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PERIODIC ORBITS OF QUADRATIC POLYNOMIALS OF PERIODS SIX AND SEVEN

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ABSTRACT: The dynamics of quadratic polynomials is commonly studied by using the family of maps $f_c(x) = x^2 + c$, where $c \in \mathbb{C}$. In this paper we form equations of periodic orbits of periods six and seven on a new (u, v)-plane and consider also the corresponding equations on the (x, y)-plane. The new (u, v)-model produces equations for the periods six and seven with significantly lower degree than the ones obtained by the previous models which enables us to find their solutions.

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1. INTRODUCTION

The dynamics of quadratic polynomials is often studied by using the family of maps $f_c(x) = x^2 + c$, where $c \in \mathbb{C}$ and $x_{i+1} = f_c(x_i) = x_i^2 + c$ (see, for example, [1], [3] and [7]). In this work a central role is played by the periodic orbits or cycles of f_c for which $f_c^n(x_0) = x_0$ for some $n \in \mathbb{N}$. Now the number n is the period of the orbit and $x_0, x_1, x_2, \dots x_{n-1}$ are periodic points of period n.

In this paper, we obtain the equations for the periodic orbits 6 and 7 of the family f_c by iterating the function (2.1) below and forming the corresponding iterating system on the (u, v)-plane by using a model introduced in [5] (see Section 2). The function G in (2.1) is a two-dimensional quadratic polynomial map which is defined in the complex 2-space $\mathbb{C}^2$ and its iteration reveals the dynamics of f_c. We will also compare the (u, v)-plane model to the (x, y)-plane model, which was introduced by

Erkama in [4]. The purpose of developing this new model in [5] was that using it we obtain equations of periodic orbits of lower degree, which are easier to handle. The solutions of the periods 6 and 7 obtained in the current paper is a good example of the usefulness of this method.

When x is the periodic point of the period n in the (x, c)-plane, then $f_c^n(x) = x$. Correspondingly, when (u, v) is the periodic point of the period n in the (u, v)-plane, then $G^n(u, v) = (u, v)$. The one significant difference between these previous two models is that the (u, v)-plane model describes information about both the parameter and dynamical spaces for quadratic polynomial dynamics in a single dynamical system on $\mathbb{C}^2$. Also Erkama's model ([4]) can be used to study the dynamics of f_c very effectively. In the same way, when (x, y) is the periodic point of the period n in the (x, y)-plane, then $F^n(x, y) = (x, y)$, where $F(x, y) = (y, y^2 + y - x^2)$.

Previously the equation of period four orbits has been presented in the literature before the article [5] twice; T. Erkama [4] presented nearly the same form as in [5] at 2006 and Morton [6] presented a corresponding solution first, but in a more complicated form at 1998. In the article [5] we obtained equations of period $1 - 5$ orbits on the (u, v)-plane by iterating the function G. This equation of the period five orbits in the article [5] is the first explicit presentation of the model with two variables. In the articles [1] and [6] Brown and Morton have also studied period five and six cases and they have formed the so called *trace* formulas in both cases using c and the sum of period cycle points as parameters. Respectively E. V. Flynn, Bjorn Poonen and Edward F. Schaefer have studied the same parameter formula in the case of period five in the article [3]. In Sections 3 and 4 we present first explicit equations of period six and seven orbits on the new (u, v)-plane. Moreover, we present all the central critical points and their real cycles of period six and seven cases. We see the curves of period six orbits on the (u, v)-plane in the Figure 2 and on the (x, y)-plane in the Figure 1. The curves of the period six orbits and curves of eigenvalues $|\lambda_6(x, y)| = 1$ and $|\lambda_6(u, v)| = 1$ are presented in the Figures 3 and 4.

2. THE (U, V)-MODEL

In the article [5] we studied the dynamics of quadratic polynomials by iterating the function

$$G(u, v) = \left(\frac{-u + v + uv}{u}, \frac{u^2 - u + v - u^2 v - uv + uv^2 + v^2}{u} \right), \tag{2.1}$$

and forming corresponding iterating system on the (u, v)-plane. We obtained a new model by using the change of variables

$$\begin{cases} u = x + y \\ v = x + y^2 + y - x^2 \end{cases} \tag{2.2}$$

to the (x, y)-plane model ([4]), when

$$x = \frac{u^2 + u - v}{2u}, \qquad y = \frac{u^2 - u + v}{2u}. \tag{2.3}$$

The new iteration system was defined as follows:

$$\begin{aligned} G(u, v) &= (R(u, v), Q(u, v)) \\ &= \Big(\frac{-u + v + uv}{u}, \frac{u^2 - u + v - u^2 v - uv + uv^2 + v^2}{u} \Big), \end{aligned}$$

when recursively

$$\begin{cases} (R_0(u, v), Q_0(u, v)) = (u, v), \\ \\ (R_{n+1}(u, v), Q_{n+1}(u, v)) = G(R_n(u, v), Q_n(u, v)), \end{cases} \tag{2.4}$$

where

$$\begin{cases} R_{n+1}(u, v) = Q_n(u, v) - 1 + Q_n(u, v)/R_n(u, v) \\ \\ Q_{n+1}(u, v) = R_{n+1}(u, v)(1 + Q_n(u, v) - R_n(u, v)), \end{cases} \tag{2.5}$$

with $n \in \mathbb{N} \cup \{0\}$.

Now (u, v) is fixed G^n, so $G^n(u, v) = (u, v)$, if and only if $(R_n(u, v), Q_n(u, v))=(u, v)$. The set of such points is the union of all orbits, whose period divides n, and the set of periodic points of period n are the points with exact period dividing n. As mentioned in the introduction, the main motivation to develop the new (u, v)-model is that the equations of periodic orbits obtained through it are of significantly lower degree than before.

When we solve the pair of equations (2.5) to higher periods, the situation gets complicated very fast due to the growth of degree in the equations. The following result from the article [5], Theorem 3.1, is very useful in sections $3 - 4$ below.

Theorem 2.1. *Let $n \in \mathbb{N} \setminus \{1\}$. Any point of exact period $n + 1$ satisfies*

$$\begin{cases} R_0 R_1 R_2 \cdots R_n = & 1 \\ Q_0 Q_1 Q_2 \cdots Q_n = & 1. \end{cases} \tag{2.6}$$

When we denote $T_{n+1}(u, v) = 1 + Q_n(u, v) - R_n(u, v)$ the eigenvalue $\lambda_n(u, v)$ of the periodic point of the period n by function $G^n(u, v)$ is

$$\lambda_n(u, v) = u\Big(\frac{\partial R_n}{\partial u} \frac{\partial T_n}{\partial v} - \frac{\partial R_n}{\partial v} \frac{\partial T_n}{\partial u} \Big), \tag{2.7}$$

where $n = 1, 2, \ldots$. On the (u, v)-plane fixed points and periodic points are classified in the following way:

Definition 2.2. Let us assume that $(u,v) \in \mathbb{C}^2$ is a periodic point of period n of the function $G(u,v)$. In that case (u,v) is

1. attracting, if $0 < |\lambda_n(u,v)| < 1$,
2. super-attracting or critical, if $\lambda_n(u,v) = 0$,
3. repulsive, if $|\lambda_n(u,v)| > 1$,
4. indifferent (neutral), if $|\lambda_n(u,v)| = 1$,

when $n \in \mathbb{N}$.

3. PERIODIC ORBITS OF PERIOD SIX

By (2.5) periodic orbits of the period six satisfy the pair of equations

$$\begin{cases} R_6(u,v) = & u \\ Q_6(u,v) = & v. \end{cases} \tag{3.1}$$

By Theorem 2.1 we also know that the pair of equations

$$\begin{cases} R_0R_1R_2R_3R_4R_5 = & 1 \\ Q_0Q_1Q_2Q_3Q_4Q_5 = & 1 \end{cases} \tag{3.2}$$

satisfies the equation of the period six orbits. Therefore, the equation

$$1 = R_0R_1R_2R_3R_4R_5 \tag{3.3}$$

includes now orbits of the period six. When we calculate formulas R_1, R_2, R_3, R_4 and R_5 by using (2.4) and then substitute these formulas to the equation (3.3) and factorize the result, we obtain

$$\frac{(1+u)(uv+1+v)P_6L_6}{u^{15}} = 0, \tag{3.4}$$

where

$$\begin{aligned} P_6 \quad = \quad & a_{13}u^{13} + a_{12}u^{12} + a_{11}u^{11} + a_{10}u^{10} + a_9u^9 + a_8u^8 + a_7u^7 \\ & + a_6u^6 + a_5u^5 + a_4u^4 + a_3u^3 + a_2u^2 + a_1u + a_0, \end{aligned} \tag{3.5}$$

$$\begin{aligned}
a_{13} &= -4\,v^4 + v^3 - 4\,v^6 + v^7 + 6\,v^5 \\
a_{12} &= v^2 - 7\,v^8 - 3\,v^4 - 4\,v^3 + 30\,v^5 + 30\,v^7 - 47\,v^6 \\
a_{11} &= -18\,v^5 - 15\,v^3 + v^2 + 174\,v^7 + 46\,v^4 - 107\,v^6 + 21\,v^9 - 102\,v^8 \\
a_{10} &= v - 35\,v^{10} + 8\,v^3 - 7\,v^2 + 206\,v^9 - 402\,v^8 + 120\,v^6 + 251\,v^7 - 208\,v^5 + 66\,v^4
\end{aligned}$$

$$
\begin{aligned}
a_9 &= -270\,v^{10} + 47\,v^3 + 546\,v^6 + 640\,v^9 - 310\,v^7 - 449\,v^8 - 160\,v^5 - 73\,v^4 - v \\
&\quad -5\,v^2 + 35\,v^{11} \\
a_8 &= -726\,v^{10} - 145\,v^4 + 10\,v^2 + 645\,v^9 + 278\,v^6 + 452\,v^8 - 21\,v^{12} + 7\,v^3 - 964\,v^7 \\
&\quad -v + 234\,v^{11} + 231\,v^5 \\
a_7 &= 2\,v^4 + v^2 + 278\,v^5 + 3\,v - 1 - 33\,v^3 - 729\,v^{10} - 417\,v^6 - 130\,v^{12} + 582\,v^{11} \\
&\quad +7\,v^{13} - 378\,v^9 + 1218\,v^8 - 401\,v^7 \\
a_6 &= -314\,v^{12} - 16\,v^5 - 4\,v^3 - 1116\,v^9 + 493\,v^8 - 4\,v^2 + 63\,v^4 + 118\,v^{10} + 488\,v^7 \\
&\quad -371\,v^6 - v^{14} + 42\,v^{13} + 613\,v^{11} \\
a_5 &= -489\,v^9 + 8\,v^4 - 355\,v^{12} + 720\,v^{10} + 3\,v^3 - 77\,v^5 + 106\,v^{11} - 6\,v^{14} + 102\,v^{13} \\
&\quad -384\,v^8 + 20\,v^6 + 360\,v^7 \\
a_4 &= 125\,v^{13} - 296\,v^{11} - 257\,v^8 - 15\,v^{14} + 62\,v^6 - 13\,v^7 + 198\,v^9 - v^4 - 148\,v^{12} \\
&\quad +365\,v^{10} - 8\,v^5 \\
a_3 &= -191\,v^{11} + 136\,v^9 - 20\,v^{14} + 7\,v^8 - 33\,v^7 + 52\,v^{12} + 4\,v^6 + 75\,v^{13} - 56\,v^{10} \\
a_2 &= 12\,v^{13} + 62\,v^{12} + 11\,v^8 - 54\,v^{10} - 15\,v^{14} - 2\,v^{11} - v^7 - 5\,v^9 \\
a_1 &= 16\,v^{11} + 9\,v^{12} + 3\,v^{10} - 6\,v^{14} - 8\,v^{13} - 2\,v^9 \\
a_0 &= -v^{14} - 3\,v^{12} - 3\,v^{13} - v^{11},
\end{aligned}
$$

and

$$
\begin{aligned}
L_6 &= b_{16}u^{16} + b_{15}u^{15} + b_{14}u^{14} + b_{13}u^{13} + b_{12}u^{12} + b_{11}u^{11} + b_{10}u^{10} \\
&\quad +b_9u^9 + b_8u^8 + b_7u^7 + b_6u^6 + b_5u^5 + b_4u^4 + b_3u^3 + b_2u^2 + b_1u + b_0
\end{aligned} \tag{3.6}
$$

where

$$
\begin{aligned}
b_{16} &= v^4 - 4\,v^5 + v^8 + 6\,v^6 - 4\,v^7 \\
b_{15} &= 36\,v^8 - 8\,v^9 + 2\,v^3 - 6\,v^5 - 60\,v^7 - 6\,v^4 + 42\,v^6 \\
b_{14} &= v^2 - 31\,v^4 + 2\,v^6 - 148\,v^9 - 212\,v^7 + 28\,v^{10} + 78\,v^5 + 282\,v^8 \\
b_{13} &= -440\,v^6 + 66\,v^7 + 694\,v^8 - 56\,v^{11} + 196\,v^5 + 364\,v^{10} + 3\,v^2 - 24\,v^3 - 820\,v^9 \\
&\quad +17\,v^4 \\
b_{12} &= 1472\,v^7 - 138\,v^5 - 201\,v^8 + 1630\,v^{10} - 748\,v^6 - 28\,v^3 - 588\,v^{11} - 1648\,v^9 \\
&\quad +70\,v^{12} + 183\,v^4 + v - 5\,v^2 \\
b_{11} &= 2012\,v^7 - 2304\,v^{11} - 3288\,v^8 - 15\,v^2 + 53\,v^3 + 127\,v^4 - 753\,v^5 + 2954\,v^{10} \\
&\quad +490\,v^6 + 136\,v^9 + 644\,v^{12} - 56\,v^{13} \\
b_{10} &= 558\,v^{10} - 392\,v^5 + 5142\,v^9 + 5\,v^2 - 3\,v - 4067\,v^8 - 238\,v^4 - 476\,v^{13} + 28\,v^{14} \\
&\quad -958\,v^7 + 1997\,v^6 - 4008\,v^{11} + 84\,v^3 + 2330\,v^{12} \\
b_9 &= 19\,v^2 - 5640\,v^{10} - 23\,v^3 - 1806\,v^{11} + 6284\,v^9 - 3696\,v^7 + 981\,v^8 + 922\,v^6
\end{aligned}
$$

$$
\begin{aligned}
& \quad +228\,v^{14} + 4042\,v^{12} + 621\,v^{5} - 8\,v^{15} - 1652\,v^{13} - 276\,v^{4} \\
b_8 &= -2924\,v^{13} + 604\,v^{5} + 4104\,v^{11} + 45\,v^{4} + v^{16} + 3\,v - 64\,v^{15} + 2720\,v^{12} \\
& \quad -66\,v^{9} + 4938\,v^{8} + 780\,v^{14} - 1 - 1684\,v^{7} - 7358\,v^{10} - 60\,v^{3} - 1050\,v^{6} \\
b_7 &= -1384\,v^{10} + 117\,v^{4} - 6\,v^{2} + 8\,v^{16} - 5\,v^{3} - 42\,v^{5} - 2498\,v^{13} + 2356\,v^{8} \\
& \quad -220\,v^{15} - 1562\,v^{12} - 4783\,v^{9} - 927\,v^{6} + 6352\,v^{11} + 1428\,v^{14} + 1198\,v^{7} \\
b_6 &= 28\,v^{16} + 15\,v^{4} + 3265\,v^{10} - 232\,v^{13} - 152\,v^{5} - 3822\,v^{12} + 1024\,v^{7} - 420\,v^{15} \\
& \quad +4\,v^{6} - 2486\,v^{9} - 919\,v^{8} + 2262\,v^{11} + 1434\,v^{14} + 7\,v^{3} \\
b_5 &= -1416\,v^{11} + 442\,v^{9} + 137\,v^{6} - 1980\,v^{12} - 4\,v^{4} + 1956\,v^{10} - 19\,v^{5} + 35\,v^{7} \\
& \quad +624\,v^{14} + 1398\,v^{13} + 56\,v^{16} - 476\,v^{15} - 825\,v^{8} \\
b_4 &= 1048\,v^{13} + 490\,v^{9} - 1128\,v^{11} - 41\,v^{8} + 70\,v^{16} - 88\,v^{10} + v^{5} + 223\,v^{12} \\
& \quad -162\,v^{14} - 88\,v^{7} + 13\,v^{6} - 308\,v^{15} \\
b_3 &= -5\,v^{7} + 454\,v^{12} - 300\,v^{14} - 84\,v^{15} + 21\,v^{9} + 56\,v^{16} - 48\,v^{11} - 220\,v^{10} \\
& \quad +40\,v^{8} + 146\,v^{13} \\
b_2 &= 54\,v^{12} - 102\,v^{14} + v^{8} + 78\,v^{11} - 3\,v^{10} + 28\,v^{16} + 20\,v^{15} - 108\,v^{13} - 12\,v^{9} \\
b_1 &= -26\,v^{13} - 2\,v^{11} - 22\,v^{12} + 4\,v^{14} + 20\,v^{15} + 2\,v^{10} + 8\,v^{16} \\
b_0 &= u + v^{16} + 4\,v^{15} + 6\,v^{14} + 4\,v^{13} + v^{12}.
\end{aligned}
$$

We see that (4.8) includes also the periodic orbits of period two and period three. From these the factor $P_6 = 0$ satisfies the equation of period six orbits (Figure 2), because it is satisfied also by period three orbits in the bifurcation points (see [5]). This curve is of degree thirteen with respect to variable u and of degree fourteen with respect to variable v. Respectively on the (x, y)-plane the value of degree of the period six equation is 26 as for both factors. We obtain this equation of period six by using formulas (2.2) to the earlier expression P_6. It is of the form

$$
\begin{aligned}
& c_{27}x^{27} + c_{26}x^{26} + c_{25}x^{25} + c_{24}x^{24} + c_{23}x^{23} + c_{22}x^{22} + c_{21}x^{21} + c_{20}x^{20} + c_{19}x^{19} \\
& +c_{18}x^{18} + c_{17}x^{17} + c_{16}x^{16} + c_{15}x^{15} + c_{14}x^{14} + c_{13}x^{13} + c_{12}x^{12} + c_{11}x^{11} + c_{10}x^{10} \\
& +c_{9}x^{9} + c_{8}x^{8} + c_{7}x^{7} + c_{6}x^{6} + c_{5}x^{5} + c_{4}x^{4} + c_{3}x^{3} + c_{2}x^{2} + c_{1}x + c_{0} = 0,
\end{aligned}
\tag{3.7}
$$

where

$$
\begin{aligned}
c_{27} &= 1 \\
c_{26} &= -y - 1 \\
c_{25} &= -6 - 13\,y^{2} - 12\,y \\
c_{24} &= 25\,y^{2} + 18\,y + 7 + 13\,y^{3} \\
c_{23} &= 14 + 78\,y^{4} + 66\,y + 144\,y^{3} + 132\,y^{2} \\
c_{22} &= -222\,y^{4} - 276\,y^{3} - 78\,y^{5} - 92\,y - 210\,y^{2} - 20
\end{aligned}
$$

$$
\begin{aligned}
c_{21} &= -18 - 886\,y^3 - 1056\,y^4 - 454\,y^2 - 792\,y^5 - 286\,y^6 - 140\,y \\
c_{20} &= 286\,y^7 + 2008\,y^4 + 1848\,y^5 + 1078\,y^6 + 224\,y + 720\,y^2 + 1472\,y^3 + 33 \\
c_{19} &= 2640\,y^7 + 2120\,y^3 + 4010\,y^4 + 17 + 4620\,y^6 + 5230\,y^5 + 752\,y^2 + 715\,y^8 \\
&\quad +162\,y \\
c_{18} &= -40 - 7060\,y^4 - 3355\,y^8 - 7260\,y^7 - 10070\,y^6 - 1364\,y^2 - 715\,y^9 \\
&\quad -329\,y - 9900\,y^5 - 3692\,y^3 \\
c_{17} &= -12870\,y^8 - 11 - 5940\,y^9 - 17970\,y^6 - 1287\,y^{10} - 2674\,y^3 - 12972\,y^5 \\
&\quad -737\,y^2 - 136\,y - 18090\,y^7 - 6918\,y^4 \\
c_{16} &= 7227\,y^{10} + 31455\,y^8 + 34632\,y^6 + 1692\,y^2 + 355\,y + 13207\,y^4 + 5481\,y^3 \\
&\quad +24300\,y^5 + 38040\,y^7 + 18810\,y^9 + 1287\,y^{11} + 38 \\
c_{15} &= 2152\,y^3 + 40980\,y^9 + 76\,y + 24552\,y^{10} + 1716\,y^{12} + 44256\,y^7 + 16232\,y^5 \\
&\quad +9504\,y^{11} + 30496\,y^6 + 2 + 6692\,y^4 + 49260\,y^8 + 512\,y^2 \\
c_{14} &= -15528\,y^4 - 1556\,y^2 - 11220\,y^{12} - 61248\,y^6 - 298\,y - 34056\,y^{11} - 66324\,y^{10} \\
&\quad -102696\,y^8 - 1716\,y^{13} - 34276\,y^5 - 94200\,y^9 - 26 - 5624\,y^3 - 88432\,y^7 \\
c_{13} &= -12088\,y^5 - 64092\,y^{11} - 52360\,y^7 - 33264\,y^{12} - 1716\,y^{14} - 79156\,y^8 \\
&\quad -4284\,y^4 - 214\,y^2 - 11088\,y^{13} - 1 - 1172\,y^3 - 89628\,y^{10} - 95256\,y^9 \\
&\quad -8\,y - 27828\,y^6 \\
c_{12} &= 1090\,y^2 + 98280\,y^{12} + 183\,y + 114292\,y^7 + 12964\,y^4 + 65768\,y^6 + 44352\,y^{13} \\
&\quad +31940\,y^5 + 166586\,y^8 + 4294\,y^3 + 17 + 12804\,y^{14} + 200256\,y^{10} + 159264\,y^{11} \\
&\quad +1716\,y^{15} + 201712\,y^9 \\
c_{11} &= 102620\,y^9 + 32472\,y^{14} + 15760\,y^6 + 70812\,y^{13} + 1 + 112308\,y^{12} + 1706\,y^4 \\
&\quad +9504\,y^{15} + 65566\,y^8 + 136752\,y^{11} + 131712\,y^{10} + 10\,y^2 + 5884\,y^5 + 296\,y^3 \\
&\quad +34952\,y^7 + 1287\,y^{16} \\
c_{10} &= -21282\,y^5 - 93608\,y^7 - 7986\,y^4 - 231438\,y^9 - 48068\,y^6 - 289352\,y^{10} - 12 \\
&\quad -2430\,y^3 - 103\,y - 188664\,y^{13} - 1287\,y^{17} - 10791\,y^{16} - 266952\,y^{12} \\
&\quad -158232\,y^8 - 305256\,y^{11} - 586\,y^2 - 104076\,y^{14} - 41976\,y^{15} \\
c_9 &= -97860\,y^{14} - 31690\,y^8 - 5940\,y^{17} - 59240\,y^9 - 145292\,y^{12} + 2 - 129052\,y^{11} \\
&\quad +16\,y^3 - 22770\,y^{16} - 1488\,y^5 - 95182\,y^{10} - 14500\,y^7 - 5454\,y^6 - 133560\,y^{13} \\
&\quad -55380\,y^{15} + 11\,y^2 - 179\,y^4 - 715\,y^{18} \\
c_8 &= 5 + 58\,y + 6655\,y^{18} + 28710\,y^{17} + 3603\,y^4 + 280\,y^2 + 246000\,y^{14} + 1099\,y^3 \\
&\quad +332094\,y^{12} + 52494\,y^7 + 97680\,y^8 + 24874\,y^6 + 300254\,y^{11} + 312872\,y^{13} \\
&\quad +10221\,y^5 + 78645\,y^{16} + 160930\,y^9 + 715\,y^{19} + 157200\,y^{15} + 234720\,y^{10} \\
c_7 &= 88032\,y^{15} - 32\,y^5 + 61496\,y^{11} + 636\,y^6 + 106208\,y^{14} + 58470\,y^{16} - 32\,y^3
\end{aligned}
$$

$$
\begin{aligned}
&+37536\,y^{10}+2640\,y^{19}+105224\,y^{13}+286\,y^{20}+19940\,y^{9}+11220\,y^{18}\\
&-16\,y^{2}-10\,y+3152\,y^{7}+8966\,y^{8}+30090\,y^{17}+87252\,y^{12}-70\,y^{4}\\
c_6 \;=\;&-114\,y^{2}-154692\,y^{16}-40526\,y^{8}-20\,y-13860\,y^{19}-8988\,y^{6}-3\\
&-116988\,y^{10}-41530\,y^{18}-251872\,y^{14}-20244\,y^{7}-3606\,y^{5}-286\,y^{21}\\
&-215600\,y^{15}-424\,y^{3}-252116\,y^{13}-2926\,y^{20}-169480\,y^{11}-1304\,y^{4}\\
&-219832\,y^{12}-90540\,y^{17}-72606\,y^{9}\\
c_5 \;=\;&-919\,y^{8}+16\,y^{2}-3368\,y^{9}+52\,y^{4}-53992\,y^{15}-22910\,y^{18}-15676\,y^{11}\\
&+38\,y^{3}+96\,y^{5}-49626\,y^{16}+1-3696\,y^{20}-49412\,y^{14}-26252\,y^{12}\\
&-10830\,y^{19}-78\,y^{22}-38632\,y^{13}-8066\,y^{10}-792\,y^{21}-37596\,y^{17}+134\,y^{6}\\
c_4 \;=\;&63536\,y^{18}+33\,y^{2}+4488\,y^{21}+78\,y^{23}+36390\,y^{10}+116\,y^{3}+20961\,y^{9}\\
&+5254\,y^{7}+57522\,y^{11}+368\,y^{4}+34400\,y^{19}+14592\,y^{20}+972\,y^{5}\\
&+127128\,y^{14}+121813\,y^{16}+95952\,y^{17}+10951\,y^{8}+870\,y^{22}+9\,y+1\\
&+2352\,y^{6}+82716\,y^{12}+132996\,y^{15}+108004\,y^{13}\\
c_3 \;=\;&-1-2\,y-2\,y^{2}+13\,y^{24}+144\,y^{23}+732\,y^{22}+2326\,y^{21}+5314\,y^{20}+9416\,y^{19}\\
&+13456\,y^{18}+15890\,y^{17}-32\,y^{4}-8\,y^{3}-107\,y^{8}-96\,y^{7}-64\,y^{6}-46\,y^{5}\\
&+602\,y^{10}+32\,y^{9}+3790\,y^{12}+1800\,y^{11}+15833\,y^{16}+13528\,y^{15}+10064\,y^{14}\\
&+6612\,y^{13}\\
c_2 \;=\;&1-3\,y-8\,y^{2}-13\,y^{25}-157\,y^{24}-876\,y^{23}-3070\,y^{22}-7772\,y^{21}-15396\,y^{20}\\
&-24972\,y^{19}-34116\,y^{18}-40177\,y^{17}-65\,y^{4}-26\,y^{3}-1852\,y^{8}-904\,y^{7}\\
&-172\,y^{5}-6522\,y^{10}-3571\,y^{9}-17126\,y^{12}-11022\,y^{11}-41568\,y^{16}\\
&-32044\,y^{14}-24470\,y^{13}-414\,y^{6}-38360\,y^{15}\\
c_1 \;=\;&-1+2\,y+y^{2}-y^{26}-12\,y^{25}-66\,y^{24}-226\,y^{23}-554\,y^{22}-1052\,y^{21}+8\,y^{5}\\
&-2050\,y^{19}-2201\,y^{18}-2032\,y^{17}+y^{4}+2\,y^{3}+26\,y^{8}+18\,y^{7}+16\,y^{6}\\
&+31\,y^{10}+32\,y^{9}-149\,y^{12}-16\,y^{11}-1635\,y^{16}-1164\,y^{15}-730\,y^{14}-384\,y^{13}\\
&-1614\,y^{20}\\
c_0 \;=\;&1-y+y^{2}+y^{27}+13\,y^{26}+78\,y^{25}+293\,y^{24}+792\,y^{23}+1672\,y^{22}+3\,y^{3}\\
&+2892\,y^{21}+4219\,y^{20}+5313\,y^{19}+5892\,y^{18}+5843\,y^{17}+7\,y^{4}+927\,y^{11}\\
&+155\,y^{8}+76\,y^{7}+35\,y^{6}+17\,y^{5}+536\,y^{10}+298\,y^{9}+1525\,y^{12}+2331\,y^{13}\\
&+5258\,y^{16}+4346\,y^{15}+3310\,y^{14}.
\end{aligned}
$$

Curves of the period six (3.7) are presented on the (x, y)-plane in Figure 1. The eigenvalue of the period six by (2.7) on the (u, v)-plane is

$$\lambda_6(u,v) = u\Big(\frac{\partial R_6}{\partial u}\frac{\partial T_6}{\partial v} - \frac{\partial R_6}{\partial v}\frac{\partial T_6}{\partial u}\Big), \tag{3.8}$$

where $T_6(u,v) = 1 + Q_5(u,v) - R_5(u,v)$. We obtain

$$\lambda_6(u,v) = \frac{a_6 b_6 c_6 d_6 e_6 f_6}{u^{10}}, \tag{3.9}$$

where

$$\begin{aligned}
a_6 &= u^2 + u - v \\
b_6 &= u^2 - u + v \\
c_6 &= -2\,vu - v + u + u^2 \\
d_6 &= -2\,v^2u - 2\,v^2 - v + 2\,vu + u + 2\,vu^2 - u^2 \\
e_6 &= -2\,vu^4 + 4\,vu^2 + 6\,v^2u^3 + vu - 4\,v^2u + 4\,v^4u - 8\,v^3u^2 - 2\,v^3u + 2\,v^4 \\
&\quad +2\,u^4v^2 + 2\,u^2v^4 - 4\,u^3v^3 - u^2 - u^3 + 2\,v^3 \\
f_6 &= vu^3 - 4\,vu^4 + 8\,v^2u^3 + 62\,v^5u^3 + 8\,v^8u + 2\,v^4u - 6\,v^3u^2 - 30\,v^6u^2 + 4\,v^7u \\
&\quad -12\,v^6u + 2\,v^6 + 20\,u^2v^5 - 22\,u^3v^4 - 6\,uv^5 + 2\,u^8v^2 - 2\,u^7v^2 + 12\,u^6v^3 \\
&\quad +46\,u^5v^3 - 70\,u^4v^4 + 2\,u^6v - 4\,u^5v + 12\,u^4v^2 + 12\,u^2v^4 + 2\,u^7v - 16\,u^6v^2 \\
&\quad -8\,u^5v^2 + 16\,u^4v^3 - 16\,u^3v^3 - 8\,u^5v^7 + 2\,u^4v^8 + 8\,u^3v^8 + 2\,u^8v^4 - 20\,u^2v^7 \\
&\quad -8\,u^7v^5 + 12\,u^6v^6 - 4\,u^8v^3 + 20\,u^7v^4 - 44\,u^6v^5 + 52\,u^5v^6 - 32\,u^4v^7 \\
&\quad -12\,u^7v^3 + 34\,u^6v^4 - 60\,u^5v^5 + 68\,u^4v^6 - 44\,u^3v^7 + 12\,u^2v^8 - 22\,u^5v^4 \\
&\quad +12\,u^4v^5 + 12\,u^3v^6 - u^4 + u^5 + 4\,v^7 + 2\,v^8.
\end{aligned}$$

The bifurcation points are produced by the pair of equations

$$\begin{cases} P_6(u,v) &= 0 \\ |\lambda_6(u,v)| &= 1, \end{cases}$$

for which the Singular program ([2]) gives altogether 520 solutions with multiplicity. In the Figure 4 we see real bifurcation points by the intersection points of the period six and eigenvalue curves $|\lambda_6(u,v)| = 1$. The corresponding points on the (x,y)-plane are depicted in Figure 3. We obtain all critical points of the period six by solving

$$\begin{cases} P_6(u,v) &= 0 \\ |\lambda_6(u,v)| &= 0, \end{cases} \tag{3.10}$$

which has 260 solutions by Singular. Because $v = u^2 + u$ is the critical curve on the (u,v)-plane (see [5]), we obtain the central critical points from the pair of equations

$$\begin{cases} P_6(u,v) = 0 \\ v = u^2 + u, \end{cases}$$

which is equivalent with the equation

$$-u^{34} - 13\,u^{33} - 78\,u^{32} - 293\,u^{31} - 792\,u^{30} - 1672\,u^{29} - 2892\,u^{28} - 4219\,u^{27}$$

$$-5313\,u^{26} - 5892\,u^{25} - 5843\,u^{24} - 5258\,u^{23} - 4346\,u^{22} - 3310\,u^{21} - 2331\,u^{20}$$
$$-1525\,u^{19} - 927\,u^{18} - 536\,u^{17} - 298\,u^{16} - 155\,u^{15} - 76\,u^{14} - 35\,u^{13} - 17\,u^{12}$$
$$-7\,u^{11} - 3\,u^{10} - u^{9} + u^{8} - u^{7} = 0.$$

The central critical point is a critical point of the critical cycle, which is located on the critical curve. So by calculating central critical points first, we obtain other points of the cycle by iterating. A great advantage of this method is that the system of equations is of significantly lower degree than system (3.10). Since $u = 0$ is the point of singularity (see [5], Theorem 2.1), the real central critical points by a numerical approximation of six decimals are (see [8] Table p.106 or [1])

$$\begin{aligned}
(u,v) &\approx (-1.476015, 0.702605),\\
(u,v) &\approx (-1.772893, 1.370256),\\
(u,v) &\approx (-1.907280, 1.730437),\\
(u,v) &\approx (-1.966773, 1.901424),\\
(u,v) &\approx (-1.996376, 1.989142).
\end{aligned}$$

Table 3.1 includes these points with their cycles. These points has been ordered according to the action of the map so the reader can easily see the orbits. The complex solutions are

$$\begin{aligned}
(u,v) \approx &(0.389007 + 0.215851i, 0.493742 + 0.383785i),\\
(u,v) \approx &(0.443326 + 0.372962i, 0.500762 + 0.703650i),\\
(u,v) \approx &(0.396535 + 0.604182i, 0.188739 + 1.083340i),\\
(u,v) \approx &(0.359893 + 0.684762i, 0.020516 + 1.177644i),\\
(u,v) \approx &(-0.015570 + 1.020497i, -1.056743 + .988718i),\\
(u,v) \approx &(-0.113419 + 0.860570i, -0.841135 + 0.665360i),\\
(u,v) \approx &(-0.163598 + 1.097781i, -1.341956 + 0.738591i),\\
(u,v) \approx &(-0.217527 + 1.114454i, -1.412217 + 0.629607i),\\
(u,v) \approx &(-0.596892 + 0.662981i, -0.680155 - 0.128475i),\\
(u,v) \approx &(-1.284085 + 0.427269i, 0.182230 - 0.670030i),\\
(u,v) \approx &(-1.138001 + 0.240332i, 0.099285 - 0.306664i),\\
(u,v) \approx &(-1.138001 - 0.240332i, 0.099285 + 0.306664i)\\
(u,v) \approx &(-1.284085 - 0.427269i, 0.182230 + 0.670030i),\\
(u,v) \approx &(-0.596892 - 0.662981i, -0.680155 + 0.128475i),\\
(u,v) \approx &(-0.217527 - 1.114455i, -1.412217 - 0.629607i),\\
(u,v) \approx &(-0.163598 - 1.097781i, -1.341956 - 0.738591i),\\
(u,v) \approx &(-0.113419 - 0.860569i, -0.841135 - 0.665360i),
\end{aligned}$$

$$\begin{aligned}
(u,v) &\approx (-0.015570 - 1.020497i, -1.056743 - 0.988718i),\\
(u,v) &\approx (0.359893 - 0.684762i, 0.020516 - 1.177644i),\\
(u,v) &\approx (0.396535 - 0.604182i, 0.188739 - 1.083340i),\\
(u,v) &\approx (0.443326 - 0.372962i, 0.500762 - 0.703650i),\\
(u,v) &\approx (0.389007 - 0.2158507i, 0.493742 - 0.383785i).
\end{aligned}$$

There are altogether 162 critical points.

4. PERIODIC ORBITS OF PERIOD SEVEN

By (2.5) periodic points of the period seven satisfy the pair of equations

$$\begin{cases} R_7(u,v) = & u \\ Q_7(u,v) = & v. \end{cases} \tag{4.1}$$

By Theorem 2.1 we now know that the pair of equations

$$\begin{cases} R_0R_1R_2R_3R_4R_5 = & 1 \\ Q_0Q_1Q_2Q_3Q_4Q_5 = & 1 \end{cases} \tag{4.2}$$

must satisfies the equation of the period seven orbits and so the equation

$$1 = R_0R_1R_2R_3R_4R_5R_6 \tag{4.3}$$

includes the orbits of the period seven. After calculating R_1, R_2, R_3, R_4, R_5 and R_6 by using (2.4) we substitute these formulas to the equation (4.3) and factorize it. We obtain

$$\frac{P_7L_7}{u^{31}} = 0, \tag{4.4}$$

where

$$\begin{aligned}
P_7 \quad = \quad & a_{31}u^{31} + a_{30}u^{30} + a_{29}u^{29} + a_{28}u^{28} + a_{27}u^{27} + a_{26}u^{26} + a_{25}u^{25} + a_{24}u^{24} \\
& + a_{23}u^{23} + a_{22}u^{22} + a_{21}u^{21} + a_{20}u^{20} + a_{19}u^{19} + a_{18}u^{18} + a_{17}u^{17} + a_{16}u^{16} \\
& + a_{15}u^{15} + a_{14}u^{14} + a_{13}u^{13} + a_{12}u^{12} + a_{11}u^{11} + a_{10}u^{10} + a_9u^9 + a_8u^8 \\
& + a_7u^7 + a_6u^6 + a_5u^5 + a_4u^4 + a_3u^3 + a_2u^2 + a_1u + a_0,
\end{aligned} \tag{4.5}$$

and

$$\begin{aligned}
L_7 \quad = \quad & b_{32}u^{32} + b_{31}u^{31} + b_{30}u^{30} + b_{29}u^{29} + b_{28}u^{28} + b_{27}u^{27} + b_{26}u^{26} + b_{25}u^{25} \\
& + b_{24}u^{24} + b_{23}u^{23} + b_{22}u^{22} + b_{21}u^{21} + b_{20}u^{20} + b_{19}u^{19} + b_{18}u^{18} + b_{17}u^{17} \\
& + b_{16}u^{16} + b_{15}u^{15} + b_{14}u^{14} + b_{13}u^{13} + b_{12}u^{12} + b_{11}u^{11} + b_{10}u^{10} + b_9u^9
\end{aligned} \tag{4.6}$$

Table 3.1.

Real critical points of period six on the (u,v)-plane.

$(u,v)\approx(-1.4760153, 0.702605)$	$(u,v)\approx(-1.772893, 1.370256)$
$(u,v)\approx(-0.773410, -2.458376)$	$(u,v)\approx(-0.402637, -1.668183)$
$(u,v)\approx(-0.279757, 0.191624)$	$(u,v)\approx(1.474966, -0.391672)$
$(u,v)\approx(-1.493342, -2.197275)$	$(u,v)\approx(-1.657219, 1.436210)$
$(u,v)\approx(-1.725894, -0.510981)$	$(u,v)\approx(-0.430429, -1.761929)$
$(u,v)\approx(-1.214913, -2.690928)$	$(u,v)\approx(1.331500, -0.441393)$
$(u,v)\approx(-1.907280, 1.730437)$	$(u,v)\approx(-1.966773, 1.901424)$
$(u,v)\approx(-0.176843, -0.820147)$	$(u,v)\approx(-0.065350, -0.318134)$
$(u,v)\approx(2.817570, 1.005015)$	$(u,v)\approx(3.550062, 2.652660)$
$(u,v)\approx(0.361711, -0.293910)$	$(u,v)\approx(2.399875, 0.246222)$
$(u,v)\approx(-2.106465, -0.725422)$	$(u,v)\approx(-0.651180, 0.751237)$
$(u,v)\approx(-1.381043, -3.288323)$	$(u,v)\approx(-1.402417, -3.369190)$
$(u,v)\approx -1.996376, 1.989142)$	
$(u,v)\approx(-0.007235, -0.036068)$	
$(u,v)\approx(3.949450, 3.835573)$	
$(u,v)\approx(3.806739, 3.373240)$	
$(u,v)\approx(3.259363, 1.846431)$	
$(u,v)\approx(1.412932, -0.583445)$	

$$+b_8u^8+b_7u^7+b_6u^6+b_5u^5+b_4u^4+b_3u^3+b_2u^2+b_1u+b_0.$$

The explicit forms of the coefficients a_0–a_{31} and b_0–b_{32} are very lengthy and they have been presented in the Appendix.

From these factors $P_7=0$ satisfies the equation of period seven orbits, since, for example, the point $(u,v)=(-1.574889, 0.905387)$ is an approximation of a seven-periodic point under iteration of function $G(u,v)$, and it is also the central critical point of the period seven (see below) so it satisfies the equation $P_7(u,v)=0$. Instead, this point is not a solution of the equation $L_7(u,v)=0$ and hence $L_7(u,v)=0$ cannot be the equation of period seven orbits.

The curve $P_7=0$ is of degree 31 regarding variable u and variable v, and the curve $L_7=0$ is of degree 32 regarding both variables. Respectively on the (x,y)-plane the value of degree of the period seven equation is 63 for both factors and it is of the form

$$c_{63}x^{63}+c_{62}x^{62}+c_{61}x^{61}+c_{60}x^{60}+c_{59}x^{59}+c_{58}x^{58}+c_{57}x^{57}+c_{56}x^{56}+c_{55}x^{55}$$
$$+c_{54}x^{54}+c_{53}x^{53}+c_{52}x^{52}+c_{51}x^{51}+c_{50}x^{50}+c_{49}x^{49}+c_{48}x^{48}+c_{47}x^{47}+c_{46}x^{46}$$

$$
\begin{aligned}
&+c_{45}x^{45}+c_{44}x^{44}+c_{43}x^{43}+c_{42}x^{42}+c_{41}x^{41}+c_{40}x^{40}+c_{39}x^{39}+c_{38}x^{38}+c_{37}x^{37} \\
&+c_{36}x^{36}+c_{35}x^{35}+c_{34}x^{34}+c_{33}x^{33}+c_{32}x^{32}+c_{31}x^{31}+c_{30}x^{30}+c_{29}x^{29}+c_{28}x^{28} \\
&+c_{27}x^{27}+c_{26}x^{26}+c_{25}x^{25}+c_{24}x^{24}+c_{23}x^{23}+c_{22}x^{22}+c_{21}x^{21}+c_{20}x^{20}+c_{19}x^{19} \\
&+c_{18}x^{18}+c_{17}x^{17}+c_{16}x^{16}+c_{15}x^{15}+c_{14}x^{14}+c_{13}x^{13}+c_{12}x^{12}+c_{11}x^{11}+c_{10}x^{10} \\
&+c_9x^9+c_8x^8+c_7x^7+c_6x^6+c_5x^5+c_4x^4+c_3x^3+c_2x^2+c_1x+c_0=0,
\end{aligned} \tag{4.7}
$$

where the coefficients c_0-c_{63} have been presented in the Appendix.

The eigenvalue of the period seven by (2.7) on the (u,v)-plane is

$$\lambda_7(u,v)=\frac{a_7b_7c_7}{u^{18}}, \tag{4.8}$$

where

$$
\begin{aligned}
a_7 &= u^2+u-v \\
b_7 &= u^2-u+v \\
c_7 &= -2\,vu-v+u+u^2 \\
d_7 &= -2\,uv^2-2\,v^2-v+2\,vu+u+2\,u^2v-u^2 \\
e_7 &= 2\,v^3+2\,u^2v^4-4\,u^3v^3+2\,v^4+2\,u^4v^2+6\,u^3v^2-8\,u^2v^3-2\,uv^3-4\,uv^2 \\
&\quad -2\,u^4v-u^3+4\,u^2v-u^2+vu+4\,v^4u \\
f_7 &= 2\,v^6+12\,u^2v^4+2\,u^7v-16\,u^6v^2-8\,u^5v^2+16\,u^4v^3-16\,u^3v^3+u^5+2\,u^6v \\
&\quad -4\,u^5v+12\,u^4v^2+8\,u^3v^2-6\,u^2v^3-4\,u^4v+u^3v-u^4+8\,u^3v^8-8\,u^5v^7 \\
&\quad +2\,u^4v^8+2\,u^8v^4-8\,u^7v^5+12\,u^6v^6+20\,u^7v^4-44\,u^6v^5+52\,u^5v^6-32\,u^4v^7 \\
&\quad -4\,u^8v^3+20\,u^2v^5-22\,u^3v^4-70\,u^4v^4+2\,u^8v^2-2\,u^7v^2+12\,u^6v^3+46\,u^5v^3 \\
&\quad -6\,uv^5-12\,u^7v^3+4\,v^7+2\,v^8+34\,u^6v^4-60\,u^5v^5-22\,u^5v^4+68\,u^4v^6 \\
&\quad +12\,u^4v^5-44\,u^3v^7+12\,u^3v^6+62\,u^3v^5+4\,uv^7-12\,uv^6+12\,v^8u^2+8\,v^8u \\
&\quad +2\,v^4u-20\,u^2v^7-30\,u^2v^6 \\
g_7 &= 2\,v^{12}+2\,v^{16}+8\,v^{15}+8\,v^{13}-u^9+6\,u^8v+u^7v+40\,uv^{15}+6\,u^{13}v^2-56\,u^{12}v^3 \\
&\quad +254\,u^{11}v^4-784\,u^{10}v^5+1844\,u^9v^6-3368\,u^8v^7+4712\,u^7v^8-4972\,u^6v^9 \\
&\quad +3912\,u^5v^{10}-2256\,u^4v^{11}+908\,u^3v^{12}-216\,u^2v^{13}+106\,u^{11}v^3-476\,u^{10}v^4 \\
&\quad +1242\,u^9v^5-2100\,u^8v^6+2396\,u^7v^7-1838\,u^6v^8+884\,u^5v^9-176\,u^4v^{10} \\
&\quad -96\,u^3v^{11}+108\,u^2v^{12}-52\,uv^{13}+4524\,u^6v^{11}+2\,u^{14}v^2-48\,u^{13}v^3 \\
&\quad +366\,u^{12}v^4-1506\,u^{11}v^5+3994\,u^{10}v^6-7392\,u^9v^7+9876\,u^8v^8-9566\,u^7v^9 \\
&\quad +6530\,u^6v^{10}-2832\,u^5v^{11}+446\,u^4v^{12}-10\,u^{12}v^2+34\,u^{13}v^4-276\,u^{12}v^5 \\
&\quad +980\,u^{11}v^6-1916\,u^{10}v^7+1962\,u^9v^8-132\,u^8v^9-2768\,u^7v^{10}-12\,u^{15}v^4 \\
&\quad +156\,u^{14}v^5-880\,u^{13}v^6+2944\,u^{12}v^7-6576\,u^{11}v^8+10284\,u^{10}v^9+4\,u^{15}v^3 \\
&\quad -62\,u^{14}v^4+392\,u^{13}v^5-1496\,u^{12}v^6+4024\,u^{11}v^7-8134\,u^{10}v^8+12568\,u^9v^9
\end{aligned}
$$

$$
\begin{aligned}
&-14716\,u^8v^{10} - u^8 + 12\,v^{14} - 8\,u^{16}v^5 + 84\,u^{15}v^6 - 424\,u^{14}v^7 + 2\,u^{16}v^4\\
&-12\,u^{15}v^5 + 4\,u^{14}v^6 + 132\,u^{13}v^7 - 402\,u^{12}v^8 + 12\,u^{16}v^6 - 16\,u^9v^{15} + 2\,u^8v^{16}\\
&+16\,u^7v^{16} - 112\,u^{13}v^{11} + 140\,u^{12}v^{12} - 112\,u^{11}v^{13} + 56\,u^{10}v^{14} + 1288\,u^{11}v^{12}\\
&-952\,u^{10}v^{13} + 456\,u^9v^{14} - 128\,u^8v^{15} - 3304\,u^9v^{13} + 1560\,u^8v^{14} - 440\,u^7v^{15}\\
&+56\,u^6v^{16} + 2856\,u^7v^{14} - 840\,u^6v^{15} + 112\,u^5v^{16} - 952\,u^5v^{15} + 140\,u^4v^{16}\\
&+112\,u^3v^{16} - 600\,u^3v^{14} + 40\,u^2v^{15} + 2\,u^{16}v^8 - 16\,u^{15}v^9 + 56\,u^{14}v^{10}\\
&+72\,u^{15}v^8 - 296\,u^{14}v^9 + 728\,u^{13}v^{10} - 1176\,u^{12}v^{11} - 1640\,u^{13}v^9 + 3260\,u^{12}v^{10}\\
&-4608\,u^{11}v^{11} + 4660\,u^{10}v^{12} + 5908\,u^{11}v^{10} - 8016\,u^{10}v^{11} + 8084\,u^9v^{12}\\
&-5848\,u^8v^{13} - 3612\,u^9v^{11} + 5440\,u^8v^{12} - 4996\,u^7v^{13} + 2868\,u^6v^{14}\\
&-3124\,u^7v^{12} - 464\,u^6v^{13} + 1248\,u^5v^{14} - 616\,u^4v^{15} + 2796\,u^5v^{13} - 324\,u^4v^{14}\\
&-168\,u^3v^{15} + 56\,u^2v^{16} - 8\,u^{16}v^7 - 120\,u^{15}v^7 + 564\,u^{14}v^8 + 1388\,u^{13}v^8\\
&-3296\,u^{12}v^9 + 272\,u^{11}v^9 + 1116\,u^{10}v^{10} - 11280\,u^9v^{10} + 8208\,u^8v^{11}\\
&+12704\,u^7v^{11} - 7644\,u^6v^{12} - 3960\,u^5v^{12} + 2096\,u^4v^{13} + 292\,u^3v^{13}\\
&-204\,u^2v^{14} + 8\,uv^{14} - 440\,u^3v^{10} + 156\,u^2v^{11} - 44\,uv^{12} + 42\,u^3v^9 - 6\,u^2v^{10}\\
&+80\,u^3v^8 - 24\,u^2v^9 + 4\,uv^{10} - 1854\,u^7v^6 + 2048\,u^6v^7 - 1650\,u^5v^8 + 980\,u^4v^9\\
&+70\,u^5v^7 - 82\,u^4v^8 - 30\,u^{11}v^2 + 168\,u^{10}v^3 - 552\,u^9v^4 + 1208\,u^8v^5 - 46\,u^9v^3\\
&+90\,u^8v^4 - 84\,u^7v^5 + 8\,u^6v^6 + 234\,u^7v^4 - 304\,u^6v^5 + 274\,u^5v^6 - 176\,u^4v^7\\
&+2\,u^{12}v + 10\,u^{10}v^2 + 38\,u^9v^2 - 120\,u^8v^3 - 6\,u^{10}v - 12\,u^7v^2 + 14\,u^6v^3\\
&-4\,uv^{11} - 10\,u^7v^3 + 30\,u^6v^4 - 38\,u^5v^5 - 8\,u^5v^4 + 26\,u^4v^6 + 2\,u^4v^5 - 10\,u^3v^7\\
&+2\,v^8u^2 + 16\,v^{16}u.
\end{aligned}
$$

We obtain the central critical points from the pair of equations

$$
\begin{cases} P_7(u,v) = 0\\ v = u^2 + u,\end{cases}
$$

which is equivalent with the equation

$$
\begin{aligned}
&-u^{78} - 32\,u^{77} - 496\,u^{76} - 4976\,u^{75} - 36440\,u^{74} - 208336\,u^{73} - 971272\,u^{72}\\
&-3807704\,u^{71} - 12843980\,u^{70} - 37945904\,u^{69} - 99582920\,u^{68} - 234813592\,u^{67}\\
&-502196500\,u^{66} - 981900168\,u^{65} - 1766948340\,u^{64} - 2943492972\,u^{63}\\
&-4562339774\,u^{62} - 6609143792\,u^{61} - 8984070856\,u^{60} - 11500901864\,u^{59}\\
&-13910043524\,u^{58} - 15941684776\,u^{57} - 17357937708\,u^{56} - 17999433372\,u^{55}\\
&-17813777994\,u^{54} - 16859410792\,u^{53} - 15286065700\,u^{52} - 13299362332\,u^{51}\\
&-11120136162\,u^{50} - 8948546308\,u^{49} - 6939692682\,u^{48} - 5193067630\,u^{47}\\
&-3754272037\,u^{46} - 2625062128\,u^{45} - 1777171560\,u^{44} - 1166067016\,u^{43}
\end{aligned}
$$

$$-742179284\,u^{42} - 458591432\,u^{41} - 275276716\,u^{40} - 160617860\,u^{39}$$
$$-91143114\,u^{38} - 50323496\,u^{37} - 27049196\,u^{36} - 14162220\,u^{35} - 7228014\,u^{34}$$
$$-3598964\,u^{33} - 1749654\,u^{32} - 831014\,u^{31} - 385741\,u^{30} - 175048\,u^{29}$$
$$-77684\,u^{28} - 33708\,u^{27} - 14290\,u^{26} - 5916\,u^{25} - 2398\,u^{24} - 950\,u^{23} - 365\,u^{22}$$
$$-132\,u^{21} - 42\,u^{20} - 14\,u^{19} - 5\,u^{18} - 2\,u^{17} - u^{16} - u^{15} = 0.$$

Since $u = 0$ is the point of singularity, the real central critical points using a numerical approximation of six decimals are

$$\begin{aligned}
(u, v) &\approx (-1.574889, .905387),\\
(u, v) &\approx (-1.674066, 1.128431),\\
(u, v) &\approx (-1.832315, 1.525064),\\
(u, v) &\approx (-1.884804, 1.667681),\\
(u, v) &\approx (-1.927148, 1.786751),\\
(u, v) &\approx (-1.953706, 1.863261),\\
(u, v) &\approx (-1.977180, 1.932060),\\
(u, v) &\approx (-1.991814, 1.975510),\\
(u, v) &\approx (-1.999096, 1.997288).
\end{aligned}$$

Table 4.1 includes these points with their cycles and they have again been ordered according to the action of the map. The complex solutions are

$$\begin{aligned}
(u, v) \approx &(0.376009 + 0.144749i, 0.496439 + 0.253603i),\\
(u, v) \approx &(0.432376 + 0.226760i, 0.567905 + 0.422851i),\\
(u, v) \approx &(0.456823 + 0.347759i, 0.544575 + 0.665487i),\\
(u, v) \approx &(0.452774 + 0.396170i, 0.500828 + 0.754922i),\\
(u, v) \approx &(0.386539 + 0.569325i, 0.211821 + 1.009457i),\\
(u, v) \approx &(0.412916 + 0.614807i, 0.205428 + 1.122534i),\\
(u, v) \approx &(0.376893 + 0.678569i, 0.058486 + 1.190065i),\\
(u, v) \approx &(0.352483 + 0.698337i, -0.010948 + 1.190641i),\\
(u, v) \approx &(0.121193 + 0.610612i, -0.236966 + 0.758615i),\\
(u, v) \approx &(0.014895 + 0.848149i, -0.704239 + 0.873416i),\\
(u, v) \approx &(-0.006984 + 1.003604i, -1.014156 + 0.989586i),\\
(u, v) \approx &(-0.014233 + 1.032915i, -1.080944 + 1.003511i),\\
(u, v) \approx &(-0.127500 + 0.987461i, -1.086323 + 0.735658i),\\
(u, v) \approx &(-0.157516 + 1.109007i, -1.362600 + 0.759634i),\\
(u, v) \approx &(-0.174578 + 1.071428i, -1.292058 + 0.697332i),\\
(u, v) \approx &(-0.207284 + 1.117481i, -1.413081 + 0.654209i),
\end{aligned}$$

$$
\begin{aligned}
(u,v) \approx& (-0.224916 + 1.116260i, -1.420366 + 0.614131i),\\
(u,v) \approx& (-0.272102 + .842365i, -0.907641 + 0.383946i),\\
(u,v) \approx& (-0.530828 + .668289i, -0.695659 - 0.041204i),\\
(u,v) \approx& (-0.623532 + 0.681064i, -0.698588 - 0.168267i),\\
(u,v) \approx& (-0.622436 + 0.424878i, -0.415531 - 0.104041i),\\
(u,v) \approx& (-1.028194 + 0.361377i, -0.101604 - 0.381754i),\\
(u,v) \approx& (-1.292558 + 0.438199i, 0.186130 - 0.694596i),\\
(u,v) \approx& (-1.262287 + 0.408104i, 0.164533 - 0.622185i),\\
(u,v) \approx& (-1.252736 + 0.342471i, 0.199325 - 0.515580i),\\
(u,v) \approx& (-1.408446 + 0.1361720i, 0.556732 - 0.247410i),\\
(u,v) \approx& (-1.769262 + 0.056920i, 1.357785 - 0.144491i),\\
(u,v) \approx& (-1.769262 - 0.056920i, 1.357785 + 0.144491i),\\
(u,v) \approx& (-1.408446 - 0.136172i, 0.556732 + 0.247410i),\\
(u,v) \approx& (-1.252736 - 0.342471i, 0.199325 + 0.515580i),\\
(u,v) \approx& (-1.262287 - 0.408104i, 0.164533 + 0.622185i),\\
(u,v) \approx& (-1.292558 - 0.438199i, 0.186130 + 0.694596i),\\
(u,v) \approx& (-1.028194 - 0.361377i, -0.101604 + 0.381754i),\\
(u,v) \approx& (-0.622436 - 0.424878i, -0.415531 + 0.104041i),\\
(u,v) \approx& (-0.623532 - 0.681064i, -0.698588 + 0.168267i),\\
(u,v) \approx& (-0.530828 - 0.668289i, -0.695659 + 0.041204i),\\
(u,v) \approx& (-0.272102 - 0.842365i, -0.907641 - 0.383946i),\\
(u,v) \approx& (-0.224916 - 1.116260i, -1.420366 - 0.614131i),\\
(u,v) \approx& (-0.207284 - 1.117481i, -1.413081 - 0.654209i),\\
(u,v) \approx& (-0.174578 - 1.071428i, -1.292058 - 0.697332i),\\
(u,v) \approx& (-0.157516 - 1.109007i, -1.362600 - 0.759634i),\\
(u,v) \approx& (-0.127500 - 0.987461i, -1.086323 - 0.735658i),\\
(u,v) \approx& (-0.014233 - 1.032915i, -1.080944 - 1.003511i),\\
(u,v) \approx& (-0.006984 - 1.003604i, -1.014156 - 0.989586i),\\
(u,v) \approx& (0.014895 - 0.848149i, -0.704239 - 0.873416i),\\
(u,v) \approx& (0.121193 - 0.610612i, -0.236966 - 0.758615i),\\
(u,v) \approx& (0.352483 - 0.698337i, -0.010948 - 1.190641i),\\
(u,v) \approx& (0.376893 - 0.678569i, 0.058486 - 1.190065i),\\
(u,v) \approx& (0.412916 - 0.614807i, 0.205428 - 1.122534i),
\end{aligned}
$$

$$(u,v) \approx (0.386539 - 0.569325i, 0.211821 - 1.009457i),$$
$$(u,v) \approx (0.452774 - 0.396170i, 0.500828 - 0.754922i),$$
$$(u,v) \approx (0.456823 - 0.347759i, 0.544575 - 0.665487i),$$
$$(u,v) \approx (0.432376 - 0.226760i, 0.567905 - 0.422851i),$$
$$(u,v) \approx (0.376009 - 0.144749i, 0.496439 - 0.253603i).$$

There are altogether 441 critical points.

ACKNOWLEDGMENTS

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"The world around us is very complicated.
The tools at our disposal to describe
it are very weak."

Benoit Mandelbrot

Table 4.1.

Real critical points of period seven on the (u, v)-plane.

$(u, v) \approx (-1.574889, .905387)$	$(u, v) \approx (-1.674066, 1.128431)$
$(u, v) \approx (-0.669502, -2.330053)$	$(u, v) \approx (-0.545635, -2.074775)$
$(u, v) \approx (0.150223, -0.099230)$	$(u, v) \approx (0.727722, -0.385067)$
$(u, v) \approx (-1.759780, -1.320799)$	$(u, v) \approx (-1.914207, 0.215902)$
$(u, v) \approx (-1.570252, -2.259562)$	$(u, v) \approx (-0.896887, -2.807355)$
$(u, v) \approx (-1.820581, -0.565635)$	$(u, v) \approx (-0.677246, 0.616611)$
$(u, v) \approx (-1.254946, -2.829835)$	$(u, v) \approx (-1.293857, -2.967923)$
$(u, v) \approx (-1.832315, 1.525064)$	$(u, v) \approx (-1.884804, 1.667681)$
$(u, v) \approx (-0.307251, -1.338811)$	$(u, v) \approx (-0.217123, -0.988447)$
$(u, v) \approx (2.018568, -0.063705)$	$(u, v) \approx (2.564037, 0.586332)$
$(u, v) \approx (-1.095264, 1.185375)$	$(u, v) \approx (-0.184993, 0.180869)$
$(u, v) \approx (-0.896898, -2.942399)$	$(u, v) \approx (-1.796837, -2.454231)$
$(u, v) \approx (-0.661760, 0.691871)$	$(u, v) \approx (-2.088369, -0.715487)$
$(u, v) \approx (-1.353630, -3.185946)$	$(u, v) \approx (-1.372882, -3.257685)$
$(u, v) \approx (-1.927148, 1.786751)$	$(u, v) \approx (-1.953706, 1.863261)$
$(u, v) \approx (-0.140397, -0.661818)$	$(u, v) \approx (-0.090445, -0.435671)$
$(u, v) \approx (3.052081, 1.460663)$	$(u, v) \approx (3.381296, 2.213985)$
$(u, v) \approx (0.939242, -0.555484)$	$(u, v) \approx (1.868759, -0.312663)$
$(u, v) \approx (-2.146902, 1.062130)$	$(u, v) \approx (-1.479974, 1.748475)$
$(u, v) \approx (-0.432597, -1.820814)$	$(u, v) \approx (-0.432948, -1.830698)$
$(u, v) \approx (1.388217, -0.538930)$	$(u, v) \approx (1.397750, -0.555956)$
$(u, v) \approx (-1.977180, 1.932060)$	$(u, v) \approx (-1.991814, 1.975510)$
$(u, v) \approx (-0.045120, -0.221505)$	$(u, v) \approx (-0.016305, -0.080990)$
$(u, v) \approx (3.687734, 3.037273)$	$(u, v) \approx (3.886333, 3.634943)$
$(u, v) \approx (2.860888, 0.999991)$	$(u, v) \approx (3.570257, 2.672728)$
$(u, v) \approx (0.349530, -0.300909)$	$(u, v) \approx (2.421337, 0.248117)$
$(u, v) \approx (-2.161806, -0.755684)$	$(u, v) \approx (-0.649412, 0.761904)$
$(u, v) \approx (-1.406122, -3.383302)$	$(u, v) \approx (-1.411317, -3.403131)$
$(u, v) \approx (-1.999096, 1.997288)$	
$(u, v) \approx (-0.001808, -0.009033)$	
$(u, v) \approx (3.987352, 3.958544)$	
$(u, v) \approx (3.951319, 3.837491)$	
$(u, v) \approx (3.808684, 3.375150)$	
$(u, v) \approx (3.261322, 1.847427)$	
$(u, v) \approx (1.413893, -0.585203)$	

5. APPENDIX

Figures of periodic orbit curves and curves of eigenvalues on the (u, v)- and the (x, y)-plane.

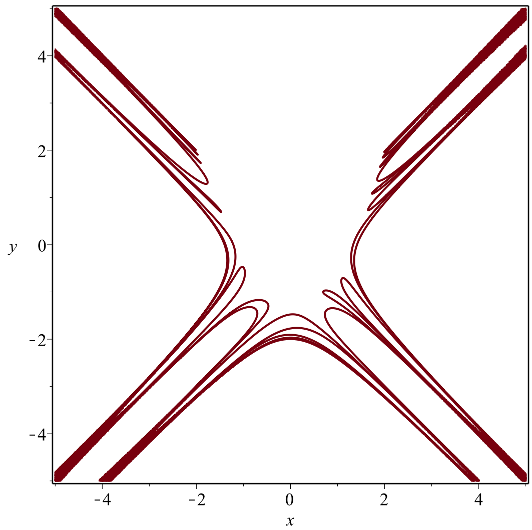

Figure 1: Curves of the period six orbits on the (x, y)-plane.

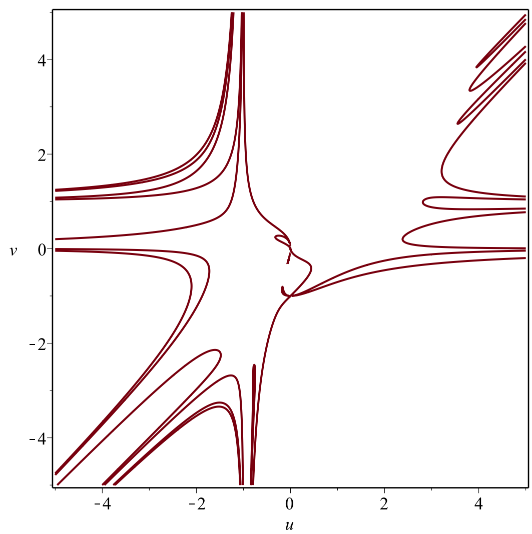

Figure 2: Curves of the period six orbits on the (u, v)-plane.

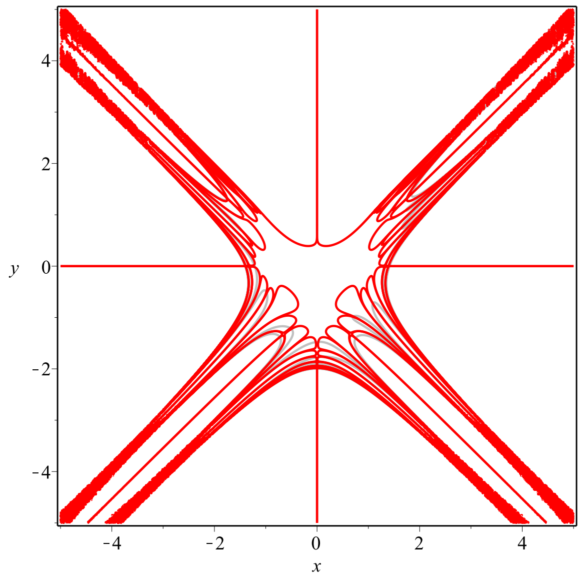

Figure 3: Curves of the period six orbits and eigenvalues $|\lambda_6(x, y)| = 1$.

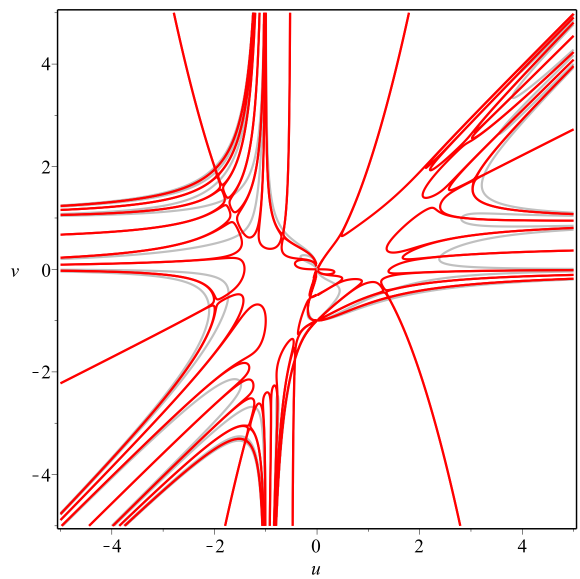

Figure 4: Curves of the period six orbits and eigenvalues $|\lambda_6(u, v)| = 1$.

The coefficients of equations (4.5), (4.6) and (4.7) in period seven case.

$$a_{31} = -8v^{15} - 56v^{11} - 56v^{13} + v^8 - 8v^9 + 70v^{12} + 28v^{14} + v^{16} + 28v^{10}$$

$$a_{30} = 1004v^{14} - 15v^{17} - 1302v^{13} + 128v^{16} - 28v^8 - 448v^{11} - 476v^{15} + 4v^7 + 36v^{10} + 1036v^{12} + 61v^9$$

$$a_{29} = -34v^8 + 3864v^{16} - 1637v^{10} + 105v^{18} + 11742v^{14} - 8668v^{15} + 6v^6 - 968v^{17} + 3170v^{12} + 608v^9 + 1140v^{11} - 9296v^{13} - 32v^7$$

$$a_{28} = -17670v^{12} + 54376v^{14} + 47904v^{16} - 455v^{19} - 202v^7 + 18087v^{11} - 5778v^{10} - 6v^6 - 19888v^{17} - 68446v^{15} + 998v^8 + 4592v^{18} + 4v^5 - 766v^9 - 12750v^{13}$$

$$a_{27} = v^4 + 30066v^{14} + 72660v^{18} - 12988v^9 - 123365v^{12} + 14v^5 + 15492v^{10} - 234920v^{15} - 15288v^{19} + 131486v^{13} - 189920v^{17} + 2405v^8 + 1365v^{20} + 289688v^{16} - 208v^6 + 466v^7 + 33046v^{11}$$

$$a_{26} = 101142v^{10} - 232v^6 - 33638v^{15} - 134550v^{11} - 199836v^{19} - 646494v^{14} + 803096v^{16} - 946624v^{17} + 37856v^{20} - 131968v^{12} - 15309v^9 - 79v^5 + 573388v^{18} + 601165v^{13} - 3003v^{21} - 8502v^8 + 3578v^7 + 10v^4$$

$$a_{25} = 956v^7 + 743982v^{12} - 549110v^{11} + 2v^3 - 1365108v^{19} - 72072v^{21} + 2338382v^{15} - 7540v^{16} + 1857v^6 + 5005v^{22} - 318v^5 + 428064v^{20} + 2475844v^{18} + 80104v^9 - 2262372v^{17} + 61121v^{10} - v^4 - 33331v^8 + 413592v^{13} - 2259057v^{14}$$

$$a_{24} = 5103v^8 + 5370092v^{18} - 6554460v^{16} + 271v^5 - 728728v^{21} - 35140v^{17} + 2260378v^{12} + 3874v^6 + 2617416v^{20} - 21145v^7 + 107536v^{22} - 169477v^{11} + 205721v^9 - 488420v^{10} + 6v^3 - 5296032v^{19} - 1152656v^{14} - 2964152v^{13} + 6846367v^{15} - 6435v^{23} - 119v^4$$

$$a_{23} = -937353v^{10} - 4091208v^{21} + 996996v^{22} - 10845324v^{19} + 829388v^{18} - 77263v^9 + 9030784v^{14} + 149197v^8 - 4989v^6 + 6435v^{24} - 100v^4 + 14631112v^{17} - 18v^3 + 2011v^5 - 7423354v^{13} - 27482v^7 + 9380882v^{20} + 387297v^{12} + 2143092v^{11} + 3157728v^{15} + v^2 - 17180848v^{16} - 126984v^{23}$$

$$a_{22} = 43575v^7 - 3816896v^{19} - 8521504v^{16} - 13841806v^{21} + 18666340v^{20} - 7179724v^{12} + 3365459v^{11} - 17755v^6 - v^2 + 473v^5 + 441131v^{10} - 21712088v^{15} + 136502v^8 + 499v^4 - 49v^3 + 20064244v^{14} - 26321700v^{18} - 5005v^{25} - 1101100v^{23} + 5243260v^{22} - 739474v^9 + 118976v^{24} - 1028735v^{13} + 36205378v^{17}$$

$$a_{21} = 3549145v^{14} + 69v^3 + 10559090v^{20} - 45427268v^{15} + 38113392v^{19} - 64411298v^{18} + 21088408v^{17} - 9909897v^{12} - 5445v^5 - 1577987v^{11} - 5512628v^{23} + 980408v^{24} + 19018372v^{13} - 10v^2 + 2764411v^{10}$$

$$
\begin{aligned}
&- 239348\, v^{8} - 27295632\, v^{21} + 17031938\, v^{22} + 41783680\, v^{16} + 102773\, v^{7} \\
&+ 158\, v^{6} - 527838\, v^{9} + 3003\, v^{26} - 88088\, v^{25} + 592\, v^{4} \\
a_{20} =& - v - 40627714\, v^{14} + 189\, v^{3} - 43601822\, v^{20} - 11838681\, v^{15} + 96634514\, v^{19} \\
&- 44947970\, v^{18} - 64308896\, v^{17} + 3907089\, v^{12} - 3903\, v^{5} - 8136439\, v^{11} \\
&- 17423322\, v^{23} + 4734544\, v^{24} + 24483979\, v^{13} + 2\, v^{2} + 1693084\, v^{10} \\
&- 435680\, v^{8} - 21002066\, v^{21} + 33714296\, v^{22} + 86855316\, v^{16} - 10502\, v^{7} \\
&+ 34857\, v^{6} + 929402\, v^{9} - 1365\, v^{27} + 50960\, v^{26} - 698880\, v^{25} - 991\, v^{4} \\
a_{19} =& - v - 51307375\, v^{14} - 80\, v^{3} - 121452630\, v^{20} + 70570902\, v^{15} + 80029422\, v^{19} \\
&+ 77472658\, v^{18} - 140393194\, v^{17} + 19355098\, v^{12} + 6949\, v^{5} - 4647032\, v^{11} \\
&- 34898416\, v^{23} + 14714308\, v^{24} - 6664985\, v^{13} + 34\, v^{2} - 2709254\, v^{10} \\
&+ 49943\, v^{8} + 37302526\, v^{21} + 32060458\, v^{22} + 32130528\, v^{16} - 153547\, v^{7} \\
&+ 18305\, v^{6} + 1436462\, v^{9} + 455\, v^{28} - 22568\, v^{27} + 393484\, v^{26} - 3290928\, v^{25} \\
&- 1518\, v^{4} \\
a_{18} =& 3\, v + 6538121\, v^{14} - 324\, v^{3} - 117627052\, v^{20} + 91458619\, v^{15} - 68162532\, v^{19} \\
&+ 190984568\, v^{18} - 69036680\, v^{17} + 10997458\, v^{12} + 7652\, v^{5} + 6122512\, v^{11} \\
&- 38641518\, v^{23} + 29968400\, v^{24} - 37782254\, v^{13} + 10\, v^{2} - 3810848\, v^{10} \\
&+ 503980\, v^{8} + 126227076\, v^{21} - 19696926\, v^{22} - 99352564\, v^{16} - 68377\, v^{7} \\
&- 30722\, v^{6} - 116662\, v^{9} - 105\, v^{29} + 7392\, v^{28} - 171108\, v^{27} + 1822964\, v^{26} \\
&- 10141796\, v^{25} + 699\, v^{4} \\
u_{17} =& 3\, v + 60951286\, v^{14} - 36\, v^{3} + 32832036\, v^{20} + 1924473\, v^{15} - 216375728\, v^{19} \\
&+ 118469442\, v^{18} + 111153274\, v^{17} - 10878884\, v^{12} - 3101\, v^{5} + 8294076\, v^{11} \\
&- 1532018\, v^{23} + 37170392\, v^{24} - 22338406\, v^{13} - 39\, v^{2} + 91121\, v^{10} \\
&+ 210743\, v^{8} + 141821436\, v^{21} - 105994786\, v^{22} - 138387138\, v^{16} + 95916\, v^{7} \\
&- 28019\, v^{6} - 1287560\, v^{9} + 15\, v^{30} - 1688\, v^{29} + 55440\, v^{28} - 785740\, v^{27} \\
&+ 5608288\, v^{26} - 21066500\, v^{25} + 1662\, v^{4} \\
a_{16} =& - 3\, v + 38648946\, v^{14} + 199\, v^{3} + 200246592\, v^{20} - 81251972\, v^{15} \\
&- 164024172\, v^{19} - 93395916\, v^{18} + 176716376\, v^{17} - 14970644\, v^{12} - 5715\, v^{5} \\
&+ 355049\, v^{11} + 68706502\, v^{23} + 16524272\, v^{24} + 15158994\, v^{13} - 19\, v^{2} \\
&+ 2625390\, v^{10} - 224065\, v^{8} + 15740502\, v^{21} - 139138104\, v^{22} - 22457228\, v^{16} \\
&+ 79396\, v^{7} + 8563\, v^{6} - 536892\, v^{9} + 240\, v^{30} - 12600\, v^{29} + 253896\, v^{28} \\
&- 2426284\, v^{27} + 11904060\, v^{26} - 28525280\, v^{25} - v^{31} + 111\, v^{4} \\
a_{15} =& - 1 - 3\, v - 16095562\, v^{14} + 65\, v^{3} + 183972136\, v^{20} - 56575654\, v^{15} \\
&+ 48245628\, v^{19} - 188584370\, v^{18} + 51804868\, v^{17} - 1648193\, v^{12} - 391\, v^{5} \\
&- 4334750\, v^{11} + 109427632\, v^{23} - 30628580\, v^{24} + 22534164\, v^{13} + 12\, v^{2} \\
&+ 1125784\, v^{10} - 179075\, v^{8} - 145772816\, v^{21} - 54807110\, v^{22} + 88750297\, v^{16}
\end{aligned}
$$

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\begin{aligned}
&- 15593\, v^{7} + 14455\, v^{6} + 403601\, v^{9} + 1792\, v^{30} - 57840\, v^{29} + 790380\, v^{28}\\
&- 5269460\, v^{27} + 17293612\, v^{26} - 20394332\, v^{25} - 16\, v^{31} - 616\, v^{4}\\
a_{14} =& - 28336650\, v^{14} - 26\, v^{3} + 5316014\, v^{20} + 11764102\, v^{15} + 165450096\, v^{19}\\
&- 78657862\, v^{18} - 77692282\, v^{17} + 5839218\, v^{12} + 1335\, v^{5} - 1940602\, v^{11}\\
&+ 67829118\, v^{23} - 67007248\, v^{24} + 3908723\, v^{13} + 6\, v^{2} - 567876\, v^{10} + 17740\, v^{8}\\
&- 166440628\, v^{21} + 76249250\, v^{22} + 69683704\, v^{16} - 28161\, v^{7} + 1266\, v^{6}\\
&+ 325840\, v^{9} + 8280\, v^{30} - 182168\, v^{29} + 1758120\, v^{28} - 8116388\, v^{27}\\
&+ 15696324\, v^{26} + 5119990\, v^{25} - 120\, v^{31} - 157\, v^{4}\\
a_{13} =& - 6563925\, v^{14} - 7\, v^{3} - 115795986\, v^{20} + 29723798\, v^{15} + 90078860\, v^{19}\\
&+ 51622457\, v^{18} - 71808864\, v^{17} + 2755485\, v^{12} + 281\, v^{5} + 623344\, v^{11}\\
&- 19727526\, v^{23} - 55712752\, v^{24} - 6430862\, v^{13} - 482126\, v^{10} + 43318\, v^{8}\\
&- 43482370\, v^{21} + 119499168\, v^{22} - 3184426\, v^{16} - 3138\, v^{7} - 2137\, v^{6}\\
&- 7274\, v^{9} + 26460\, v^{30} - 415352\, v^{29} + 2847880\, v^{28} - 8548176\, v^{27}\\
&+ 4946900\, v^{26} + 29998092\, v^{25} - 560\, v^{31} + 36\, v^{4}\\
a_{12} =& 5769064\, v^{14} - 80900268\, v^{20} + 8496737\, v^{15} - 21813289\, v^{19} + 61424066\, v^{18}\\
&- 5854568\, v^{17} - 523051\, v^{12} - 39\, v^{5} + 583643\, v^{11} - 65522026\, v^{23}\\
&- 9179068\, v^{24} - 3234021\, v^{13} - 15984\, v^{10} + 5754\, v^{8} + 60497808\, v^{21}\\
&+ 54055608\, v^{22} - 25868010\, v^{16} + 2572\, v^{7} - 385\, v^{6} - 53383\, v^{9} + 61880\, v^{30}\\
&- 703612\, v^{29} + 3329872\, v^{28} - 5220012\, v^{27} - 8158108\, v^{26} + 32823380\, v^{25}\\
&- 1820\, v^{31} + 4\, v^{4}\\
a_{11} =& 3148867\, v^{14} - 295688\, v^{20} - 4164772\, v^{15} - 43037982\, v^{19} + 11396578\, v^{18}\\
&+ 18466340\, v^{17} - 581252\, v^{12} - v^{5} + 40916\, v^{11} - 42476176\, v^{23} - 4368\, v^{31}\\
&+ 24868670\, v^{24} + 314411\, v^{13} + 53257\, v^{10} - 2333\, v^{8} + 57327534\, v^{21}\\
&- 18991186\, v^{22} - 8785970\, v^{16} + 401\, v^{7} + 32\, v^{6} - 7898\, v^{9} + 109200\, v^{30}\\
&- 891072\, v^{29} + 2656290\, v^{28} + 3164\, v^{27} - 13578104\, v^{26} + 14427864\, v^{25}\\
a_{10} =& - 103599\, v^{14} + 24091192\, v^{20} - 2551683\, v^{15} - 11998580\, v^{19} - 10567488\, v^{18}\\
&+ 7386354\, v^{17} - 53468\, v^{12} - 43445\, v^{11} - 1617460\, v^{23} + 23553308\, v^{24}\\
&+ 478825\, v^{13} + 8253\, v^{10} - 310\, v^{8} + 9996760\, v^{21} - 31477458\, v^{22}\\
&+ 2350018\, v^{16} - 18\, v^{7} + 1581\, v^{9} + 147576\, v^{30} - 830258\, v^{29} + 1145144\, v^{28}\\
&+ 3453120\, v^{27} - 9094932\, v^{26} - 4268952\, v^{25} - 8008\, v^{31}\\
a_{9} =& 6328152\, v^{24} - 8991576\, v^{25} + 3439688\, v^{27} + 4601992\, v^{19} - 11440\, v^{31}\\
&+ 29332\, v^{12} - 959216\, v^{17} + 6\, v^{8} - 1678614\, v^{26} - 328098\, v^{14} - 6647\, v^{11}\\
&+ 153010\, v^{30} - 780\, v^{10} - 536536\, v^{29} - 5091762\, v^{18} + 9113936\, v^{20}\\
&- 27735\, v^{15} + 1725350\, v^{16} - 10198304\, v^{21} + 48776\, v^{13} - 211640\, v^{28}\\
&- 10035264\, v^{22} + 12644454\, v^{23} + 172\, v^{9}
\end{aligned}
$$

$$
\begin{aligned}
a_8 =& -16626\,v^{13} - 12870\,v^{31} + 2766896\,v^{22} - 975308\,v^{17} + 1524362\,v^{27} - v^{9}\\
&+ 2885166\,v^{19} + 68316\,v^{16} - 716976\,v^{28} - 1298628\,v^{20} + 4148\,v^{12}\\
&+ 188122\,v^{15} - 5317002\,v^{21} + 1960588\,v^{26} - 192764\,v^{29} - 33468\,v^{14}\\
&- 3074416\,v^{24} + 6162960\,v^{23} + 204116\,v^{18} + 259\,v^{11} + 120120\,v^{30}\\
&- 4245668\,v^{25} - 66\,v^{10}\\
a_7 =& -43\,v^{12} + 31556\,v^{27} + 27632\,v^{21} - 55936\,v^{23} + 21840\,v^{29} + 461494\,v^{18}\\
&- 55754\,v^{17} + 61660\,v^{19} - 90884\,v^{16} - 11440\,v^{31} - 1340044\,v^{20} - 39500\,v^{25}\\
&+ 1612380\,v^{26} + 17632\,v^{15} + 2403422\,v^{22} - 2002\,v^{13} + 8006\,v^{14}\\
&- 2610751\,v^{24} - 516516\,v^{28} + 68640\,v^{30} + 16\,v^{11}\\
a_6 =& 74256\,v^{29} + 738\,v^{14} + 217698\,v^{22} - 168168\,v^{28} - 328580\,v^{27} - 88466\,v^{20}\\
&- 3274\,v^{15} + 505624\,v^{21} - 7042\,v^{16} + 386764\,v^{26} + 25480\,v^{30} + 724351\,v^{25}\\
&- 8008\,v^{31} - 396536\,v^{24} + 30598\,v^{18} - 182902\,v^{19} + 37301\,v^{17} - 817698\,v^{23}\\
&- 6\,v^{13} - 2\,v^{12}\\
a_5 =& -12536\,v^{19} + 1110\,v^{16} - 4368\,v^{31} + 226372\,v^{25} + 3276\,v^{30} + 60820\,v^{20}\\
&- 158480\,v^{27} - 151912\,v^{22} + 45304\,v^{29} - 146870\,v^{23} + 188024\,v^{24} - 202\,v^{15}\\
&- 90615\,v^{26} - 13092\,v^{18} + 51602\,v^{21} + 2002\,v^{17} + 5\,v^{14} + 5208\,v^{28}\\
a_4 =& -17162\,v^{21} + 59508\,v^{24} + 13132\,v^{29} - 17367\,v^{27} - 324\,v^{18} + 26320\,v^{28}\\
&- 67204\,v^{26} - 2520\,v^{30} + 3900\,v^{19} - v^{15} - 1820\,v^{31} - 21112\,v^{22} - 16244\,v^{25}\\
&+ 38\,v^{16} - 296\,v^{17} + 34410\,v^{23} + 3846\,v^{20}\\
a^3 =& 9180\,v^{27} + 6872\,v^{23} + 608\,v^{29} - 4\,v^{17} - 6748\,v^{26} + 8911\,v^{28} - 1840\,v^{30}\\
&+ 4276\,v^{22} - 788\,v^{21} + 56\,v^{18} - 16256\,v^{25} - 4674\,v^{24} - 941\,v^{20} - 12\,v^{19}\\
&- 560\,v^{31}\\
a_2 =& -300\,v^{25} + 552\,v^{28} - 861\,v^{29} - 1892\,v^{24} + 48\,v^{22} - 600\,v^{30} - 998\,v^{23}\\
&+ 2576\,v^{26} - 6\,v^{19} + 20\,v^{20} - 120\,v^{31} + 161\,v^{21} + 2828\,v^{27}\\
a_1 =& 208\,v^{24} + 28\,v^{23} - 28\,v^{27} - 13\,v^{22} - 105\,v^{30} + 420\,v^{25} - 16\,v^{31} + 350\,v^{26}\\
&- 4\,v^{21} - 312\,v^{28} - 272\,v^{29}\\
a_0 =& -8\,v^{30} - 28\,v^{29} - 56\,v^{28} - 70\,v^{27} - 56\,v^{26} - 28\,v^{25} - v^{31} - v^{23} - 8\,v^{24},\\
b_{32} =& 28\,v^{14} - 56\,v^{13} + 70\,v^{12} + 28\,v^{10} - 8\,v^{9} + v^{8} + v^{16} - 56\,v^{11} - 8\,v^{15}\\
b_{31} =& -504\,v^{15} + 44\,v^{10} - 16\,v^{17} + 1092\,v^{12} + 4\,v^{7} - 1372\,v^{13} - 28\,v^{8} - 476\,v^{11}\\
&+ 1060\,v^{14} + 136\,v^{16} + 60\,v^{9}\\
b_{30} =& -32\,v^{7} + 6\,v^{6} + 120\,v^{18} - 1096\,v^{17} + 1104\,v^{11} - 38\,v^{8} - 9672\,v^{15} + 3618\,v^{12}\\
&+ 13044\,v^{14} - 1698\,v^{10} - 10332\,v^{13} + 636\,v^{9} + 4340\,v^{16}\\
b_{29} =& -560\,v^{19} + 19724\,v^{11} + 63672\,v^{14} + 1030\,v^{8} + 5560\,v^{18} - 6386\,v^{10} - 6\,v^{6}\\
&- 18810\,v^{12} - 80188\,v^{15} - 23752\,v^{17} - 732\,v^{9} - 15920\,v^{13} - 208\,v^{7} + 4\,v^{5}\\
&+ 56572\,v^{16}
\end{aligned}
$$

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\begin{aligned}
b_{28} =& 42816\,v^{14} + 472\,v^{7} + 1820\,v^{20} + 2607\,v^{8} - 212\,v^{6} + 16258\,v^{10} - 19880\,v^{19}\\
&+ 14\,v^{5} + 38824\,v^{11} - 141452\,v^{12} + 149156\,v^{13} - 13986\,v^{9} + v^{4} - 289296\,v^{15}\\
&- 237824\,v^{17} + 92548\,v^{18} + 358134\,v^{16}\\
b_{27} =& - 8968\,v^{8} + 53144\,v^{20} + 724530\,v^{13} - 63704\,v^{15} - 246\,v^{6} - 777980\,v^{14}\\
&+ 114130\,v^{10} - 4368\,v^{21} + 10\,v^{4} + 763308\,v^{18} - 165014\,v^{12} - 272496\,v^{19}\\
&- 17714\,v^{9} - 1236312\,v^{17} + 1038016\,v^{16} - 150042\,v^{11} - 80\,v^{5} + 3786\,v^{7}\\
b_{26} =& 1188\,v^{7} + 26098\,v^{16} + 878532\,v^{12} - 328\,v^{5} + 88606\,v^{9} - 109928\,v^{21}\\
&+ 545560\,v^{13} + 2\,v^{3} + 1936\,v^{6} - 1938496\,v^{19} - v^{4} + 3422468\,v^{18}\\
&- 650252\,v^{11} - 2860222\,v^{14} + 627900\,v^{20} + 8008\,v^{22} + 2984876\,v^{15}\\
&+ 76430\,v^{10} - 3065468\,v^{17} - 36909\,v^{8}\\
b_{25} =& - 3708134\,v^{13} - 27600\,v^{17} - 1566248\,v^{14} + 6\,v^{3} + 3982524\,v^{20} + 4147\,v^{8}\\
&+ 179608\,v^{22} + 9105424\,v^{15} - 23002\,v^{7} + 4192\,v^{6} - 121\,v^{4} - 1156792\,v^{21}\\
&+ 272\,v^{5} - 11440\,v^{23} - 568524\,v^{10} - 7771876\,v^{19} + 239052\,v^{9} - 8892842\,v^{16}\\
&- 230598\,v^{11} + 2809488\,v^{12} + 7632464\,v^{18}\\
b_{24} =& 864528\,v^{18} + 4310384\,v^{15} - 82366\,v^{9} - 24027215\,v^{16} - 18\,v^{3} + 12870\,v^{24}\\
&+ 2631512\,v^{11} - 234520\,v^{23} + v^{2} - 5260\,v^{6} + 11994936\,v^{14} + 21185572\,v^{17}\\
&- 106\,v^{4} + 2130\,v^{5} - 6708624\,v^{21} - 16215416\,v^{19} + 1725724\,v^{22}\\
&+ 14676914\,v^{20} - 1143074\,v^{10} - 31356\,v^{7} + 170342\,v^{8} + 556774\,v^{12}\\
&- 9683732\,v^{13}\\
b_{23} =& - 40952812\,v^{18} + 53386226\,v^{17} - 11679232\,v^{16} - 9322816\,v^{12} + 517\,v^{4}\\
&- 4646284\,v^{19} - 50\,v^{3} - 2098096\,v^{23} - 11440\,v^{25} + 245960\,v^{24} - 888671\,v^{9}\\
&+ 9334468\,v^{22} + 27487598\,v^{14} - 30742872\,v^{15} - 1416032\,v^{13} - v^{2} - 19766\,v^{6}\\
&+ 48564\,v^{7} - 23222688\,v^{21} + 518394\,v^{10} + 163984\,v^{8} + 4302812\,v^{11}\\
&+ 29511664\,v^{20} + 573\,v^{5}\\
b_{22} =& - 2019118\,v^{11} - 664340\,v^{9} - 10755888\,v^{23} + 2081508\,v^{24} + 8008\,v^{26}\\
&- 207064\,v^{25} - 13275356\,v^{12} + 70\,v^{3} + 29609912\,v^{17} + 26198096\,v^{13}\\
&+ 4577880\,v^{14} - 65491512\,v^{15} + 63495768\,v^{16} - 100616676\,v^{18}\\
&+ 14375986\,v^{20} - 45961972\,v^{21} + 30873744\,v^{22} - 282923\,v^{8} + 120528\,v^{7}\\
&- 315\,v^{6} - 10\,v^{2} - 5944\,v^{5} + 641\,v^{4} + 3503885\,v^{10} + 64435092\,v^{19}\\
b_{21} =& - v - 10900850\,v^{11} + 1168750\,v^{9} - 34455260\,v^{23} + 10247172\,v^{24} - 4368\,v^{27}\\
&+ 139048\,v^{26} - 1679288\,v^{25} + 5485076\,v^{12} + 199\,v^{3} - 106092576\,v^{17}\\
&+ 34393876\,v^{13} - 59646086\,v^{14} - 15387826\,v^{15} + 132282584\,v^{16} - 4495\,v^{5}\\
&- 66036378\,v^{18} + 161045812\,v^{19} - 81715214\,v^{20} - 31561156\,v^{21} - 1060\,v^{4}\\
&+ 61009928\,v^{22} - 538453\,v^{8} - 10660\,v^{7} + 40302\,v^{6} + 2\,v^{2} + 2220922\,v^{10}\\
b_{20} =& - v - 6340116\,v^{11} + 1872142\,v^{9} - 68612712\,v^{23} + 32137630\,v^{24} + 1820\,v^{28}
\end{aligned}
$$

$$- 73528\,v^{27} + 1092364\,v^{26} - 8025472\,v^{25} + 27491537\,v^{12} - 82\,v^{3}$$
$$- 227248510\,v^{17} - 10572074\,v^{13} - 75791354\,v^{14} + 111198616\,v^{15}$$
$$+ 43969209\,v^{16} + 141781554\,v^{18} + 124977392\,v^{19} - 218087144\,v^{20}$$
$$+ 80904348\,v^{21} + 53062524\,v^{22} + 60445\,v^{8} - 188404\,v^{7} + 22208\,v^{6}$$
$$+ 35\,v^{2} + 7940\,v^{5} - 1707\,v^{4} - 3638656\,v^{10}$$

$$b_{19} = 3\,v + 8831766\,v^{11} - 166605\,v^{9} - 70701976\,v^{23} + 64866816\,v^{24} - 560\,v^{29}$$
$$+ 29960\,v^{28} - 564592\,v^{27} + 5113892\,v^{26} - 24856104\,v^{25} + 15644490\,v^{12}$$
$$- 358\,v^{3} - 101167208\,v^{17} - 57137352\,v^{13} + 13203106\,v^{14} + 142765994\,v^{15}$$
$$- 169923466\,v^{16} + 331377762\,v^{18} - 145635190\,v^{19} - 197656474\,v^{20}$$
$$+ 247679706\,v^{21} - 56999452\,v^{22} + 657527\,v^{8} - 86682\,v^{7} - 37671\,v^{6}$$
$$+ 11\,v^{2} + 9170\,v^{5} + 779\,v^{4} - 5247310\,v^{10}$$

$$b_{18} = 3\,v + 12104924\,v^{11} - 1791540\,v^{9} + 18164908\,v^{23} + 75811910\,v^{24} + 120\,v^{30}$$
$$- 9080\,v^{29} + 226548\,v^{28} - 2608704\,v^{27} + 15750084\,v^{26} - 51034900\,v^{25}$$
$$- 17001396\,v^{12} - 46\,v^{3} + 210505838\,v^{17} - 33335864\,v^{13} + 98733540\,v^{14}$$
$$- 4613648\,v^{15} - 229845757\,v^{16} + 187506122\,v^{18} - 407360296\,v^{19}$$
$$+ 259448488\,v^{21} - 232221862\,v^{22} + 279120\,v^{8} + 126638\,v^{7} - 35671\,v^{6}$$
$$- 3800\,v^{5} + 1986\,v^{4} + 207783\,v^{10} + 100994568\,v^{20} - 42\,v^{2}$$

$$b_{17} = 1 - 3\,v + 263928\,v^{11} - 747635\,v^{9} + 174701288\,v^{23} + 18056290\,v^{24} - 16\,v^{31}$$
$$+ 1928\,v^{30} - 68040\,v^{29} + 1039636\,v^{28} - 8034572\,v^{27} + 32970560\,v^{26}$$
$$- 65695672\,v^{25} - 23264720\,v^{12} + 238\,v^{3} + 315103514\,v^{17} + 26037878\,v^{13}$$
$$+ 60987352\,v^{14} - 142203258\,v^{15} - 24381701\,v^{16} - 204549190\,v^{18}$$
$$- 282493614\,v^{19} + 416622320\,v^{20} - 17091534\,v^{21} - 280959540\,v^{22} + 147\,v^{4}$$
$$- 319981\,v^{8} + 107415\,v^{7} + 11664\,v^{6} - 22\,v^{2} - 7377\,v^{5} + 3912950\,v^{10}$$

$$b_{16} = - 1 - 3\,v - 6960140\,v^{11} + v^{32} + 627666\,v^{9} + 248565736\,v^{23} - 99335082\,v^{24}$$
$$- 256\,v^{31} + 14392\,v^{30} - 311736\,v^{29} + 3216664\,v^{28} - 17173520\,v^{27}$$
$$+ 45818892\,v^{26} - 36918604\,v^{25} - 2003242\,v^{12} + 84\,v^{3} + 74262096\,v^{17}$$
$$+ 37504808\,v^{13} - 31254556\,v^{14} - 95224600\,v^{15} + 170002269\,v^{16}$$
$$+ 141641544\,v^{19} + 347996308\,v^{20} - 346019408\,v^{21} - 70547612\,v^{22}$$
$$- 24156\,v^{7} + 20170\,v^{6} + 15\,v^{2} - 502\,v^{5} - 815\,v^{4} + 1662676\,v^{10}$$
$$- 365300746\,v^{18} - 258471\,v^{8}$$

$$b_{15} = - 3066386\,v^{11} + 16\,v^{32} + 504915\,v^{9} + 122636228\,v^{23} - 176434880\,v^{24}$$
$$- 1912\,v^{31} + 66120\,v^{30} - 972548\,v^{29} + 7027580\,v^{28} - 25410000\,v^{27} - 222\,v^{4}$$
$$+ 36090656\,v^{26} + 35748570\,v^{25} + 10173968\,v^{12} - 38\,v^{3} - 166442579\,v^{17}$$
$$+ 5556916\,v^{13} - 50870814\,v^{14} + 27859664\,v^{15} + 126259358\,v^{16} + 1951\,v^{5}$$
$$- 130462730\,v^{18} + 354034466\,v^{19} - 42929614\,v^{20} - 350412764\,v^{21}$$

$$
\begin{aligned}
&+ 222022066\, v^{22} + 33333\, v^{8} - 42616\, v^{7} + 1657\, v^{6} + 9\, v^{2} - 971477\, v^{10}\\
b_{14} =& 1191220\, v^{11} + 120\, v^{32} - 25014\, v^{9} - 95976776\, v^{23} - 123541870\, v^{24} - 8840\, v^{31}\\
&+ 208628\, v^{30} - 2173472\, v^{29} + 10964268\, v^{28} - 24244500\, v^{27} - 173090\, v^{26}\\
&+ 97005340\, v^{25} + 4696087\, v^{12} - 13\, v^{3} - 141492568\, v^{17} - 12270080\, v^{13}\\
&- 10472648\, v^{14} + 58060448\, v^{15} - 14948528\, v^{16} + 129314739\, v^{18}\\
&- 281246082\, v^{20} - 48798384\, v^{21} + 285939796\, v^{22} + 71479\, v^{8} - 4404\, v^{7}\\
&- 3472\, v^{6} + 438\, v^{5} + 62\, v^{4} - 807966\, v^{10} + 168736722\, v^{19}\\
b_{13} =& 1065769\, v^{11} + 560\, v^{32} - 96701\, v^{9} - 185021194\, v^{23} + 10548458\, v^{24}\\
&- 28280\, v^{31} + 477232\, v^{30} - 3551492\, v^{29} + 11878048\, v^{28} - 10166912\, v^{27}\\
&- 38156200\, v^{26} + 88536132\, v^{25} - 1146395\, v^{12} - 2670142\, v^{17} - 5989506\, v^{13}\\
&+ 12199926\, v^{14} + 15060662\, v^{15} - 55591808\, v^{16} + 133232930\, v^{18}\\
&- 73435746\, v^{19} - 170979128\, v^{20} + 176293794\, v^{21} + 97537978\, v^{22} + 8892\, v^{8}\\
&+ 4709\, v^{7} - 666\, v^{6} - 75\, v^{5} + 11\, v^{4} - 8710\, v^{10}\\
b_{12} =& 56900\, v^{11} + 1820\, v^{32} - 13652\, v^{9} - 96531784\, v^{23} + 90390696\, v^{24}\\
&- 66248\, v^{31} + 812812\, v^{30} - 4220944\, v^{29} + 7876302\, v^{28} + 8161272\, v^{27}\\
&- 46401484\, v^{26} + 23606932\, v^{25} - 1164895\, v^{12} + 44334350\, v^{17} + 837462\, v^{13}\\
&+ 6382888\, v^{14} - 9933836\, v^{15} - 17282707\, v^{16} + 17251146\, v^{18} - 104462048\, v^{19}\\
&+ 21517601\, v^{20} + 138227802\, v^{21} - 79488994\, v^{22} - 4905\, v^{8} + 786\, v^{7} + 71\, v^{6}\\
&- 5\, v^{5} + 106640\, v^{10}\\
b_{11} =& - 96702\, v^{11} + 4368\, v^{32} + 3914\, v^{9} + 17373726\, v^{23} + 66029484\, v^{24} + 16151\, v^{10}\\
&- 117208\, v^{31} + 1038648\, v^{30} - 3486548\, v^{29} + 1141980\, v^{28} + 17031224\, v^{27}\\
&- 23522796\, v^{26} - 29137622\, v^{25} - 94384\, v^{12} + 16172324\, v^{17} + 1060077\, v^{13}\\
&- 418010\, v^{14} - 5700550\, v^{15} + 6514790\, v^{16} - 29033828\, v^{18} - 23395158\, v^{19}\\
&+ 67129174\, v^{20} + 10292448\, v^{21} - 88804992\, v^{22} - 711\, v^{8} - 50\, v^{7} + v^{6}\\
b_{10} =& - 14900\, v^{11} + 8008\, v^{32} + 482\, v^{9} + 44121912\, v^{23} + 7945612\, v^{24} - 159016\, v^{31}\\
&+ 983268\, v^{30} - 1681680\, v^{29} - 3664760\, v^{28} + 12534620\, v^{27} + 2590338\, v^{26}\\
&- 32544884\, v^{25} + 72777\, v^{12} - 3309234\, v^{17} + 102244\, v^{13} - 806923\, v^{14}\\
&+ 75864\, v^{15} + 4277033\, v^{16} - 12478116\, v^{18} + 15169480\, v^{19} + 21112516\, v^{20}\\
&- 34289496\, v^{21} - 20032024\, v^{22} + 24\, v^{8} - 2361\, v^{10}\\
b_{9} =& - 14430938\, v^{21} - 165880\, v^{31} - 15718870\, v^{24} - 82244\, v^{14} - 10573820\, v^{25}\\
&+ 656656\, v^{30} - 45958\, v^{13} - 4156664\, v^{28} + 7976928\, v^{19} + 1039\, v^{11}\\
&- 2700658\, v^{17} + 10952164\, v^{26} + 3202976\, v^{27} + 96051\, v^{16} + 18876\, v^{29}\\
&+ 11440\, v^{32} + 516220\, v^{15} - 5900620\, v^{20} - 7\, v^{9} + 12965200\, v^{22} - 238\, v^{10}\\
&+ 10795\, v^{12} + 16198224\, v^{23} + 1163332\, v^{18}\\
b_{8} =& 51100\, v^{15} + 5858048\, v^{26} - 124070\, v^{17} + 7720424\, v^{22} + 1326260\, v^{21} - 302\, v^{12}
\end{aligned}
$$

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\begin{aligned}
&- 131560\,v^{31} + 24632\,v^{14} + 3034916\,v^{25} + 261404\,v^{30} - 2040878\,v^{28}\\
&+ 1436802\,v^{18} + v^{10} - 4225210\,v^{20} - 1929032\,v^{27} + 738816\,v^{29} - 2822832\,v^{23}\\
&- 142456\,v^{19} + 82\,v^{11} - 279006\,v^{16} + 12870\,v^{32} - 8773711\,v^{24} - 6150\,v^{13}\\
b_7 =&590772\,v^{29} + 1845668\,v^{21} - 3221120\,v^{23} + 3640\,v^{30} + 86352\,v^{18} + 2740\,v^{14}\\
&+ 37\,v^{13} - 340600\,v^{24} - 24674\,v^{16} - 11280\,v^{15} + 128185\,v^{17} + 190066\,v^{22}\\
&- 1940960\,v^{27} - 76648\,v^{31} - 18\,v^{12} + 426264\,v^{26} - 150126\,v^{20} + 11440\,v^{32}\\
&+ 3335102\,v^{25} - 199724\,v^{28} - 644396\,v^{19}\\
b_6 =&243722\,v^{20} - 657536\,v^{22} + 140068\,v^{21} + 4384\,v^{16} - 70980\,v^{30} - 940\,v^{15}\\
&+ 213472\,v^{29} - 814966\,v^{26} + 8008\,v^{32} - 43134\,v^{19} + 1005722\,v^{24} - 50393\,v^{18}\\
&+ 9044\,v^{17} - 545244\,v^{27} - 364568\,v^{23} + 11\,v^{14} + 333788\,v^{28} - 29848\,v^{31}\\
&+ 622908\,v^{25} + 2\,v^{13}\\
b_5 =&184800\,v^{28} - 5096\,v^{31} + 240\,v^{16} - 47824\,v^{30} + 73248\,v^{27} + 186322\,v^{23}\\
&+ 7924\,v^{29} + 16382\,v^{20} - 1406\,v^{17} - 72714\,v^{22} - 204268\,v^{25} + 16992\,v^{19}\\
&+ 4368\,v^{32} - 2326\,v^{18} + 206378\,v^{24} - 77982\,v^{21} - 6\,v^{15} - 293576\,v^{26}\\
b_4 =&26278\,v^{28} - 42\,v^{17} + 1960\,v^{31} - 4634\,v^{21} + 312\,v^{19} - 4841\,v^{20} - 25712\,v^{29}\\
&+ 27984\,v^{23} + 21438\,v^{22} - 75764\,v^{25} + 352\,v^{18} + 76384\,v^{27} - 14972\,v^{30}\\
&+ 9496\,v^{26} + 1820\,v^{32} + v^{16} - 39084\,v^{24}\\
b_3 =&- 62\,v^{19} + 4374\,v^{25} + 4\,v^{18} - 5274\,v^{23} + 836\,v^{22} + 1720\,v^{31} - 9772\,v^{29}\\
&+ 32\,v^{20} - 1208\,v^{30} - 8764\,v^{24} + 9576\,v^{27} + 560\,v^{32} - 8628\,v^{28} + 1102\,v^{21}\\
&+ 18832\,v^{26}\\
b_2 =&- 2604\,v^{27} - 174\,v^{22} + 1206\,v^{24} + 6\,v^{20} - 824\,v^{29} + 650\,v^{26} + 584\,v^{31}\\
&- 3140\,v^{28} - 24\,v^{21} + 2312\,v^{25} + 120\,v^{32} - 20\,v^{23} + 756\,v^{30}\\
b_1 =&4\,v^{22} - 420\,v^{27} + 264\,v^{30} - 476\,v^{26} - 28\,v^{28} + 284\,v^{29} + 104\,v^{31} - 36\,v^{24}\\
&- 236\,v^{25} + 12\,v^{23} + 16\,v^{32}\\
b_0 =&v^{32} + v^{24} + 8\,v^{31} + 28\,v^{30} + 56\,v^{29} + 70\,v^{28} + 56\,v^{27} + 28\,v^{26} + 8\,v^{25},\\
c_{63} =&1\\
c_{62} =&y\\
c_{61} =&- 16 - 32\,y - 31\,y^2\\
c_{60} =&- 16\,y - 32\,y^2 - 31\,y^3\\
c_{59} =&960\,y^2 + 120 + 960\,y^3 + 496\,y + 465\,y^4\\
c_{58} =&496\,y^2 + 960\,y^4 + 465\,y^5 + 120\,y + 960\,y^3\\
c_{57} =&- 20880\,y^4 - 568 - 13920\,y^5 - 3600\,y - 10680\,y^2 - 4495\,y^6 - 18864\,y^3\\
c_{56} =&- 3600\,y^2 - 568\,y - 10680\,y^3 - 18864\,y^4 - 13920\,y^6 - 20880\,y^5 - 4495\,y^7\\
c_{55} =&259840\,y^6 + 1932 + 66416\,y^2 + 16472\,y + 279440\,y^4 + 129920\,y^7 + 31465\,y^8\\
&+ 165760\,y^3 + 326816\,y^5
\end{aligned}
$$

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\begin{aligned}
c_{54} =& 66416\,y^3 + 165760\,y^4 + 31465\,y^9 + 279440\,y^5 + 326816\,y^6 + 259840\,y^7 \\
&+ 1932\,y + 129920\,y^8 + 16472\,y^2 \\
c_{53} =& - 275268\,y^2 - 2192400\,y^8 - 169911\,y^{10} - 3946320\,y^6 - 884680\,y^3 \\
&- 876960\,y^9 - 54096\,y - 1986208\,y^4 - 5096 - 3254496\,y^5 - 3505824\,y^7 \\
c_{52} =& - 169911\,y^{11} - 884680\,y^4 - 3946320\,y^7 - 2192400\,y^9 - 1986208\,y^5 \\
&- 3505824\,y^8 - 3254496\,y^6 - 5096\,y - 275268\,y^3 - 54096\,y^2 - 876960\,y^{10} \\
c_{51} =& 736281\,y^{12} + 19106984\,y^5 + 30702672\,y^6 + 26456976\,y^9 + 13680576\,y^{10} \\
&+ 3279360\,y^3 + 838768\,y^2 + 9140404\,y^4 + 137592\,y + 36658440\,y^8 + 10948 \\
&+ 4560192\,y^{11} + 38228736\,y^7 \\
c_{50} =& 30702672\,y^7 + 38228736\,y^8 + 36658440\,y^9 + 19106984\,y^6 + 736281\,y^{13} \\
&+ 3279360\,y^4 + 838768\,y^3 + 137592\,y^2 + 26456976\,y^{10} + 9140404\,y^5 \\
&+ 4560192\,y^{12} + 10948\,y + 13680576\,y^{11} \\
c_{49} =& - 239798520\,y^7 - 304686200\,y^8 - 30215528\,y^4 - 76260464\,y^5 - 19788 \\
&- 66502800\,y^{12} - 151361756\,y^6 - 150630480\,y^{11} - 2002644\,y^2 - 2629575\,y^{14} \\
&- 309145200\,y^9 - 246846600\,y^{10} - 9144408\,y^3 - 284648\,y - 19000800\,y^{13} \\
c_{48} =& - 309145200\,y^{10} - 151361756\,y^7 - 2002644\,y^3 - 304686200\,y^9 - 284648\,y^2 \\
&- 66502800\,y^{13} - 246846600\,y^{11} - 76260464\,y^6 - 19000800\,y^{14} - 19788\,y \\
&- 150630480\,y^{12} - 2629575\,y^{15} - 239798520\,y^8 - 9144408\,y^4 - 30215528\,y^5 \\
c_{47} =& 7888725\,y^{16} + 1005396736\,y^7 + 527024064\,y^6 + 2034153680\,y^9 \\
&+ 224838432\,y^5 + 20243552\,y^3 + 3911880\,y^2 + 2156030240\,y^{10} \\
&+ 1575645864\,y^8 + 675904320\,y^{13} + 1857273600\,y^{11} + 76666576\,y^4 \\
&+ 65145600\,y^{15} + 494700\,y + 30782 + 1275674400\,y^{12} + 260582400\,y^{14} \\
c_{46} =& 30782\,y + 1275674400\,y^{13} + 20243552\,y^4 + 675904320\,y^{14} + 2034153680\,y^{10} \\
&+ 1005396736\,y^8 + 260582400\,y^{15} + 1857273600\,y^{12} + 527024064\,y^7 \\
&+ 494700\,y^2 + 76666576\,y^5 + 2156030240\,y^{11} + 3911880\,y^3 + 1575645864\,y^9 \\
&+ 65145600\,y^{16} + 224838432\,y^6 + 7888725\,y^{17} \\
c_{45} =& - 41944 - 12651024880\,y^{11} - 11565415680\,y^{12} - 517756288\,y^5 \\
&- 3069905696\,y^7 - 5674585584\,y^8 - 2461942080\,y^{15} - 36935732\,y^3 \\
&- 738768\,y - 8690136000\,y^{13} - 842821200\,y^{16} - 1387485456\,y^6 - 6437842\,y^2 \\
&- 8815886944\,y^9 - 20160075\,y^{18} - 187293600\,y^{17} - 156342280\,y^4 \\
&- 5250679200\,y^{14} - 11527653176\,y^{10} \\
c_{44} =& - 5250679200\,y^{15} - 12651024880\,y^{12} - 517756288\,y^6 - 2461942080\,y^{16} \\
&- 842821200\,y^{17} - 11527653176\,y^{11} - 3069905696\,y^8 - 1387485456\,y^7 \\
&- 8690136000\,y^{14} - 11565415680\,y^{13} - 156342280\,y^5 - 8815886944\,y^{10} \\
&- 5674585584\,y^9 - 187293600\,y^{18} - 36935732\,y^4 - 20160075\,y^{19}
\end{aligned}
$$

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\begin{aligned}
&- 6437842\,y^{3} - 738768\,y^{2} - 41944\,y\\
c_{43} =& 264696582\,y^{4} + 15262104264\,y^{8} + 42393023200\,y^{10} + 9115712\,y^{2} + 964712\,y\\
&+ 2896080616\,y^{6} + 968788028\,y^{5} + 7227407968\,y^{7} + 60648896880\,y^{13}\\
&+ 48925059040\,y^{14} + 457828800\,y^{19} + 56009848576\,y^{11} + 27488903728\,y^{9}\\
&+ 7433403120\,y^{17} + 32685818880\,y^{15} + 57014784\,y^{3} + 63235287544\,y^{12}\\
&+ 17659956600\,y^{16} + 44352165\,y^{20} + 2289144000\,y^{18} + 50788\\
c_{42} =& 7227407968\,y^{8} + 50788\,y + 63235287544\,y^{13} + 964712\,y^{2} + 48925059040\,y^{15}\\
&+ 60648896880\,y^{14} + 56009848576\,y^{12} + 7433403120\,y^{18} + 15262104264\,y^{9}\\
&+ 44352165\,y^{21} + 17659956600\,y^{17} + 2896080616\,y^{7} + 9115712\,y^{3}\\
&+ 2289144000\,y^{19} + 42393023200\,y^{11} + 27488903728\,y^{10} + 457828800\,y^{20}\\
&+ 264696582\,y^{5} + 57014784\,y^{4} + 968788028\,y^{6} + 32685818880\,y^{16}\\
c_{41} =& - 961440480\,y^{21} - 235104125872\,y^{12} - 13663739684\,y^{7} - 4958853354\,y^{6}\\
&- 18891752880\,y^{19} - 231728994640\,y^{15} - 1117336\,y - 176458354800\,y^{11}\\
&- 115163329512\,y^{10} - 32174130384\,y^{8} - 271728739744\,y^{13}\\
&- 101056235280\,y^{17} - 11289684\,y^{2} - 167711503960\,y^{16} - 75898184\,y^{3}\\
&- 49439590200\,y^{18} - 84672315\,y^{22} - 1515101520\,y^{5} - 55308 - 380782360\,y^{4}\\
&- 65357234096\,y^{9} - 5287922640\,y^{20} - 271122347496\,y^{14}\\
c_{40} =& - 49439590200\,y^{19} - 5287922640\,y^{21} - 167711503960\,y^{17} - 101056235280\,y^{18}\\
&- 1515101520\,y^{6} - 75898184\,y^{4} - 380782360\,y^{5} - 18891752880\,y^{20}\\
&- 115163329512\,y^{11} - 65357234096\,y^{10} - 55308\,y - 231728994640\,y^{16}\\
&- 271122347496\,y^{15} - 961440480\,y^{22} - 13663739684\,y^{8} - 176458354800\,y^{12}\\
&- 32174130384\,y^{9} - 4958853354\,y^{7} - 84672315\,y^{23} - 11289684\,y^{3}\\
&- 1117336\,y^{2} - 235104125872\,y^{13} - 271728739744\,y^{14}\\
c_{39} =& 54746 + 1161468\,y + 12395472\,y^{2} + 141120525\,y^{24} + 1748073600\,y^{23}\\
&+ 10488441600\,y^{22} + 40877796960\,y^{21} + 116781865200\,y^{20}\\
&+ 261025564800\,y^{19} + 475001326800\,y^{18} + 722381883080\,y^{17}\\
&+ 473596296\,y^{4} + 88466928\,y^{3} + 55058724882\,y^{8} + 21364460416\,y^{7}\\
&+ 7138798144\,y^{6} + 2021478160\,y^{5} + 242729271928\,y^{10} + 857514144032\,y^{13}\\
&+ 123496243892\,y^{9} + 639258711056\,y^{12} + 419669647264\,y^{11}\\
&+ 934774965124\,y^{16} + 1042368845056\,y^{15} + 1010698496192\,y^{14}\\
c_{38} =& 54746\,y + 1161468\,y^{2} + 141120525\,y^{25} + 1748073600\,y^{24} + 10488441600\,y^{23}\\
&+ 40877796960\,y^{22} + 116781865200\,y^{21} + 261025564800\,y^{20}\\
&+ 475001326800\,y^{19} + 722381883080\,y^{18} + 934774965124\,y^{17}\\
&+ 88466928\,y^{4} + 12395472\,y^{3} + 21364460416\,y^{8} + 7138798144\,y^{7}\\
&+ 2021478160\,y^{6} + 473596296\,y^{5} + 123496243892\,y^{10} + 55058724882\,y^{9}
\end{aligned}
$$

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\begin{aligned}
&+ 419669647264\, y^{12} + 242729271928\, y^{11} + 1042368845056\, y^{16}\\
&+ 1010698496192\, y^{15} + 857514144032\, y^{14} + 639258711056\, y^{13}\\
c_{37} =&- 49700 - 1094920\, y - 12201950\, y^{2} - 206253075\, y^{26} - 2767783200\, y^{25}\\
&- 17990590800\, y^{24} - 75951788640\, y^{23} - 235118410800\, y^{22}\\
&- 570067898400\, y^{21} - 1127467621280\, y^{20} - 1868663373720\, y^{19}\\
&- 2644651365356\, y^{18} - 3239549996144\, y^{17} - 516409928\, y^{4}\\
&- 91427092\, y^{3} - 78464517736\, y^{8} - 28228295952\, y^{7} - 8799133768\, y^{6}\\
&- 2337402240\, y^{5} - 411333491870\, y^{10} - 191153981104\, y^{9}\\
&- 1335956846424\, y^{12} - 785717717036\, y^{11} - 3468792768056\, y^{16}\\
&- 3270491250208\, y^{15} - 2729493910864\, y^{14} - 2023781557056\, y^{13}\\
c_{36} =&- 49700\, y - 1094920\, y^{2} - 206253075\, y^{27} - 2767783200\, y^{26}\\
&- 17990590800\, y^{25} - 75951788640\, y^{24} - 235118410800\, y^{23}\\
&- 570067898400\, y^{22} - 1127467621280\, y^{21} - 1868663373720\, y^{20}\\
&- 2644651365356\, y^{19} - 3239549996144\, y^{18} - 3468792768056\, y^{17}\\
&- 91427092\, y^{4} - 12201950\, y^{3} - 28228295952\, y^{8} - 8799133768\, y^{7}\\
&- 2337402240\, y^{6} - 516409928\, y^{5} - 191153981104\, y^{10} - 78464517736\, y^{9}\\
&- 785717717036\, y^{12} - 411333491870\, y^{11} - 3270491250208\, y^{16}\\
&- 2729493910864\, y^{15} - 2023781557056\, y^{14} - 1335956846424\, y^{13}\\
c_{35} =&41658 + 944300\, y + 10875816\, y^{2} + 265182525\, y^{28} + 3832315200\, y^{27}\\
&+ 26826206400\, y^{26} + 121944710160\, y^{25} + 406506227400\, y^{24}\\
&+ 1062065222400\, y^{23} + 2266467672560\, y^{22} + 4061277591800\, y^{21}\\
&+ 6230647907484\, y^{20} + 8300632978176\, y^{19} + 9704772508816\, y^{18}\\
&+ 10037591056584\, y^{17} + 499392190\, y^{4} + 84663024\, y^{3} + 94890173796\, y^{8}\\
&+ 32030345872\, y^{7} + 9418433808\, y^{6} + 2372198484\, y^{5} + 575308646016\, y^{10}\\
&+ 247809863848\, y^{9} + 2224609904090\, y^{12} + 1194203144832\, y^{11}\\
&+ 9240215595324\, y^{16} + 7605354581664\, y^{15} + 5615059610136\, y^{14}\\
&+ 3726298760324\, y^{13}\\
c_{34} =&41658\, y + 944300\, y^{2} + 265182525\, y^{29} + 3832315200\, y^{28} + 26826206400\, y^{27}\\
&+ 121944710160\, y^{26} + 406506227400\, y^{25} + 1062065222400\, y^{24}\\
&+ 2266467672560\, y^{23} + 4061277591800\, y^{22} + 6230647907484\, y^{21}\\
&+ 8300632978176\, y^{20} + 9704772508816\, y^{19} + 10037591056584\, y^{18}\\
&+ 9240215595324\, y^{17} + 84663024\, y^{4} + 10875816\, y^{3} + 32030345872\, y^{8}\\
&+ 9418433808\, y^{7} + 2372198484\, y^{6} + 499392190\, y^{5} + 247809863848\, y^{10}\\
&+ 94890173796\, y^{9} + 1194203144832\, y^{12} + 575308646016\, y^{11}\\
&+ 5615059610136\, y^{15} + 3726298760324\, y^{14} + 2224609904090\, y^{13}
\end{aligned}
$$

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\begin{aligned}
&+ 7605354581664\,y^{16}\\
c_{33} =&- 32398 - 749844\,y - 8849490\,y^2 - 300540195\,y^{30} - 4653525600\,y^{29}\\
&- 34901442000\,y^{28} - 169949964240\,y^{27} - 606843808200\,y^{26}\\
&- 1698913048560\,y^{25} - 3888387037080\,y^{24} - 7483593343720\,y^{23}\\
&- 12355689365396\,y^{22} - 17758908907344\,y^{21} - 22468474332920\,y^{20}\\
&- 25237557100392\,y^{19} - 25335391302956\,y^{18} - 22849211615160\,y^{17}\\
&- 432498180\,y^4 - 70894108\,y^3 - 98786488380\,y^8 - 31612018812\,y^7\\
&- 8856044570\,y^6 - 2135472168\,y^5 - 677034433908\,y^{10} - 273541049032\,y^9\\
&- 3031077040872\,y^{12} - 1507215783752\,y^{11} - 18587267839548\,y^{16}\\
&- 13678571283612\,y^{15} - 9124132183414\,y^{14} - 5521648486832\,y^{13}\\
c_{32} =&- 32398\,y - 749844\,y^2 - 300540195\,y^{31} - 4653525600\,y^{30} - 34901442000\,y^{29}\\
&- 169949964240\,y^{28} - 606843808200\,y^{27} - 1698913048560\,y^{26}\\
&- 3888387037080\,y^{25} - 7483593343720\,y^{24} - 12355689365396\,y^{23}\\
&- 17758908907344\,y^{22} - 22468474332920\,y^{21} - 25237557100392\,y^{20}\\
&- 25335391302956\,y^{19} - 22849211615160\,y^{18} - 18587267839548\,y^{17}\\
&- 70894108\,y^4 - 8849490\,y^3 - 31612018812\,y^8 - 8856044570\,y^7\\
&- 2135472168\,y^6 - 432498180\,y^5 - 273541049032\,y^{10} - 98786488380\,y^9\\
&- 1507215783752\,y^{12} - 677034433908\,y^{11} - 13678571283612\,y^{16}\\
&- 9124132183414\,y^{15} - 5521648486832\,y^{14} - 3031077040872\,y^{13}\\
c_{31} =&23461 + 550766\,y + 6612316\,y^2 + 300540195\,y^{32} + 4963760640\,y^{31}\\
&+ 39710085120\,y^{30} + 206210021760\,y^{29} + 785108606400\,y^{28}\\
&+ 2343979061760\,y^{27} + 5724440317120\,y^{26} + 11768038794720\,y^{25}\\
&+ 20784246546544\,y^{24} + 32017471806976\,y^{23} + 43517224083328\,y^{22}\\
&+ 52656189618240\,y^{21} + 57125944786848\,y^{20} + 55882561771456\,y^{19}\\
&+ 49516167229392\,y^{18} + 39883640561464\,y^{17} + 338249088\,y^4\\
&+ 54081072\,y^3 + 89545258664\,y^8 + 27418986624\,y^7 + 7385214272\,y^6\\
&+ 1720386848\,y^5 + 680135364160\,y^{10} + 260380673904\,y^9\\
&+ 3445948191456\,y^{12} + 1606265391040\,y^{11} + 29281089322476\,y^{16}\\
&+ 19630028733952\,y^{15} + 12028540910848\,y^{14} + 6737207143360\,y^{13}\\
c_{30} =&23461\,y + 550766\,y^2 + 4963760640\,y^{32} + 39710085120\,y^{31}\\
&+ 206210021760\,y^{30} + 785108606400\,y^{29} + 2343979061760\,y^{28}\\
&+ 5724440317120\,y^{27} + 11768038794720\,y^{26} + 20784246546544\,y^{25}\\
&+ 32017471806976\,y^{24} + 43517224083328\,y^{23} + 52656189618240\,y^{22}\\
&+ 57125944786848\,y^{21} + 55882561771456\,y^{20} + 49516167229392\,y^{19}\\
&+ 39883640561464\,y^{18} + 29281089322476\,y^{17} + 300540195\,y^{33}
\end{aligned}
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\begin{aligned}
&+ 54081072\,y^4 + 6612316\,y^3 + 27418986624\,y^8 + 7385214272\,y^7\\
&+ 1720386848\,y^6 + 338249088\,y^5 + 260380673904\,y^{10} + 89545258664\,y^9\\
&+ 1606265391040\,y^{12} + 680135364160\,y^{11} + 19630028733952\,y^{16}\\
&+ 12028540910848\,y^{15} + 6737207143360\,y^{14} + 3445948191456\,y^{13}\\
c_{29} =& - 15864 - 375376\,y - 4555563\,y^2 - 39554967600\,y^{32} - 218191512960\,y^{31}\\
&- 882244267200\,y^{30} - 2797321603200\,y^{29} - 7257693203200\,y^{28}\\
&- 15862083433120\,y^{27} - 29816683632336\,y^{26} - 48957289784384\,y^{25}\\
&- 71053495586208\,y^{24} - 92003796675648\,y^{23} - 107080614147104\,y^{22}\\
&- 112699479483904\,y^{21} - 107790255861456\,y^{20} - 94064767870216\,y^{19}\\
&- 75137868974516\,y^{18} - 55073622939168\,y^{17} - 265182525\,y^{34}\\
&- 4653525600\,y^{33} - 240447060\,y^4 - 37778802\,y^3 - 71327065296\,y^8\\
&- 21079547040\,y^7 - 5505704480\,y^6 - 1249565808\,y^5 - 589797314744\,y^{10}\\
&- 215898531488\,y^9 - 3316370807392\,y^{12} - 1463762798544\,y^{11}\\
&- 37104707404400\,y^{16} - 23000462824896\,y^{15} - 13120524730080\,y^{14}\\
&- 6883455615616\,y^{13}\\
c_{28} =& - 15864\,y - 375376\,y^2 - 218191512960\,y^{32} - 882244267200\,y^{31}\\
&- 2797321603200\,y^{30} - 7257693203200\,y^{29} - 15862083433120\,y^{28}\\
&- 29816683632336\,y^{27} - 48957289784384\,y^{26} - 71053495586208\,y^{25}\\
&- 92003796675648\,y^{24} - 107080614147104\,y^{23} - 112699479483904\,y^{22}\\
&- 107790255861456\,y^{21} - 94064767870216\,y^{20} - 75137868974516\,y^{19}\\
&- 55073622939168\,y^{18} - 37104707404400\,y^{17} - 265182525\,y^{35}\\
&- 4653525600\,y^{34} - 39554967600\,y^{33} - 37778802\,y^4 - 4555563\,y^3\\
&- 21079547040\,y^8 - 5505704480\,y^7 - 1249565808\,y^6 - 240447060\,y^5\\
&- 215898531488\,y^{10} - 71327065296\,y^9 - 1463762798544\,y^{12}\\
&- 589797314744\,y^{11} - 23000462824896\,y^{16} - 13120524730080\,y^{15}\\
&- 6883455615616\,y^{14} - 3316370807392\,y^{13}\\
c_{27} =& 10068 + 237960\,y + 2901504\,y^2 + 861824917800\,y^{32} + 2891533209600\,y^{31}\\
&+ 7939784758080\,y^{30} + 18373993554080\,y^{29} + 36600446098384\,y^{28}\\
&+ 63755034646016\,y^{27} + 98303363102784\,y^{26} + 135461150729632\,y^{25}\\
&+ 168115942730736\,y^{24} + 189102273515456\,y^{23} + 193798659736592\,y^{22}\\
&+ 181743697614808\,y^{21} + 156524571863548\,y^{20} + 124160351514112\,y^{19}\\
&+ 90915946208640\,y^{18} + 61553704697040\,y^{17} + 206253075\,y^{36}\\
&+ 3832315200\,y^{35} + 34490836800\,y^{34} + 201401378640\,y^{33} + 156030381\,y^4\\
&+ 24248704\,y^3 + 50301165160\,y^8 + 14464607280\,y^7 + 3692732284\,y^6\\
&+ 822736158\,y^5 + 445393105888\,y^{10} + 157195789520\,y^9
\end{aligned}
$$

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\begin{aligned}
&+ 2731352951576\,y^{12} + 1151711923264\,y^{11} + 38569662181960\,y^{16}\\
&+ 22374507261760\,y^{15} + 12012381259328\,y^{14} + 5962478863376\,y^{13}\\
c_{26} =&10068\,y + 237960\,y^2 + 2891533209600\,y^{32} + 7939784758080\,y^{31}\\
&+ 18373993554080\,y^{30} + 36600446098384\,y^{29} + 63755034646016\,y^{28}\\
&+ 98303363102784\,y^{27} + 135461150729632\,y^{26} + 168115942730736\,y^{25}\\
&+ 189102273515456\,y^{24} + 193798659736592\,y^{23} + 181743697614808\,y^{22}\\
&+ 156524571863548\,y^{21} + 124160351514112\,y^{20} + 90915946208640\,y^{19}\\
&+ 61553704697040\,y^{18} + 38569662181960\,y^{17} + 206253075\,y^{37}\\
&+ 3832315200\,y^{36} + 34490836800\,y^{35} + 201401378640\,y^{34}\\
&+ 861824917800\,y^{33} + 24248704\,y^4 + 2901504\,y^3 + 14464607280\,y^8\\
&+ 3692732284\,y^7 + 822736158\,y^6 + 156030381\,y^5 + 157195789520\,y^{10}\\
&+ 50301165160\,y^9 + 1151711923264\,y^{12} + 445393105888\,y^{11}\\
&+ 22374507261760\,y^{16} + 12012381259328\,y^{15} + 5962478863376\,y^{14}\\
&+ 2731352951576\,y^{13}\\
c_{25} =& - 6036 - 140952\,y - 1711748\,y^2 - 7497840382760\,y^{32} - 18307707855840\,y^{31}\\
&- 38500788815216\,y^{30} - 70862893244864\,y^{29} - 115577732666144\,y^{28}\\
&- 168697385650976\,y^{27} - 222115579375536\,y^{26} - 265542242629664\,y^{25}\\
&- 289833757036928\,y^{24} - 290151785862120\,y^{23} - 267452722477668\,y^{22}\\
&- 227729992194848\,y^{21} - 179594286555632\,y^{20} - 131450824320400\,y^{19}\\
&- 89431228294824\,y^{18} - 56609342891504\,y^{17} - 141120525\,y^{38}\\
&- 2767783200\,y^{37} - 26293940400\,y^{36} - 162029846160\,y^{35}\\
&- 731482151400\,y^{34} - 2588688538800\,y^{33} - 92661976\,y^4\\
&- 14328872\,y^3 - 31589613164\,y^8 - 8905006466\,y^7 - 2237847235\,y^6\\
&- 492686736\,y^5 - 294908965224\,y^{10} - 101142237120\,y^9\\
&- 1940993334032\,y^{12} - 788267379728\,y^{11} - 33353648804696\,y^{16}\\
&- 18289630975280\,y^{15} - 9327903364424\,y^{14} - 4419115551904\,y^{13}\\
c_{24} =& - 6036\,y - 140952\,y^2 - 18307707855840\,y^{32} - 38500788815216\,y^{31}\\
&- 70862893244864\,y^{30} - 115577732666144\,y^{29} - 168697385650976\,y^{28}\\
&- 222115579375536\,y^{27} - 265542242629664\,y^{26} - 289833757036928\,y^{25}\\
&- 290151785862120\,y^{24} - 267452722477668\,y^{23} - 227729992194848\,y^{22}\\
&- 179594286555632\,y^{21} - 131450824320400\,y^{20} - 89431228294824\,y^{19}\\
&- 56609342891504\,y^{18} - 33353648804696\,y^{17} - 141120525\,y^{39}\\
&- 2767783200\,y^{38} - 26293940400\,y^{37} - 162029846160\,y^{36}\\
&- 731482151400\,y^{35} - 2588688538800\,y^{34} - 7497840382760\,y^{33}\\
&- 14328872\,y^4 - 1711748\,y^3 - 8905006466\,y^8 - 2237847235\,y^7
\end{aligned}
$$

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\begin{aligned}
&- 492686736\,y^{6} - 92661976\,y^{5} - 101142237120\,y^{10} - 31589613164\,y^{9}\\
&- 788267379728\,y^{12} - 294908965224\,y^{11} - 18289630975280\,y^{16}\\
&- 9327903364424\,y^{15} - 4419115551904\,y^{14} - 1940993334032\,y^{13}\\
c_{23} =\,&3434 + 78468\,y + 939080\,y^{2} + 34698266843604\,y^{32} + 67244843649536\,y^{31}\\
&+ 115581600821120\,y^{30} + 177976221503808\,y^{29} + 247527653786784\,y^{28}\\
&+ 313044440817472\,y^{27} + 362051571096048\,y^{26} + 384774038456808\,y^{25}\\
&+ 377305281107300\,y^{24} + 342567709116672\,y^{23} + 288828205184896\,y^{22}\\
&+ 226682829211488\,y^{21} + 165925005540528\,y^{20} + 113432524543520\,y^{19}\\
&+ 72495738192736\,y^{18} + 43337018585352\,y^{17} + 84672315\,y^{40}\\
&+ 1748073600\,y^{39} + 17480736000\,y^{38} + 113364582240\,y^{37}\\
&+ 538429273200\,y^{36} + 2004180447360\,y^{35} + 6105039384080\,y^{34}\\
&+ 15679936536680\,y^{33} + 50417880\,y^{4} + 7806416\,y^{3} + 17734396617\,y^{8}\\
&+ 4933332096\,y^{7} + 1227992256\,y^{6} + 268745712\,y^{5} + 172079552524\,y^{10}\\
&+ 57768036230\,y^{9} + 1197375768160\,y^{12} + 471909608688\,y^{11}\\
&+ 24233161211084\,y^{16} + 12670853369728\,y^{15} + 6189870335424\,y^{14}\\
&+ 2821260753888\,y^{13}\\
c_{22} =\,&3434\,y + 78468\,y^{2} + 67244843649536\,y^{32} + 115581600821120\,y^{31}\\
&+ 177976221503808\,y^{30} + 247527653786784\,y^{29} + 313044440817472\,y^{28}\\
&+ 362051571096048\,y^{27} + 384774038456808\,y^{26} + 377305281107300\,y^{25}\\
&+ 342567709116672\,y^{24} + 288828205184896\,y^{23} + 226682829211488\,y^{22}\\
&+ 165925005540528\,y^{21} + 113432524543520\,y^{20} + 72495738192736\,y^{19}\\
&+ 43337018585352\,y^{18} + 24233161211084\,y^{17} + 84672315\,y^{41}\\
&+ 1748073600\,y^{40} + 17480736000\,y^{39} + 113364582240\,y^{38}\\
&+ 538429273200\,y^{37} + 2004180447360\,y^{36} + 6105039384080\,y^{35}\\
&+ 15679936536680\,y^{34} + 34698266843604\,y^{33} + 7806416\,y^{4}\\
&+ 939080\,y^{3} + 4933332096\,y^{8} + 1227992256\,y^{7} + 268745712\,y^{6}\\
&+ 50417880\,y^{5} + 57768036230\,y^{10} + 17734396617\,y^{9} + 471909608688\,y^{12}\\
&+ 172079552524\,y^{11} + 12670853369728\,y^{16} + 6189870335424\,y^{15}\\
&+ 2821260753888\,y^{14} + 1197375768160\,y^{13}\\
c_{21} =\,&- 1860 - 41208\,y - 481390\,y^{2} - 98205182723032\,y^{32} - 158975487711040\,y^{31}\\
&- 232679061489952\,y^{30} - 310041634001792\,y^{29} - 378312376903472\,y^{28}\\
&- 424824424882648\,y^{27} - 440914214605564\,y^{26} - 424505680464736\,y^{25}\\
&- 380335644625872\,y^{24} - 317952218824032\,y^{23} - 248556355004464\,y^{22}\\
&- 182021476061568\,y^{21} - 125037946100144\,y^{20} - 80649214256600\,y^{19}\\
&- 48871493922372\,y^{18} - 27829911367280\,y^{17} - 44352165\,y^{42}
\end{aligned}
$$

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\begin{aligned}
&- 961440480\,y^{41} - 10095125040\,y^{40} - 68726417760\,y^{39} - 342554612400\,y^{38}\\
&- 1337717340960\,y^{37} - 4274354404320\,y^{36} - 11515763991160\,y^{35}\\
&- 26737946431196\,y^{34} - 54393771958064\,y^{33} - 25155248\,y^{4} - 3933420\,y^{3}\\
&- 8913416664\,y^{8} - 2461329648\,y^{7} - 610337816\,y^{6} - 133547584\,y^{5}\\
&- 88749319431\,y^{10} - 29355926672\,y^{9} - 643793038084\,y^{12}\\
&- 247995308890\,y^{11} - 14890744366584\,y^{16} - 7482590828128\,y^{15}\\
&- 3527920071680\,y^{14} - 1558496855536\,y^{13}\\
c_{20} =&- 1860\,y - 41208\,y^{2} - 158975487711040\,y^{32} - 232679061489952\,y^{31}\\
&- 310041634001792\,y^{30} - 378312376903472\,y^{29} - 424824424882648\,y^{28}\\
&- 440914214605564\,y^{27} - 424505680464736\,y^{26} - 380335644625872\,y^{25}\\
&- 317952218824032\,y^{24} - 248556355004464\,y^{23} - 182021476061568\,y^{22}\\
&- 125037946100144\,y^{21} - 80649214256600\,y^{20} - 48871493922372\,y^{19}\\
&- 27829911367280\,y^{18} - 14890744366584\,y^{17} - 44352165\,y^{43}\\
&- 961440480\,y^{42} - 10095125040\,y^{41} - 68726417760\,y^{40} - 643793038084\,y^{13}\\
&- 342554612400\,y^{39} - 1337717340960\,y^{38} - 4274354404320\,y^{37}\\
&- 11515763991160\,y^{36} - 26737946431196\,y^{35} - 54393771958064\,y^{34}\\
&- 98205182723032\,y^{33} - 3933420\,y^{4} - 481390\,y^{3} - 2461329648\,y^{8}\\
&- 610337816\,y^{7} - 133547584\,y^{6} - 25155248\,y^{5} - 29355926672\,y^{10}\\
&- 8913416664\,y^{9} - 247995308890\,y^{12} - 88749319431\,y^{11}\\
&- 7482590828128\,y^{16} - 3527920071680\,y^{15} - 1558496855536\,y^{14}\\
c_{19} =&958 + 20460\,y + 231152\,y^{2} + 184072524213116\,y^{32} + 257524494604608\,y^{31}\\
&+ 330288464159280\,y^{30} + 390331660078856\,y^{29} + 426932172806836\,y^{28}\\
&+ 433845313567488\,y^{27} + 410966437011968\,y^{26} + 363928613524112\,y^{25}\\
&+ 302008912761192\,y^{24} + 235340174568928\,y^{23} + 172488228305888\,y^{22}\\
&+ 119061638600344\,y^{21} + 77473784139620\,y^{20} + 47555285107616\,y^{19}\\
&+ 27546629792112\,y^{18} + 15059190513160\,y^{17} + 20160075\,y^{44}\\
&+ 457828800\,y^{43} + 5036116800\,y^{42} + 35910354480\,y^{41} + 187411824600\,y^{40}\\
&+ 766058092800\,y^{39} + 2561529770800\,y^{38} + 7221452461080\,y^{37}\\
&+ 17547533041324\,y^{36} + 37370557075456\,y^{35} + 70667860599952\,y^{34}\\
&+ 119898127453832\,y^{33} + 11532806\,y^{4} + 1839264\,y^{3} + 4007479388\,y^{8}\\
&+ 1104644976\,y^{7} + 274478488\,y^{6} + 60458908\,y^{5} + 40475460160\,y^{10}\\
&+ 13269333080\,y^{9} + 302227968993\,y^{12} + 114532596480\,y^{11}\\
&+ 7767883706028\,y^{16} + 3778587358192\,y^{15} + 1731687401452\,y^{14}\\
&+ 746622879894\,y^{13}\\
c_{18} =&958\,y + 20460\,y^{2} + 257524494604608\,y^{32} + 330288464159280\,y^{31}
\end{aligned}
$$

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\begin{aligned}
&+ 390331660078856\, y^{30} + 426932172806836\, y^{29} + 433845313567488\, y^{28} \\
&+ 410966437011968\, y^{27} + 363928613524112\, y^{26} + 302008912761192\, y^{25} \\
&+ 235340174568928\, y^{24} + 172488228305888\, y^{23} + 119061638600344\, y^{22} \\
&+ 77473784139620\, y^{21} + 47555285107616\, y^{20} + 27546629792112\, y^{19} \\
&+ 15059190513160\, y^{18} + 7767883706028\, y^{17} + 20160075\, y^{45} \\
&+ 457828800\, y^{44} + 5036116800\, y^{43} + 35910354480\, y^{42} + 302227968993\, y^{13} \\
&+ 187411824600\, y^{41} + 766058092800\, y^{40} + 2561529770800\, y^{39} \\
&+ 7221452461080\, y^{38} + 17547533041324\, y^{37} + 37370557075456\, y^{36} \\
&+ 70667860599952\, y^{35} + 119898127453832\, y^{34} + 184072524213116\, y^{33} \\
&+ 1839264\, y^{4} + 231152\, y^{3} + 1104644976\, y^{8} + 274478488\, y^{7} \\
&+ 60458908\, y^{6} + 11532806\, y^{5} + 13269333080\, y^{10} + 4007479388\, y^{9} \\
&+ 114532596480\, y^{12} + 40475460160\, y^{11} + 3778587358192\, y^{16} \\
&+ 1731687401452\, y^{15} + 746622879894\, y^{14} \\
c_{17} = &- 470 - 9580\, y - 104030\, y^{2} - 239995902220932\, y^{32} - 297452787946808\, y^{31} \\
&- 341592891568364\, y^{30} - 364919215746912\, y^{29} - 363903464593872\, y^{28} \\
&- 339772161667024\, y^{27} - 297806217436296\, y^{26} - 245578506693520\, y^{25} \\
&- 190884200458392\, y^{24} - 140069671803336\, y^{23} - 97153052600812\, y^{22} \\
&- 63758016163248\, y^{21} - 39618645403896\, y^{20} - 23322138259880\, y^{19} \\
&- 13009335747612\, y^{18} - 6876470437704\, y^{17} - 7888725\, y^{46} - 187293600\, y^{45} \\
&- 2153876400\, y^{44} - 16053214320\, y^{43} - 87541997400\, y^{42} - 373773800400\, y^{41} \\
&- 1305134270440\, y^{40} - 3841741051720\, y^{39} - 9747162009156\, y^{38} \\
&- 21678813590544\, y^{37} - 42828036093656\, y^{36} - 75952348723368\, y^{35} \\
&- 121961553432684\, y^{34} - 178605863828952\, y^{33} - 4865820\, y^{4} - 798700\, y^{3} \\
&- 1607047332\, y^{8} - 444849428\, y^{7} - 111553074\, y^{6} - 24949832\, y^{5} \\
&- 16284818508\, y^{10} - 5320278232\, y^{9} - 123743046968\, y^{12} - 46403526648\, y^{11} \\
&- 3443385293384\, y^{16} - 1632526042378\, y^{15} - 732079699215\, y^{14} - 310045351120\, y^{13} \\
c_{16} = &- 470\, y - 9580\, y^{2} - 297452787946808\, y^{32} - 341592891568364\, y^{31} \\
&- 364919215746912\, y^{30} - 363903464593872\, y^{29} - 339772161667024\, y^{28} \\
&- 297806217436296\, y^{27} - 245578506693520\, y^{26} - 190884200458392\, y^{25} \\
&- 140069671803336\, y^{24} - 97153052600812\, y^{23} - 63758016163248\, y^{22} \\
&- 39618645403896\, y^{21} - 23322138259880\, y^{20} - 13009335747612\, y^{19} \\
&- 6876470437704\, y^{18} - 3443385293384\, y^{17} - 7888725\, y^{47} - 187293600\, y^{46} \\
&- 2153876400\, y^{45} - 16053214320\, y^{44} - 87541997400\, y^{43} - 373773800400\, y^{42} \\
&- 1305134270440\, y^{41} - 3841741051720\, y^{40} - 9747162009156\, y^{39} \\
&- 21678813590544\, y^{38} - 42828036093656\, y^{37} - 75952348723368\, y^{36}
\end{aligned}
$$

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\begin{aligned}
&- 121961553432684\, y^{35} - 178605863828952\, y^{34} - 239995902220932\, y^{33} \\
&- 798700\, y^{4} - 104030\, y^{3} - 444849428\, y^{8} - 111553074\, y^{7} - 24949832\, y^{6} \\
&- 4865820\, y^{5} - 5320278232\, y^{10} - 1607047332\, y^{9} - 46403526648\, y^{12} \\
&- 16284818508\, y^{11} - 1632526042378\, y^{16} - 732079699215\, y^{15} \\
&- 310045351120\, y^{14} - 123743046968\, y^{13}
\end{aligned}
$$

$$
\begin{aligned}
c_{15} =& 221 + 4230\, y + 43696\, y^{2} + 224453875410934\, y^{32} + 251204504233472\, y^{31} \\
&+ 262741215297792\, y^{30} + 257628708088256\, y^{29} + 237475576589280\, y^{28} \\
&+ 206270655047872\, y^{27} + 169176128332608\, y^{26} + 131244284158128\, y^{25} \\
&+ 96449234110088\, y^{24} + 67224175268736\, y^{23} + 44483237082816\, y^{22} \\
&+ 27967868049376\, y^{21} + 16717751610432\, y^{20} + 9504532480656\, y^{19} \\
&+ 5140527848084\, y^{18} + 2644832316938\, y^{17} + 2629575\, y^{48} + 65145600\, y^{47} \\
&+ 781747200\, y^{46} + 6078646080\, y^{45} + 34571815200\, y^{44} + 153893625600\, y^{43} \\
&+ 560065128480\, y^{42} + 1717893903440\, y^{41} + 4541584786024\, y^{40} \\
&+ 10526132702976\, y^{39} + 21675724202176\, y^{38} + 40083624525984\, y^{37} \\
&+ 67150920705424\, y^{36} + 102659859883488\, y^{35} + 144113043662568\, y^{34} \\
&+ 186755849437596\, y^{33} + 1886008\, y^{4} + 321512\, y^{3} + 572231076\, y^{8} \\
&+ 160253440\, y^{7} + 40907456\, y^{6} + 9375440\, y^{5} + 5749591888\, y^{10} \\
&+ 1882331432\, y^{9} + 43987583888\, y^{12} + 16410546464\, y^{11} + 1294108820623\, y^{16} \\
&+ 601787533568\, y^{15} + 265670307712\, y^{14} + 111164450976\, y^{13}
\end{aligned}
$$

$$
\begin{aligned}
c_{14} =& 221\, y + 4230\, y^{2} + 251204504233472\, y^{32} + 262741215297792\, y^{31} \\
&+ 257628708088256\, y^{30} + 237475576589280\, y^{29} + 206270655047872\, y^{28} \\
&+ 169176128332608\, y^{27} + 131244284158128\, y^{26} + 96449234110088\, y^{25} \\
&+ 67224175268736\, y^{24} + 44483237082816\, y^{23} + 27967868049376\, y^{22} \\
&+ 16717751610432\, y^{21} + 9504532480656\, y^{20} + 5140527848084\, y^{19} \\
&+ 2644832316938\, y^{18} + 1294108820623\, y^{17} + 2629575\, y^{49} + 65145600\, y^{48} \\
&+ 781747200\, y^{47} + 6078646080\, y^{46} + 34571815200\, y^{45} + 153893625600\, y^{44} \\
&+ 560065128480\, y^{43} + 1717893903440\, y^{42} + 4541584786024\, y^{41} \\
&+ 10526132702976\, y^{40} + 21675724202176\, y^{39} + 40083624525984\, y^{38} \\
&+ 67150920705424\, y^{37} + 102659859883488\, y^{36} + 144113043662568\, y^{35} \\
&+ 186755849437596\, y^{34} + 224453875410934\, y^{33} + 321512\, y^{4} + 43696\, y^{3} \\
&+ 160253440\, y^{8} + 40907456\, y^{7} + 9375440\, y^{6} + 1886008\, y^{5} \\
&+ 1882331432\, y^{10} + 572231076\, y^{9} + 16410546464\, y^{12} + 5749591888\, y^{11} \\
&+ 601787533568\, y^{16} + 265670307712\, y^{15} + 111164450976\, y^{14} + 43987583888\, y^{13}
\end{aligned}
$$

$$
\begin{aligned}
c_{13} =& - 100 - 1768\, y - 17083\, y^{2} - 153233296391224\, y^{32} - 157250659653568\, y^{31} \\
&- 151878629674464\, y^{30} - 138406407201664\, y^{29} - 119267409857632\, y^{28}
\end{aligned}
$$

$$
\begin{aligned}
&- 97368253050768\, y^{27} - 75431670307928\, y^{26} - 55531823760672\, y^{25} \\
&- 38896049239568\, y^{24} - 25947141564512\, y^{23} - 16499352006432\, y^{22} \\
&- 10007856635088\, y^{21} - 5793562425276\, y^{20} - 3202132928406\, y^{19} \\
&- 1690028073569\, y^{18} - 851676813040\, y^{17} - 736281\, y^{50} - 19000800\, y^{49} \\
&- 237510000\, y^{48} - 1923405120\, y^{47} - 11389341600\, y^{46} - 52765702080\, y^{45} \\
&- 199792087040\, y^{44} - 637437406320\, y^{43} - 1752659764856\, y^{42} \\
&- 4224896000864\, y^{41} - 9049932306800\, y^{40} - 17413503368352\, y^{39} \\
&- 30366290882256\, y^{38} - 48348103454336\, y^{37} - 70726238869416\, y^{36} \\
&- 95575763081092\, y^{35} - 119875889776986\, y^{34} - 140131546291984\, y^{33} \\
&- 668072\, y^{4} - 119178\, y^{3} - 179857704\, y^{8} - 51395408\, y^{7} - 13472664\, y^{6} \\
&- 3191560\, y^{5} - 1766676236\, y^{10} - 583385904\, y^{9} - 13459829376\, y^{12} \\
&- 5022155192\, y^{11} - 409652090600\, y^{16} - 187926274976\, y^{15} \\
&- 82125226896\, y^{14} - 34131247424\, y^{13}
\end{aligned}
$$

$$
\begin{aligned}
c_{12} = &- 100\, y - 1768\, y^2 - 157250659653568\, y^{32} - 151878629674464\, y^{31} \\
&- 138406407201664\, y^{30} - 119267409857632\, y^{29} - 97368253050768\, y^{28} \\
&- 75431670307928\, y^{27} - 55531823760672\, y^{26} - 38896049239568\, y^{25} \\
&- 25947141564512\, y^{24} - 16499352006432\, y^{23} - 10007856635088\, y^{22} \\
&- 5793562425276\, y^{21} - 3202132928406\, y^{20} - 1690028073569\, y^{19} \\
&- 851676813040\, y^{18} - 409652090600\, y^{17} - 736281\, y^{51} - 19000800\, y^{50} \\
&- 237510000\, y^{49} - 1923405120\, y^{48} - 11389341600\, y^{47} - 13459829376\, y^{13} \\
&- 52765702080\, y^{46} - 199792087040\, y^{45} - 637437406320\, y^{44} \\
&- 1752659764856\, y^{43} - 4224896000864\, y^{42} - 9049932306800\, y^{41} \\
&- 17413503368352\, y^{40} - 30366290882256\, y^{39} - 48348103454336\, y^{38} \\
&- 70726238869416\, y^{37} - 95575763081092\, y^{36} - 119875889776986\, y^{35} \\
&- 140131546291984\, y^{34} - 153233296391224\, y^{33} - 119178\, y^{4} - 17083\, y^{3} \\
&- 51395408\, y^{8} - 13472664\, y^{7} - 3191560\, y^{6} - 668072\, y^{5} \\
&- 583385904\, y^{10} - 179857704\, y^{9} - 5022155192\, y^{12} - 1766676236\, y^{11} \\
&- 187926274976\, y^{16} - 82125226896\, y^{15} - 34131247424\, y^{14}
\end{aligned}
$$

$$
\begin{aligned}
c_{11} = &42 + 700\, y + 6216\, y^2 + 76808425621716\, y^{32} + 73185606168640\, y^{31} \\
&+ 66018061921600\, y^{30} + 56493484492368\, y^{29} + 45941140332152\, y^{28} \\
&+ 35558795229248\, y^{27} + 26231648793312\, y^{26} + 18465315874512\, y^{25} \\
&+ 12416392346504\, y^{24} + 7982466313872\, y^{23} + 4910432839540\, y^{22} \\
&+ 2892162420314\, y^{21} + 1631773224327\, y^{20} + 882200588160\, y^{19} \\
&+ 457073117824\, y^{18} + 226902468024\, y^{17} + 169911\, y^{52} + 4560192\, y^{51} \\
&+ 59282496\, y^{50} + 499196880\, y^{49} + 3072724200\, y^{48} + 14792494080\, y^{47}
\end{aligned}
$$

$$
\begin{aligned}
&+ 58180609760\, y^{46} + 192763266800\, y^{45} + 550294423224\, y^{44} \\
&+ 1377218757376\, y^{43} + 3063057109728\, y^{42} + 6120734583600\, y^{41} \\
&+ 11087868148168\, y^{40} + 18346259909600\, y^{39} + 27904269575304\, y^{38} \\
&+ 39228726807340\, y^{37} + 51219029799966\, y^{36} + 62371671684096\, y^{35} \\
&+ 71104825778240\, y^{34} + 76139818059720\, y^{33} + 214017\, y^{4} + 40400\, y^{3} \\
&+ 49420548\, y^{8} + 14531784\, y^{7} + 3945168\, y^{6} + 974334\, y^{5} + 8853161400\, y^{13} \\
&+ 466960416\, y^{10} + 156747336\, y^{9} + 3498053324\, y^{12} + 1313106496\, y^{11} \\
&+ 107874349260\, y^{16} + 49075383776\, y^{15} + 21338054064\, y^{14} \\
c_{10} =&42\, y + 700\, y^{2} + 73185606168640\, y^{32} + 66018061921600\, y^{31} \\
&+ 56493484492368\, y^{30} + 45941140332152\, y^{29} + 35558795229248\, y^{28} \\
&+ 26231648793312\, y^{27} + 18465315874512\, y^{26} + 12416392346504\, y^{25} \\
&+ 7982466313872\, y^{24} + 4910432839540\, y^{23} + 2892162420314\, y^{22} \\
&+ 1631773224327\, y^{21} + 882200588160\, y^{20} + 457073117824\, y^{19} \\
&+ 226902468024\, y^{18} + 107874349260\, y^{17} + 169911\, y^{53} + 3498053324\, y^{13} \\
&+ 4560192\, y^{52} + 59282496\, y^{51} + 499196880\, y^{50} + 3072724200\, y^{49} \\
&+ 14792494080\, y^{48} + 58180609760\, y^{47} + 192763266800\, y^{46} \\
&+ 550294423224\, y^{45} + 1377218757376\, y^{44} + 3063057109728\, y^{43} \\
&+ 6120734583600\, y^{42} + 11087868148168\, y^{41} + 18346259909600\, y^{40} \\
&+ 27904269575304\, y^{39} + 39228726807340\, y^{38} + 51219029799966\, y^{37} \\
&+ 62371671684096\, y^{36} + 71104825778240\, y^{35} + 76139818059720\, y^{34} \\
&+ 76808425621716\, y^{33} + 40400\, y^{4} + 6216\, y^{3} + 14531784\, y^{8} \\
&+ 3945168\, y^{7} + 974334\, y^{6} + 214017\, y^{5} + 156747336\, y^{10} + 8853161400\, y^{14} \\
&+ 49420548\, y^{9} + 1313106496\, y^{12} + 466960416\, y^{11} + 49075383776\, y^{16} \\
&+ 21338054064\, y^{15} \\
c_{9} =& - 14 - 252\, y - 2090\, y^{2} - 28077372499868\, y^{32} - 25098955113968\, y^{31} \\
&- 21347223917672\, y^{30} - 17303673015328\, y^{29} - 13387253668816\, y^{28} \\
&- 9898697135120\, y^{27} - 7003509266248\, y^{26} - 4746453967584\, y^{25} \\
&- 3084283289652\, y^{24} - 1923259624582\, y^{23} - 1151691937865\, y^{22} \\
&- 662684126384\, y^{21} - 366550654856\, y^{20} - 194947311000\, y^{19} \\
&- 99690728668\, y^{18} - 49004477896\, y^{17} - 31465\, y^{54} - 876960\, y^{53} \\
&- 11838960\, y^{52} - 103508496\, y^{51} - 661326120\, y^{50} - 3303409200\, y^{49} \\
&- 13476208200\, y^{48} - 46296237520\, y^{47} - 137010274856\, y^{46} \\
&- 355427925664\, y^{45} - 819405519408\, y^{44} - 1697441917808\, y^{43} \\
&- 3188473544424\, y^{42} - 5472176918320\, y^{41} - 8636341292464\, y^{40} \\
&- 12604032196532\, y^{39} - 17092716269202\, y^{38} - 21632053806736\, y^{37}
\end{aligned}
$$

$$
\begin{aligned}
& - 25646306254712\,y^{36} - 28580395790376\,y^{35} - 30029268395908\,y^{34} \\
& - 29828198869240\,y^{33} - 61164\,y^{4} - 12444\,y^{3} - 11665092\,y^{8} \\
& - 3559218\,y^{7} - 1009351\,y^{6} - 262264\,y^{5} - 104312228\,y^{10} - 1900252432\,y^{13} \\
& - 35884960\,y^{9} - 756947512\,y^{12} - 287897544\,y^{11} - 23143828508\,y^{16} \\
& - 10493317928\,y^{15} - 4562670436\,y^{14} \\
c_8 = & - 14\,y - 252\,y^{2} - 25098955113968\,y^{32} - 21347223917672\,y^{31} \\
& - 17303673015328\,y^{30} - 13387253668816\,y^{29} - 9898697135120\,y^{28} \\
& - 7003509266248\,y^{27} - 4746453967584\,y^{26} - 3084283289652\,y^{25} \\
& - 1923259624582\,y^{24} - 1151691937865\,y^{23} - 662684126384\,y^{22} \\
& - 366550654856\,y^{21} - 194947311000\,y^{20} - 99690728668\,y^{19} \\
& - 49004477896\,y^{18} - 23143828508\,y^{17} - 31465\,y^{55} - 876960\,y^{54} \\
& - 11838960\,y^{53} - 103508496\,y^{52} - 661326120\,y^{51} - 3303409200\,y^{50} \\
& - 13476208200\,y^{49} - 46296237520\,y^{48} - 137010274856\,y^{47} \\
& - 355427925664\,y^{46} - 819405519408\,y^{45} - 1697441917808\,y^{44} \\
& - 3188473544424\,y^{43} - 5472176918320\,y^{42} - 8636341292464\,y^{41} \\
& - 12604032196532\,y^{40} - 17092716269202\,y^{39} - 21632053806736\,y^{38} \\
& - 25646306254712\,y^{37} - 28580395790376\,y^{36} - 30029268395908\,y^{35} \\
& - 29828198869240\,y^{34} - 28077372499868\,y^{33} - 12444\,y^{4} - 2090\,y^{3} \\
& - 3559218\,y^{8} - 1009351\,y^{7} - 262264\,y^{6} - 61164\,y^{5} - 35884960\,y^{10} \\
& - 11665092\,y^{9} - 287897544\,y^{12} - 104312228\,y^{11} - 10493317928\,y^{16} \\
& - 4562670436\,y^{15} - 1900252432\,y^{14} - 756947512\,y^{13} \\
c_7 = & 5 + 70\,y + 616\,y^{2} + 7313081337386\,y^{32} + 6187019672704\,y^{31} \\
& + 5001900819008\,y^{30} + 3869759209760\,y^{29} + 2868764374752\,y^{28} \\
& + 2040261727824\,y^{27} + 1393577428740\,y^{26} + 915089088130\,y^{25} \\
& + 578194161219\,y^{24} + 351808762496\,y^{23} + 206277316544\,y^{22} \\
& + 116608603248\,y^{21} + 63575384600\,y^{20} + 33433959952\,y^{19} + 1776429824\,y^{15} \\
& + 16959111912\,y^{18} + 8295244308\,y^{17} + 4495\,y^{56} + 129920\,y^{55} \\
& + 1818880\,y^{54} + 16488864\,y^{53} + 109201680\,y^{52} + 565219200\,y^{51} \\
& + 2388353968\,y^{50} + 8495842680\,y^{49} + 26027492764\,y^{48} + 326893136\,y^{13} \\
& + 69884743936\,y^{47} + 166748284864\,y^{46} + 357531140512\,y^{45} + 51357240\,y^{11} \\
& + 695222438736\,y^{44} + 1235449012000\,y^{43} + 2019536911384\,y^{42} \\
& + 3053870732836\,y^{41} + 4292981378154\,y^{40} + 5634619819904\,y^{39} \\
& + 6931822075968\,y^{38} + 8020643537936\,y^{37} + 8755782372104\,y^{36} \\
& + 9042912134000\,y^{35} + 8857775392592\,y^{34} + 8247140069852\,y^{33} \\
& + 15316\,y^{4} + 3408\,y^{3} + 2295159\,y^{8} + 733024\,y^{7} + 219248\,y^{6}
\end{aligned}
$$

$$
\begin{aligned}
&+ 60696\,y^{5} + 19109072\,y^{10} + 6790994\,y^{9} + 132235736\,y^{12} + 776871360\,y^{14} \\
&+ 3911002194\,y^{16} \\
c_6 =& 5\,y + 70\,y^{2} + 6187019672704\,y^{32} + 5001900819008\,y^{31} + 3869759209760\,y^{30} \\
&+ 2868764374752\,y^{29} + 2040261727824\,y^{28} + 1393577428740\,y^{27} \\
&+ 578194161219\,y^{25} + 351808762496\,y^{24} + 206277316544\,y^{23} \\
&+ 63575384600\,y^{21} + 33433959952\,y^{20} + 16959111912\,y^{19} + 8295244308\,y^{18} \\
&+ 3911002194\,y^{17} + 4495\,y^{57} + 129920\,y^{56} + 1818880\,y^{55} + 16488864\,y^{54} \\
&+ 109201680\,y^{53} + 565219200\,y^{52} + 2388353968\,y^{51} + 8495842680\,y^{50} \\
&+ 26027492764\,y^{49} + 69884743936\,y^{48} + 166748284864\,y^{47} \\
&+ 695222438736\,y^{45} + 1235449012000\,y^{44} + 2019536911384\,y^{43} \\
&+ 4292981378154\,y^{41} + 5634619819904\,y^{40} + 6931822075968\,y^{39} \\
&+ 8755782372104\,y^{37} + 9042912134000\,y^{36} + 8857775392592\,y^{35} \\
&+ 7313081337386\,y^{33} + 3408\,y^{4} + 616\,y^{3} + 733024\,y^{8} + 219248\,y^{7} \\
&+ 60696\,y^{6} + 15316\,y^{5} + 6790994\,y^{10} + 2295159\,y^{9} + 51357240\,y^{12} \\
&+ 19109072\,y^{11} + 1776429824\,y^{16} + 776871360\,y^{15} + 326893136\,y^{14} \\
&+ 132235736\,y^{13} + 915089088130\,y^{26} + 116608603248\,y^{22} \\
&+ 3053870732836\,y^{42} + 8020643537936\,y^{38} + 8247140069852\,y^{34} + 357531140512\,y^{46} \\
c_5 =& - 2 - 20\,y - 135\,y^{2} - 1291240176900\,y^{32} - 1041748018848\,y^{31} \\
&- 806281172416\,y^{30} - 599429462736\,y^{29} - 428584659180\,y^{28} \\
&- 295028984862\,y^{27} - 195735333933\,y^{26} - 125273542192\,y^{25} \\
&- 77410559816\,y^{24} - 46217249904\,y^{23} - 26676063960\,y^{22} - 491718996\,y^{16} \\
&- 14891452032\,y^{21} - 8042059472\,y^{20} - 4202120092\,y^{19} - 99677176\,y^{14} \\
&- 2124447526\,y^{18} - 1039137368\,y^{17} - 465\,y^{58} - 13920\,y^{57} - 7062174\,y^{11} \\
&- 201840\,y^{56} - 1894816\,y^{55} - 12991440\,y^{54} - 69588960\,y^{53} - 17671720\,y^{12} \\
&- 304194016\,y^{52} - 1119005992\,y^{51} - 3544114548\,y^{50} - 42689912\,y^{13} \\
&- 9836099344\,y^{49} - 24256384264\,y^{48} - 53753140896\,y^{47} - 225093584\,y^{15} \\
&- 108038522256\,y^{46} - 198480699200\,y^{45} - 335496053496\,y^{44} - 1007064\,y^{9} \\
&- 524756939068\,y^{43} - 763293723014\,y^{42} - 1037044904688\,y^{41} - 2719985\,y^{10} \\
&- 1321226300232\,y^{40} - 1583998322704\,y^{39} - 1792664023752\,y^{38} \\
&- 1920611818240\,y^{37} - 1952903458120\,y^{36} - 1888915844532\,y^{35} \\
&- 1741500968654\,y^{34} - 1533253370056\,y^{33} - 3204\,y^{4} - 770\,y^{3} \\
&- 357052\,y^{8} - 120568\,y^{7} - 38564\,y^{6} - 11592\,y^{5} \\
c_4 =& - 2\,y - 20\,y^{2} - 1041748018848\,y^{32} - 806281172416\,y^{31} - 599429462736\,y^{30} \\
&- 428584659180\,y^{29} - 295028984862\,y^{28} - 195735333933\,y^{27} \\
&- 77410559816\,y^{25} - 46217249904\,y^{24} - 26676063960\,y^{23} - 14891452032\,y^{22}
\end{aligned}
$$

$$
\begin{aligned}
&- 8042059472\, y^{21} - 4202120092\, y^{20} - 2124447526\, y^{19} - 1039137368\, y^{18}\\
&- 491718996\, y^{17} - 465\, y^{59} - 13920\, y^{58} - 201840\, y^{57} - 1894816\, y^{56}\\
&- 12991440\, y^{55} - 69588960\, y^{54} - 304194016\, y^{53} - 1119005992\, y^{52}\\
&- 9836099344\, y^{50} - 24256384264\, y^{49} - 53753140896\, y^{48} - 108038522256\, y^{47}\\
&- 198480699200\, y^{46} - 335496053496\, y^{45} - 524756939068\, y^{44}\\
&- 1037044904688\, y^{42} - 1321226300232\, y^{41} - 1583998322704\, y^{40}\\
&- 1920611818240\, y^{38} - 1952903458120\, y^{37} - 1888915844532\, y^{36}\\
&- 1533253370056\, y^{34} - 1291240176900\, y^{33} - 770\, y^{4} - 135\, y^{3} - 120568\, y^{8}\\
&- 38564\, y^{7} - 11592\, y^{6} - 3204\, y^{5} - 1007064\, y^{10} - 357052\, y^{9} - 7062174\, y^{12}\\
&- 2719985\, y^{11} - 225093584\, y^{16} - 99677176\, y^{15} - 42689912\, y^{14}\\
&- 125273542192\, y^{26} - 3544114548\, y^{51} - 763293723014\, y^{43} - 17671720\, y^{13}\\
&- 1741500968654\, y^{35} - 1792664023752\, y^{39}
\end{aligned}
$$

$$
\begin{aligned}
c_3 =& 1 + 6\, y + 28\, y^{2} + 139134657962\, y^{32} + 107772280144\, y^{31} + 80374075940\, y^{30}\\
&+ 57780713586\, y^{29} + 40086413227\, y^{28} + 26866822144\, y^{27} + 17412522304\, y^{26}\\
&+ 10922249064\, y^{25} + 6635841796\, y^{24} + 3907393968\, y^{23} + 2231027256\, y^{22}\\
&+ 1235719404\, y^{21} + 664148942\, y^{20} + 346461216\, y^{19} + 175470704\, y^{18}\\
&+ 86311084\, y^{17} + 31\, y^{60} + 960\, y^{59} + 14400\, y^{58} + 139824\, y^{57} + 1650\, y^{5}\\
&+ 991320\, y^{56} + 5488896\, y^{55} + 24792208\, y^{54} + 94201160\, y^{53} + 1622763\, y^{12}\\
&+ 308074372\, y^{52} + 882664192\, y^{51} + 2246800496\, y^{50} + 5139121208\, y^{49}\\
&+ 10661724164\, y^{48} + 20219969056\, y^{47} + 35289119800\, y^{46} + 3802026\, y^{13}\\
&+ 57003970868\, y^{45} + 85655656018\, y^{44} + 120260511360\, y^{43} + 41250862\, y^{16}\\
&+ 158389551488\, y^{42} + 196385714120\, y^{41} + 229965381556\, y^{40} + 672720\, y^{11}\\
&+ 255056476176\, y^{39} + 268633279376\, y^{38} + 269307919028\, y^{37} + 14352\, y^{7}\\
&+ 257525633614\, y^{36} + 235343516912\, y^{35} + 205895850024\, y^{34} + 5000\, y^{6}\\
&+ 172721468332\, y^{33} + 495\, y^{4} + 120\, y^{3} + 39586\, y^{8} + 8657072\, y^{14}\\
&+ 270536\, y^{10} + 105324\, y^{9} + 19164440\, y^{15}
\end{aligned}
$$

$$
\begin{aligned}
c_2 =& y + 6\, y^{2} + 107772280144\, y^{32} + 80374075940\, y^{31} + 57780713586\, y^{30}\\
&+ 40086413227\, y^{29} + 26866822144\, y^{28} + 17412522304\, y^{27} + 10922249064\, y^{26}\\
&+ 6635841796\, y^{25} + 3907393968\, y^{24} + 2231027256\, y^{23} + 1235719404\, y^{22}\\
&+ 664148942\, y^{21} + 346461216\, y^{20} + 175470704\, y^{19} + 86311084\, y^{18}\\
&+ 41250862\, y^{17} + 31\, y^{61} + 960\, y^{60} + 14400\, y^{59} + 139824\, y^{58}\\
&+ 991320\, y^{57} + 5488896\, y^{56} + 24792208\, y^{55} + 94201160\, y^{54}\\
&+ 308074372\, y^{53} + 882664192\, y^{52} + 2246800496\, y^{51} + 5139121208\, y^{50}\\
&+ 10661724164\, y^{49} + 20219969056\, y^{48} + 35289119800\, y^{47} + 57003970868\, y^{46}\\
&+ 85655656018\, y^{45} + 120260511360\, y^{44} + 158389551488\, y^{43}
\end{aligned}
$$

$$
\begin{aligned}
&+229965381556\,y^{41}+255056476176\,y^{40}+268633279376\,y^{39}\\
&+257525633614\,y^{37}+235343516912\,y^{36}+205895850024\,y^{35}\\
&+139134657962\,y^{33}+120\,y^{4}+28\,y^{3}+14352\,y^{8}+5000\,y^{7}+1650\,y^{6}\\
&+495\,y^{5}+105324\,y^{10}+39586\,y^{9}+672720\,y^{12}+270536\,y^{11}\\
&+19164440\,y^{16}+8657072\,y^{15}+3802026\,y^{14}+1622763\,y^{13}\\
&+196385714120\,y^{42}+269307919028\,y^{38}+172721468332\,y^{34}\\
c_1 = &-1-2\,y-5\,y^{2}-6939692682\,y^{32}-5193067630\,y^{31}-3754272037\,y^{30}\\
&-2625062128\,y^{29}-1777171560\,y^{28}-1166067016\,y^{27}-742179284\,y^{26}\\
&-458591432\,y^{25}-275276716\,y^{24}-160617860\,y^{23}-91143114\,y^{22}-132\,y^{5}\\
&-50323496\,y^{21}-27049196\,y^{20}-14162220\,y^{19}-7228014\,y^{18}-5916\,y^{9}\\
&-3598964\,y^{17}-y^{62}-32\,y^{61}-496\,y^{60}-4976\,y^{59}-175048\,y^{13}-14290\,y^{10}\\
&-36440\,y^{58}-208336\,y^{57}-971272\,y^{56}-3807704\,y^{55}-385741\,y^{14}\\
&-12843980\,y^{54}-37945904\,y^{53}-99582920\,y^{52}-234813592\,y^{51}-365\,y^{6}\\
&-502196500\,y^{50}-981900168\,y^{49}-1766948340\,y^{48}-2943492972\,y^{47}\\
&-4562339774\,y^{46}-6609143792\,y^{45}-8984070856\,y^{44}-831014\,y^{15}\\
&-11500901864\,y^{43}-13910043524\,y^{42}-15941684776\,y^{41}-77684\,y^{12}\\
&-17357937708\,y^{40}-17999433372\,y^{39}-17813777994\,y^{38}-33708\,y^{11}\\
&-16859410792\,y^{37}-15286065700\,y^{36}-13299362332\,y^{35}-1749654\,y^{16}\\
&-11120136162\,y^{34}-8948546308\,y^{33}-42\,y^{4}-14\,y^{3}-2398\,y^{8}-950\,y^{7}\\
c_0 = &-1-y-2\,y^{2}-5193067630\,y^{32}-3754272037\,y^{31}-2625062128\,y^{30}\\
&-1777171560\,y^{29}-1166067016\,y^{28}-742179284\,y^{27}-458591432\,y^{26}\\
&-275276716\,y^{25}-160617860\,y^{24}-91143114\,y^{23}-50323496\,y^{22}\\
&-27049196\,y^{21}-14162220\,y^{20}-7228014\,y^{19}-3598964\,y^{18}-1749654\,y^{17}\\
&-y^{63}-32\,y^{62}-496\,y^{61}-4976\,y^{60}-36440\,y^{59}-208336\,y^{58}-971272\,y^{57}\\
&-3807704\,y^{56}-12843980\,y^{55}-37945904\,y^{54}-99582920\,y^{53}-175048\,y^{14}\\
&-502196500\,y^{51}-981900168\,y^{50}-1766948340\,y^{49}-2943492972\,y^{48}\\
&-6609143792\,y^{46}-8984070856\,y^{45}-11500901864\,y^{44}-13910043524\,y^{43}\\
&-15941684776\,y^{42}-17357937708\,y^{41}-17999433372\,y^{40}-17813777994\,y^{39}\\
&-16859410792\,y^{38}-15286065700\,y^{37}-13299362332\,y^{36}-11120136162\,y^{35}\\
&-8948546308\,y^{34}-6939692682\,y^{33}-14\,y^{4}-5\,y^{3}-950\,y^{8}-365\,y^{7}\\
&-132\,y^{6}-42\,y^{5}-5916\,y^{10}-2398\,y^{9}-33708\,y^{12}-14290\,y^{11}-77684\,y^{13}\\
&-831014\,y^{16}-385741\,y^{15}-234813592\,y^{52}-4562339774\,y^{47}.
\end{aligned}
$$