Neural, Parallel, and Scientific Computations 17 (2009) 205 - 214

AN ESTIMATION OF STRUCTURE PARAMETERS OF ROTATIONALLY AND TIDALLY DISTORTED POLYTROPIC MODELS OF STARS IN THE PRESENCE OF CORIOLIS FORCE

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ABSTRACT

In this paper we propose suitable modifications in the concept of Roche equipotential to account for the effects of Coriolis force on its equipotential surface besides the usual effects of gravitational and centrifugal forces and use this in conjunction with Kippenhahn and Thomas, 1970 approach in a manner earlier used by Mohan et al, 1990 to incorporate the effects of Coriolis force in the equations of stellar structures. The proposed method has been used to compute the equilibrium structures, shapes and other observable parameters of rotationally and tidally distorted polytropic models of stars. The results thus obtained have been compared with the corresponding results earlier available in literature in which the effects of Coriolis force had not been taken into account.

Keywords: Binary systems – Rotating stars – Roche equipotentials – Coriolis force – Equilibrium structures

1. INTODUCTION

The mathematical problem of determining the effect of rotational and tidal forces on the equilibrium structure of a star is quite complex. Therefore, attempts have been made in literature to carry out the study of such problems in some approximate way. In one such approximation (Mohan and Saxena, 1983; Mohan et al, 1990 and Lal et al 2006) the actual equipotential surface of a rotationally and tidally distorted star is approximated by equivalent rotationally and tidally distorted Roche equipotential. In this approximation Kippenhahn and Thomas, 1970 averaging approach and results of the Roche equipotentials obtained by Kopal, 1972 are used to incorporate the rotational and tidal effects upto second order of smallness in the stellar structure equations.

In 1933, Chandrasekhar developed a theory of distorted polytropes. Since then several authors such as James, 1964; Monaghan, 1967; Kopal, 1972; Linnell, 1981;

Received February 10, 2009

Mohan and Saxena, 1983; Mohan et al, 1990 and Lal et al, 2006 have addressed themselves to these problems. Kopal, 1972 discussed the Roche equipotential of rotationally and/or tidally distorted stars by taking into account the effect of gravitational potential which arises due to primary and secondary components of binary system and centrifugal force due to rotation of binary system. However the effect of Coriolis force was ignored completely while determining the Roche equipotential surface of rotationally and tidally distorted stellar models. Later Mohan and Saxena, 1983, Mohan et al, 1990 and Lal et al, 2006 used the Kopal's results of Roche equipotentials in conjunction with the averaging technique as discussed by Kippenhahn and Thomas, 1970 to incorporate the rotational and tidal effects in the stellar structure equations. The method has then been used to compute the equilibrium structures and periods of small oscillations of certain rotationally and tidally distorted stellar models. While computing the eigenfrequencies of g – modes of nonradial oscillations of rotationally and tidally distorted stellar models, Mohan and Saxena, 1985 and Mohan et al, 1998 observed that eigenfrequencies of g – modes decreases in presence of rotation which is contrary to the results earlier obtained by some other authors (Clement, 1984). We suspect that the discrepancy in the eigenfrequencies of g – modes might be due to Coriolis force which has not been taken into account by the authors. Therefore, it will be of our interest to see how the equilibrium structures of rotationally and tidally distorted stars get affected with the inclusion of Coriolis force in the earlier studies.

In this paper we propose to introduce the effects of Coriolis force in addition to that of centrifugal and gravitational forces to study the equilibrium structures and various shapes of Roche equipotential surfaces of rotationally and tidally distorted primary components of binary systems. The methodology thus developed has been used to determine the equilibrium structures of rotationally and tidally distorted polytropic models of stars.

The paper is organized as follows: The expression for the modified Roche equipotential of the rotationally and tidally distorted stars is obtained in section 2. The modified Roche equipotential is next used in section 3 to develop second order differential equation governing the equilibrium structures of rotationally and tidally distorted polytropic models of stars. In section 4, the numerical results for the inner structure and shapes of rotationally and tidally distorted polytropic models are obtained for the polytropic indices N =1.5 and 3.0. Analysis of the results and discussion are finally presented in section 5.

2. MODIFIED ROCHE EQUIPOTENTIAL

Following Kopal, 1972, let M_0 and M_1 be the masses of the two components of a close binary system separated by a distance D. The primary component of mass M_0 of this binary system is supposed to be very large than its companion star of mass M_1 which is regarded as a point mass $(M_0 >>> M_1)$. Suppose that the position of the two components is referred to a rectangular system of cartesian coordinates with origin at the center of gravity of mass M_0, the $X-$ axis along the line joining the mass centers of the two components and $Z-$axis perpendicular to the plane of the orbit of the two components (Fig. 1). For such a system, the total potential ψ due to gravitational and centrifugal forces (here the Coriolis force has been ignored), acting at an arbitrary point P(x, y, z) which lie inside the primary component is given by:

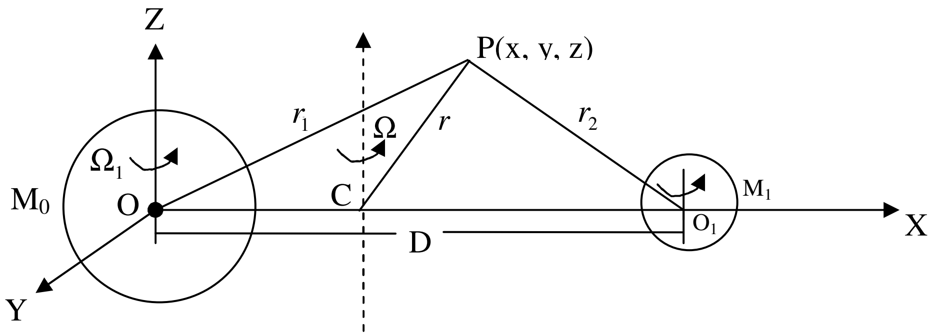

Fig.1 Axes of Reference

$$\psi = \frac{GM_0}{|\vec{r}_1|} + \frac{GM_1}{|\vec{r}_2|} + \vec{r}_1 \cdot [\vec{\Omega}\times(\vec{\Omega}\times\vec{d}_1)] - \frac{\vec{r}_1}{2}\cdot[\vec{\Omega}\times(\vec{\Omega}\times\vec{r}_1)] \quad (1)$$

which in cartesian coordinate form can be written as:

$$\psi = \frac{GM_0}{r_1} + \frac{GM_1}{r_2} - x\Omega^2 d_1 + \frac{\Omega^2}{2}(x^2+y^2) \quad (2)$$

where $r_1^{\,2} = x^2+y^2+z^2$, $r_2^2 = (D-x)^2+y^2+z^2$ and $r^2 = (x-d_1)^2+y^2+z^2$ represent the squares of the distances of point P from the center of gravity of the two components and center of gravity C$(d_1, 0, 0)$ (where $d_1 = \dfrac{M_1 D}{M_0+M_1}$) of the system respectively. Also Ω denotes the angular velocity of rotation of the system about an axis perpendicular to the XY- plane and passing through the center of gravity of the system and G is the constant of gravitation. It has been assumed that $\vec{\Omega} = \vec{\Omega}_1 = \vec{\Omega}_k$ (where $\vec{\Omega}_1$ is the angular velocity of rotation of primary component of the system and $\vec{\Omega}_k$ is the keplerian angular velocity).

The first, second and third terms on the right hand side of Eqn 1 represents the potential arising due to the mass M_0 of the primary component, the disturbing potential of its companion of mass M_1 and the potential arising from the centrifugal force respectively. Here the potential due to Coriolis force has not been considered. So in order to find out the expression for potential at point P (x, y, z) which also includes the effect of Coriolis force we start from the basic equation of motion in corotating frame of reference.

Following Kruszewski, 1966 the equation of motion for a system as described above is given by:

$$\frac{d^2 r_1}{dt^2} = \frac{d^2 r}{dt^2} - \frac{d^2 R_1}{dt^2} - \vec{\Omega} \times (\vec{\Omega} \times \vec{r}_1) - 2\vec{\Omega} \times \frac{d\vec{r}_1}{dt} \tag{3}$$

which on simplification finally changes to

$$\frac{d^2 r_1}{dt^2} = \nabla \psi - 2\vec{\Omega} \times \frac{d\vec{r}_1}{dt} \tag{4}$$

where

$$\psi = \frac{GM_0}{r_1} + \frac{GM_1}{r_2} - x\Omega^2 d_1 + \frac{1}{2}\Omega^2 (x^2 + y^2) \tag{5}$$

which is potential at point P (x, y, z) due to gravitational force and centrifugal force only and is same as Eqn (2). Here $\frac{d\vec{r}_1}{dt} = \vec{V}$ is the velocity of unit mass at point P (x, y, z) in rotating frame of reference whose origin is fixed at the centre of primary component with X - axis along the line joining the centre of two components of the system and Z - axis is parallel to the axis of rotation of orbital plane of the system. Clearly Eqn 5 does not include the factor due to Coriolis force because the term $2\vec{\Omega} \times \frac{d\vec{r}_1}{dt}$ which arises due to Coriolis force in equation of motion has not been included in potential function in Eqn (4). Now to obtain the factor due to Coriolis force in potential function in Eqn (4), we proceed as follows:

Since $\vec{r}_1 = x\hat{i} + y\hat{j} + z\hat{k}$ so $\vec{V} = \dot{x}\hat{i} + \dot{y}\hat{j}$ (as the rate change along z-axis is zero).

Hence

$$-2\vec{\Omega} \times \vec{V} = 2\Omega(\dot{y}\hat{i} - \dot{x}\hat{j}) \tag{6}$$

Let $\dot{y} = \Omega x$ and $\dot{x} = -\Omega y$, then

$$-2\vec{\Omega} \times \vec{V} = \nabla[\Omega^2 (x^2 + y^2)] \tag{7}$$

Using Eqn (7) in (4), we get

$$\frac{d^2 r_1}{dt^2} = \nabla(\psi_M)$$

where

$$\psi_M = \frac{GM_0}{r_1} + \frac{GM_1}{r_2} - x\Omega^2 d_1 + \frac{1}{2}\Omega^2 (x^2 + y^2) + \Omega^2 (x^2 + y^2)$$

which can be also written as

$$\psi_M = \frac{GM_0}{r_1} + \frac{GM_1}{r_2} - x\Omega^2 d_1 + \frac{3}{2}\Omega^2 (x^2 + y^2) \tag{8}$$

is the potential at point P(x, y, z) which also includes the effect of Coriolis force. In vector form, Eqn (8) can be represented as:

$$\psi_M = \frac{GM_0}{r_1} + \frac{GM_1}{r_2} + \vec{r}_1 \cdot [\vec{\Omega}\times(\vec{\Omega}\times\vec{d}_1)] - \frac{\vec{r}_1}{2}\cdot[\vec{\Omega}\times(\vec{\Omega}\times\vec{r}_1)] + \vec{V}\cdot(\vec{\Omega}\times\vec{r}) \tag{9}$$

where

$$\vec{V}\cdot(\vec{\Omega}\times\vec{r}) = \Omega^2 (x^2 + y^2) \tag{10}$$

is the potential at point P due to Coriolis force.

Following Kopal, 1972, Eqn (8) in nondimensional form can be expressed as:

$$\psi_M{}^* = \frac{1}{r_1^*} + q[\frac{1}{\sqrt{1 - 2\lambda r_1^* + r_1^{*2}}} - \lambda r_1^*] + 3nr_1^{*2}(1 - v^2) \tag{11}$$

where

$$\psi_M{}^* = \frac{D\psi}{GM_0}$$

is the nondimensional form of the total potential ψ_M which includes gravitational force, centrifugal force and Coriolis force. Here $r_1^* = r_1/D$ is nondimensional form of r_1, $\lambda = \sin\theta\cos\varphi$, $\mu = \sin\theta\sin\varphi$, $\upsilon = \cos\theta$ (r_1, θ, φ being the polar spherical coordinates of the point P). Also $q = M_1/M_0$ is a nondimensional parameter representing the ratio of the mass of the secondary over primary and $\omega^2 = 2n$ represents the square of the normalized angular velocity Ω. For a binary system in synchronous rotation, the angular velocity Ω is identical with Keplerian angular velocity so that $\Omega^2 = G(M_0 + M_1)/D^3$ which in nondimensional variables becomes $n = (q+1)/2$.

The surfaces generated by setting $\psi_M{}^*$ = constant on the left hand side of Eqn (11) are referred to as Roche equipotentials. In general we can represent the above equation as

$$\psi_M{}^* = \frac{1}{r_1^*} + q[\frac{1}{\sqrt{1 - 2\lambda r_1^* + r_1^{*2}}} - \alpha\lambda r_1^*] + \beta nr_1^{*2}(1 - v^2) \tag{12}$$

where α and β are positive real constants. For the synchronous binary systems, we take the value α = 1 and β = 3 whereas for uniformly rotating stars we substitute the values $q=0$ and $\beta=3$. On substituting α = 1 and β = 1, Eqn (12) reduces to the expression for Roche equipotential earlier used by Kopal, 1972 and other authors.

3. EQUILIBRIUM STRUCTURES OF ROTATIONALLY AND TIDALLY DISTORTED POLYTROPIC MODELS OF STARS

Following the approach of section 2 and assuming primary component of a binary system as a uniformly rotating polytropic model, the equilibrium structure of a primary component will be rotationally and tidally distorted model. If P_ψ denotes the pressure and ρ_ψ the density on the equipotential surface $\psi_M{}^*$ = constant, then ρ_ψ and P_ψ are connected through the polytropic relations

$$P_\psi = P_{c\psi}\,\theta_\psi{}^{N+1} \text{ and } \rho_\psi = \rho_{c\psi}\,\theta_\psi{}^{N} \tag{13}$$

where N is the polytropic index, $P_{c\psi}$ and $\rho_{c\psi}$ are the values of P_ψ and ρ_ψ at the center and θ_ψ represents some average value of the parameter θ on the equipotential surface $\psi_M{}^*$ = constant. Following Mohan and Saxena, 1983 the differential equation governing the equilibrium structure of a rotationally and tidally distorted primary component with polytropic structure which incorporates the effects of Coriolis force can be written in nondimensional form as:

$$\frac{d}{dr_0}[A(r_0,n,q)\frac{d\theta_\psi}{dr_0}] = -\frac{\xi_u^2}{K^2}\theta_\psi^N r_0^2 B(r_0,n,q) \tag{14}$$

where

$$A(r_0,\, n,\, q) = r_0{}^2\,[1-(\frac{3q^2}{5}+\frac{6nq}{5}+\frac{12n^2}{5})r_0^6 - \frac{6q^2}{7}r_0^8 - \frac{10q^2}{9}r_0^{10} + \ldots]$$

$$B(r_0,\, n,\, q) = [1+12n\,r_0{}^3 + (\frac{36q^2}{5}+\frac{72nq}{5}+\frac{864n^2}{5})\,r_0{}^6 + \frac{55q^2}{7}\,r_0{}^8 + \frac{26q^2}{3}r_0^{10} + \ldots]$$

$$r_0 = \frac{1}{\psi_M{}^* - q}$$

In above expressions terms up to second order of smallness in n, q and upto r_0^{10} in r_0 are retained. The dimensionless constant K in Eqn (14) is the ratio of the undistorted radius R_ψ of the primary to the separation D between the centers of the primary and secondary star. Also, we can write

$$\frac{D}{\ell} = \frac{D\xi_u}{\ell\xi_u} = \frac{D}{R_\psi}\xi_u = \frac{1}{K}\xi_u \tag{15}$$

where

$$\ell^2 = \frac{(N+1)P_{c\psi}}{4\pi G\,\rho_{c\psi}^2},$$

and ξ_u is the value of ξ at the outer surface of the undistorted polytropic model. The boundary conditions which Eqn (14) has to satisfy are

$$\theta_\psi = 1,\ \frac{d\theta_\psi}{dr_0} = 0, \text{at the center } r_0 = 0 \tag{16a}$$

$$\theta_\psi = 0, \text{ at the surface } r_0 = r_{0s} \tag{16b}$$

where r_{0s} is the value of r_0 at surface. Eqn (14) subject to these boundary conditions determines the equilibrium structures of rotationally and tidally distorted polytropic models incorporating the effects of Coriolis force on its equipotential surfaces. On setting $q=0$ and $K=1$, Eqn (14) can be used to determine the equilibrium structure of a polytropic model distorted by rotation alone. Also, by setting $n=(q+1)/2$, Eqn (14) can be used to determine the equilibrium structures of synchronously rotating primary components of binary systems.

To obtain the inner structure, the shape, volume and surface area of a rotationally and tidally distorted polytropic model, Eqn (14) has been integrated numerically for specified values of the parameters N, ξ_u, n, q and K. Starting with a series solution near the centre, the integration is carried outward numerically using a fourth order Runge –Kutta routines with a step size of 0.005. The integration is continued till θ_ψ first becomes zero and finally the value of r_0 at surface, that is, r_{os} is obtained. This value of r_{os} is further used to determine the volume, surface area and shapes of rotationally and tidally distorted polytropic models.

4. COMPUTATION OF VOLUMES, SURFACE AREAS AND SHAPES OF ROCHE EQUIPOTENTIAL SURFACES OF ROTATIONALLY AND TIDALLY DISTORTED POLYTROPIC MODELS OF STARS

Following Kopal, 1972 and Mohan et al, 1990, the explicit expressions for volume V_ψ and surface area S_ψ of rotationally and tidally distorted polytropic models which incorporates the effects of Coriolis force are:

$$V_\psi = \frac{4\pi}{3}\left(\frac{\ell\xi}{K}\right)^3 r_{0s}^3\,[1+6nr_{0s}^3+\left(\frac{12q^2}{5}+\frac{24nq}{5}+\frac{288n^2}{5}\right)r_{0s}^6+\frac{15q^2}{7}r_{0s}^8+2q^2r_{0s}^{10}+\ldots] \tag{17}$$

$$S_\psi = 4\pi\left(\frac{\ell\xi}{K}\right)^2 r_{os}{}^2[1+4nr_{0s}^3+\left(\frac{7q^2}{5}+\frac{14nq}{5}+\frac{168n^2}{5}\right)r_{0s}^6+\frac{9q^2}{7}r_{0s}^8+\frac{11q^2}{9}r_{0s}^{10}+\ldots] \tag{18}$$

The distance r of a point on the equipotential surface from the centre of the star is given by

$$r=(\frac{\ell\xi}{K})r_{0s}\ [1+a_0 r_{0s}{}^3+(qP_3)r_{0s}{}^4+(q\,P_4)r_{0s}{}^5+(qP_5+3a_0{}^2)r_{0s}{}^6+(qP_6+7\,a_0 qP_3)r_{0s}{}^7$$
$$+\ (qP_7+8\,a_0 q\,P+4q^2P_3^2)r_{0s}{}^8+(q\,P_8+9\,a_0 qP_5+9\,q^2P_3\,P_4)r_{0s}{}^9 \tag{19}$$
$$+(q\,P_9+10\,a_0 q\,P_6+5q^2\{P_4^2+2\,P_3\,P_5\})\ r_{0s}{}^{10}+...]$$

where $a_0 = q\,P_2 + 3n(1-\upsilon^2)$ and $P_j = P_j\,(\lambda)$ are Legendre polynomials. Again, in the above expressions terms up to second order of smallness in n, q and upto r_{0s}^{10} in r_{0s} are retained. The relation (19) can be used to obtain the various shapes of Roche equipotential surfaces.

5. COMPARISON AND ANALYSIS OF RESULTS

The results of volume and surface area obtained for different values of the input parameters are tabulated in Table 1. We have taken the value of parameter $\alpha=1$ and $\beta=3$. The value of parameter K has been taken as one for the rotationally distorted model and 0.5 for the tidally distorted or rotationally and tidally distorted models (this value of K provides the outer-most surfaces of the models well within Roche lobe for each considered case).

Table 1 shows that with the inclusion of Coriolis force, there is increase in the values of volume and surface area of polytropic models in all the cases (uniformly rotating, synchronously and non – synchronously rotating binaries). Among all the cases, rotational distortion shows maximum percentage increase in the values of volume and surface area. Polytropic models with indices $N = 3.0$ shows more percentage increase than polytropes with $N = 1.5$. Also, the percentage increase in the values of volume is more than the values of surface area in both the cases of $N = 1.5$ and $N = 3.0$.

In Table 2 we have considered rotational case for different values of the rotation parameter $v = \Omega^2/(2\pi G\rho_c)$ which is connected with our nondimensional parameter n through the relation $v = 4n\overline{\rho}/(3\rho_c)$, where $\overline{\rho}$ is the mean density and ρ_c is the central density of the undistorted polytropic model. We have compared our present results with the results earlier obtained by Mohan and Saxena, 1983; Chandrasekhar, 1933; James, 1964 and Linnell, 1981.

From Table 2, it is clear that when the effect of Coriolis force is taken into account then with increase in the value of n the value of volume and surface area of polytropic model of star increases and also these values are greater than the values earlier obtained by Mohan and Saxena, 1983; Chandrasekhar, 1933; James, 1964 and Linnell, 1981.

Thus, our present study has shown that the volume and surface area of rotating stars and stars in binary stars increases in the presence of Coriolis force.

Table 1 Volume and Surface area of rotationally and tidally distorted polytropic models

Type of model	n	q	Volume $V_\psi \times 10^{-2}$ Present values	Mohan et al (1983)	% Increase	Surface Area $S_\psi \times 10^{-2}$ Present values	Mohan et al (1983)	% Increase
Polytropic index $N = 1.5$								
Undistorted	0.0	0.0	2.043	2.043	0	1.678	1.678	0.0
Uniformly rotating	0.02	0.0	2.188	2.090	4.7	1.756	1.704	3.1
Synchronous Binary	0.525	0.05	2.537	2.203	15.2	1.950	1.765	10.5
	0.55	0.1	2.560	2.221	15.3	1.964	1.774	10.7
Nonsynchronous Binary	0.05	0.2	2.089	2.062	1.3	1.702	1.688	0.8
	0.1	0.2	2.135	2.077	2.8	1.727	1.696	1.8
Polytropic index $N = 3.0$								
			Volume $V_\psi \times 10^{-3}$			**Surface Area $S_\psi \times 10^{-2}$**		
Undistorted	0.0	0.0	1.374	1.374	0.0	5.977	5.977	0.0
Uniformly rotating	0.02	0.0	1.516	1.419	6.8	6.382	6.106	4.5
Synchronous Binary	0.525	0.05	1.921	1.530	25.6	7.535	6.424	17.3
	0.55	0.1	1.952	1.549	26.0	7.622	6.475	17.7
Nonsynchronous Binary	0.05	0.2	1.417	1.391	1.9	6.101	6.026	1.2
	0.1	0.2	1.462	1.405	4.1	6.229	6.066	2.7

Table 2 Comparison of the volumes and surface areas of uniformly rotating polytropes

v	Volumes: Present values	Mohan et al (1983)	Chandra sekhar (1933)	Linnell (1981)	James (1964)	Surface areas: Present values	Mohan et al (1983)	Chandra sekhar (1933)	Linnell (1981)
Polytropic index $N = 1.5$									
0.000	2.04	2.04	2.04	2.04	2.04	1.68	1.68	1.68	1.68
0.008	2.31	2.13	2.13	2.14	2.14	1.82	1.73	1.72	1.73
0.016	2.58	2.23	2.21	2.26	2.25	1.98	1.78	1.77	1.79
0.024	2.80	2.33	2.30	2.41	2.40	2.11	1.83	1.82	1.87
0.032	2.96	2.44	2.39	2.62	2.59	2.22	1.89	1.86	1.97
0.040	3.08	2.55	2.47	2.91	2.92	2.30	1.95	1.91	2.11
Polytropic index $N = 3.0$									
0.0000	1.37	1.37	1.37	1.37	1.37	5.98	5.98	5.98	5.98
0.0008	1.62	1.45	1.44	1.45	1.45	6.67	6.19	6.17	6.19
0.0016	1.91	1.53	1.51	1.54	1.54	7.52	6.44	6.36	6.44
0.0024	2.22	1.63	1.57	1.65	1.66	8.39	6.71	6.55	6.76
0.0032	2.50	1.74	1.64	1.80	1.83	9.24	7.02	6.74	7.17

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