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COMPUTATIONS AND DETAILING OF PRESSURE VESSELS HAVING UNUSUAL REQUIREMENTS in DESIGN

ANIL PATNAIK[1], ASHENAFI HEGANA[2] AND T.S. SRIVATSAN[2]

[1]Department of Civil Engineering
The University of Akron, Akron, OH44325, USA
E-Mail: Patnaik@uakron.edu

[2]Department of Mechanical Engineering
The University of Akron, Akron, OH44325, USA
E-Mail: tsrivatsan@uakron.edu

Abstract. The provisions of Australian Standard AS 1210-1989 for design and detailing of unfired pressure vessels can be applied to commonly encountered design situations where the shells have small openings. The technique of finite element analysis is a convenient and powerful tool for the purpose of computational stress analysis when the design requirements are such that either the openings are large, or where concentrated loads and/or thermal movements are present. This paper provides a summary of one such practical design of a pressure vessel. While the computational technique using finite element analysis simplifies the stress analysis of pressure vessels considerably, the study revealed a key need to standardize the different factors affecting the stresses developed during the design of a pressure vessel.

1. INTRODUCTION

The guidelines provided in design standards, such as, Australian Standard (AS 1210-1989) [1], the British Standard (BS 5500) [2], and American Society of Mechanical Engineers (ASME) Section VIII [3] are applicable to normally encountered design situations. However, the analysis procedure given in the design standard AS 1210-1989 for pressure vessels is not applicable to design of a shell whose dimensions at an opening is greater than 1000 mm. Further, these procedures become inapplicable when size of an opening in a pressure vessel shell is greater than half the diameter of the shell (see Clause 3.18.4.1 (a & b)). Over the years, several handbooks and text books [4-7] have been put forth that deal with both the theoretical and practical design aspects of pressure vessels. The use of classical shell theory for the purpose of stress analysis of pressure vessels becomes difficult, if not impossible, when the design requirements are essentially far from usual. It is therefore advantageous to put to use advanced computational stress analysis techniques, such as the finite element analysis (FEA). Finite element analysis has been in use for well over 25 plus years for the purpose of both identifying and establishing stresses at complicated nozzle-shell junctures [5]. However, it does become challenging to use finite element analysis primarily because judgment from an engineering perspective coupled with experience of the individual designer is called upon to decide on both the compliance and overall structural safety of the design. Currently,

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there are no guidelines available in the design standards to deal with stress analysis procedures along with acceptance criteria for 'local' stress concentrations when the technique of finite element analysis (FEA) is used for the purpose of analysis of pressure vessels.

One such pressure vessel design that can be classified as unusual is outlined in the following sections. This design highlights the acute need for developing some standards in the procedures that are used for an analysis of stress when the finite element method (FEM) is used. The design is that of a large existing pressure vessel that required modifications to install a steam pot at its top. The junctures that arise due to the attachment of the steam pot are complicated. It is believed that a numerical computational technique, such as finite element analysis, can provide both a simplistic and rapid solution for the purpose of computational stress analysis. In the following section, the general expressions for the determination of stresses in shell elements are briefly outlined.

2. EQUILIBRIUM EQUATIONS FOR SHELL ELEMENTS

For a shell with two radii of curvature, let r_1 be the radius of curvature of the meridian, and r_2 the radius measured to the central axis measured normal to the meridian. The horizontal radius when the axis is vertical is r. Further details of the sign convention are given in Spence and Tooth, 1994 [4]. Six equations of equilibrium can be established by considering the force and moment equilibrium of the element in the three orthogonal directions. These six equations in the simplified form are as follows [4]:

$$r_1 \frac{\partial N_{\theta\emptyset}}{\partial\theta} + \frac{\partial(rN_\emptyset)}{\partial\emptyset} - r_1 N_\theta cos\emptyset - Q_\emptyset r + p_\emptyset r r_1 = 0 \tag{1}$$

$$\frac{\partial(rN_{\emptyset\theta})}{\partial\emptyset} + r_1 \frac{\partial N_\theta}{\partial\theta} - r_1 N_{\theta\emptyset} cos\emptyset - Q_\emptyset r_1 \, sin\emptyset + p_\theta r r_1 = 0 \tag{2}$$

$$N_\theta r_1 \, sin\emptyset + rN_\emptyset + \frac{\partial Q_\theta}{\partial\theta} r_1 + \frac{\partial(rQ_\emptyset)}{\partial\emptyset} - p_r r r_1 = 0 \tag{3}$$

$$r_1 \frac{\partial M_{\theta\emptyset}}{\partial\theta} + \frac{\partial(rM_\emptyset)}{\partial\emptyset} - r_1 M_\theta cos\emptyset - Q_\emptyset r r_1 = 0 \tag{4}$$

$$\frac{\partial(rM_{\emptyset\theta})}{\partial\emptyset} + r_1 \frac{\partial M_\theta}{\partial\theta} + r_1 M_{\theta\emptyset} cos\emptyset - Q_\theta r r_1 = 0 \tag{5}$$

$$N_{\emptyset\theta} - N_{\theta\emptyset} + \frac{M_{\theta\emptyset}}{r_2} - \frac{M_{\emptyset\theta}}{r_1} = 0 \tag{6}$$

For special cases, such as, the membrane shells, the effects arising from bending can be small primarily because of the small thickness of the shell when compared to its radius. Bending stresses can also be neglected when designing the shell portions that are located remotely from discontinuities. In such cases, equations (1) to (6) can be reduced to the following three equations:

$$r_1 \frac{\partial N_{\theta\emptyset}}{\partial\theta} + \frac{\partial(rN_\emptyset)}{\partial\emptyset} - r_1 N_\theta cos\emptyset + p_\emptyset r r_1 = 0 \tag{7}$$

$$\frac{\partial(rN_{\theta\emptyset})}{\partial\emptyset} + r_1\frac{\partial N_\theta}{\partial\theta} - r_1 N_{\theta\emptyset} cos\emptyset + p_\theta r r_1 = 0 \quad (8)$$

$$\frac{N_\theta}{r_2} - \frac{N_\emptyset}{r_1} - p_r = 0 \quad (9)$$

Equations (1) to (9) are general equations for the determination of stresses in curved shells. If a cylindrical shell is long compared to its radius, simplified stress equations can be used. The uniform internal pressure will often tend to cause a radial force that is assumed to be uniformly distributed along the circumference of the shell. A uniform enlargement of the cylindrical shell will take place under internal pressure. Equilibrium of the shell can be established by cutting a section across any diameter of the shell. A free body diagram as shown in **Figure 1** can be used to determine the force per unit length of the shell in terms of the internal pressure (q) acting on the circumference of the shell whose radius is "*r*". This force will be equal to $qrd\phi$ [5].

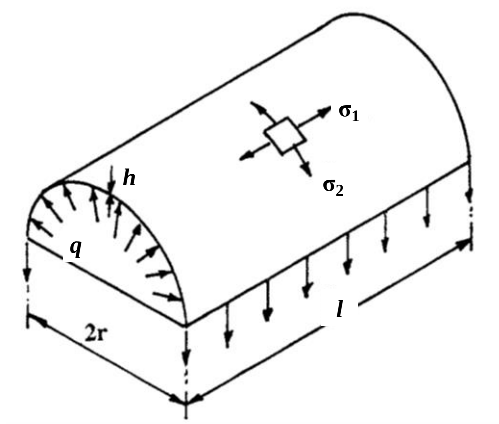

Figure 1. Cylindrical Shell cut into Half

The sum total of the vertical components of these forces gives an equation of equilibrium, which is (**Figure 2**):

$$2F = 2\int_0^{\frac{\pi}{2}} qr \sin\emptyset \;\; d\emptyset \;\; = \;\; 2\,qr$$

$$F = qr \quad (10)$$

and the hoop stress is:

$$\sigma_2 = \frac{pr}{h} \quad (11)$$

where, p is the internal pressure acting on the pressure vessel, and h the thickness of the shell **(Figure 2).**

Similarly, the longitudinal stress is calculated based on internal pressure (p) acting at the end of the cylindrical shell (**Figure 3**). The longitudinal stress is:

$$\sigma_1 = \frac{pr}{2h} \tag{12}$$

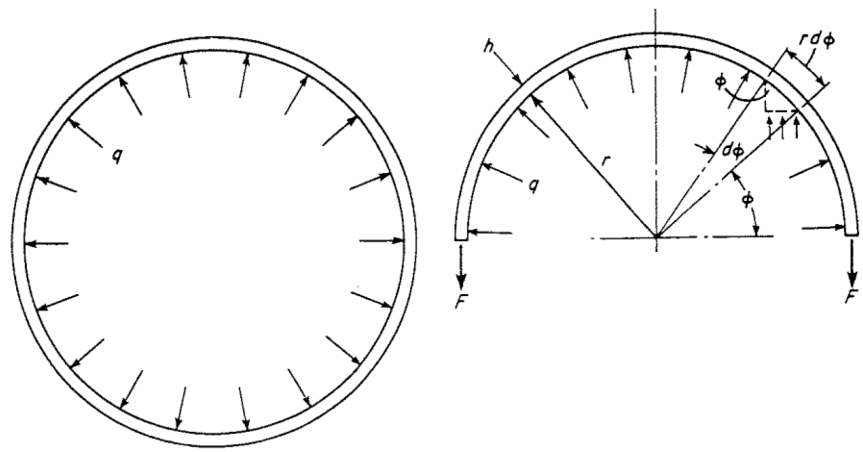

Figure 2. Radial Internal Pressure and Hoop Stresses in a Cylindrical Shell

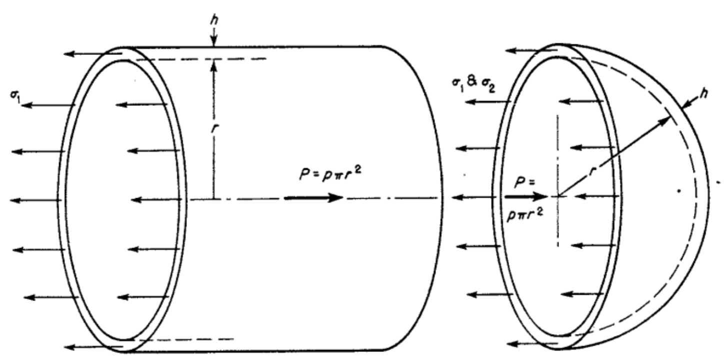

Figure 3. Longitudinal Stress in a Cylindrical Shell

The simplified equations (Eq. 1 to Eq. 12) can be safely applied for the purpose of design of pressure vessels assuming that the vessel is normally proportioned and does not have any large cutouts. However, for the design that is described in this paper, the condenser (pressure vessel) is classified as an unusual pressure vessel due to its large openings coupled with a nozzle that is fitted at an off-axis alignment.

3. DESCRIPTION OF THE CONDENSER

The modification of a condenser having a diameter of 3.5 meter, a length of 4.7 meter was designed. Schematic details of the condenser prior to its modification are shown in **Figure 4.** The modified condenser is as shown in **Figure 5**. It was required to make a cut-out in the existing condenser shell and attach a steam separator pot to the main shell as shown in the figure. The diameter of the steam pot that was to be attached was 1300 mm. The unusual nature of modification is not quite obvious and intersection of the steam pot with the main shell is far from simple. The steam pot intersects with both the steam inlet and main shell of the condenser. The centre-line of the steam pot does not pass through the longitudinal axis of the cylindrical shell. Also, the dished ends of the condenser and the steam inlet have an offset with respect to the longitudinal axis of the main shell. Therefore, an analysis of stresses in the condenser shell arising from the desired modification is certainly unusual.

A water collector is fixed at the bottom of the main shell and having appropriate connections to the condensate pump. The condenser was provided with a compensator between the main shell and one of the tube plates with the primary intent of minimizing thermal effects. The two circular tube plates were bolted on to the end plates of the main shell.

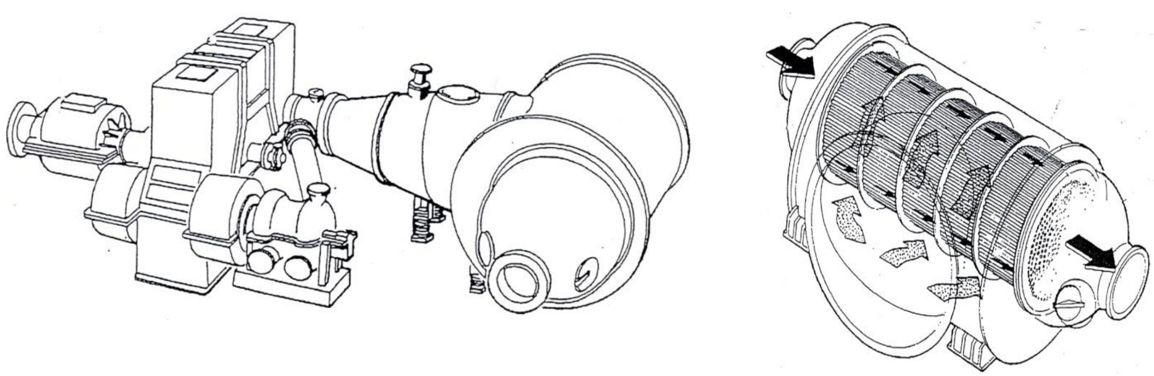

Figure 4. Schematic showing a detailed overview of the condenser prior to modification

The scope of modification in design of the condenser unit was to:

(a) Conduct a stress analysis on the main condenser shell with the steam pot attached.

(b) Design the steam pot and the nozzles to include the steam flow deflectors and its connection to the main condenser shell.

(c) Check the structural safety of the condenser for the amount of external load that is expected to be applied on flanges of the pot nozzles.

(d) Develop fabrication details and the welding procedure for welding the pot on to the existing condenser shell. The procedure was required to be approved by the ABS (American Bureau of Shipping) since the condenser was supported on a vessel. The welding procedure was to be developed so as to ensure there occurs no visible or measurable distortion of the turbine flange connection.

2.1 Conceptual Planning of the Modification

The size of both the steam pot and nozzle were specified for the modification. Steam flow deflectors were required to provide smooth flow of steam into the condenser through the inlet nozzles. Deflectors of this large size were not readily available with suppliers and therefore had to be custom designed. The shape of the deflectors was designed to facilitate a smooth transition. The deflectors were attached to the blind flange and the whole unit was thus made removable. This removable type of arrangement does facilitate an inspection of the inside of the condenser. The flanges of this unit were noticeably larger than the standard sizes available and had to be custom designed to ensure both the pot and the nozzles were both air tight and well-sealed. The shell of the steam pot protrudes 50 mm all around, inside the main condenser shell. This protrusion enables the welding to be conveniently done while concurrently providing additional stiffness around the opening.

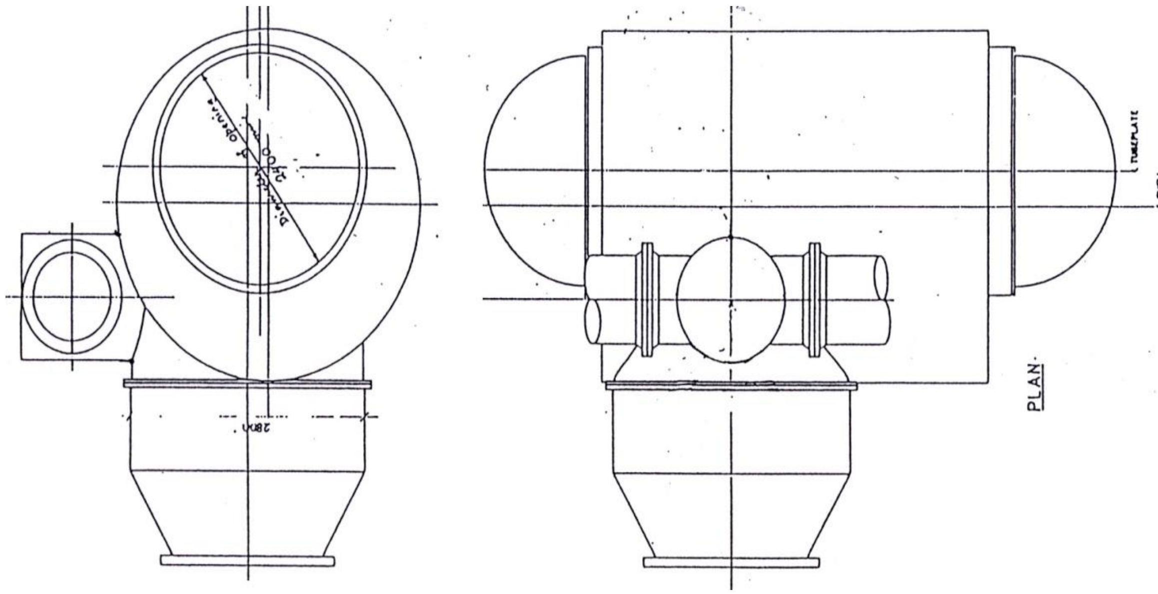

Figure 5. Details of the required modification

2.2 Design Data

The following design standards were used:

● Main shell and steam pot design:	AS1210-1989 Unfired Pressure Vessels
● Steel material:	AS 1548 Grade 7-430
● Bolts:	AS 1111 Grade 4.6
● Allowance for corrosion:	3 mm (see AS 1210-1989)

The material chosen for the condenser shell was evaluated to be equivalent to steel conforming to AS 1548 Grade 7-430 with an allowable tensile stress of 108 MPa.
Thickness of main shell of the condenser was 16.1 mm (to include an allowance of 3 mm for corrosion). Thickness of the inlet pipe was 10 mm.

The gravity loads arising from assembly of the tube were estimated from data provided by the vendor. An allowance for a maximum height of standing water of 1 meter collected in the condenser shell was made so as to allow for possible malfunctioning of the condensate pump.

2.3 Design Approach

The design was carried out in five stages:

1. *Global analysis*: stress analysis of the condenser main shell with and without the steam pot attached
2. *Local Analysis:* stress analysis of the steam pot and the nozzles
3. *Component analysis:* stress analysis of the steam deflectors
4. *Detailing:* development of details
5. *Welding procedures:* development of welding procedures

These design details are outlined in the following sections.

2.4 Global Analysis - Main Condenser Shell and Steam Pot

Finite element analysis was conducted for purpose of stress analysis on the main shell of the condenser. This is because the analysis procedure of AS 1210-1989 is not applicable to a cut-out in the shell having this size. Three dimensional numerical computational stress analysis of the condenser shell was conducted using a commercial finite element analysis program called ANSYS [8]. Two models were developed for an analysis of the main shell. The first model was considered without the attachment of the pot while the second model was considered with the pot attached. The stresses developed in the two cases are compared so as to investigate and concurrently establish the effect making the cut-out in the condenser shell and attaching the steam pot at the top.

2.4.1 Finite element models

A finite element model with 3-D elastic shell elements (Shell63) of ANSYS was developed for purpose of analysis of the condenser.

(a) Four-node quadrilateral elements were used in the models to the extent possible.
(b) Three-node triangular elements were used both at the transition region and intersections.

The intersections, both at the steam inlet and the steam pot with the main shell, were developed using the Boolean functions of ANSYS. The geometry conformed to the details shown in **Figure 4** and **Figure 5.**

2.4.2 Material properties

The following were the properties of the materials chosen for the condenser shell:

- Modulus of elasticity — 210 GPa
- Shear modulus — 80 GPa
- Poisson's ratio — 0.3
- Coefficient of thermal expansion — 11.7×10^{-6} per degree C
- Mass density — 7,850 kg/m^3

- Yield stress 216 MPa
- Ultimate tensile strength 363 to 441 MPa
- Allowable tensile stress 108 MPa at 120ºC

2.4.3 Boundary conditions

The main shell is supported on four saddles. The saddles were such that the shell is firmly supported for one-full meter along the longitudinal axis at each end of the main shell and 0.35 meter on both sides of the longitudinal axis. Therefore, all of the nodes falling within the supported area were considered to be fixed in both the translational direction and the rotational direction.

2.4.4 Additional modeling features

The flange plates of the steam inlet, flat end plates of the main condenser shell, the tube plates, and the two dished ends provided to enclose the condenser tubes along with the condenser tube bundle assembly provide considerable rigidity to the system. The effect of these components was introduced into the finite element model by providing rigid plates in their place since primary objective of the stress analysis was to determine stress concentrations both at and around the steam pot. The total weight of the condenser tubes was lumped at the nodes located within the end plates at the appropriate locations.

2.4.5 Results for condenser shell without the steam pot

The model representing the condenser shell, without the pot, consisted of 860 nodes and 945 elements. The equivalent peak stress (Von Mises stress) in the shell due to a suction pressure of 0.1 MPa around the steam inlet was about 65 MPa. The equivalent peak stress due to gravity loads was about 18 MPa, which is about the same as the case for condenser with the pot attached. The peak stress in the condenser shell due to a combination of suction pressure and gravity load was less than 65 MPa, which is well within the allowable stress limit of 108 MPa.

2.4.6 Results for condenser shell with the steam pot

The finite element model of condenser shell (3.5 m diameter x 4.7 m length), steam-inlet (2800 mm diameter x 750 mm length), steam-pot (1300 diameter x 1250 mm length) and flanges was developed using elastic 16mm thick shell-181 element of ANSYS as shown in **Figure 6**. The shell-181 element is a four-node element with six degrees of freedom at each node: translations in the x, y, and z directions, and rotations about the x, y, and z-axes. However, the four saddles of condenser shell are modeled using 10 node tetrahedral solid-186 element of ANSYS. The model representing the condenser shell with the pot consisted of 7,442 elements.

From the operating conditions of the condenser unit, the following three basic load cases were considered:

(a) An internal suction pressure in the condenser of 0.1 MPa.
(b) Self weight of the condenser tubes, which is applied as a concentrated load within the end plates at location of center of gravity of the tube assembly.
(c) Self weight of the shell, and the weight of 1 meter of condensate (standing water) that is collected at the bottom of the condenser shell without suction pressure.

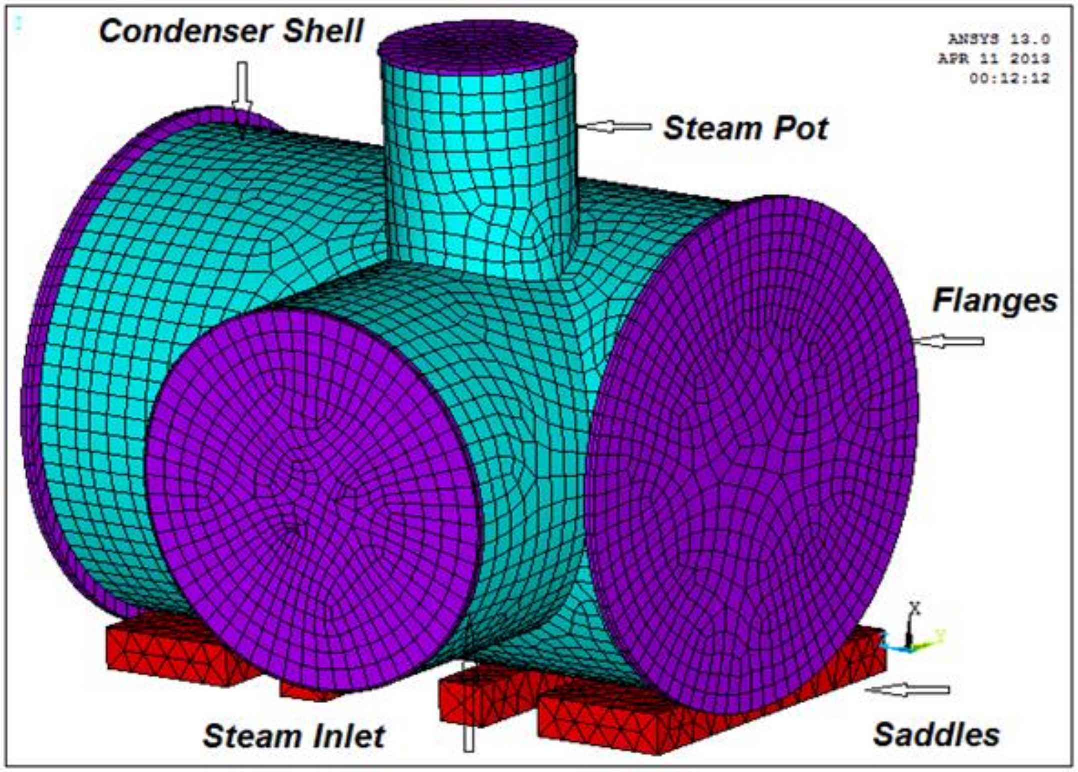

Figure 6. Finite Element Model of the Shell with Steam Pot Attached

The equivalent peak stresses in the main shell for a combination of the three load cases were developed. Large stress concentrations of up to 120 MPa were observed in a very small isolated area near the steam inlet and the pot. This stress is greater than the allowable stress of 108 MPa. However, the stress concentration may not be entirely due to the load applied to the condenser shell model. This could have resulted from the numerical inaccuracies associated with the use of triangular elements. The stress concentration was disregarded since the area of this high stress is small and in the immediate vicinity of a number of triangular elements. The rest of the shell was stressed to less than about 80 MPa. Therefore, the existing shell was concluded to be safe upon attaching the steam pot.

Typical contours for displacements under the combined loading of internal suction pressure and the gravity loads are shown in **Figure 7**. The displacement contours are shown in meter units. Similarly, the stress contours under the same loading condition are shown in **Figure 8**. The contours in the figure are in Pascal units (Newton per square meter).

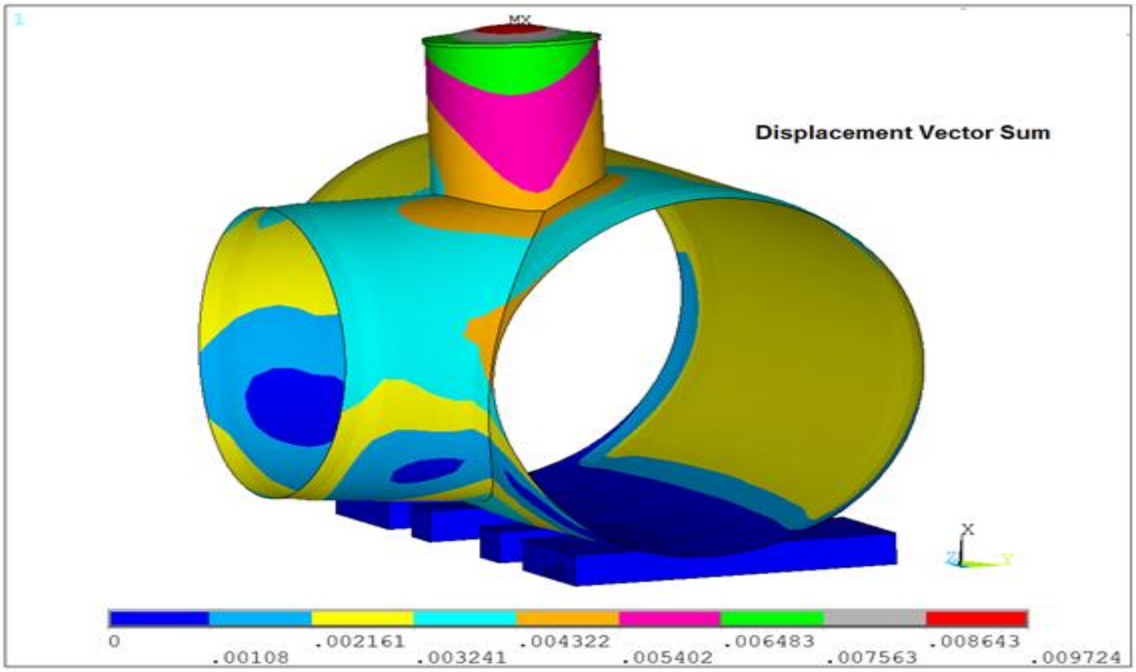

Figure 7. Typical Displacement Contours for the Combined Load Case of Internal Suction Pressure and Gravity Loading

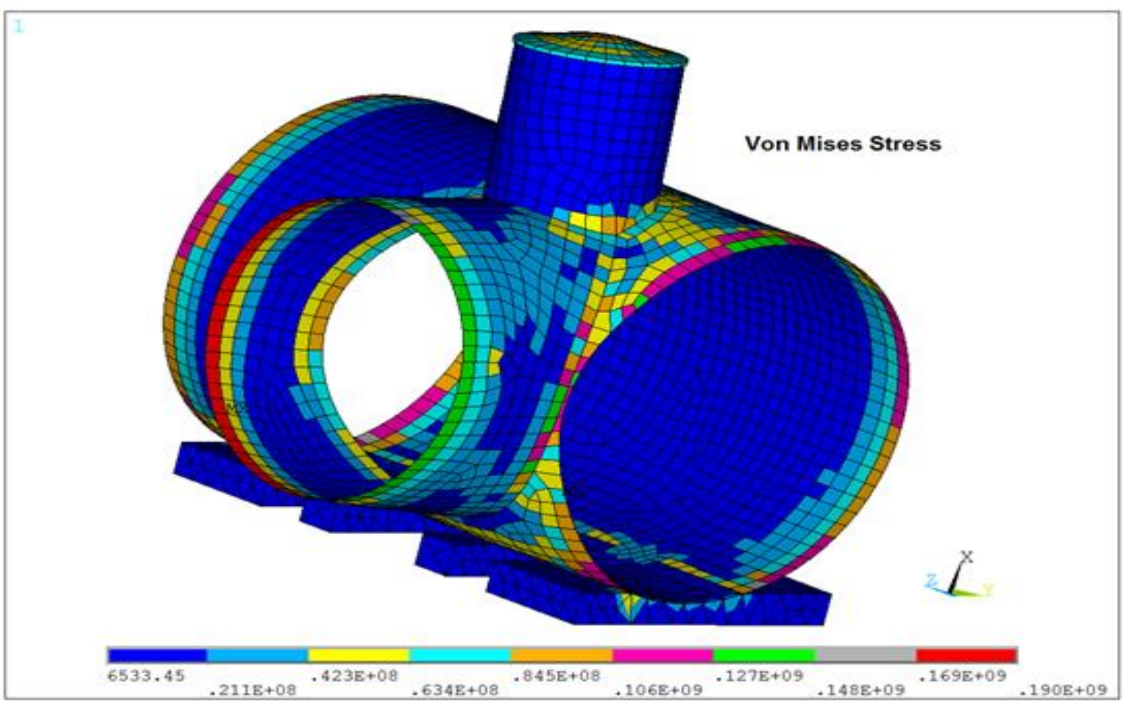

Figure 8. Typical Stress Contours for the Combined Load Case of Internal Suction Pressure and Gravity Loads

A comparative summary of the stresses obtained from the finite element analysis, with and without the steam pot, is given in **Table 1**. The locations for peak stresses under suction pressure do not coincide well with locations for peak stresses under gravity loading. Therefore, an algebraic summation of the peak stresses for the two individual loading conditions did not result in the resultant peak stresses for the combined loading case which is beneficial.

Table 1 Comparison of the Peak Stresses with and without Steam Pot

Loading Condition	Location of Stress Concentration	Without Steam Pot	With Steam Pot
Suction Pressure	Around the steam inlet	50 to 65 MPa	120 MPa
Gravity Loading	Around the steam inlet	10 MPa	12 MPa
Combined Suction Pressure and Gravity Loading	Around the steam inlet	50 to 65 MPa	127 MPa

2.4.7 Comments on analysis

The results of the stress analyses were carefully studied. Isolated cases of high stress concentrations were observed at the following two locations:

(i) In the vicinity of pot-shell junctures, and
(ii) At the location of supports.

The supports provided in the actual structure are evenly spread on six plates over a two meter length along the longitudinal axis of the main condenser shell. The support plates provide an even support by virtue of their 0.35 meter length across the longitudinal axis on both sides of the longitudinal axis. However, in the finite element analysis, the supports were idealized to be a limited number of discrete points. Some loss of accuracy can be expected due to the use of coarse triangular elements that are known to introduce an error of around ten percent. An increase in stress was observed in the computation finite element model of the condenser upon the introduction of the cut-out for the steam pot. However, the final stresses in the condenser main shell were still well below the allowable limit. Therefore, it was not felt necessary to provide any reinforcement (or compensating plates) around the cut-out.

2.4.8 Stresses due to nozzle loads

Maximum nozzle loads that can be expected on the condenser were supplied by the end user. These loads were applied to the condenser model along with pressure, gravity, and condensate loads. The stresses were found to be within the limits for the specified loads.

2.4.9 Deformations near the cut-out in the main shell during the modification

When the cut-out is made for attaching the steam pot, the shape of the cut-out may be distorted due to self-weight of the condenser shell. This was studied by observing the nodes defining the cut-out for the pot when the gravity loads are applied to the finite element model having an opening in the main shell. The peak deformation (a change in dimension of the opening) was found to be less than 0.12 mm, which is theoretically acceptable. However, from the view point of practical considerations, additional temporary timber supports around the bottom half of the shell are recommended to be provided during the modifications in order to prevent distortion of the cut-out due to unforeseen loads the shell may have to resist during modification.

2.5 Local Analysis - Steam Pot and Nozzles

A separate model with fine mesh was developed for analysis of the pot and the nozzle connections. The steam pot had a diameter of 1295 mm and the nozzles attached to the pot was 914 mm in diameter. The procedures outlined in the Australian Standard AS 1210-1989 [1] for analysis is not applicable for such a case since size of the opening is greater than half the diameter of the steam pot and 500 mm (see Clause 3.18.4.1.(a)). Therefore, numerical computational finite element analysis was separately conducted on both the steam pot and the nozzles using a fine mesh and the stress levels were determined from the analysis. Further, it is not practical to introduce nozzles in the finite element model developed for the main condenser shell since the model would be both large and complicated to handle. Both the accuracy and focus of the stress analysis will be diverted away from the cut-out (for steam pot). Therefore, it was necessary to separately analyze both the steam pot and the nozzles.

The peak stresses resulting from a suction pressure of 0.1 MPa for 16 mm thick pot and nozzles (3 mm allowance for corrosion) were less than the allowable stress of 108 MPa. Therefore, the pot and nozzles were finalized to be fabricated with a 16 mm plate. No compensating plates were provided around the cut-outs for the nozzles since the stresses were well within the limit.

2.6 Component Analysis - Steam Flow Detector

The shape of the deflector plates was specified by the process engineer who was working on a related project. It was required to design the thickness of the deflector plates. Shape of the deflector plate makes it impossible for sound theoretical analysis. However, a finite element model of the deflector plate was easily developed and numerical computational analysis performed without much difficulty.

2.7 Details of Removable Deflector Unit

The deflector unit consisted of a blind flange that covers the top of the steam pot, two steam deflectors and an arrangement to hang the deflectors from the blind flange. The shape of the deflectors is based on characteristics of the steam flow. The structural details of this unit are shown in **Figure 9**.

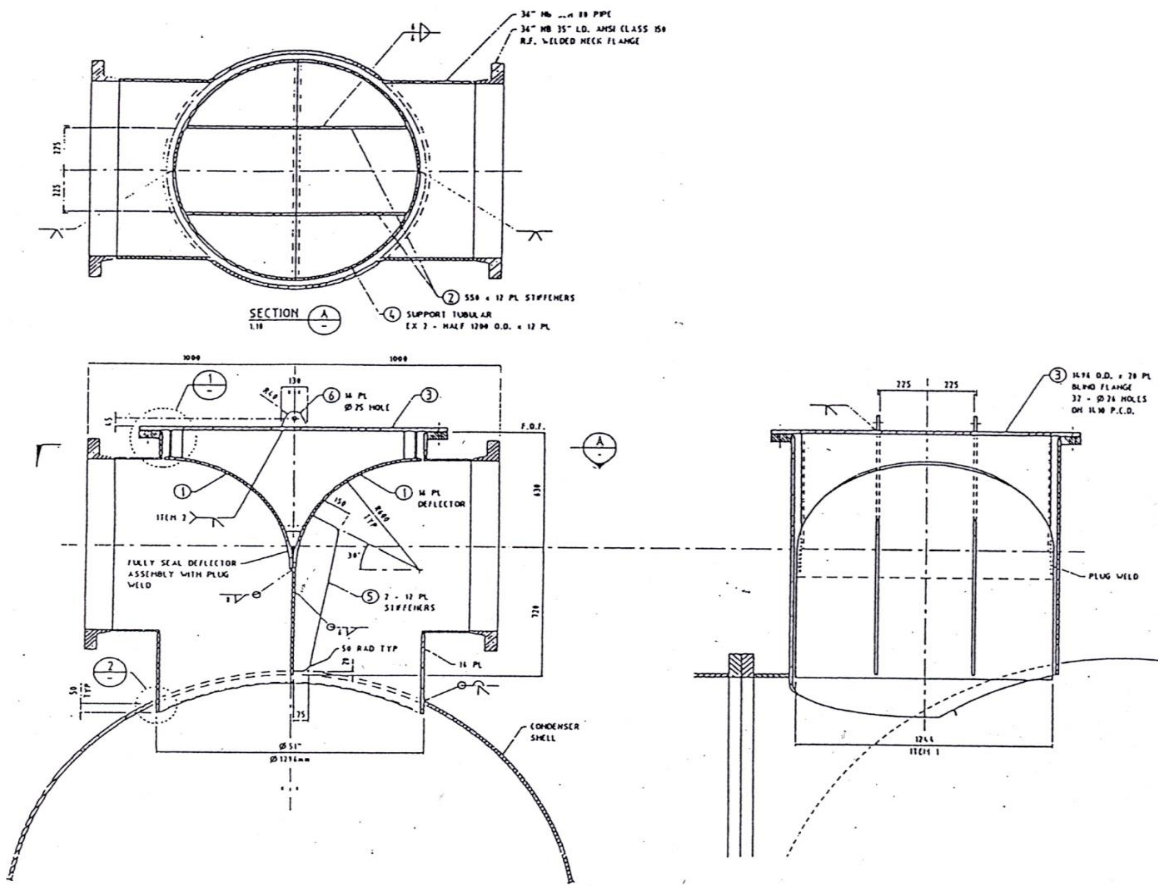

Figure 9 Final details of the modification

2.8 Welding Procedures

The details of the welding procedures were developed by a specialist to minimize and/or reduce any distortion arising due to the introduction of a cut-out and subsequent welding of the steam pot. A detailed method is specified having well defined tolerances and precautions.

2.9 Summary

Modifications to the condenser were done very much in conformance with the details outlined in the preceding sections and it was successfully commissioned. The entire condenser shell and the steam pot arrangement were structurally safe and functionally acceptable. Use of finite element method for analysis of the stress made this modification in design both simple and rapid.

4. DISCUSSION and the NEED FOR DESIGN GUIDELINES

The design and analysis adopted in the pressure vessel described raises questions as to what is an acceptable design when finite element analysis is used for the purpose of numerical computational of stress in pressure vessels. The stress concentrations exhibited in an analysis may not necessarily represent the true stresses near the junctures and intersections. This is because numerical inaccuracies can creep into the results of finite element analysis. There is a need to define and develop guidelines on the allowable stresses at the locations of high stress concentration in relation to both size of the element and shape of the element. It is well recognized that the use of numerical computations for stress analysis, to include the effects of non-linear and/or local buckling behavior, is not practical in day-to-day design of pressure vessels. Guidelines on the size limits of the element are also needed for the purpose of optimizing practical design. Furthermore, it is both essential and required to define what the most suitable and appropriate element size is for both an acceptable and reasonable accuracy of the results. The stresses close to the intersection of various components of the pressure vessel increase significantly (but the area of high stresses decreases) when size of the element is reduced near the intersections. Also, the triangular elements that are introduced by Boolean operations in ANSYS cause substantially greater stress concentration. The generally accepted aspect ratio that is easy to follow for straight edge thin wall structures (like a box-girder bridge) are not applicable for pressure vessel shells as they tend to have curved intersections. It is difficult to avoid large element aspect ratios for such shapes. A large aspect ratio, particularly for the triangular elements, will often result in distorted values that are difficult to justify. In some cases, limits on deflections, namely: deformation or ovalization of the shells from their original circular shape, must be specified.

Some standards dealing with structural design requirements for thin walled steel structures such as box girders, chimneys and silos (for example, Euro code Part 3) have recently introduced a few guidelines and recommendations for the purpose of design when the finite element method is used. Similar guidelines for the analysis and design of pressure vessels will be helpful particularly to practicing engineers who are lacking in experience with pressure vessel design and finite element analysis. In the absence of such guidelines, decisions on the design are left to the individual designer who has to make a number of decisions based on his or her engineering judgment and personal experience. A unified approach for stress analysis must be specified in the standards so that commercial aspects do not take precedence over structural safety and functional requirements of pressure vessels.

The geometry definition and the Boolean functions for intersections between complex surfaces particularly when there are more than two surfaces involved in the intersection is a research need that experts in mathematics would find fascinating. Furthermore, a closed-form mathematical solution for pressure vessels with large opening is currently unavailable. That is one of the primary reasons for engineers to resort to the finite element method of stress analysis, which is a cost involved approach necessitating the need for highly skilled stress analysts. Any efforts by the mathematics community to facilitate the development of a closed-form solution for this kind of problem makes the solution cost-effective and would be of mutual interest to both design engineers and mathematicians.

5. CONCLUSIONS

The following are the key findings on this comprehensive study on analysis of a pressure vessel with unusual requirements from the standpoint of design:

1. The national standards on design of pressure vessels do not specify requirements for unusual design situations.

2. Finite element analysis is a powerful tool for stress analysis in such situations.

3. It is demonstrated with the help of a practical design as to how computational stress analysis can be made both simple and rapid when the finite element analysis is used.

4. There is a distinct need for the development of standards, to include recommendations on various aspects of computational stress analysis, when a finite element analysis is used.

5. Some of the areas in which guidelines are required are identified. In the absence of such guidelines, stress analysis and design of pressure vessels are based on individual judgment and experience.

6. This paper identifies a need for collaborative research between mathematicians and design engineers to develop a suitable closed-form solution for geometries defined by complex curvatures at the intersections of multiple elements.

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