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PREMIUM AND BENEFIT LEVELS IN THE PUBLIC PENSION SYSTEM: A MARTINGALE APPROACH

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ABSTRACT: In this paper we propose a relation between the premium and the benefit levels in the Public Pension Systems (PPS) targeting the generational fairness. First we introduce the actuarial projection model used in the actuarial valuation for the PPS, in which it is possible to control the premium and the benefit levels. In order to match up the premium and the benefit levels, we focus on the generational fairness. For the concept of the generational fairness we remind the martingale, and notice the generational accounting. On the detail construction of the martingale sequence, we utilize the cash flow of the premium and the benefit from the output of the actuarial projection model and the historical data, and iterate the method of the generational accounting on the consecutive future time. From this approach, we find the fair relation between the premium and the benefit levels according to the generations. Through our result we expect to propose a new framework of the relation in the premium and the benefit levels in the PPS.

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Key Words: public pension system, premium and benefit levels, martingale, generational accounting, actuarial projection model

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1. INTRODUCTION

Many countries have legally administered the Public Pension System (PPS) as a welfare policy for the basic old-age income security. In the operation of the PPS, the premium and the benefit levels are determined by the law. For the more wealthy old age, it is needed to ensure the high benefit level. To be like that, it must be supported by the high premium level. However, it is not clear in the relation between the premium and the benefit levels unlike the private pension. In the private sectors these relationship can be controlled by the equivalence principle, which can be also used as the fundamental tool for evaluating the financial status in the total risk management of the private company. Whereas, it is usual for the representative member under the PPS to receive the more benefit compared to the payed premium in the expectation sense. This is why the equivalence principle does not work in the PPS. This imbalanced design in the premium and the benefit levels is the main issue causing the financial insolvency in the PPS. In this reason it is recommended to examine the financial status of the PPS periodically based on the actuarial valuation (IAA [6], ILO [7, 8], Iyer [9]). In case of the risky financial status from the actuarial valuation, the reform of the PPS must be driven targeting the adjustment for the premium and the benefit levels. In this time, the chief purpose for the reform of the PPS is naturally to diminish the financial unbalance in the gross. For this purpose, there are basically two kinds of schemes. One is to decrease the benefit level, the other to increase the premium level. Whichever scheme is taken, it is inevitable to suffer the serious trouble because of the complexity in the interests along all generations.

Motivated by these problems we suggest a novel method to determine the premium and the benefit levels in the PPS. When the reform is targeted on the financial balance, it is unavoidable to undergo the generational affliction due to the differences of the profit from the PPS. In order to focus on the generational fairness, we note the concept of the martingale, which is natural since a martingale is characterized as the fair game in the probability theory. Then, the question is how to link the martingale with the premium and the benefit levels in the PPS. Our idea is started from the generational accounting known as the powerful method measuring the effects of the fiscal policy.

Since Auerbach et al. introduced the generational accounting in the early 1990s, it is broadly utilized to access the sustainability of the fiscal policy and to measure the fiscal burdens of the current and future generations (Auerbach et al. [2]). This is oriented from the economic theories such as the overlapping generations and the general equilibrium model. So, it is easy to examine the economic value on the generations and make an implication in the collective point. Besides the immediate applications on the fiscal division, it is also applied on the division of the pension (Ponds [12], Gollier [4], Hoevenaars and Ponds [5], Cui et al. [3]), in which it is focussed on the issue of the intergenerational risk sharing such that they find out the optimality for the welfare or the funding ratio. And, in the separated way from the generational accounting there are similar literatures investigating the money's worth across the cohorts and the legacy debt by the total transfers (Leimer [10], [11]). It is applied with the real public pension, the Old Age and Survivors Insurance (OASI, the typical social security program of the US) such that it is investigated from the historical data and the official projection result from the Trustees Report[1].

In the referred literatures, it is examined under some scenarios such that it revealed the actual intergenerational imbalances. However, we propose the method to determine the future premium and benefit levels in the PPS with the intergenerational balances applying the martingale. In order to take the martingale sequence, we generate the time sequence by the successive calculations of the net burden including the amount of the past occurrence along the generations at the consecutive future points as it has been done at the only present time in the generational accounting, and control the calculated time sequence to have the martingale property. In the process of the detail calculation, we utilize the official assumptions, the actuarial projection result of the PPS and the possible historical data in the same manner of Leimer. From this approach we obtain our desirable result.

This paper is organized as follows. In section 2 we introduce the actuarial projection model used in the actuarial valuation for the PPS. From this model, we can utilize the cash flow of the premium and the benefit by cohorts and control the premium and the benefit levels for the future needed in the next section. In section 3 we consider the generational fairness on the PPS so that derive the relation between the premium and the benefit levels based on the generational accounting and the martingale property. In section 4 we provide some numerical example to illustrate our result. The conclusions are presented in section 5.

2. ACTUARIAL PROJECTION MODEL

As we pointed out in the previous section the government administrating the PPS has enforced the periodic actuarial valuation such that the actuarial valuation report is usually required to contain the demographic and financial projections results. In this reason the actuarial projection model is utilized. In this section, we introduce the actuarial projection model. According to the PPS, the corresponding actuarial projection model is made up reflecting its own scheme. So, there is no unique model encompassing all PPS. Taking this into consideration, we try to introduce the generalized model reflecting the common schemes in the PPS. Let's start with some notations.

$P_{t,a,i}$: the premium revenue by i-th participant with age a in year t,

$B_{t,a,j}$: the benefit expenditure by j-th beneficiary with age a in year t,

[1]It is annually published including the assumptions for the long-term projection period, 75 years.

$N_{t,a}^{(p)}$: the number of the participants with age a in year t,

$N_{t,a}^{(b)}$: the number of the beneficiaries with age a in year t,

F_t : the fund amount at the end of year t,

Z_t : the fund return during year t,

a_e : the entrance age,

a_r : the retirement age

For the simplicity, we assume that the cash flow occurs only at the end of year. Then the following identity holds in the PPS:

$$F_{t+1} = F_t\left(1+Z_{t+1}\right) + \sum_{a=a_e}^{a_r-1}\sum_{i=1}^{N_{t+1,a}^{(p)}} P_{t+1,a,i} - \sum_{a=a_r}^{\infty}\sum_{j=1}^{N_{t+1,a}^{(b)}} B_{t+1,a,j} \tag{1}$$

where $t \geq t_0$ for the present year t_0. In practice, there are more something to be considered in the PPS such as the survivors insurance, the disability insurance and the early old age pension, which are mainly related to the benefit condition. As we referred, the individual PPS has the various specific schemes in accordance with the own circumstances. However, we restrict our model to the main scheme of the PPS, the income security after the normal retirement age.

Next, we take the more detail modeling in (1) highlighting the premium and the benefit forms. In general, the premium in the PPS is proportional to the participant's salary. So, we model the premium $P_{t,a,i}$ as

$$P_{t,a,i} = p_t S_{t,a,i} \tag{2}$$

where $S_{t,a,i}$ is the salary of the i-th participant with age a in year t and p_t is a constants reflecting the premium level in year t.

In the benefit modeling, it is needed to consider two kinds of the groups, the beginning one and the others. In the PPS it is general to calculate the first pension amount reflecting the benefit level and escalate the previous amount with respect to some rate after the second benefit. So, we divide the benefit part in (1) according to the ages and model as follows

$$\begin{aligned}\sum_{a=a_r}^{\infty}\sum_{j=1}^{N_{t+1,a}^{(b)}} B_{t+1,a,j} &= \sum_{a=a_r}^{a_r}\sum_{j=1}^{N_{t+1,a}^{(b)}} B_{t+1,a,j} + \sum_{a=a_r+1}^{\infty}\sum_{j=1}^{N_{t+1,a}^{(b)}} B_{t+1,a,j} \\ &= \sum_{j=1}^{N_{t+1,a_r}^{(b)}} b_t S_{t+1,a_r,j}^{(Av)} + \sum_{a=a_r+1}^{\infty}\sum_{j=1}^{N_{t+1,a}^{(b)}} B_{t,a-1,j} \times (1+C_{t+1}) \end{aligned} \tag{3}$$

where b_t is a constant reflecting the benefit level, $S_{t,a_r,j}^{(Av)}$ is the revalued average salary of the j-th beneficiary with age a_r in year t during the working ages, and C_t is the escalating factor in year t. As the escalating factor, it is typical to take the Consumer Price Index (CPI). By (2) and (3) we can rewrite the identity (1) as follows

$$\begin{aligned} F_{t+1} &= F_t(1+Z_{t+1}) + p_t \sum_{a=a_e}^{a_r-1} \sum_{i=1}^{N^{(p)}_{t+1,a}} S_{t+1,a,i} \\ &\qquad -b_t \sum_{j=1}^{N^{(b)}_{t+1,a_r}} S^{(Av)}_{t+1,a_r,j} - \sum_{a=a_r+1}^{\infty} \sum_{j=1}^{N^{(b)}_{t+1,a}} B_{t,a-1,j} \times (1+C_{t+1}). \end{aligned} \tag{4}$$

Note that the variables in (4) are all random for $t > t_0$ except the ages a, the premium levels p_t and the benefit levels b_t. In particular, the premium levels p_t and the benefit levels b_t are the controllable variables so that these are main variables to evaluate the fairness in our method.

For the concrete baseline calculation, it is standard to make the estimation in the sense of the expectation in (4). That is, the population size by ages, the economic values for the salary and the escalating factor are implemented from the auxiliary result of the demographic projection and the economic forecasting, which are described as the demographic and the economic assumptions in the actuarial report. Based on these assumptions, we figure out the random values such as $N^{(p)}_{t,a}$, $N^{(b)}_{t,a}$, $S_{t,a,i}$, $S^{(Av)}_{t,a,i}$, Z_t, C_t considering the detail schemes of the PPS. Related to the detail schemes, it is another important part of the assumptions for the actuarial projection. For instance, it is linked with the specific benefit condition of the PPS to project the number of the beneficiaries by ages and revalue the average salary $S^{(Av)}_{t+1,a,j}$. Through these procedure, we can have the estimated future processes of the premium revenue, the benefit expenditure and the fund amount for the actuarial valuation. The Table 1 is the practical example of the actuarial projection result of Korean PPS.

(unit : billion KRW)

Year	Accumulated fund	Revenue			Expenditure		Balance
		Total	Premium	Return	Total	Benefit	
2013	417,727	52,217	32,135	20,082	14,556	14,032	37,661
2020	847,171	109,098	54,073	55,025	33,923	33,487	75,175
2030	1,732,381	186,913	95,041	91,872	89,953	89,176	96,960
2040	2,494,494	258,427	141,595	116,832	213,773	212,563	44,654
2050	2,200,519	309,781	203,282	106,498	414,088	412,288	-104,308
2060	-280,716	263,375	263,375	0	657,820	655,155	-394,445
2070	-	358,101	358,101	0	948,255	944,311	-590,154
2080	-	477,892	477,892	0	1,263,650	1,257,811	-785,757

Table 1: Basic projection result on 2013 actuarial valuation of Korean PPS

3. MARTINGALE RELATION BETWEEN PREMIUM AND BENEFIT

As we commented previously, the reform for the premium and the benefit levels is subject to accompany the severe stress along all generations as the member of the

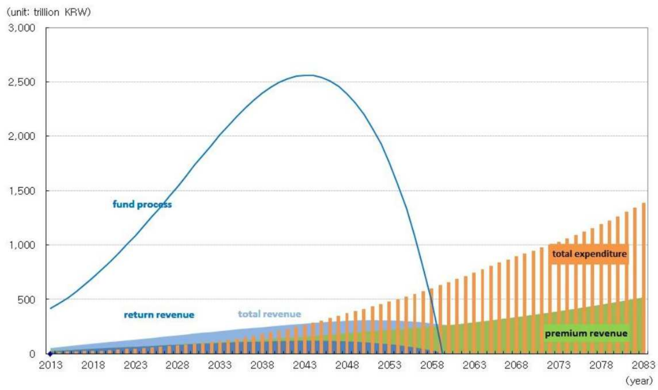

Figure 1: Basic projection result on 2013 actuarial valuation of Korean PPS

PPS. This is due to the property of the generational dependency in the financing for the PPS. In this point of view we attempt to derive the rational relationship between the premium and the benefit levels focusing on the generational fairness applying the martingale. So, the key point is how we construct a martingale sequence reflecting the premium and the benefit of the PPS. In fact, it is not simple to argue the fairness over all generations. Fortunately, we can find a clue from the generational accounting.

In the generational accounting it is possible to measure the relative fiscal burdens falling upon different generations by the fiscal and tax policy of the government. Roughly speaking in the point of the PPS, for each generation the net profit is calculated by subtracting the premium amount from the benefit one. This net profit is corresponding to the relative fiscal burdens in the original generational accounting. The counted amount in the generational accounting is restricted to the occurred one after the present. But in the stance of the PPS it must be also counted for the entire life time, that is the premium and the benefit amount occurred before the present must be included in the calculation of the net profit for each generation. In this sense there are some attempts based on the OASI by Leimer [10], [11], in which some money's worth such as the lifetime benefit/premium[2] ratio and lifetime net transfer has been calculated including the past occurred premium and benefit. However Leimer just calculated these according to the present law policy and two alternative policies. In the present law policy the present premium and benefit levels are supported with any adjustment and in the alternative policies the premium level is raised up or the benefit level is reduced after the reserve fund is exhausted. So, there is no

[2]Instead of the premium, it is used with the 'tax' in OASDI. But, we use the 'premium' for the consistency of the context.

information about the generational fairness from such adjustments of the premium and the benefit levels. Now, let's introduce some notations for the detail arguments.

$$P_y(g_y; p_y) = \text{premium revenue at year } y \text{ by age } g_y \text{ with the premium level } p_y \quad (5)$$

$$B_y(g_y; b_y) = \text{benefit expenditure at year } y \text{ by age } g_y \text{ with the benefit level } b_y \quad (6)$$

$$Pv\_P_t(g_t; \tilde{p}_t) = \sum_y P_y(g_t + y - t; p_y) \times DR_t(y) \quad (7)$$

$$Pv\_B_t(g_t; \tilde{b}_t) = \sum_y B_y(g_t + y - t; b_y) \times DR_t(y) \quad (8)$$

$$NP_t(g_t; \tilde{p}_t, \tilde{b}_t) = Pv\_B_t(g_t; \tilde{b}_t) - Pv\_P_t(g_t; \tilde{p}_t) \quad (9)$$

where $\tilde{p}_t$ and $\tilde{b}_t$ are the collection of the variables p_y and b_y with $y = \cdots, t-1, t, t+1, \cdots$, respectively, $DR_t(y)$ is a discount factor for the present value at the baseline t as

$$DR_t(y) = \begin{cases} \prod_{i=y}^{t}(1+R_i) & \text{if } y < t \\ 1 & \text{if } y = t \\ \prod_{i=t+1}^{y} \frac{1}{1+R_i} & \text{if } y > t \end{cases},$$

R_i is an interest rate at year i.

Note that the premium and the benefit levels of the PPS are determined by the law. So, for the future y, p_y and b_y are the controllable variables so that these are non-random. But, for any future year y, $P_y(g_y; p_y)$ and $B_y(g_y; b_y)$ are random variables reflecting the future population structure and economic situation.

We explain the notations briefly. $Pv\_P(t, g_t; \tilde{p}_t)$ and $Pv\_B(t, g_t; \tilde{b}_t)$ are the present values on year t of the total lifetime premium and the total lifetime benefit reflecting the premium level collection $\tilde{p}_t$ and the benefit level collection $\tilde{b}_t$ of age g_t, respectively, $NP_t(g; \tilde{b}, \tilde{p})$ is the net profit of age g at year t under the premium level collection $\tilde{p}$ and the benefit level collection $\tilde{b}$.

Now, we define a time sequence M_t as

$$M_t = DR_{t_0}(t) \cdot \frac{\sum_{g=0}^{\infty} NP_t(g; \tilde{b}, \tilde{p})}{\sum_a N_{t,a}^{(p)} + \sum_{a'} N_{t,a'}^{(b)}} \quad \text{for all } t \geq t_0 \quad (10)$$

where t_0 is the present time, $N_{t,a}^{(p)}$ and $N_{t,a'}^{(b)}$ are the numbers of the participants with age a and beneficiaries with age a' at time t, respectively, as the same notations in section 2.

Note that M_t is the individual net profit of all generations alive at time t from the PPS with the premium level collection $\tilde{p}$ and the benefit level collection $\tilde{b}$, and the composition of generations included in M_t are changed following the time t. In this sense, we can regard that the premium level collection $\tilde{p}$ and the benefit level collection $\tilde{b}$ are fair if M_t has the martingale property.

Proposition 1. *The generational fair relation between the premium level and the benefit level in the PPS can be proposed the sequence M_t defined in (10) satisfies the following martingale condition*

$$M_t \quad = \quad E[M_{t+1}|\mathcal{F}_t] \quad \text{for all } t \geq t_0 \tag{11}$$

where $\mathcal{F}_t$ is a filtration reflecting all circumstances such as the population structure and the economic situation at time t, and t_0 is the present time.

Remark 3.1. Related to the notion of the generation, we can have two points. One is the unique cohort itself and another the collective cohorts at the unique time. We adopt the latter in our approach for the design of the martingale sequence. In fact, it may be favorable to take the former since it is more tight condition. However, it is more unfeasible due to the existing imbalance in the PPS. If it is possible to make up every cohort itself fair, it may be not needed to reform the PPS.

Remark 3.2. In the generational accounting it is calculated for the amount in the future as we noticed. So, it is excluded from the past premium and benefit effects, which is very important component in the PPS since the history of the past effects is the main reason to cause the financial insolvency. In relation of the detail calculation, the past effects parts are already fixed. So, it is not influenced from the expectation, i.e., non-random. On the other hand, the future premium and the benefit amounts of $P_y(g_y; p_y)$ and $B_y(g_y; b_y)$ for $y = t, t+1, ...$ are depended on the usual actuarial projection model of the PPS so that the expectation of M_t is taken from the assumption corresponding to the base line scenario of the official actuarial report.

4. EMPIRICAL ANALYSIS

In this section, we present an numerical example for the application of our method. For the detail PPS and the necessary data, it is based on the Korean PPS, National Pension Service (NPS) and the 2013 3rd actuarial valuation of the NPS. The NPS has begun with 3% of the premium level and 70%[3] of the benefit level since the year 1988. Through the several reforms the premium level is fixed with 9% after the year 2005 and the benefit level will be fixed with 40% after the year 2028. As we described it is

[3]This is provided that the contributed period is 40 years.

tried with the possible data of the NPS for the historical value and the 3rd actuarial projection result under the baseline assumptions for the future value. For the needed extra future of the beyond official projection period, we extend the final assumptions of the official ones the same as the case of Leimer ([10], [11]).

As the official assumptions for the 3rd actuarial valuation of the NPS, the principal economic variables are supposed as follows. In particular, we apply the discount factor

Table 2: Principal economic assumptions *

	2011 ∼2020	2021 ∼2030	2031 ∼2040	2041 ∼2050	2051 ∼2060	2061 ∼2070	2071 ∼2083
Real wage growth rate	2.7	3.1	2.4	2.1	2.0	2.0	2.0
Real interest rate	2.6	2.7	2.5	2.4	2.5	2.6	2.7
Inflation rate	3.2	2.8	2.2	2.0	2.0	2.0	2.0

* : the presented figures are the average of the assumed values during the each designated period.
unit : %

with the nominal wage growth rate.

Based on the above descriptions, it is calculated as the Table 3 indicating the premium and benefit amounts by the cohorts. This table is a kind of the generational accounting at the basis time of 2013 year. In the point of M_t in (10), it corresponds to

Table 3: Premium and benefit by cohort

cohort	Premium(P)			Benefit(B)			B - P
(age)	subtotal	history	future	subtotal	history	future	
1933(80)	204	204	-	943	693	251	740
1943(70)	2,471	2,471	-	8,289	3,736	4,553	5,818
1953(60)	8,367	8,367	-	19,248	526	18,722	10,880
1963(50)	22,417	13,956	8,461	39,712	-	39,712	17,295
1973(40)	32,962	13,123	19,839	60,138	-	60,138	27,176
1983(30)	33,985	4,384	29,600	62,628	-	62,628	28,643
1993(20)	35,820	75	35,745	65,434	-	65,434	29,614
2003(10)	27,273	-	27,273	50,531	-	50,531	23,259
2013(0)	26,372	-	26,372	49,575	-	49,575	23,203
2023	26,685	-	26,685	51,083	-	51,083	24,399
2033	23,520	-	23,520	45,829	-	45,829	22,309
2043	19,598	-	19,598	38,684	-	38,684	19,086
2053	18,915	-	18,915	37,333	-	37,333	18,419
2063	18,092	-	18,092	35,840	-	35,840	17,748
2073	17,337	-	17,337	34,192	-	34,192	16,855

unit : billion in 2013 year / age : in 2013 year
Benefit(B) : including only old age pension

the vertical sum in the last column 'B - P' restricted to the current cohorts including the dropped cohorts in Table 3 with $t = 2013$. In order to take a time sequence M_t, we iterate the calculation like Table 3 changing the basis time, $t = 2014, 2015, ...$, and take the present value at time 2013 for the vertical sum in the last column for each changed basis time. In practice, it is impossible to calculate at the infinite basis time. So, we restrict to 70th basis time as the number of iterations in our example, which is the official projection period[4] of the NPS. Then, it is left to find a pair of the premium level collection $\tilde{p}$ and the benefit level collection $\tilde{b}$ satisfying the definition of the martingale for the calculated sequence M_t. Numerically, it is hard to obtain the perfect equality in the definition of the martingale. So, we take the following approximation.

Let RM_t be the ratio as

$$RM_t = \frac{M_{t+1}}{M_t}$$

and δ the pre-determined positive constant close to zero. If $r_t := |RM_t - 1| < \delta$ for each t, then we label this sequence as a semi-martingale. And, we obtain the fair premium level collection $\tilde{p}$ and benefit level collection $\tilde{b}$ from the best semi-martingale $\widehat{M_t}$ defined by

$$\widehat{M_t} = \min_{\tilde{p},\tilde{b}} \left\{ \sum r_t \middle| M_t \ \text{ are semi-martingales} \right\}$$

In Figure 2 the best semi-martingale is lined with the corresponding ratio RM_t and

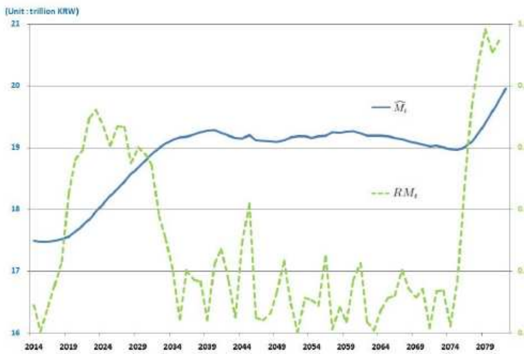

Figure 2: Best semi-martingale

$\delta = 1.0\%$. In more technically brief, we preserve the current benefit level schedule of the NPS, which is decreasing by 0.5% point every year down from the year 2008 with 50% and fixed with 40% after the year 2028. At the same time, for $t > 2013$ we replace the one premium level so that we find the desirable semi-martingale $\widehat{M_t}$. Besides, we utilize the wage increasing rate for the discount factor because the premium and the benefit are linked to the wage increasing rate and the combination of the wage increasing rate and the escalating factor in the NPS, respectively. As a result, the fair premium level is approximately 14.76%.

[4]In general, it is common to take the projection period as one generation, about 70 years.

5. CONCLUDING REMARKS

In this paper we explore on the topic to determine the premium and the benefit levels in the PPS. In the existing literatures on this topic it is mostly depended on the financial equilibrium so that it is not easy to assist the generational conflict in case of the reform for the PPS. In this point of view we propose a novel method focussing on the generational fairness.

For the notion of the fairness we apply the martingale so called as the fair game. In the construction of the martingale sequence, it is mainly implicated from two motivations. One is from the generational accounting introduced by Auerbach et al. and the other from Leimer's research based on OASDI, the PPS of the US.

The generational accounting is useful to measure the fiscal burdens of the current and the future generations. For the part of the past it is made up from the historical data as it has done by Leimer. However these have been calculated only at the present time, $t = 0$. Since we need some time sequence for the martingale we iterate the same calculation at the successive time after the present, i.e., $t = 1, 2, \ldots$.

There are surely some limitations in our approach. We may have some different result in accordance with the choice of the discount factor. And, it seems to be somewhat loose design for the generational fairness as we referred in the Remark 3.1. However it is expected to be a useful framework of the relation in the premium and the benefit levels in the PPS so that it contributes to the policy decision administrating the PPS considering the generational fairness with the consistency of the official actuarial valuation result.

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