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Design of Adaptive Feedback Linearization Control for Spacecraft with Flexible Appendages using Model Estimator

Jinli Chen[1], Donghai Li[2], Wenxiao Shan[2] and Xianfang Sun[1]
[1]School of Automation Science & Electrical Engineering, Beijing University of Aeronautics & Astronautics, Beijing, China 100083
[2]Department of Thermal Engineering, Tsinghua University, Beijing, China 100084

Abstract

Base on a kind of model estimator, this paper presents an adaptive feedback linearization control for the large-angle rotational maneuver and vibration suppression of a flexible spacecraft. The model estimator provides the approximate model information through the measure of system input variable, output variable (pitch angle) and its time derivative. The integral actions included can not only compensate of the entire dynamics of the system which is assumed to be unknown, but also ensure that the steady state error in the regulation of pitch angle is equal to zero. In addition, the control law is easy to implement. Simulation results are presented to show that, compared with differential geometric feedback linearization control and variable structure adaptive control, the adaptive feedback linearization control designed is superior in resisting external disturbances and adapting the uncertainties of system model. It improves robustness and adaptability of the flexible spacecraft attitude control systems greatly.

Keywords - adaptive feedback linearization control, flexible spacecraft, model estimator

1. INTRODUCTION

Spacecraft with flexible appendages plays an important role in communication, weather service, electronic reconnaissance and a variety of space researches. The model under consideration in this paper is representative of a relative large class of flexible spacecraft. It consists of a rigid central hub, which represents the spacecraft body, and a pair of flexible appendages, which represent antennae, solar arrays, or any other flexible structure attached to the rigid body (Karray, et al., 1997). Such spacecraft are often required to undergo large-angle rapid rotational maneuvers in order to track different "targets" on Earth or in space. The maneuvers inevitably cause elastic deflections of flexible appendages. The system stability maybe deteriorates if the serious vibration can not be suppressed effectively. Generally speaking, the models of spacecraft, which have both rigid and flexible mode interaction, involve complex dynamics characterized by large dimensions of state space, nonlinearities, and perturbations in system parameters. Further complications occur with the undesired disturbances effect. Designing a reliable and easy-to-implement controller to ensure high performance of tracking behavior is one of the major difficulties for the flexible spacecraft.

In recent years, several studies have been carried out to study the dynamics of such flexible spacecraft and design linear and nonlinear control systems for the attitude control and vibration suppression. Differential geometric feedback linearization approach has been used to obtain inverse control laws for the maneuvers of such flexible spacecraft (Karray, et al., 1997). But it is assumed that the entire dynamics of the system is precisely known. However, there unavoidably exist un-modeled dynamics, parameters perturbations and unknown internal or external noises, etc. The mismatch between the true spacecraft model and the formulated mathematical model used in inverse control laws may degrade the control performance and would lead to serious stability problems. Many adaptive and robust control schemes, such as output feedback variable structure adaptive control (Zeng, et al., 1999), adaptive output feedback control (Singh & Zhang, 2004)-(Zhang & Singh, 2001) and some other control have been investigated to deal

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with the uncertain problem of rotational maneuvers of spacecraft. These control schemes can greatly improve system stability, robustness, and dynamic performance. However, the expressions of these controllers are relative complex and are not easy to implement widely in control engineering practice. Thus there is an urgent need to design simple and easy-to-realize control system for large maneuvers of the flexible spacecraft in the presence of un-modeled uncertainties and parameters perturbations.

Reference (Tornambe & Valigi, 1994) has designed a kind of simple model estimator (ME) for a class of SISO/MIMO dynamical systems, whose entire dynamics is assumed unknown and its state variables not completed measurable. The model estimator provides the approximate model information through the measure of system input variable, output variable and its time derivative. With suitable parameters, the integral action included in model estimator can not only compensate of the entire dynamics of the system which is assumed to be unknown, but also ensure that the steady state error is equal to zero.

Using the model estimator in (Tornambe & Valigi, 1994), this paper investigates an adaptive feedback linearization control law (AFLC) for large-angle rotational maneuver and vibration suppression of the flexible spacecraft. The design of proposed controller does not require the details of the flexible spacecraft system. It has a simple structure and possesses an adaptive capability to deal with un-modeled dynamics, uncertainty in system parameters, and disturbance input. Simulation results show precise attitude control and vibration suppression.

The paper is organized as follows. In Section 2, flexible spacecraft mathematical model and the pitch angle trajectory control problem under consideration are briefly described. Next, in Section 3, the adaptive feedback linearization control law for flexible spacecraft based on model estimator is designed and stability analysis is also considered. Various simulation experiments are carried out in Section 4. Finally, Section 5 contains concluding remarks.

2. FLEXIBLE SPACECRAFT MATHEMATICAL MODEL

Figure 1 shows the model of the flexible spacecraft under consideration.

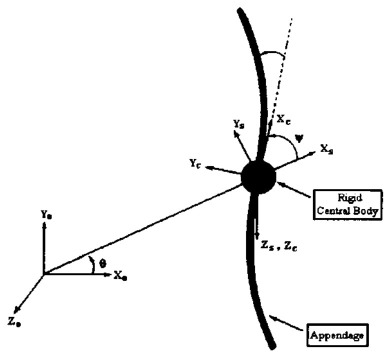


Figure 1. Spacecraft with flexible appendages

The three coordinate frames are described detailed in (Karray, et al., 1997). Let θ be the true anomaly and ψ be the pitch angle which is controlled using a torque generating device located at the center of the hub.

In the study, control of pitch maneuver limited to the orbit normal axis is considered. It is assumed that the pitch maneuver excites the two flexible appendages antisymmetrically. For the configuration of spacecraft, this assumption of antisymmetric deformation is reasonable and was validated in (Karray, et al., 1997).

The nonlinear differential equations describing the rigid hub pitch motion and elastic dynamics are given by [see (Karray, et al., 1997)]

$$\begin{aligned}&(J + 2J_1 + \boldsymbol{p}^T \boldsymbol{M}_{pp}\boldsymbol{p})\ddot{\psi} + \boldsymbol{m}_{\psi p}^T \ddot{\boldsymbol{p}} + 2(\dot{\psi} + \dot{\theta})\dot{\boldsymbol{p}}^T \boldsymbol{M}_{pp}\boldsymbol{p} \\ &\quad + 3\dot{\theta}^2 \sin(2\psi)(J_1 - 0.5\boldsymbol{p}^T \boldsymbol{M}_{pp}\boldsymbol{p}) + 3\dot{\theta}^2 \cos(2\psi)\boldsymbol{m}_{\psi p}^T \boldsymbol{p} = u,\end{aligned} \tag{1}$$

$$\begin{aligned}&\boldsymbol{M}_{pp}\ddot{\boldsymbol{p}} + \boldsymbol{m}_{\psi p}\ddot{\psi} + 1.5\dot{\theta}^2 \sin(2\psi)\boldsymbol{m}_{\psi p} + \boldsymbol{C}_{pp}\dot{\boldsymbol{p}} \\ &\quad + [\boldsymbol{K}_{pp} - (\dot{\psi}^2 + 2\dot{\psi}\dot{\theta} + 3\dot{\theta}^2 \sin^2\psi)\boldsymbol{M}_{pp}]\boldsymbol{p} = \boldsymbol{0},\end{aligned} \tag{2}$$

where $p=(p_1,...,p_N)^T$ is the vector of appendage flexibility generalized coordinates, J and J_1 are the mass moments of inertia of the central hub and each appendage, respectively, $J_t=J+2J_1$, u is the control torque, $\dot{\theta}$ is the orbital rate, and $\boldsymbol{M}_{\mathbf{pp}}$, $\boldsymbol{m}_{\mathbf{\psi p}}$, $\boldsymbol{C}_{\mathbf{pp}}$ and $\boldsymbol{K}_{\mathbf{pp}}$ are the following modal integrals (Singh & Zhang, 2004):

$$[M_{pp}]_{ij}=2\int_r^{r+L}\phi_i(l-r)\phi_j(l-r)dm, \tag{3}$$

$$[m_{\psi p}]_i=2r\int_r^{r+L}l\phi_i(l-r)dm, \tag{4}$$

$$[C_{pp}]_{ij}=2\int_r^{r+L}CI\phi_i''(l-r)\phi_j''(l-r)dl, \tag{5}$$

$$[K_{pp}]_{ij}=2\int_r^{r+L}EI\phi_i''(l-r)\phi_j''(l-r)dl, \tag{6}$$

where $i=1,\cdots,N,\ j=1,\cdots,N$, r is the radius of the hub, L is the appendage length, l is the distance of a point chosen on the appendage from the center of the hub, ϕ_i are the chosen admissible functions, $\phi_i''=(\partial^2\phi_i/\partial l^2)$, C and E are the damping coefficient and modulus of elasticity for the appendages, respectively, and I is the sectional area moment of inertia with respect to the appendage bending axis.

System (1) and (2) can be written in a compact form as

$$\boldsymbol{M}\begin{bmatrix}\ddot{\psi}\\ \ddot{\boldsymbol{p}}\end{bmatrix}+\boldsymbol{C}\begin{bmatrix}\dot{\psi}\\ \dot{\boldsymbol{p}}\end{bmatrix}+\boldsymbol{K}\begin{bmatrix}\psi\\ \boldsymbol{p}\end{bmatrix}=\begin{bmatrix}h_1\\ \boldsymbol{h}_2\end{bmatrix}+\begin{bmatrix}1\\ \mathbf{0}_{N\times1}\end{bmatrix}u, \tag{7}$$

where

$$\boldsymbol{M}=\begin{pmatrix}J_t+\boldsymbol{p}^T\boldsymbol{M}_{\mathbf{pp}}\boldsymbol{p} & \boldsymbol{m}_{\mathbf{\psi p}}^T\\ \boldsymbol{m}_{\mathbf{\psi p}} & \boldsymbol{M}_{\mathbf{pp}}\end{pmatrix}, \tag{8}$$

$$\boldsymbol{C}=\mathrm{diag}(0,\boldsymbol{C}_{\mathbf{pp}}), \tag{9}$$

$$\boldsymbol{K}=\mathrm{diag}(0,\boldsymbol{K}_{\mathbf{pp}}), \tag{10}$$

$$\begin{aligned}h_1=&-2(\dot{\psi}+\dot{\theta})\dot{\boldsymbol{p}}^T\boldsymbol{M}_{\mathbf{pp}}\boldsymbol{p}-3\dot{\theta}^2\sin(2\psi)\\ &\times(J_1-0.5\boldsymbol{p}^T\boldsymbol{M}_{\mathbf{pp}}\boldsymbol{p})-3\dot{\theta}^2\cos(2\psi)\boldsymbol{m}_{\mathbf{\psi p}}^T\boldsymbol{p},\end{aligned} \tag{11}$$

$$\begin{aligned}\boldsymbol{h}_2=&(\dot{\psi}^2+2\dot{\psi}\dot{\theta}+3\dot{\theta}^2\sin^2\psi)\boldsymbol{M}_{\mathbf{pp}}\boldsymbol{p}\\ &-1.5\dot{\theta}^2\sin(2\psi)\boldsymbol{m}_{\mathbf{\psi p}}.\end{aligned} \tag{12}$$

In order to state the control problem clearly, we define $\boldsymbol{x}_1=(\psi,\boldsymbol{p}^T)^T\in\mathbf{R}^{N+1}$, $\boldsymbol{x}_2=(\dot{\psi},\dot{\boldsymbol{p}}^T)^T\in\mathbf{R}^{N+1}$, and the state vector $\boldsymbol{x}=(\boldsymbol{x}_1^T,\boldsymbol{x}_2^T)^T$, then the system (7) can be rearranged as the following form [see (Singh & Zhang, 2004)],

$$\dot{\boldsymbol{x}}=\boldsymbol{f}(\boldsymbol{x})+\boldsymbol{g}(\boldsymbol{x})u, \tag{13}$$

where

$$\boldsymbol{f}(\boldsymbol{x})=\begin{pmatrix}\boldsymbol{x}_2\\ \boldsymbol{M}^{-1}(-\boldsymbol{C}\boldsymbol{x}_2-\boldsymbol{K}\boldsymbol{x}_1+(h_1,\boldsymbol{h}_2^T)^T)\end{pmatrix}, \tag{14}$$

$$\boldsymbol{g}(\boldsymbol{x})=\begin{pmatrix}\mathbf{0}_{(N+1)\times1}\\ \boldsymbol{M}^{-1}(1,\mathbf{0}_{N\times1}^T)^T\end{pmatrix}, \tag{15}$$

$$\boldsymbol{M}^{-1}=\begin{bmatrix}(J_a-\boldsymbol{m}_{\mathbf{\psi p}}^T\boldsymbol{M}_{\mathbf{pp}}^{-1}\boldsymbol{m}_{\mathbf{\psi p}})^{-1} & -J_a^{-1}\boldsymbol{m}_{\mathbf{\psi p}}^T\Delta^{-1}\\ -J_a^{-1}\Delta^{-1}\boldsymbol{m}_{\mathbf{\psi p}} & \Delta^{-1}\end{bmatrix}, \tag{16}$$

$$\Delta=(\boldsymbol{M}_{\mathbf{pp}}-\boldsymbol{m}_{\mathbf{\psi p}}J_a^{-1}\boldsymbol{m}_{\mathbf{\psi p}}^T), \tag{17}$$

$$J_a=J_t+\boldsymbol{p}^T\boldsymbol{M}_{\mathbf{pp}}\boldsymbol{p}, \tag{18}$$

We consider the pitch angle control, that is

$$y=\psi. \tag{19}$$

Let $\psi_m(t)$ be a smooth reference trajectory converging to the target pitch angle ψ^*. The designed control law $u(t)$ should cause $e(t)=\psi(t)-\psi_m(t)$ asymptotically tends to zero and the elastic modes remain bounded during the maneuver. Specially, for a constant set value control, it is desired that the flexible modes converge to zero. Desired pitch angle control is obtained by suitable choice of ψ^* and $\psi_m(t)$.

3. FEEDBACK LINEARIZTION CONTROL LAW

3.1. Exact feedback linearization control law

Let us consider a single-input single-output (SISO) affine nonlinear dynamic system described by:

$$\begin{cases} \dot{x}(t) = f(x(t)) + g(x(t))u(t), \\ y(t) = h(x(t)), \end{cases} \tag{20}$$

where $x(t) \in \mathbf{R}^n$ is the state vector, $u(t) \in \mathbf{R}$ is the control vector, $y(t) \in \mathbf{R}$ is the output vector, $f(x(t))$ and $g(x(t))$ are the n-dimensional vector fields in the state space, $h(x(t))$ is the scalar function of $x(t)$, n is the dimension of the state vector. The output of SISO nonlinear system (20) $y(t)$ track a bounded reference signal $y_m(t)$ and smoothly reach the terminal value y^*, which is a constant.

Definition 1 [see (Isidori, 1995)]: The SISO nonlinear system (20) is said to have relative degree r at a point x^0 if

(i) $L_g L_f^k h(x(t)) = 0$ for all x in a neighborhood of x^0 and all $k < r-1$.

(ii) $L_g L_f^{r-1} h(x^0) \neq 0$.

Suppose that:

(a) The relative degree r is finite and known.

(b) The zero dynamics is locally asymptotically stable when $r < n$.

In the input/output linearization procedure, the output $y(t)$ is differentiated, with respect to time, r times until the control input $u(t)$ appears explicitly. Then the input-output mapping form is

$$y^{(r)}(t) = a(x(t)) + b(x(t))u(t), \tag{21}$$

where $a(x(\mathrm{t})) = L_f^r h(x(t))$ and $b(x(\mathrm{t})) = L_g L_f^{r-1} h(x(t))$, which can be derived from the expressions of $f(x(t))$, $g(x(t))$, $h(x(t))$.

Defining new state variables

$$z = (z_1, z_2, \cdots, z_r)^T = (y(t), y^{(1)}(t), \cdots, y^{(r-1)}(t))^T,$$

system (21) can be rewritten as

$$\dot{z} = Az + B(a(x(t)) + b(x(t))u(t)), \tag{22}$$

where

$$A = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ 0 & 0 & \cdots & 0 \end{bmatrix}_{r\times r}, \quad B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}_{r\times 1}.$$

Suppose that:

(c) The output variable $y(t)$ and its time derivatives $y^{(i)}(t)\ (i = 1, \cdots, r-1)$ are measurable.

(d) The reference trajectory $y_m(t)$ and r of its time derivatives are available i.e. $y_m(t), y_m^{(1)}(t), \cdots, y_m^{(r)}(t)$ are known.

(e) Nonzero Function $b(x(\mathrm{t}))$ is continuous, which is always positive or negative, and its sign function $\mathrm{sgn}(b(x(\mathrm{t})))$ is known. Without loss of generality, we may assume $b(x(\mathrm{t}))$ always positive in order to facilitate analysis here.

We can obtain the following exact feedback linearization control law according to (Isidori, 1995)

$$u(t) = b^{-1}(x(t))(-a(x(t)) + y_m^{(r)}(t) - \sum_{i=0}^{r-1} l_i e^{(i)}(t)), \tag{23}$$

where tracking error $e(t) = y(t) - y_m(t)$.

Substituting the expression of control law (23) into (21), we get

$$e^{(r)}(t)+l_{r-1}e^{(r-1)}(t)+\cdots+l_0e(t)=0, \tag{24}$$

where $e(t)$ will converges to zero as t tends to ∞ if suitable positive parameters $l_i\,(i=0,\cdots,r-1)$ are provided such that system (24) is asymptotically stable.

3.2. Design of ME and AFLC

It can be seen that control law (23) is accomplished only if the model information is completely known. But in practice, there inevitably exists un-modeled dynamics, uncertainty in system parameters and disturbance input, then functions $a(x(t))$ and $b(x(t))$ are not known exactly. Control law (23) can not be actually implemented. For overcoming this different, (Tornambe & Valigi, 1994) designs a kind of model estimator to substitute real system information.

Equation (21) can be rewritten as

$$y^{(r)}(t)=d+u(t), \tag{25}$$

where the extended state $d=a(x(t))+(b(x(t))-1)u(t)$ is considered as generalized disturbance, which includes all model information, both the internal dynamics with uncertainties as well as the external disturbances. The idea in this paper is to estimate d in real time by ME and cancel its effect using the approximate model information $\hat{d}$ in the feedback linearization control law.

Suppose that:

(f) The extended state d and its derivation $\dot{d}$ are Lipschitz in their arguments and bounded over the domain of interest.

Adding this extended state into the state vector z, we obtain $z_e=(z,d)^T$, then system (25) can be rewritten as

$$\begin{cases}\dot{z}_e=A_ez_e+B_eu+E\dot{d}\\ y\;=Cz_e\end{cases}, \tag{26}$$

where

$$A_e=\begin{bmatrix}0&1&0&\cdots&0\\0&0&1&\cdots&0\\\vdots&\vdots&\vdots&\ddots&\vdots\\0&0&0&\cdots&1\\0&0&0&\cdots&0\end{bmatrix}_{(r+1)\times(r+1)},\quad B_e=\begin{bmatrix}0\\0\\\vdots\\1\\0\end{bmatrix}_{(r+1)\times1},\quad E=\begin{bmatrix}0\\0\\\vdots\\0\\1\end{bmatrix}_{(r+1)\times1},\quad C=\begin{bmatrix}1&0&0&\cdots&0\end{bmatrix}_{1\times(r+1)}.$$

Based on the assumption of (a)-(f), a ME is designed as

$$\begin{cases}\hat{d}=\xi+\sum\limits_{i=0}^{r-1}k_ie^{(i)}(t),\\ \dot{\xi}=-k_{r-1}\xi-k_{r-1}\sum\limits_{i=0}^{r-1}k_ie^{(i)}(t)-\sum\limits_{i=0}^{r-2}k_ie^{(i+1)}(t)-k_{r-1}u,\end{cases} \tag{27}$$

where $k_{r-1}=\mathrm{sgn}(b(x(t)))\mu$,with μ being a suitable positive constant, which dominates the system stability, and $k_i\,(i=0,\cdots,r-2)$ are arbitrary constants, which have certain influence on the system dynamic performance. The integral actions included in ME can compensate for the entire dynamics of the system and ensure no steady state error. Under the assumption of (e), we have $k_{r-1}=\mu$.

Using the output of ME, the AFLC is designed as the following form

$$u(t)=y_m^{(r)}(t)-\sum_{i=0}^{r-1}l_ie^{(i)}(t)-\hat{d}\,. \tag{28}$$

where parameters $l_i\,(i=0,\cdots,r-1)$ are suitable positive constants such that system (24) is asymptotically stable.

Furthermore, for achieving the design specifications on dynamic and static performance, the desired reference trajectory equation of the closed-loop system is selected as follows:

$$y_m^{(r)}(t)+\alpha_{r-1}y_m^{(r-1)}(t)+\cdots+\alpha_1y_m^{(1)}(t)+\alpha_0y_m(t)=\alpha_0y^* \tag{29}$$

By the choice of the command input y^* in (29), one can generate desired output reference trajectory. The positive constant $\alpha_i\;(i=0,\cdots,r-1)$ are selected such that all roots of

$$s^r + \alpha_{r-1}s^{r-1} + \cdots + \alpha_1 s + \alpha_0 = 0 \tag{30}$$

are placed suitably in the open left half-plane.

On the whole, the design steps of AFLC for SISO affine nonlinear system (20) can be summarized as follows:

Step 1: Analyze the system characteristics and relationship between input variable $u(t)$ and output variable $y(t)$, compute the relative degree r, and then examine whether or not the six assumptions (a)-(f) can be satisfied.

Step 2: Specify the desired smooth reference trajectory equation (29).

Step 3: Design ME (27) and AFLC (28). Choose the suitable parameters k_i $(i = 0,\cdots,r-1)$ and l_i $(i=0,\cdots,r-1)$.

Step 4: Simulate the closed-loop system to examine whether or not the desired performance is satisfied in spite of the presence of un-modeled dynamics, uncertainty in system parameters, and disturbance input.

3.3. Stability Analysis

The following theorem ensures the estimation error of ME is bounded.

Theorem 1: ME has a construction estimation error $\tilde{d} = d - \hat{d}$ with uniform ultimate bound for any bounded $\dot{d}$. The estimation error satisfies $|\tilde{d}| < |\varepsilon|/\mu$, where $\varepsilon = \mu y_m^{(r)} + \dot{d}$.

Proof: Let us compute the time derivative of $\hat{d}$,

$$\begin{aligned}\dot{\hat{d}} &= \dot{\xi} + \sum_{i=0}^{r-1} k_i e^{(i+1)}(t) \\ &= -k_{r-1}\xi - k_{r-1}\sum_{i=0}^{r-1} k_i e^{(i)}(t) - \sum_{i=0}^{r-2} k_i e^{(i+1)}(t) - k_{r-1}u \\ &\quad + \sum_{i=0}^{r-2} k_i e^{(i+1)}(t) + k_{r-1}(d + u - y_m^{(r)}) \\ &= k_{r-1}(d - \xi - \sum_{i=0}^{r-1} k_i e^{(i)}(t) - y_m^{(r)}) \\ &= k_{r-1}\tilde{d} - k_{r-1}y_m^{(r)}\end{aligned} \tag{31}$$

Hence we have

$$\begin{aligned}\dot{\tilde{d}} &= \dot{d} - \dot{\hat{d}} \\ &= -k_{r-1}\tilde{d} + k_{r-1}y_m^{(r)} + \dot{d} \\ &= -\mu\tilde{d} + \mu y_m^{(r)} + \dot{d}\end{aligned} \tag{32}$$

From the assumption of (d) and (f), we can know that $y_m^{(r)}$ and $\dot{d}$ are both bounded, so the introduced variable $\varepsilon = \mu y_m^{(r)} + \dot{d}$ is also bounded. Then (32) can be rewritten as

$$\dot{\tilde{d}} = -\mu\tilde{d} + \varepsilon \tag{33}$$

Let us consider

$$V = \frac{1}{2}\tilde{d}^2$$

as a Lyapunov function candidate, then

$$\begin{aligned}\dot{V} &= \tilde{d}\dot{\tilde{d}} \\ &= -\mu\tilde{d}^2 + \varepsilon\tilde{d} \\ &\le -\mu|\tilde{d}|^2 + |\varepsilon|\cdot|\tilde{d}|\end{aligned} \tag{34}$$

It is straightforward to check that if

$$|\tilde{d}| > |\varepsilon|/\mu, \tag{35}$$

then $\dot{V} < 0$. It implies that the absolute value of estimation error $|\tilde{d}|$ will decrease for any $\tilde{d}$ in accord with the condition of (35). Hence, ME has a construction error with uniform ultimate bound: $|\tilde{d}| < |\varepsilon|/\mu$.

Furthermore, based on the conclusion of theorem 1, we also get the following theorem.

Theorem 2: The closed loop system is BIBO stable under the designed ME (27) and AFLC (28) for any bounded $\dot{d}$.

Proof: Substituting controller (28) into system (25), we obtain:

$$\begin{aligned} y^{(r)}(t) &= d + y_m^{(r)}(t) - \sum_{i=0}^{r-1} l_i e^{(i)}(t) - \hat{d} \\ &= y_m^{(r)}(t) - \sum_{i=0}^{r-1} l_i e^{(i)}(t) + \tilde{d}, \end{aligned} \tag{36}$$

From (36), we can compute the expression of $e^{(r)}(t)$ as follows,

$$e^{(r)}(t) = -\sum_{i=0}^{r-1} l_i e^{(i)}(t) + \tilde{d}\ . \tag{37}$$

Let us introduce a new state vector

$$\boldsymbol{e} = (e_1, e_2, \cdots, e_r)^T = (e(t), e^{(1)}(t), \cdots, e^{(r-1)}(t))^T\ ,$$

then (37) can be rewritten as the following form,

$$\dot{\boldsymbol{e}} = \boldsymbol{F}\boldsymbol{e} + \boldsymbol{B}\tilde{d}, \tag{38}$$

where $\boldsymbol{F} = \begin{bmatrix} 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \\ -l_0 & -l_1 & \dots & -l_{r-1} \end{bmatrix}$, $\boldsymbol{B} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}$. Because (24) is asymptotically stable, matrix $\boldsymbol{F}$ is Hurwitz.

Combining (33) and (38), the close-loop system including controller and model estimator described in compact form as follows:

$$\dot{\bar{\boldsymbol{e}}} = \boldsymbol{G}\bar{\boldsymbol{e}} + \boldsymbol{\eta}\ . \tag{39}$$

where $\bar{\boldsymbol{e}} = \begin{bmatrix} \boldsymbol{e} \\ \tilde{d} \end{bmatrix}_{(r+1)\times 1}$, $\boldsymbol{G} = \begin{bmatrix} \boldsymbol{F} & \boldsymbol{B} \\ \boldsymbol{0} & -\mu \end{bmatrix}_{(r+1)\times(r+1)}$ and $\boldsymbol{\eta} = \begin{bmatrix} \boldsymbol{0} \\ \varepsilon \end{bmatrix}_{(r+1)\times 1}$.

Because $\boldsymbol{F}$ is Hurwitz and μ is a suitable positive constant, $\boldsymbol{G}$ is also Hurwitz. Moreover, it is known from theorem 1 that vector $\boldsymbol{\eta}$ is bounded for any bounded $\dot{d}$.

The Lyapunov function candidate is:

$$V = \bar{\boldsymbol{e}}^T \boldsymbol{P} \bar{\boldsymbol{e}}\ ,$$

where $\boldsymbol{P}$ is the unique positive definite solution of the Lyapunov equation $\boldsymbol{G}^T\boldsymbol{P} + \boldsymbol{P}\boldsymbol{G} = -\boldsymbol{Q}$ and $\boldsymbol{Q}$ is a symmetrical positive definite matrix.

Then we have

$$\begin{aligned} \dot{V} &= \dot{\bar{\boldsymbol{e}}}^T \boldsymbol{P} \bar{\boldsymbol{e}} + \bar{\boldsymbol{e}}^T \boldsymbol{P} \dot{\bar{\boldsymbol{e}}} \\ &= \bar{\boldsymbol{e}}^T (\boldsymbol{G}^T\boldsymbol{P} + \boldsymbol{P}\boldsymbol{G})\bar{\boldsymbol{e}} + 2\boldsymbol{\eta}^T \boldsymbol{P}\bar{\boldsymbol{e}} \\ &= -\bar{\boldsymbol{e}}^T \boldsymbol{Q} \bar{\boldsymbol{e}} + 2\boldsymbol{\eta}^T \boldsymbol{P}\bar{\boldsymbol{e}} \\ &\le -\lambda_Q \|\bar{\boldsymbol{e}}\|^2 + 2\|\boldsymbol{\eta}\|\|\boldsymbol{P}\|\|\bar{\boldsymbol{e}}\|, \end{aligned} \tag{40}$$

where λ_Q is the smallest eigenvalue of $\boldsymbol{Q}$, $\|\bullet\|$ denotes the standard Euclidean norm.

It can be checked easily that if

$$\|\bar{\boldsymbol{e}}\| > 2\|\boldsymbol{\eta}\|\|\boldsymbol{P}\| / \lambda_Q\ , \tag{41}$$

then $\dot{V}$ is negative definite. That implies that $\|\bar{\boldsymbol{e}}\|$ decreases for any $\bar{\boldsymbol{e}}$ that satisfies (41). Hence $\bar{\boldsymbol{e}}$ is uniform ultimate bounded. The bound will reduces with the value of λ_Q increasing.

The above analysis shows that the closed-loop system (39) is BIBO stable under the designed ME (27) and AFLC (28) for any bounded $\dot{d}$.

3.4. AFLC for pitch angle control of flexible spacecraft

Now we design AFLC for the flexible spacecraft (13). Based on the forementioned analysis, the structure of the control law is shown in Figure 2.

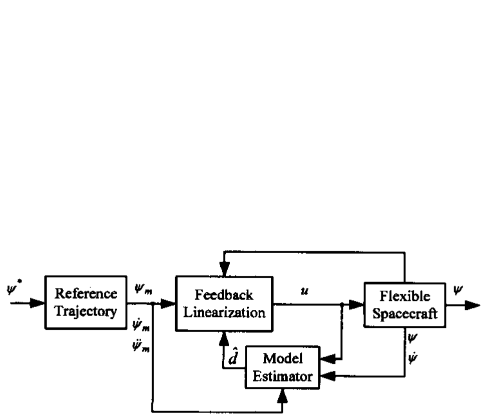


Figure 2. AFLC Structure for Flexible Spacecraft

Since $y=\psi$ has relative degree two, the closed-loop system has zero dynamics of dimension 2N. Reference (Singh & Zhang, 2004) has proved that, the zero dynamics of this system is always asymptotically stable. So assumptions of (a) and (b) are satisfied. We also note that pitch angle ψ and pitch rate $\dot{\psi}$ can both be measurable, and reference trajectory ψ_m and its time derivatives $\dot{\psi}_m$, $\ddot{\psi}_m$ can be computed easily. Moreover, it is verified that all the boundary conditions given in assumptions (f) exist for the pitch angle control.

Hence AFLC scheme based on ME is suitable for flexible spacecraft system, which can satisfy all assumptions.

By employing the estimation of generalized disturbance $\hat{d}$ to eliminate the effect of system unknown dynamics and uncertainties, the AFLC designed according to (27) and (28) for the pitch angle control of flexible spacecraft is as follows:

$$u=\ddot{\psi}_m-l_0e-l_1\dot{e}-\hat{d}, \tag{42}$$

$$\hat{d}=\xi+k_0e+k_1\dot{e}, \tag{43}$$

$$\dot{\xi}=-k_1\xi-k_1(k_0e+k_1\dot{e})-k_0\dot{e}-k_1u. \tag{44}$$

where $e=\psi-\psi_m$.

4. Simulations and results

In this section, we investigate the performance of AFLC for pitch angle control of flexible spacecraft through simulation and comparison with differential geometric feedback linearization control (DFLC) and variable structure adaptive control (VSAC). Some of the results of digital simulation are presented.

The spacecraft is assumed to be placed in a circular, low Earth orbit at an attitude of 400 km. The physical parameters of the spacecraft given in (Karray, et al., 1997) are used for simulation. The nominal parameters are hub inertia J=3972 kg m^2, appendage inertia J_1=500 kg m^2, density ρ=6/30 kg/m, appendage structural damping CI=545 kg m^3/s, appendage stiffness EI=1500 kg m^3/s^2, appendage length L=30 m, hub radius r=1 m, and three modes are chosen for the simulation (i.e., N=3). Higher modes have insignificant effect on the responses. The mode shapes given in (Turner & Chun, 1984) of the form (i=1,2,3)

$$\phi_i(l-r)=1-\cos\left(\frac{i\pi(l-r)}{L}\right)+\frac{1}{2}(-1)^{i+1}\left(\frac{i\pi(l-r)}{L}\right)^2 \tag{45}$$

are used to compute the modal integrals in (3)-(6).

For achieving smooth large-angle rotational maneuver, the desired reference trajectory equation is selected as follows:

$$\frac{\psi_m}{\psi^*}=\frac{0.01}{s^2+0.2s+0.01}, \tag{46}$$

i.e. set $\alpha_0=0.01$, $\alpha_1=0.2$ in (29). The time-varying reference trajectory $\psi_m(t)$ is generated using (46) and it converges to the constant target pitch angle ψ^* smoothly. Here the command input ψ^* is chosen to be 1 rad for the attitude nonlinear control of the flexible spacecraft.

The parameters in control law (42)-(44) are chosen as $l_0=1$, $l_1=2$, $k_0=0$, $k_1=6000$.

4.1. Responses in nominal condition

Simulation is done when all parameters of the flexible spacecraft are in the nominal condition and there is no un-modeled dynamics, uncertainty in system parameters, and disturbance input. The initial conditions are assumed to be

$\psi(0)=\dot{\psi}(0)=0$, $p(0)=(0,0,0)^T$, $\dot{p}(0)=(0.5,0,0)^T$ .And all the initial conditions of the reference trajectory are set to zero.

Using the measurements feedback of pitch angle $\psi(t)$ and pitch rate $\dot{\psi}(t)$, ME gives the approximate model information $\hat{d}$. The response of estimation error $\tilde{d}$ is shown in Figure 3.

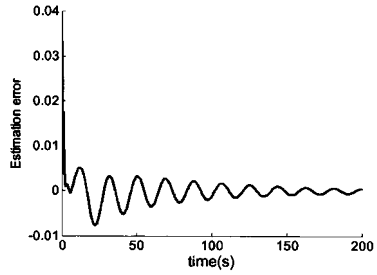
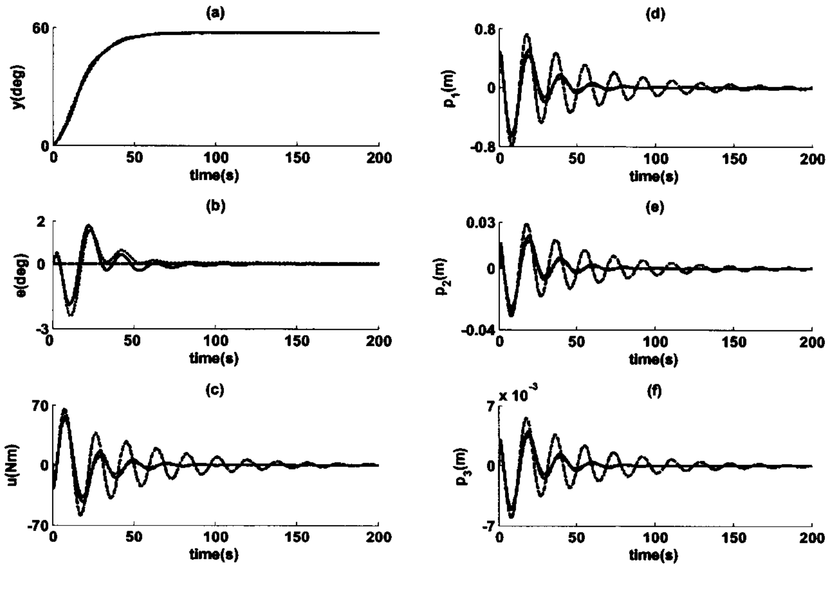


Figure 3. Estimation error $\tilde{d}$

It is clearly demonstrated that ME with suitable parameters has excellent estimation performance. The estimation error $\tilde{d}$ is bounded and converges to zero as $t \to \infty$. The maximum value of $|\tilde{d}|$ is just 0.0334 during the transient process.

Selected responses adopting AFLC are shown in Figure 4. We also give the responses in same condition using DFLC and VSAC [see (Zeng, et al., 1999)] to compare their tracking performance.



Solid line-AFLC, Dashed line-DFLC, Dotted line-VSAC
(a)Pitch angle ψ ; (b) Tracking error e ; (c) Control input u;
(d)Elastic mode p_1; (e) Elastic mode p_2; (f) Elastic mode p_3
Figure 4. Pitch angle control and elastic modes responses

Here we use the following notations for simplicity:

$$u_m = \max|u|,\ e_m = \max|e|,\ p_{1m} = \max|p_1|,\ p_{2m} = \max|p_2|,\ p_{3m} = \max|p_3|,$$

where the notations represent the maximum absolute value of control magnitude, the maximum absolute value of output tracking error , three maximum absolute values of elastic modes magnitude during the transient response, respectively.

As can be seen from Figure 4, smooth control of pitch angle in about 200 seconds is observed. The entire closed-loop system is stabilized. DFLC has the least tracking error, can fully accomplish the rapid precise attitude control. However, it needs relatively large control torque and longer time to suppress the elastic vibration; the magnitudes of elastic modes p_1, p_2, p_3 are greater than the other two control laws. The steady state error is 0.076 deg using VSAC, thus it can not be used when high precision is needed although its control torque and elastic vibration are both have smaller value.

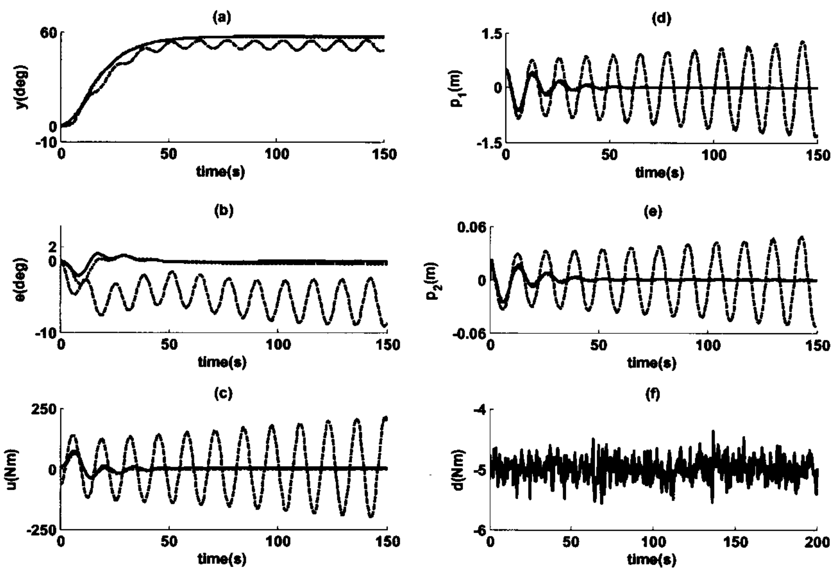
Compared with these two control approaches, AFLC has no steady state error, and its values of control torque and elastic vibration locate between the corresponding value of DFLC and VSAC. The tendencies of state variables are in accord with VSAC despite of some small differences in magnitude. Some relative data is shown in Table 1.

Table 1. Experimental data in nominal condition

Control Law	u_m(N m)	e_m(deg)	p_{1m}(m)	p_{2m}(m)	p_{3m}(m)
AFLC	57.4395	1.6133	0.6871	0.0276	0.0053
DFLC	64.7346	9.4736e-8	0.7952	0.0313	0.0061
VSAC	51.8838	2.3320	0.6356	0.0256	0.0049

4.2. Responses in existing external disturbances and parameters perturbation condition

In order to examine the robustness of AFLC against uncertain parameters and external disturbances, ±50% perturbations in the parameter J_t, μ and CI are introduced: $\Delta J_t = +50\%$, $\Delta\mu = -50\%$, $\Delta CI = -50\%$, where μ is the appendage mass density. And at the same time, the random disturbance torque $d(t)$ is acting on the spacecraft through a filter of transfer function $H(s) = 1/(0.5s+1)$. Its mean value and the standard deviation are $d_{AV} = -5\,Nm$ and $\sigma_d^2 = 0.2^2\,Nm^2$, respectively. The other conditions remain the same with case 4.1. The selected simulation results are shown in Figure 5.



Solid line-AFLC, Dashed line-DFLC, Dotted line-VSAC
(a)Pitch angle ψ ; (b) Tracking error e ; (c) Control input u;
(d)Elastic mode p_1; (e) Elastic mode p_2; (f) Disturbance torque d.

Figure 5. Pitch angle control and elastic modes responses with external disturbances and parameters perturbation

Figure 5 shows DFLC is unable to satisfy the control performance any more. It is difficult to realize the needed control torque in practical application, and the periodical steady state error degrades the control precise seriously. What is more, the whole system turns out to be unstable due to the elastic vibration can not be suppressed. All these phenomena are caused by the control method itself because it highly relies on the plant exact model. Although having comparative advantages in suppressing the vibration and realizing control torque, VSAC exists a -5.7805 deg steady state error and thus can not accomplish precise attitude control.

In contrast, AFLC demonstrates its predominant control effect with a high tracking precision, appropriate control torque and good ability of suppressing elastic vibration when external disturbances and parameters perturbation exist.

5. CONCLUSIONS

In this paper, we present an adaptive feedback linearization control law for the large-angle rotational maneuver and vibration suppression of a flexible spacecraft base on a model estimator. A 2-degree simple and easy-to-realize controller, which can accomplish compensation of entire unknown dynamics appearing in the feedback linearization control law, has been designed and evaluated. Simulation results show that adaptive feedback linearization control law designed in this paper is well done for nonlinear flexible spacecraft system. The main advantages include simple structure, easy to realize, good tracking property with zero steady-state error to command input, ability of suppressing elastic vibration, and robustness performance under uncertainty conditions. Adaptive feedback linearization control can also be used to the other complex nonlinear industry plant and it supply us a new method to design adaptive control system.

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