(t - \tau_2) + 2 x^T(t)[P_i A_{1j} + Q_{1i} A_{1j} + A_{0j}^T Q_{3i}^T] x(t - \tau_1)$$

$$+ 2 x^T(t)[-Q_{1i} + A_{0j}^T Q_{4i}^T] \dot{x}(t) + 2 x^T(t)[P_i C_j + Q_{1i} C_j + A_{0j}^T Q_{5i}^T] \dot{x}(t - \tau_2)$$

$$- x^T(t - \tau_2) S_i x(t - \tau_2) + 2 x^T(t - \tau_2) Q_{2i} A_{1j}(t - \tau_1) - 2 x^T(t - \tau_2) Q_{2i} \dot{x}(t)$$

$$+ x^T(t - \tau_2) Q_{2i} \dot{x}(t - \tau_2) + x^T(t - \tau_1)[-H_i - K_i + Q_{3i} A_{1j} + A_{1j}^T Q_{3i}^T] x(t - \tau_1)$$

$$+ 2 x^T(t - \tau_1)(A_{1j}^T Q_{4i}^T - Q_{3i}) \dot{x}(t) + 2 x^T(t - \tau_1)(A_{1j}^T Q_{5i}^T + Q_{3i} C_j) \dot{x}(t - \tau_2)$$

$$+ \dot{x}^T(t)(T_i + \tau_1^2 H_i - Q_{4i} - Q_{4i}^T) \dot{x}(t) + 2 \dot{x}^T(t)(-Q_{5i}^T + Q_{4i} C_j) \dot{x}(t - \tau_2)$$

$$+ \dot{x}^T(t - \tau_2)(-T_i + Q_{5i} C_j + C_j^T Q_{5i}^T) \dot{x}(t - \tau_2) \Big) \Big]$$

$$= \sum_{i=1}^p \xi_i^2 \Big[x^T(t)[P_i A_{0i} + A_{0i}^T P_i + S_i - H_i + K_i + Q_{1i} A_{0i} + A_i^T Q_{1i}^T] x(t)$$

$$- 2 x^T(t) A_{0i}^T Q_{2i}^T x(t - \tau_2) + 2 x^T(t)[P_i A_{1i} + Q_{1i} A_{1i} + A_{0i}^T Q_{3i}^T] x(t - \tau_1)$$

$$+ 2 x^T(t)[-Q_{1i} + A_{0i}^T Q_{4i}^T] \dot{x}(t) + 2 x^T(t)[P_i C_i + Q_{1i} C_i + A_{0i}^T Q_{5i}^T] \dot{x}(t - \tau_2)$$

$$- x^T(t - \tau_2) S_i x(t - \tau_2) + 2 x^T(t - \tau_2) Q_{2i} A_{1i}(t - \tau_1) - 2 x^T(t - \tau_2) Q_{2i} \dot{x}(t)$$

$$+ x^T(t - \tau_2) Q_{2i} \dot{x}(t - \tau_2) + x^T(t - \tau_1)[-H_i - K_i + Q_{3i} A_{1i} + A_{1i}^T Q_{3i}^T] x(t - \tau_1)$$

$$+ 2 x^T(t - \tau_1))(A_{1i}^T Q_{4i}^T - Q_{3i}) \dot{x}(t) + 2 x^T(t - \tau_1)(A_{1i}^T Q_{5i}^T + Q_{3i} C_i) \dot{x}(t - \tau_2)$$

$$+ \dot{x}^T(t)(T_i + \tau_1^2 H_i - Q_{4i} - Q_{4i}^T) \dot{x}(t) + 2 \dot{x}^T(t)(-Q_{5i}^T + Q_{4i} C_i) \dot{x}(t - \tau_2)$$

$$+ \dot{x}^T(t - \tau_2)(-T_i + Q_{5i} C_i + C_i^T Q_{5i}^T) \dot{x}(t - \tau_2) \Big]$$

$$+ \sum_{i=1}^p \sum_{j=i+1}^{p-1} \xi_i \xi_j \Big[\Big(x^T(t)[P_i A_{0j} + A_{0j}^T P_i + S_i - H_i + K_i + Q_{1i} A_{0j} + A_j^T Q_{1i}^T] x(t)$$

$$- 2x^T(t)A_j^T Q_{2i}^T x(t - \tau_2) + 2x^T(t)[P_i A_{1j} + Q_{1i} A_{1j} + A_{0j}^T Q_{3i}^T]x(t - \tau_1)$$
$$+ 2x^T(t)[-Q_{1i} + A_{0j}^T Q_{4i}^T]\dot{x}(t) + 2x^T(t)[P_i C_j + Q_{1i} C_j + A_j^T Q_{5i}^T]\dot{x}(t - \tau_2)$$
$$- x^T(t - \tau_2)S_i x(t - \tau_2) + 2x^T(t - \tau_2)Q_{2i} A_{1j}(t - \tau_1) - 2x^T(t - \tau_2)Q_{2i}\dot{x}(t)$$
$$+ x^T(t - \tau_2)Q_{2i}\dot{x}(t - \tau_2) + x^T(t - \tau_1)[-H_i - K_i + Q_{3i} A_{1j} + A_{1j}^T Q_{3i}^T]x(t - \tau_1)$$
$$+ 2x^T(t - \tau_1)(B_j^T Q_{4i}^T - Q_{3i})\dot{x}(t) + 2x^T(t - \tau_1)(B_j^T Q_{5i}^T + Q_{3i} C_j)\dot{x}(t - \tau_2)$$
$$+ \dot{x}^T(t)(T_i + \tau_1^2 H_i - Q_{4i} - Q_{4i}^T)\dot{x}(t) + 2\dot{x}^T(t)(-Q_{5i}^T + Q_{4i} C_j)\dot{x}(t - \tau_2)$$
$$+ \dot{x}^T(t - \tau_2)(-T_i + Q_{5i} C_j + C_j^T Q_{5i}^T)\dot{x}(t - \tau_2)\Big)$$
$$+ \Big(x^T(t)[P_j A_{0i} + A_{0i}^T P_j + S_j - H_j + K_j + Q_{1j} A_{0i} + A_{0i}^T Q_{1j}^T]x(t)$$
$$- 2x^T(t)A_{0i}^T Q_{2j}^T x(t - \tau_2) + 2x^T(t)[P_j A_{1i} + Q_{1j} A_{1i} + A_{0i}^T Q_{3j}^T]x(t - \tau_1)$$
$$+ 2x^T(t)[-Q_{1j} + A_{0i}^T Q_{4j}^T]\dot{x}(t) + 2x^T(t)[P_i C_i + Q_{1j} C_i + A_{0i}^T Q_{5j}^T]\dot{x}(t - \tau_2)$$
$$- x^T(t - \tau_2)S_j x(t - \tau_2) + 2x^T(t - \tau_2)Q_{2j} A_{1i}(t - \tau_1(t)) - 2x^T(t - \tau_2)Q_{2j}\dot{x}(t)$$
$$+ x^T(t - \tau_2)Q_{2j}\dot{x}(t - \tau_2) + x^T(t - \tau_1)[-H_j - K_j + Q_{3j} A_{1i} + A_{1i}^T Q_{3j}^T]x(t - \tau_1)$$
$$+ 2x^T(t - \tau_1))(A_{1i}^T Q_{4j}^T - Q_{3j})\dot{x}(t) + 2x^T(t - \tau_1)(A_{1i}^T Q_{5j}^T + Q_{3j} C_i)\dot{x}(t - \tau_2)$$
$$+ \dot{x}^T(t)(T_j + \tau_1^2 H_j - Q_{4j} - Q_{4j}^T)\dot{x}(t) + 2\dot{x}^T(t)(-Q_{5j}^T + Q_{4j} C_i)\dot{x}(t - \tau_2)$$
$$+ \dot{x}^T(t - \tau_2)(-T_j + Q_{5j} C_i + C_i^T Q_{5j}^T)\dot{x}(t - \tau_2)\Big)\Big].$$

Therefore,

$$\dot{V}(t, x_t) \leq \eta^T(t)\Big[\sum_{i=1}^{p} \xi_i^2 \Xi_i(P_i, T_i, H_i, S_i, K_i, \mathcal{Q}_i)$$
$$+ \sum_{i=1}^{p-1}\sum_{j=i+1}^{p} \xi_i \xi_j \Big(\Xi_i(P_j, T_j, H_j, S_j, K_j, \mathcal{Q}_j) + \Xi_j(P_i, T_i, H_i, S_i, K_i, \mathcal{Q}_i)\Big)\Big]\eta(t),$$

where $\eta^T(t) = [\ x^T(t)\ \ x^T(t - \tau_2)\ \ x^T(t - \tau_1)\ \ \dot{x}^T(t)\ \ \dot{x}^T(t - \tau_2)]$.

From the conditions (3.1), we can choose a small positive number δ such that

$$\Xi_i(P_i, T_i, H_i, S_i, K_i, \mathcal{Q}_i) < -\mathbb{M} - \delta I, \quad i = 1, \dots, p,$$

and hence

$$(3.9)\quad \dot{V}(t, x_t) \leqslant \eta^T(t)\Big(-\sum_{i=1}^{p} \xi_i^2 + \frac{2}{p - 1}\sum_{i=1}^{p-1}\sum_{j=i+1}^{p} \xi_i \xi_j\Big)\mathbb{M}\eta(t) - \delta\left(\sum_{i=1}^{p} \xi_i^2\right)\|\eta(t)\|^2$$

Observe that

$$(p - 1)\sum_{i=1}^{p} \xi_i^2 - 2\sum_{i=1}^{p-1}\sum_{j=i+1}^{p} \xi_i \xi_j = \sum_{i=1}^{p-1}\sum_{j=i+1}^{p} (\xi_i - \xi_j)^2 \geqslant 0,$$

and

$$1 = \left(\sum_{i=1}^{p} \xi_i\right)^2 \leqslant p\left(\sum_{i=1}^{p} \xi_i^2\right),$$

from (3.9) we obtain

$$\dot{V}(t, x_t) < -\frac{\delta}{p}||\eta(t)||^2 \leq -\frac{\delta}{p}||x(t)||^2.$$

Since $||C_i|| < 1$, $i = 1, 2, \ldots, p$, by Proposition 2.1, the zero solution of system (2.1) is asymptotically stable. This completes the proof of the theorem. $\square$

Remark 3.1. It is worth noting that the condition (3.1) means the asymptotic stability of each ith−subsystem, while the condition (3.2) implies the asymptotic stability of the ij^{th}−subsystems and if $p = 1$ this condition is automatically removed. Thus, Theorem 3.1 includes the result of [3, 8, 12] for the neutral systems without polytope type uncertainties ($p = 1$) and of [5, 10] for polytopic time-delay nominal systems ($C = 0$).

An example. Consider system (2.1), where $p = 3, \tau_1 = 3.294, \tau_2 = 2.325$ and

$$A_{01} = \begin{bmatrix} -0.2 & 0 \\ 0 & -0.09 \end{bmatrix}, \quad A_{02} = \begin{bmatrix} -2 & -1 \\ 0 & -1 \end{bmatrix}, \quad A_{03} = \begin{bmatrix} -1.9 & 0 \\ 0 & -1 \end{bmatrix},$$

$$A_{11} = \begin{bmatrix} -0.1 & 0 \\ -0.1 & -0.1 \end{bmatrix}, \quad A_{12} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix},$$

$$A_{13} = \begin{bmatrix} -0.9 & 0 \\ -1 & -1.1 \end{bmatrix}, C_1 = C_2 = C_3 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}.$$

Using Matlab's LMI Toolbox, the LMIs (3.1), (3.2) are feasible with

$$P_1 = 10^3 \times \begin{pmatrix} 1.7286 & 0.0713 \\ 0.0713 & 0.1044 \end{pmatrix}, \quad P_2 = 10^3 \times \begin{pmatrix} 1.7286 & 0.0713 \\ 0.0713 & 0.1044 \end{pmatrix},$$

$$P_3 = 10^3 \times \begin{pmatrix} 1.7286 & 0.0713 \\ 0.0713 & 0.1044 \end{pmatrix}, \quad T_1 = \begin{pmatrix} 340.9984 & -4.0051 \\ -4.0051 & 32.0171 \end{pmatrix},$$

$$T_2 = \begin{pmatrix} 131.1557 & 93.4328 \\ 93.4328 & 267.9493 \end{pmatrix}, \quad T_3 = \begin{pmatrix} 565.8401 & 14.7002 \\ 14.7002 & 14.2196 \end{pmatrix},$$

$$S_1 = \begin{pmatrix} 10.9018 & 0.5682 \\ 0.5682 & 0.3740 \end{pmatrix}, \quad S_2 = \begin{pmatrix} 102.6679 & 27.7092 \\ 27.7092 & 12.5271 \end{pmatrix},$$

$$S_3 = \begin{pmatrix} 22.0684 & -0.0155 \\ -0.0155 & 0.0003 \end{pmatrix}, \quad K_1 = \begin{pmatrix} 126.6476 & 34.8895 \\ 34.8895 & 11.6732 \end{pmatrix},$$

$$K_2 = 10^3 \times \begin{pmatrix} 1.9646 & 0.8557 \\ 0.8557 & 1.0108 \end{pmatrix}, \quad K_3 = 10^3 \times \begin{pmatrix} 5.5205 & 0.1423 \\ 0.1423 & 0.0823 \end{pmatrix},$$

$$H_1 = \begin{pmatrix} 62.9216 & -8.3560 \\ -8.3560 & 6.5904 \end{pmatrix}, \quad H_2 = \begin{pmatrix} 2.5767 & -2.1348 \\ -2.1348 & 61.3053 \end{pmatrix},$$

$$H_3 = \begin{pmatrix} 75.1692 & 2.4741 \\ 2.4741 & 5.0096 \end{pmatrix}, \quad Q_{11} = \begin{pmatrix} -229.3060 & -251.1666 \\ 122.7765 & 43.9443 \end{pmatrix},$$

$$Q_{12} = \begin{pmatrix} -77.2918 & 484.2981 \\ -944.5658 & -629.9611 \end{pmatrix}, \quad Q_{13} = 10^3 \times \begin{pmatrix} -1.2092 & -0.3569 \\ 0.1922 & -0.0225 \end{pmatrix},$$

$$Q_{21} = Q_{22} = Q_{23} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix},$$

$$Q_{31} = \begin{pmatrix} 316.9512 & 58.0430 \\ 95.2396 & 43.3029 \end{pmatrix}, \quad Q_{32} = \begin{pmatrix} 581.9968 & 407.6488 \\ 327.0204 & -158.0781 \end{pmatrix},$$

$$Q_{33} = \begin{pmatrix} 793.9240 & -145.8548 \\ 208.6832 & -32.9185 \end{pmatrix}, \quad Q_{41} = \begin{pmatrix} 967.8075 & 30.9110 \\ 250.1379 & 191.3986 \end{pmatrix},$$

$$Q_{42} = \begin{pmatrix} 388.0444 & 454.6886 \\ -248.6346 & 604.5765 \end{pmatrix}, \quad Q_{43} = \begin{pmatrix} 938.8036 & -177.1645 \\ 224.9416 & 45.2952 \end{pmatrix},$$

$$Q_{51} = \begin{pmatrix} -17.6502 & -52.4232 \\ 2.8598 & -21.0811 \end{pmatrix}, \quad Q_{52} = \begin{pmatrix} -54.0249 & -36.5439 \\ -24.1424 & 14.0410 \end{pmatrix},$$

$$Q_{53} = \begin{pmatrix} -76.2772 & -18.1835 \\ -23.1784 & -11.2059 \end{pmatrix}, \quad M = \begin{pmatrix} 19.7472 & -0.0147 \\ -0.0147 & 0.0002 \end{pmatrix}.$$

By Theorem 3.1 the zero solution of the system is asymptotically stable.

4. CONCLUSIONS

In this paper we have presented delay-dependent sufficient conditions for the asymptotic stability of linear state-delayed neutral systems with polytope type uncertainties. The conditions are established in terms of Mondie-Kharitonov type's LMI conditions.

REFERENCES

[1] R. P. Agarwal, S. R. Grace, Asymptotic stability of certain neutral differential equations, *Math. Comput. Model.,* 31: 9–15, 2000.

[2] R. P. Agarwal, T. Diagana, H. M. Eduardo, Weighted pseudo almost periodic solutions to some partial neutral functional differential equations. *J. Nonlinear Convex Anal.,* 8: 397–415, 2007.

[3] D. Q. Cao and P. He, Sufficient conditions for stability of linear neutral systems with a single delays, *Appl. Math. Lett.,* 17: 139–144, 2004.

[4] K. Gu, An integral inequality in the stability problem of time-delay systems, In: *Proc. of 39th IEEE CDC,* Sydney, Australia, December, 2805–2810, 2000.

[5] Y. He, M. Wu, J.H. She and G.P. Liu, Parameter-Dependent Lyapunov Functional for Stability of Time-delay systems with Polytopic-type Uncertainties, *IEEE Trans. Automat.Contr.,* 49: 828–832, 2003.

[6] Kolmanovskii, V. B., and A. Myshkis,*Applied Theory of Functional Differential Equations*, Kluwer Academic Publishers, Dordrecht, Netherland,1992.

[7] S. Mondie and V. L. Kharitonov, Exponential estimates for retarded time-delay systems: An LMI approach. *IEEE Trans. Aut. Contr.,* 50: 268–273, 2005.

[8] P. T. Nam , V. N. Phat, An improved stability criterion for a class of neutral differential equations. *Appl. Math. Lett.,* 22: 31–35, 2009.

[9] A. G. Spark, Analysis of affinely parameter-varying systems using parameter dependent Lyapunov functions, In: *Proc. of 36th IEEE CDC,* California, USA, December, 990–991, 1997.

[10] V. N. Phat , P. T. Nam, Exponential stability and stabilization of uncertain linear time-varying systems using parameter dependent Lyapunov function, *Int. J. of Contr.*, 8: 1333–1341, 2007.

[11] V. N. Phat and J. Y. Park, On the Gronwall's inequality and stability of nonlinear discrete-time systems with multiple delays. *Dynamic Systems and Applications*, 1: 577–588, 2001.

[12] Y. Sun, L. Wang, Note on asymptotic stability of a class of neutral differential equations, *Appl. Math. Lett.,* 19: 949–953, 2006.