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BLOW-UP SET AND TIME FOR A SINGULAR SEMILINEAR PARABOLIC PROBLEM DUE TO A CONCENTRATED NONLINEAR SOURCE

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ABSTRACT. This article studies the following singular semilinear parabolic first initial-boundary value problem having a concentrated nonlinear source,

$$\begin{aligned} u_t - u_{xx} - \tfrac{r}{x}u_x &= a\delta(x-b)f(u(x,t)) && \text{for } 0<x<1,\ 0<t\le T,\\ u(x,0) &= \psi(x) && \text{for } 0\le x\le 1,\\ u(0,t) = 0 &= u(1,t) && \text{for } 0<t\le T, \end{aligned}$$

where r, a, b and T are real numbers such that $r<1$, $a>0$, $0<b<1$ and $T>0$, $\delta(x)$ is the Dirac delta function, and f and ψ are given functions. A criterion for u to blow up in a finite time t_b and an upper bound of t_b are given. It is established the blow-up set consists of a single point if u blows up. A computational method is devised to compute t_b.

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1. INTRODUCTION

Let r, a, b and T be real numbers such that $r<1$, $a>0$, $0<b<1$, and $T>0$, $D=(0,1)$, $\bar{D}$ be the closure of D, $\Omega = D\times(0,T]$, and $Lu = u_t - u_{xx} - r\,u_x/x$. Let us consider the following singular semilinear parabolic first initial-boundary value problem,

$$\text{(1.1)}\qquad \begin{cases} Lu = a\delta(x-b)f(u(x,t)) & \text{in } \Omega,\\ u(x,0)=\psi(x) & \text{on } \bar{D},\\ u(0,t)=0=u(1,t) & \text{for } 0<t\le T, \end{cases}$$

where $\delta(x)$ is the Dirac delta function, and f and ψ are given functions such that $f(0)\ge 0$, and $f(u)$ and its derivatives $f'(u)$ and $f''(u)$ are positive for $u>0$, and $\psi(x)$ is a nontrivial, nonnegative and continuous function such that $\psi(b)>0$, $\psi(0)=0=\psi(1)$, and

$$\text{(1.2)}\qquad \psi'' + \frac{r}{x}\psi' + a\delta(x-b)f(\psi) \ge 0 \text{ in } D.$$

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The condition (1.2) is used to show that before u blows up, u is a nondecreasing function of t. Without loss of generality, we assume that the function ψ is not a solution of the problem (1.1).

Since $u(x,t)$ is not differentiable at b, we say that a solution of the problem (1.1) is a continuous function on $\bar{\Omega}$ satisfying (1.1). A solution u of the problem (1.1) is said to blow up at the point $(\widetilde{x}, t_b)$, where $\widetilde{x} \in D$ if there exists a sequence of points $\{(x_n, t_n)\}$ in Ω such that

$$u(x_n, t_n) \to \infty \text{ as } (x_n, t_n) \to (\widetilde{x}, t_b^-).$$

If u and t are interpreted, respectively, as the temperature and the time, then t_b is the blow-up time, and $\tilde{x}$ is called the blow-up point.

Green's function (cf. Chan and Carrillo Escobar [1]) corresponding to the problem (1.1) is given by

$$G(x,t;\xi,\tau) = \sum_{i=1}^{\infty} \xi^r \phi_i(\xi)\phi_i(x)e^{-\lambda_i(t-\tau)},$$

where λ_i $(i = 1, 2, \ldots)$ are the eigenvalues of the singular Sturm-Liouville problem,

$$(x^r\phi'(x))' + \lambda x^r \phi(x) = 0, \quad \phi(0) = 0 = \phi(1), \tag{1.3}$$

with the corresponding eigenfunctions given by

$$\phi_i(x) = \frac{2^{1/2} x^\nu J_\nu\left(\lambda_i^{1/2} x\right)}{\left|J_{\nu+1}\left(\lambda_i^{1/2}\right)\right|}. \tag{1.4}$$

Here, J_ν denotes the Bessel function of the first kind of order ν.

We convert the problem (1.1) (cf. Chan and Carrillo Escobar [1]) into the nonlinear integral equation

$$u(x,t) = a\int_0^t G(x,t;b,\tau)f(u(b,\tau))d\tau + \int_D G(x,t;\xi,0)\psi(\xi)d\xi. \tag{1.5}$$

For ease of reference, let us summarize some results of Chan and Carrillo Escobar [1] in the following two theorems.

Theorem 1.1. *There exists some* $t_b\,(\leq \infty)$ *such that in* $\bar{D} \times [0, t_b)$*, the integral equation* (1.5) *has a unique continuous solution* $u \geq \psi(x)$*, and* u *is a nondecreasing function of* t*. If* t_b *is finite, then* u *is unbounded in* $[0, t_b)$*.*

By showing that the solution of the integral equation (1.5) is the solution of the problem (1.1)), they obtained the following result.

Theorem 1.2. *The problem* (1.1) *has a unique solution* u *for* $0 \leq t < t_b$*.*

In Section 2, a criterion for u to blow-up in a finite time is given. If u blows up, it is shown that the blow-up set consists of the single point b, where the concentrated source is situated. Also, an upper bound for the finite blow-up time t_b is given. In Section 3, a computational method for finding t_b is provided.

2. SINGLE BLOW-UP POINT

We modify the proof of Theorem 2.6 of Chan and Tian [2] to obtain the following result.

Theorem 2.1. *If ψ attains its maximum at $x = b$, then the solution u of the problem* (1.1) *attains its maximum at $x = b$. If in addition, t_b is finite, then $u(b, t)$ is unbounded in* $[0, t_b)$.

Proof. Let $\Omega_{0b} = (0, b) \times (0, t_b)$, and $\Omega_{b1} = (b, 1) \times (0, t_b)$. Since $u(b, t)$ is known, let us denote it by $\omega(t)$, and rewrite the problem (1.1) as the following two initial-boundary value problems:

$$\text{(2.1)} \qquad \begin{cases} Lu = 0 & \text{in } \Omega_{0b}, \\ u(x, 0) = \psi(x) & \text{for } 0 \leq x \leq b, \\ u(0, t) = 0 \text{ and } u(b, t) = \omega(t) & \text{for } 0 < t < t_b, \end{cases}$$

$$\text{(2.2)} \qquad \begin{cases} Lu = 0 & \text{in } \Omega_{b1}, \\ u(x, 0) = \psi(x) & \text{for } b \leq x \leq 1, \\ u(b, t) = \omega(t) \text{ and } u(1, t) = 0 & \text{for } 0 < t < t_b. \end{cases}$$

By Theorems 1.1 and 1.2, u is a nondecreasing function of t. Since ψ attains its maximum at $x = b$, it follows from the strong maximum principle (cf. Friedman [3, p. 34]) that the solution of the problem (2.1) attains its (absolute) maximum on the closure of $(0, b) \times (0, t)$ at $u(b^-, t)$, where $t < t_b$. Similarly, the solution of the problem (2.2) attains its (absolute) maximum on the closure of $(b, 1) \times (0, t)$ at $u(b^+, t)$, where $t < t_b$. Thus, if u blows up, it blows up at $x = b$. If in addition, t_b is finite, then by Theorem 1.1, $u(b, t)$ is unbounded in $[0, t_b)$. □

We would like to show that b is the only blow-up point.

Theorem 2.2. *If ψ attains its maximum at $x = b$, and u blows up, then b is the single blow-up point.*

Proof. By Theorem 2.1, if u blows up, then it blows up at b. To show that b is the only blow-up point, it is sufficient to show that for any arbitrarily fixed $t \in (0, t_b)$, u is concave up for $0 \leq x \leq b^-$ and for $b^+ \leq x \leq 1$.

Let us consider the problem (2.1). By Corollary 2 of Friedman [3, p. 74], u is infinitely differentiable. Hence, $Lu_t = 0$ in Ω_{0b}. From Theorem 1.1, $u_t \geq 0$ in

$D \times (0, t_b)$. If $u_t = 0$ somewhere, say at (x_4, t_4), in Ω_{0b}, then by the strong maximum principle, $u_t = 0$ in $(0, b) \times (0, t_4]$, and hence, $u = \psi$ in $(0, b) \times (0, t_4]$. This contradicts the assumption that ψ is not a solution of the problem (1.1). Thus, $u_t > 0$ in Ω_{0b}. Hence for any arbitrarily fixed $t \in (0, t_b)$, u does not have a relative maximum. By the parabolic version of the Hopf Lemma (cf. Friedman [3, p. 49]), $u_x(0, t) > 0$. If $u_{xx} < 0$ for $x \in (0, b)$, then

$$0 > \int_0^x u_{\xi\xi}(\xi, t) d\xi = u_x(x, t) - u_x(0, t),$$

which gives

$$u_x(0, t) > u_x(x, t). \tag{2.3}$$

We claim that for any $t \in (0, t_b)$,

$$u_x(0, t) < \infty. \tag{2.4}$$

Suppose for some $t_5 \in (0, t_b)$, $u_x(0, t_5) = \infty$. If $u_{xx}(0, t_5) \geq -c_4$ for some positive constant c_4, then there exists some $x_5 \in (0, b)$ such that

$$\int_0^{x_5} u_{xx}(x, t_5) dx \geq -c_4 \int_0^{x_5} dx,$$

which gives $u_x(x_5, t_5) + c_4 x_5 \geq u_x\,(0, t_5)$, and hence, $u_x(0, t_5)$ is bounded. This gives a contradiction unless

$$\lim_{x \to 0^+} u_{xx}(x, t_5) = -\infty. \tag{2.5}$$

Suppose (2.5) is true. Since $u_t > 0$, we have

$$-u_{xx} - \frac{r}{x} u_x < u_t - u_{xx} - \frac{r}{x} u_x = 0.$$

This gives

$$-u_{xx} < \frac{r}{x}\, u_x. \tag{2.6}$$

As $x \to 0$, we have a contradiction unless $r > 0$. For $r > 0$, it follows from (2.6) that

$$\begin{aligned} -\lim_{x \to 0^+} u_{xx}(x, t_5) &\leq r \lim_{x \to 0^+} \frac{1}{x^2} \lim_{x \to 0^+} \frac{u_x}{x^{-1}} \\ &= r \lim_{x \to 0^+} \frac{1}{x^2} \lim_{x \to 0^+} \frac{u_{xx}(x, t_5)}{-x^{-2}} \\ &= -r \lim_{x \to 0^+} u_{xx}(x, t_5). \end{aligned}$$

This contradiction shows that (2.4) holds. From (2.3), we now have for any $t < t_b$, there exists some constant c_5 such that $c_5 \geq u_x(0, t) > u_x(x, t)$ for any $x \in (0, b)$. Thus for any $x \in (0, b)$,

$$c_5 \int_0^x d\sigma > \int_0^x u_\sigma(\sigma, t) d\sigma.$$

Hence for any $x \in (0, b)$, $c_5 x > u(x, t)$, which implies u is bounded for any $t < t_b$. This contradicts the assumption that u blows up. Thus, $u_{xx} \geq 0$ for $x \in (0, b)$, and hence u is concave up. It remains to show that if there exists a neighborhood (x_6, b) (where $x_6 > 0$) such that $u_{xx} < 0$, then u is bounded for $x \in (x_6, b)$. The proof of this is similar to the above, and hence u is concave up in (x_6, b).

A proof analogous to the above shows that for the problem (2.2), u is concave up. Thus, if u blows up, then b is the single blow up point. □

Let $\varphi_k(x)$ be the kth eigenfunction of the problem (1.3) normalized in such a way that $\int_0^1 x^r \varphi_k(x) dx = 1$. A direct computation gives

$$\varphi_k(x) = \frac{\lambda_k^{1/2}\Gamma(\nu)}{\left(\frac{\lambda_k^{1/2}}{2}\right)^{\nu-1} + \Gamma(\nu) J_{\nu+1}(\lambda_k^{1/2})} x^{\nu} J_{\nu}(\lambda_k^{1/2} x). \tag{2.7}$$

Let

$$w(t) = \int_D x^r \varphi(x) u(x,t) dx, \tag{2.8}$$

where $\varphi(x)$ denotes $\varphi_1(x)$.

Theorem 2.3. *If ψ attains its maximum at $x = b$,*

$$\left(\frac{s}{f(s)}\right)' \leq 0, \tag{2.9}$$

$$\int_k^{\infty} \frac{ds}{f\,(s)} < \infty \text{ for any positive number } k, \tag{2.10}$$

$$\lambda w(0) < ab^r \varphi(b) f(w(0)), \tag{2.11}$$

then the solution u of the problem (1.1) *blows up in a finite time. Furthermore,*

$$0 < t_b \leq \frac{f(w(0))}{ab^r\varphi(b) f(w(0)) - \lambda w(0)} \int_{w(0)}^{\infty} \frac{d\eta}{f(\eta)}. \tag{2.12}$$

Proof. By using

$$\frac{d}{dx}\left(z^x J_\nu(x)\right) = x^\nu J_{\nu-1}(x)$$

(cf. McLachlan [4, p. 192]), a direct calculation shows that $x^r\varphi'(x)$ is bounded at $x = 0$ and $x = 1$. To show that $\lim_{x\to 0^+}(x^r u_x(x,t))$ is bounded, let us consider the problem (2.1). Since $u_t > 0$ in Ω_{0b}, we have

$$-u_{xx} - \frac{r}{x} u_x < u_t - u_{xx} - \frac{r}{x} u_x = 0,$$

which gives $-u_{xx} < r u_x / x$ for $0 < x < b$. From Theorem 2.2, u is concave up. We have $u_x\,(x,t) > 0$ for $t \in (0, t_b)$. Then,

$$-\frac{u_{xx}}{u_x} < \frac{r}{x} \text{ for } 0 < x < b.$$

For any points x_7 and x_8 between 0 and b with $x_7 < x_8$,

$$-\int_{x_7}^{x_8} \frac{u_{xx}(x,t)}{u_x(x,t)}\,dx < \int_{x_7}^{x_8} \frac{r}{x}\,dx,$$

which gives $0 \le x_7^r u_x(x_7,t) < x_8^r u_x(x_8,t)$. Therefore, $\lim_{x\to 0^+} x^r u_x(x,t)$ is bounded. Similarly, by considering the problem (2.2), we obtain $\lim_{x\to 1^-} x^r u_x(x,t)$ is bounded.

Multiplying the differential equation in (1.1) by $x^r\varphi(x)$ and integrating over D, we obtain

$$w'(t) - \int_D \varphi(x)(x^r u_x(x,t))_x dx = ab^r\varphi(b)f(u(b,t)).$$

Using integration by parts and the above properties of $x^r\varphi'(x)$ and $x^r u_x(x,t)$ at $x=0$ and $x=1$, we have

$$\int_D \varphi(x)(x^r u_x(x,t))_x dx = -\lambda w.$$

Thus,

$$w'(t) + \lambda w(t) = ab^r\varphi(b)f(u(b,t)) \text{ for } 0<t<t_b. \tag{2.13}$$

Since u attains its maximum at $x=b$, it follows from (2.8) that

$$w(t) \le \left(\int_D x^r\varphi(x)dx\right)\max_{x\in\bar{D}} u(x,t) = u(b,t). \tag{2.14}$$

Since f is increasing, it follows from (2.13) and (2.14) that

$$w'(t) \ge -\lambda w(t) + ab^r\varphi(b)f(w(t)) = f(w(t))\left(ab^r\varphi(b) - \frac{\lambda w(t)}{f(w(t))}\right). \tag{2.15}$$

By Theorem 1.1, $u(x,t) \ge \psi(x) = u(x,0)$. This gives $w(t) \ge w(0)$. From (2.9),

$$\frac{w(t)}{f(w(t))} \le \frac{w(0)}{f(w(0))}.$$

From (2.11) and (2.15),

$$\frac{dw}{f(w)} \ge \left(ab^r\varphi(b) - \frac{\lambda w(0)}{f(w(0))}\right)dt. \tag{2.16}$$

From (2.16),

$$\int_{w(0)}^{w(t)} \frac{dw}{f(w)} \ge \left(ab^r\varphi(b) - \frac{\lambda w(0)}{f(w(0))}\right)t. \tag{2.17}$$

It follows from (2.10) that there exists some $t_b\,(<\infty)$ such that $\lim_{t\to t_b} w(t) = \infty$. By (2.14), u blows up in a finite time.

From (2.17), we obtain (2.12). □

3. BLOW-UP TIME

As an illustration, let

$$\psi(x)=\begin{cases} x^2 & \text{for } 0\le x\le b,\\ \left(\frac{b}{1-b}\right)^2(1-x)^2 & \text{for } b<x\le 1.\end{cases}$$

Then,

$$x^r\psi'(x)=\begin{cases} 2x^{r+1} & \text{for } 0\le x<b,\\ -2\left(\frac{b}{1-b}\right)^2 x^r(1-x) & \text{for } b<x\le 1\end{cases}$$

is differentiable, except at $x=b$, where there is a jump of $-2b^{r+1}/\left(1-b\right)$. We have

$$\left(x^r\psi'(x)\right)'=-\frac{2b^{r+1}}{1-b}\delta(x-b)+\begin{cases} 2(r+1)x^r & \text{for } 0<x<b,\\ 0 & \text{at } x=b,\\ 2\left(\frac{b}{1-b}\right)^2 x^{r-1}\left[(r+1)x-r\right] & \text{for } b<x<1,\end{cases}$$

(cf. Stakgold [5, pp. 38–39]). Let us obtain the conditions for (1.2) to hold. For $0<x<b$,

$$\left(x^r\psi'(x)\right)'+a\delta(x-b)x^r f(\psi(x))=2(r+1)x^r\ge 0$$

holds if

$$-1\le r. \tag{3.1}$$

At $x=b$,

$$\left(x^r\psi'(x)\right)'+a\delta(x-b)x^r f(\psi(x))=b^r\left(af(b^2)-\frac{2b}{1-b}\right)\delta(x-b)\ge 0$$

holds if

$$af(b^2)\ge\frac{2b}{1-b}. \tag{3.2}$$

For $b<x<1$,

$$\left(x^r\psi'(x)\right)'+a\delta(x-b)x^r f(\psi(x))=2\left(\frac{b}{1-b}\right)^2 x^{r-1}\left[x-r\left(1-x\right)\right].$$

A sufficient condition for (1.2) to hold is

$$\left(1+r\right)b>r>0, \tag{3.3}$$

since for $r\le 0$, (1.2) holds automatically.

Let $f(u)=u^p$ where p is any number greater than 1, and $r=1/2$. Since $r/\left(1+r\right)$ attains its maximum $1/3$ for $0<r\le 1/2$, the sufficient conditions (3.1) to (3.3) for (1.2) to hold become

$$a\ge\frac{2b}{b^{2p}(1-b)},\ b>\frac{1}{3}. \tag{3.4}$$

Since $\nu=1/4$, we have $J_{1/4}\left(\lambda_1^{1/2}\right)=0$. To compute the first 10 eigenvalues to double precision, we use the subroutine gsl_sf_bessel from the GNU Scientific Library (GSL, version 1.8, April 10, 2006, stable release) for C++. We obtain the results given in the following table:

k	$\lambda_k^{1/2}$	k	$\lambda_k^{1/2}$
1	2.780887723994979410 0	6	18.461927245689263799
2	5.906142698842494099 0	7	21.602784448913070037
3	9.042383663583262177 0	8	24.743827796127696672
4	12.181341528954993336	9	27.884994603411197289
5	15.321369826012286808	10	31.026247476113038459

Table 3.1. Solutions to $J_{1/4}\left(\lambda_k^{1/2}\right) = 0$, for $k = 1, 2, 3, \ldots, 10$.

By using the eigenvalues, the subroutines gsl_gamma (to evaluate the gamma function of a double precision argument) and gsl_sf_bessel (to compute, to double precision, Bessel functions of the first kind of nonnegative real order for real positive arguments), (2.7) and (1.4), we can obtain the functions $\varphi_k(x)$ and $\phi_k(x)$ respectively. An upper bound for the blow-up time t_b can be obtained from (2.12).

From Theorem 2.3, u blows up in a finite time if

$$a > \frac{\lambda}{b^r \varphi(b) w^{p-1}(0)}, \tag{3.5}$$

and an upper bound t_u for the blow-up time t_b is given by

$$t_u = \frac{w^p(0)}{ab^r\varphi(b)w^p(0) - \lambda w(0)} \int_{w(0)}^{\infty} \eta^{-p}\, d\eta. \tag{3.6}$$

As a numerical example, let $b = 1/2$ and $p = 2$. From (3.4), $a \geq 32$. Since $\lambda = 7.733336533465930$, we can compute

$$w(0) = \int_0^b x^{r+2}\varphi(x)dx + \left(\frac{b}{1-b}\right)^2 \int_b^1 x^r(1-x)^2\varphi(x)dx.$$

We use an adaptive numerical integration, the gsl_integration (to compute, to double precision, the numerical integration based on the 61-point Gauss-Kronrod rule with an absolute error and a relative error of order 10^{-4}) to compute $w(0)$, which is 0.115265524027041. Since $\varphi(b) = 2.20858575387531$, it follows from (3.5) that $a > 42.9603896936577$, which is larger than that obtained from (3.4). Since

$$\int_{w(0)}^{\infty} \eta^{-p}\, d\eta = \lim_{t\to\infty} \int_{w(0)}^{t} \eta^{-2} d\eta = \frac{1}{w(0)},$$

it follows from (3.6) that

$$t_u = \frac{1}{ab^r\varphi(b)w(0) - \lambda}. \tag{3.7}$$

For any given $a > 42.9603896936577$, we use the following bisection procedure to determine the blow-up time t_b by taking initially its lower bound $t_l^{(0)}$ to be 0 and its upper bound $t_u^{(0)}$ to be that obtained from (3.7):

Step 1. Let $t_l^{(0)}$ and $t_u^{(0)}$ be our initial estimates of a lower bound t_l and an upper bound t_u. Then, the first estimate of t_b is

$$t_b^{(0)} = \frac{t_l^{(0)} + t_u^{(0)}}{2}.$$

Step 2. For Step k, $k = 0, 1, 2, \ldots$, if $\left| t_u^{(k)} - t_l^{(k)} \right| < \epsilon$ (a given tolerance), then

$$t_b^{(k)} = \frac{t_l^{(k)} + t_u^{(k)}}{2}$$

is accepted as the final estimate of t_b, and we stop; otherwise, we go to the next step.

Step 3. Let

$$t_m = \frac{t_l^{(k)} + t_u^{(k)}}{2},$$

where m denotes the number of subintervals of equal length $h = t_m/m$ that the interval $[0, t_m]$ is divided. Also, let

$$\tilde{G}(x, t; \xi, \tau) = \sum_{i=1}^{N} \xi^r \phi_i(\xi)\phi_i(x)e^{-\lambda_i(t-\tau)} \text{ for } t > \tau$$

be an approximation to $G(x, t; \xi, \tau)$.

Step 4. For $k = 0, 1, 2, \ldots$, we use the following iteration procedure to refine the estimate $t_b^{(k)}$ for the blow-up time. We subdivide the interval $[0, t_m]$ into m subintervals $[0, jh]$, where $j = 1, 2, 3, \ldots m$. We use the following iterative process:

$$\tilde{u}^{(0)}(b, t) = \psi(b) = b^2,$$

$$\tilde{u}^{(k)}(b, 0) = \psi(b) = b^2,\ k = 1, 2, 3, \ldots,$$

$$\begin{aligned} \tilde{u}^{(k)}(b, jh) = a \int_0^{jh} & \tilde{G}(b, jh; b, \tau) f(\tilde{u}^{(k-1)}(b, \tau)) d\tau \\ & + \int_D \tilde{G}(b, jh; \xi, 0)\psi(\xi) d\xi,\ k = 1, 2, 3, \ldots,\ j = 1, 2, 3, \ldots, m, \end{aligned}$$

where $\tilde{u}^{(k)}(b, t)$ is the interpolation of the $m + 1$ points, $\big(jh, \tilde{u}^{(k)}(b, jh)\big)$ for $j = 0, \ldots, m$, by using the subroutine gsl_spline (cubic spline interpolation in double precision). We use the same adaptive numerical integration procedure mentioned above to calculate each integral.

Step 5. We stop the calculations as follows: if $\left| \tilde{u}^{(k)}(b, t_m) - \tilde{u}^{(k-1)}(b, t_m) \right| < \delta$ (a given tolerance), then $t_l^{(k+1)} = t_m$, $t_u^{(k+1)} = t_u^{(k)}$, or else if $\left| \tilde{u}^{(k)}(b, t_m) - \tilde{u}^{(k-1)}(b, t_m) \right| > C$ (a given positive number), then $t_l^{(k+1)} = t_l^{(k)}$, $t_u^{(k+1)} = t_m$. We stop the iterative process and go to Step 2.

Table 3.2 is obtained by taking $\epsilon = 10^{-7}$, $\delta = 10^{-4}$, $C = 10^5$, $N = 10$, $m = 40$, $b = 1/2$, and $f(u) = u^2$.

a	t_b
50	0.0187088608
55	0.0157128877
60	0.0134982987
65	0.0118034160
70	0.0104694221
75	0.0093949892
80	0.0085129417
85	0.0077769411
90	0.0071542688
95	0.0066212177
100	0.0061599318

Table 3.2. Blow-up time.

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